ence of large $SU(3)$-breaking in meson-baryon coupling.[16]

The question of particle assignment and mass spectra would apply equally well for the *chiral* $SU(3)\otimes SU(3) = S\bar{W}(3)$ group, which may be thought of as being generated by the decomposition of the primitive (quark) fields into their chiral projections.[17] The mesons in this scheme have to be paired with regard to parity, though the breaking of $S\bar{W}(3)$ could lift the degeneracy between opposite parity mesons. Evidence for parity-$SU(3)$ mixing is not conclusive at the moment, although this might eventually turn out to be the case. It has been speculated that the useful symmetry group for hadrons could be as large as $SW(3)\otimes SW(3)$, with the factor $SW(3)$'s related by parity operation.[18] In such a case our discussion of meson symmetries is to be understood as being restricted to a useful subgroup of such a large symmetry (restricted to meson states with the same parity).

The interesting point that emerges from our study is the possible existence of mesons or even entire multiplets (like a vector 27-plet) which may not appear in two-meson channels. Multiplet assignments may have to await a more complete study of the boson mass spectrum.

Note added in proof. Since the submission of this paper Wu and Tuan have published a paper wherein the connection between isospin degeneracies for mesons and $SW(2)$ symmetry has been noted independently. S. F. Tuan and T. T. Wu [Phys. Rev. Letters **18**, 349 (1967)].

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We wish to thank Professor W. D. McGlinn for many stimulating conversations.

[16] For the meson coupling to $J^P=\frac{1}{2}^+$ baryons it has been suspected for a long time that kaon couplings are systematically much smaller than pion couplings. This is supported by a determination of (ΛNK) coupling constant by forward dispersion relations due to M. Lusignoli, M. Restignoli, G. Snow, and G. Violini [Phys. Letters **21**, 229 (1966)]. More recently, Yodh has analyzed the consequences of a singlet assignment for the $Y_0^*(1520)$ and found substantial deviation from $SU(3)$ predictions in the branching ratio for $\Sigma\pi$ versus $N\bar{K}$ decays of this particle [G. Yodh, Phys. Rev. Letters **18**, 810 (1967)]. As for the meson couplings connecting $J^P=\frac{3}{2}^+$ baryons with $J^P=\frac{1}{2}^+$ baryons only the pion couplings are known at present (from observed decays of $\frac{3}{2}^+$ baryons) and these are in agreement with $SU(3)$ predictions. However, it is *possible* for this situation to co-exist with sizable $SU(3)$-violations; see S. K. Bose and Y. Hara, Phys. Rev. Letters **17**, 409 (1966).

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Superconvergence Relations for Pion-Baryon Scattering

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Superconvergence relations are discussed for the spin-flip amplitudes in pion-baryon scattering at $t=0$ and at $u=0$. These sum rules are evaluated in the resonance saturation approximation, and in this case an extension of the derivation is given. This allows the sum rules to be discussed even if the relevant leading Regge trajectory lies above zero at $t=0$ for mesons or above $-\frac{1}{2}$ at $u=0$ for baryons. We consider all 14 possible sum rules, as well as the moment sum rules, which require stronger assumptions. The spin-flip sum rules are well satisfied for $\pi N\to\pi N$, and in the other cases the ρ and pion couplings to Σ, Λ, and Ξ required to satisfy the relations are in agreement with other models. The moment sum rules are less convergent and so more difficult to test, but for πN the moment sum rule for $B^{(-)}$ at $t=0$ is not satisfied with resonance saturation, and this fact can be understood.

I. INTRODUCTION

ASSUMING that the asymptotic behavior of an invariant amplitude is given by the leading Regge poles, together with possible kinematic factors if there is helicity flip, the amplitude will decrease sufficiently fast in certain cases to allow a dispersion sum rule or superconvergence relation (SCR) to be derived. Such sum rules have been discussed in strong interactions for the systems $\rho\pi$,[1] $\Sigma\pi$,[2] $N^*\pi$,[3,4] NN,[5] and pion-baryon

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[2] P. Babu, F. T. Gilman, M. Suzuki, Phys. Letters **24B**, 65 (1967).

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scattering in the $SU(3)$ limit,[6,7] all at fixed t. Use was made in many derivations of the assumption that, since no isospin two meson has been established, its Regge trajectory $\alpha(t=0)<0$; although the possibility of a $\rho+\rho$ Regge cut contributing with $\alpha>0$ has been raised.[8,9] For π-baryon scattering, these assumptions lead to only one SCR at fixed t, which is for $\Sigma\pi$ scattering and has been discussed by Babu *et al.*,[2] approximating the amplitude as a sum of resonances.

Since the Regge trajectories for baryons in general lie lower than those for mesons, there will be more SCR's for amplitudes which may be assumed to have a high-energy behavior due to Reggeized baryon exchange. We choose to derive SCR's from the fixed u dispersion relations which have contributions from s- and t-channel states. For the pion-baryon system, $u=0$ corresponds to a physically possible process for $s>2(M^2+\mu^2)$ and for $t\rightarrow\infty$. Therefore, one must analytically continue the amplitude to $u=0$ in order to evaluate the contributions from the unphysical regions. However, since the spin-flip amplitude B is not measured directly, we assume that it is well approximated by the sum of the contributions of the known resonances and bound states, and simply continue $\cos\theta$ to its unphysical values in terms of the invariant variables s, t, and u.

The resonance and bound states used to evaluate the SCR's are taken from the tables of Rosenfeld *et al.*,[10] and the t-channel contributions are assumed to be due primarily to ρ, since S-wave mesons do not contribute to the spin-flip amplitudes. The π-baryon couplings are compared with $SU(3)$ predictions and the ρ-baryon couplings are compared with those deduced from ρ dominance of the isovector form factor together with $SU(3)$ relations for the form factors. For states such as Y_2^*, no resonances are well established,[11] but we argue that this does not imply that the amplitude is negligible, since the K^+p and NN amplitudes are known to be non-negligible. Then contributions to Y_2^* may be eliminated from the SCR's and satisfactory agreement obtained.

For amplitudes corresponding to exchange of Y_2^* or $\Xi_{3/2}^*$ systems, the Regge trajectory should lie sufficiently low to allow moment SCR's, and we investigate these, although the data are inconclusive.

Taking the model, implicit in the foregoing discussion, of an amplitude as a sum of Regge poles and resonances, one may derive sum rules over the resonance spectrum[12] even if the asymptotic behavior of the whole amplitude would not allow such sum rules. We discuss these sum rules and find that they are well satisfied at $t=0$ and $u=0$ for the spin-flip π-baryon amplitudes. This argument also leads to moment and non-spin-flip SCR's, for those cases in which the Regge-pole contribution is negligible in the resonance region, and we discuss the extent to which this is satisfied.

In Sec. II we derive and discuss the necessary SCR's and in Sec. III the numerical results of resonance saturation for $\pi N\rightarrow\pi N$, $\pi\Lambda\rightarrow\pi\Lambda$, $\pi\Lambda\rightarrow\pi\Sigma$, $\pi\Sigma\rightarrow\pi\Sigma$, and $\pi\Xi\rightarrow\pi\Xi$ are presented. The conclusions are discussed in Sec. IV.

II. DERIVATION OF THE SUM RULES

For a crossing-odd amplitude $B(s,t)$ which satisfies a fixed t dispersion relation and behaves asymptotically with s such that $sB(s,t)\rightarrow 0$, the derivation of a superconvergent sum rule is well known[1,2] and leads to

$$\frac{1}{\pi}\int_{s_0}^{\infty}\mathrm{Im}B(s,t)ds=0\,, \tag{2.1}$$

where s_0 is the threshold for physical states which is $(M+\mu)^2$ for π-baryon scattering, and there may be pole terms to be added explicitly to (2.1) due to one-baryon states below this threshold.

Similarly, for an amplitude $B(s,u)$ which satisfies the fixed u dispersion relation,

$$B(s,u)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{Im}B(s',u)}{s'-s}ds'\,, \tag{2.2}$$

one has the result, if $sB(s,u)\rightarrow 0$ as $s\rightarrow\infty$, that

$$\frac{1}{\pi}\int_{-\infty}^{+\infty}\mathrm{Im}B(s,u)ds=0\,, \tag{2.3}$$

and this SCR may be given a more convenient form, which explicitly incorporates the t-channel contribution by using the crossing matrix α.

$$B^s(s,u)=\alpha B^t(t,u)\,, \tag{2.4}$$

so that (2.3) may be written

$$\frac{1}{\pi}\int_{s_0}^{\infty}\mathrm{Im}B^s(s,u)ds-\frac{1}{\pi}\int_{t_0}^{\infty}\alpha\,\mathrm{Im}B^t(t,u)dt=0\,, \tag{2.5}$$

where $t_0=4\mu^2$ for our case.

Now if one has reason to expect that $s^2B\rightarrow 0$ as $s\rightarrow\infty$, one may derive moment sum rules. At $t=0$, for pion processes, crossing symmetry restricts the sum rules of the type (2.1) to odd amplitudes, while for even amplitudes the moment sum rules take the form

$$\frac{1}{\pi}\int_{s_0}^{\infty}\tfrac{1}{2}(s-u)\,\mathrm{Im}(s,t)ds=0\,. \tag{2.6}$$

At $u=0$ there is no such crossing symmetry and so

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[12] C. Michael, Rutherford High-Energy Laboratory Report, 1967 (unpublished).

the moment sum rules are of the type

$$\frac{1}{\pi}\int_{-\infty}^{+\infty}(s-s_m)\,\mathrm{Im}B(s,u)ds=0\,, \qquad (2.7)$$

where s_m may be any value because (2.3) is also satisfied for this amplitude. We shall choose s_m so as to give a sensitive test of the moment relation which entails choosing s_m near the s values of the largest contributions to (2.3).

The fixed-u dispersion relation has discontinuities at $u=0$, which are physical for $s>2(m^2+\mu^2)$ and for $t\to\infty$, so that one must analytically continue the resonance contributions to $u=0$ outside these regions of s. This is simply done by expressing $\cos\theta$ in terms of the invariant variables s, t, and u when evaluating the contributions. An alternative would be to use the backward ($\cos\theta=-1$) dispersion relation,[13] which has contributions to the discontinuity from s, t, and u channels. In the q^2 plane this dispersion relation also has singularities at real values only, and SCR's can be deduced from it. As $s\to\infty$ at $u=0$, $\cos\theta\to-1$, so that the asymptotic behavior with s is similar, although the latter dispersion relation is valid at the point $\cos\theta=-1$ only, which is why we prefer to work with the more general fixed-u relations.

De Alfaro *et al.*[1] have emphasized that SCR's are valid for all t or all u. This gives such a strong constraint that one will need nonzero contributions from an infinite number of partial waves in order to satisfy the sum rules in general. Such high partial-wave contributions will be negligible in the sum rules we shall consider. A systematic method of exploiting the t dependence is to expand in poweres of t about $t=0$, and equate each coefficient separately to zero. In practice we find that it is only for the coefficient of t^0 that the SCR's are well approximated by known resonances and the poorer convergence found for higher coefficients renders these relations of uncertain value; hence we shall not consider them.

The Regge-pole model of high-energy behavior predicts an asymptotic behavior of $s^{\alpha(I_t)-n}$ for an amplitude B^{I_t} with a definite t-channel isospin; n is the amount of helicity flip to which the amplitude refers, and $\alpha(I_t)$, usually denoted α_{I_t}, is the intercept at $t=0$ of the Regge trajectory with the appropriate quantum numbers. α_{I_t} may be deduced either from a direct experimental study of the asymptotic behavior or else from the Chew-Frautschi plot. The values of the leading trajectory intercepts which are commonly used are $\alpha_{I_t=0}=1$, $\alpha_{I_t=0}\approx\frac{1}{2}$ and $\alpha_{I_t=2}<0$, where the last value follows from assuming that Regge cuts will not interfere with the expectations from the Chew-Frautschi plot.

For fermion Regge poles there are technical problems associated with MacDowell symmetry and the unequal-mass kinematics. The A and B amplitudes are expected[14] to have a high-energy behavior $s^{\alpha(I_u)-1/2}$ for large s at fixed u, and Freedman and Wang[15] have shown that this should continue to hold at $u=0$. The amplitude $A+MB$ is expected[14] to have a better high-energy behavior than A or B separately and possibly decreases as $s^{\alpha(I_u)-3/2}$ at $u=0$. In order to test the $u=0$ sum rules, the t-channel contribution must be known, so that the $\pi\pi$ S-wave interaction which contributes to A must be eliminated. We thus discuss primarily the B amplitude.

For baryon trajectories there is uncertainty, since a straight-line Chew-Frautschi plot is in conflict with MacDowell symmetry which relates opposite parity trajectories. The Δ(or $N_{3/2}^*$) trajectory is supposed to have $\alpha\approx0.15$,[16] the nucleon trajectory has $\alpha\approx-0.34$,[17] and the other known trajectories for Y_0^*, Y_1^*, and $\Xi_{1/2}^*$ are all assumed to have $\alpha<-\frac{1}{2}$.[18] For Y_2^* and $\Xi_{3/2}^*$ for which no resonance states are known, one may suppose that $\alpha<-\frac{3}{2}$ although this is not compatible with Regge cuts of the type $Y_1^*+\rho$, $\Xi_{1/2}^*+\rho$ or $\Delta+K^*$.

These trajectory intercepts lead to SCR's at $t=0$ for $B^{I_t=2}$ in $\Sigma\pi\to\Sigma\pi$, and at $u=0$ for $B^{I_u=1/2}$ in $\pi\Xi\to\pi\Xi$, $B^{I_u=0}$ in $\pi\Sigma\to\pi\Sigma$, and $B^{I_u=1}$ in $\pi\Lambda\to\pi\Lambda$, $\pi\Lambda\to\pi\Sigma$, and $\pi\Sigma\to\pi\Sigma$. For $B^{I_u=3/2}$ in $\pi\Xi\to\pi\Xi$ and $B^{I_u=2}$ in $\pi\Sigma\to\pi\Sigma$, one will have a moment sum rule as well.

An extension of this derivation has been proposed,[12,19,20] which may apply even when the leading Regge trajectories are such that $\alpha(t=0)>0$ and $\alpha(u=0)>-\frac{1}{2}$. The motivation for this argument is the success of the phenomenological model of Regge poles plus s-channel resonances in representing amplitudes even at quite low energies. Barger and Cline[16] have fitted π^-p backward scattering in this way and πN charge exchange has been discussed in this context down to a center-of-mass (c.m.) energy of 1500 MeV.[21] One assumes that a few low partial-wave amplitudes dominate at low energies and the Regge-pole terms dominate at high energies.

Now an amplitude may always be constructed as the sum of two parts, one of which has the asymptotic behavior and separately satisfies the dispersion relation, and a remainder or background term which will decrease faster asymptotically. This background contribution then satisfies a SCR since (i) its asymptotic behavior has been made to decrease sufficiently fast, and (ii) it satisfies the same dispersion relation as the total amplitude, because it differs from the total amplitude by a contribution explicitly constructed to satisfy the dispersion relation.

In fact, except for a modification near threshold, the simplest asymptotic form $\sum_i\beta_i s^{\alpha_i}$ will satisfy the dispersion relation, provided that the phase of β_i is correct.

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TABLE I. Pseudoscalar π-baryon couplings from $SU(3)$, where $d+f=1$.

	$f=0.25$	$f=0.33$	$f=0.40$
$g_{NN\pi}^2$	15	15	15
$g_{\Sigma\Lambda\pi}^2$	11.2	8.9	7.2
$g_{\Sigma\Lambda\pi}g_{\Sigma\Sigma\pi}$	6.5	8.2	8.3
$g_{\Sigma\Sigma\pi}^2$	3.7	6.7	9.6
$g_{\Xi\Xi\pi}^2$	3.7	1.7	0.6

For a crossing-odd amplitude, this phase must be $\beta_i=|\beta_i|(1-e^{-i\pi\alpha_i})$, as may be proved in an even more general way,[22] and is explicit in the Regge-pole theory in the signature factor.

If accurate data on the total amplitude exist, one may fit such an asymptotic form to the data and so obtain the background which will satisfy the sum rule.[19,20] However, in many cases the observed resonance spectrum will provide a good approximation to this background. This is because the asymptotic terms are known to contribute predominantly to high partial waves in order to account for the observed backward and forward peaks, whereas the low partial waves are resonance dominated and will contain only relatively small contributions from the asymptotic part. The phenomenological fitting mentioned above also supports the conjecture that the non-Regge part may be approximated by the resonance spectrum.

This discussion leads us to test SCR's for all spin-flip amplitudes, regardless of the value of α, and we find, within the rather large uncertainties of resonance parameters, that they are all equally satisfactory. Moment sum rules are also tested, and in general we do not find agreement, since the poorer convergence enlarges the contribution from the region in which the resonance and asymptotic parts are comparable, so that the subtraction procedure is less reliable.

III. APPLICATION TO PION-BARYON SCATTERING

In terms of the partial-wave amplitudes $f_{l\pm}$, referring to $J=l\pm\frac{1}{2}$, the spin-flip amplitude B and nonflip amplitude A are

$$\begin{bmatrix} B(s,\cos\theta) \\ A(s,\cos\theta) \end{bmatrix} = \frac{4\pi}{q^2}\sum_{l=1}^{\infty} P_l'(\cos\theta) \times\left\{(E+M)(f_{l-}(s)-f_{l+}(s))\begin{bmatrix} 1 \\ M-W \end{bmatrix} +(E-M)(f_{(l-1)+}(s)-f_{(l+1)-}(s))\begin{bmatrix} 1 \\ M+W \end{bmatrix}\right\}, \quad (3.1)$$

where q is the c.m. momentum, E is the baryon total energy, M is the baryon mass, and μ is the pion mass.

For the process $\pi\Lambda\to\pi\Sigma$, one generalizes (3.1) by writing both q and $(E\pm M)$ as the square root of the product of their values for $\Lambda\pi$ and $\Sigma\pi$. $t=0$ corresponds to $\cos\theta=1$ and $P_l'(1)=\frac{1}{2}l(l+1)$, while at $u=0$, $\cos\theta$ may be calculated at each value of s and P_l' evaluated from it. At $u=0$, $\cos\theta$ is physical for $s>2(M^2+\mu^2)$, which is $(1340\text{ MeV})^2$ for πN, while near threshold $\cos\theta$ becomes infinite although B and A remain finite.

Keeping in mind the large uncertainties in some of the resonance parameters, we thought it good enough to work in the zero-width approximation:

$$\operatorname{Im} f_l(s)=\operatorname{Im}\frac{\Gamma_e/2q}{M-W-\frac{1}{2}i\Gamma}=\frac{\pi\Gamma_e}{2q}\delta(M-W)\,, \quad (3.2)$$

where Γ_e is the elastic width and Γ is the total width. We use the resonance parameters as specified in the tables of Rosenfeld *et al.*[10] The normalization is as in (2.1) and (2.5) except for division by 4π to rationalize the coupling constants. This leads us to normalize the coupling constants so that $g_{NN\pi}^2=14.8$, $g_{\rho\pi\pi}^2=3.0$, etc.

The πBB couplings may be compared with $SU(3)$ with an adjustable D/F ratio. Values of f from 0.25 to 0.40 are often assumed,[23,24] and these results are summarized in Table I. For the ρBB couplings, there are vector and tensor couplings, but one may make a very simple assignment if the ρ meson is assumed to dominate the isovector baryon form factors. Then the contribution of the ρ meson to the spin-flip amplitude for the process $\pi\pi\to B\bar{B}$ will be proportional to the "spin-flip" baryon isovector form factor, which is the total magnetic moment. Then with the usual $SU(3)$ transformation properties of the photon[25] and the experimental proton and neutron magnetic moments, the isovector baryon magnetic moments are for N,Σ, $\Xi,\Lambda\Sigma$ in the ratio $4.71:3.67:-1.03:-3.31$. The similar result that the ρ contributions are in the ratio $5:4:-1:-2\sqrt{3}$ may be obtained from $SU(6)$ for the magnetic moments, or else by assuming that the ρBB vector coupling transforms under $SU(3)$ with $f=1$ and the tensor coupling with $f=0.25$.[26] The ρ contribution to the B amplitude for $\pi\pi\to N\bar{N}$ is proportional[27] to $\gamma_1+2M\gamma_2=g_{\rho\pi\pi}\frac{1}{3}(g_{\rho NN}{}^V+g_{\rho NN}{}^T)$, and assuming universal coupling of the ρ to the isospin current[28] together with dominance of the form factor leads to $\gamma_1+2M\gamma_2=4.7\gamma_1$, where $\gamma_1=-\frac{1}{3}g_{\rho\pi\pi}g_{\rho NN}{}^V$ and the factor 4.7 comes from the isovector nucleon total magnetic moment. Now from the ρ width of 160 MeV[10] one finds $g_{\rho\pi\pi}^2=3.0$ and uni-

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[23] From strong interaction dynamics, A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963) suggest $f=0.25$ and M. E. Ebel and P. B. James, Phys. Rev. Letters 15, 805 (1965) find $f>0.35$, while $SU(6)$ symmetry requires $f=0.4$.

[24] From fits to hyperon leptonic decays, N. Brene, L. Veje, M. Roos, and C. Cronström [Phys. Rev. 149, 1288 (1966)] determine $f=0.335\pm0.018$.

[25] S. Coleman and S. L. Glashow, Phys. Rev. Letters 6, 423 (1961).

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TABLE II. Contributions of particles and resonance states from Ref. 10 in the narrow-resonance approximation to the sum rules for the spin-flip πN amplitudes $B^{(+)}$ and $\frac{1}{2}(s-u)B^{(-)}$ at $t=0$ and $B^{I_u=1/2}$, $B^{I_u=3/2}$ and $\left(\frac{1}{M}A^{(-)}+B^{(-)}\right)$ at $u=0$.

	$B^{(+)}$ $(t=0)$	$\frac{1}{2}(s-u)B^{(-)}$ $(t=0)$	$B^{I_u=1/2}$ $(u=0)$	$B^{I_u=3/2}$ $(u=0)$	$\frac{1}{M}A^{(-)}+B^{(-)}$
N	14.8	−0.3	−14.8	29.6	14.8
$\Delta(1236)$	−14.6	4.6	−25.6	−6.4	1.2
$N(1400)$	2.6	2.7	−2.6	5.1	1.3
$N(1525)$	2.1	3.0	0.6	−1.1	−0.3
$N(1670)$	−0.9	−1.7	−0.6	1.3	0.2
$N(1688)$	2.5	4.9	−0.6	1.2	0.3
$\Delta(1920)$	−3.5	4.9	−5.5	−1.3	0.5
$N(2190)$	1.3	4.9	0.9	−1.9	−0.1
$\Delta(2420)$	−1.0	2.6	−2.0	−0.5	0.1
$N(2650)$	0.5	3.2	0.5	−0.9	−0.02
$\Delta(2850)$	−0.4	1.4	−0.8	−0.2	0.03
$N(3030)$	0.1	0.7	0.1	−0.2	−0.02
$\Delta(3230)$	−0.1	0.7	−0.4	−0.1	0.03
$\rho(760)$	0	0	$-28.2\gamma_1$	$14.1\gamma_1$	$10.3\gamma_1$
$f_0(1250)$	0	0	$\Delta_u(f_0)/\sqrt{6}$	$\Delta_u(f_0)/\sqrt{6}$	0

versality suggests the same value for $g^V{}_{\rho NN}{}^2$, although values from 2 to 5.6 have been obtained.[29,30] Then $\gamma_1=-1.0$ with universality and -1.4 using Ball, Scotti, and Wong's[31] value. The effect of possible Regge recurrences of the ρ will be to increase the effective magnitude of γ_1 required by the SCR's.

Other $J>0$ meson states which are known to couple to $\pi\pi$ are the $f_0(1250)$ and the $\rho(1650)$ or g meson. The coupling to nucleons of the latter state is quite unknown, so it is neglected, while for the f_0 we find that a small contribution $\Delta(f_0)$ in $\pi\pi\to N\bar{N}$ is needed. The magnitude is less than that obtained by Lyth.[32]

A. π-Nucleon

In this system, the sum rules may be investigated thoroughly since the relevant coupling constants are quite well known and many resonances have been reported. The contributions of the sum rules are shown in Table II. The SCR at $t=0$ for the amplitude $B^{(+)}=\frac{1}{3}\times(B^{I_s=1/2}+2B^{I_s=3/2})$, which corresponds to $I_t=0$ and has the P and P' Regge poles, seems to be well satisfied. There are errors arising from the resonance approximation; in the resonance parameters, the narrow-width approximation and the neglect of possible states not included in the tables. No S-wave resonances have been included since they have a factor $(E-M)/(E+M)$ relative to higher waves and are negligible. In Ref. 12, it was pointed out that parametrizing the P_{33} wave with a shape parameter tended to reduce the resonant contribution by about 20%, and this reduction would be less for the other, higher mass, narrower resonances. The agreement of 23.5 to 20.5 for the $B^{(+)}$ SCR lends some support to the arguments given in Sec. II about eliminating the Regge contribution.

This gives one confidence that the $I_u=\frac{3}{2}$ and $I_u=\frac{1}{2}$ sum rules for B should be satisfied irrespective of whether $\alpha<-\frac{1}{2}$ for the leading trajectories. These two sum rules are indeed satisfied if $\gamma_1=-1.78$ and $\Delta_u(f_0)=1.0$, both of which are reasonable values. In order to confirm these results one may use the nonflip amplitude. Then the SCR for $A^{(-)}+MB^{(-)}$ at $u=0$ should be valid without the extension we propose, since both α_N and $\alpha_\Delta<\frac{1}{2}$. This relation (Table II) requires a contribution of $\gamma_1=-1.79$ with the approximation of resonance saturation. Furthermore, the moment relation for $(s-M^2-\mu^2)(A^{(-)}+MB^{(-)})$, which requires the Regge subtraction argument, is satisfied to within 7% with this value of γ_1.

The moment sum rule for $\frac{1}{2}(s-u)B^{(-)}(t=0)$ is seen not to be satisfied, and the moment sum rules for $u=0$ are also not satisfied unless more t-channel contributions are invoked. At $t=0$ the sum rule for the forward scattering amplitude $A^{(-)}+\nu B^{(-)}$ has been discussed by Logunov *et al.*,[19] and Igi *et al.*,[20] using the cross-section data to determine the imaginary part of this amplitude. They perform the subtraction of the ρ Regge contribution explicitly and are able to obtain agreement. However, their results are sensitive to the behavior of the Regge contribution at low energies, which introduces an uncertainty. Our method crudely neglects the effect of the low-energy Regge part on the resonance contribution, so that disagreement will be expected if one tries to evaluate the more sensitive moment sum rules. The moment sum rule that should be least sensitive is that for $B^{I_u=1/2}$ since trajectories lie lower and the Regge contribution is smaller. In this case the ρ, N, and $N^*(1236)$ resonance contributions cancel and if there were contributions from Regge recurrences of the ρ to match those from the $N^*(1236)$ agreement could be obtained.

TABLE III. Contributions of particles and resonant states from Ref. 10 in the narrow-resonance approximation to the spin-flip sum rules at $t=0$ for $\pi\Lambda\to\pi\Lambda$ and at $u=0$ for $\pi\Lambda\to\pi\Lambda$ and $\pi\Lambda\to\pi\Sigma$. Γ_Λ is the partial width into the $\pi\Lambda$ channel of the relevant resonance.

	$B_\Lambda{}^{I_t=0}(t=0)$	$B_\Lambda{}^{I_u=1}(u=0)$	$B_{\Lambda\Sigma}{}^{I_u=1}(u=0)$
Σ	$g_{\Sigma\Lambda\pi}{}^2$	$g_{\Sigma\Lambda\pi}{}^2$	$\sqrt{2}g_{\Sigma\Sigma\pi}g_{\Sigma\Lambda\pi}$
$Y_1^*(1385)$	−11.4	−9.4	7.5
$Y_1^*(1660)$	$0.14\Gamma_\Lambda$	$0.052\Gamma_\Lambda$	...
$Y_1^*(1770)$	−1.1	0.1	...
$Y_1^*(1910)$	0.9	0.02	...
$Y_1^*(2035)$	−3.2	−0.8	...
$f_0(1250)$	0	$\Delta_u(f_0)$	0
$\rho(760)$	0	0	$19.9\gamma_1$

B. $\pi\Lambda\to\pi\Lambda$ and $\pi\Lambda\to\pi\Sigma$

These SCR's are evaluated in Table III and the main uncertainty is the 1660 resonance which has widely vary-

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TABLE IV. Contributions of resonances from Ref. 10 in the narrow-resonance approximation and of other states to the sum rules for the spin-flip amplitudes for the $\Sigma\pi$ system at $t=0$ with $I_t=0$ and 2 and at $u=0$ with $I_u=0$, 1, and 2. For the 1660, 1700, 2035, and 2100 MeV resonances, Armenteros *et al.* (Ref. 35) suggest partial widths Γ_Σ of 21, 25, 10, and 10 MeV, respectively, with large uncertainties.

	$B^{I_t=0}(t=0)$	$B^{I_t=2}(t=0)$	$B^{I_u=0}(u=0)$	$B^{I_u=1}(u=0)$	$B^{I_u=2}(u=0)$
Λ	$g_{\Sigma\Lambda\pi}{}^2$	$g_{\Sigma\Lambda\pi}{}^2$	$g_{\Sigma\Lambda\pi}{}^2$	$-g_{\Sigma\Lambda\pi}{}^2$	$g_{\Sigma\Lambda\pi}{}^2$
Σ	$2g_{\Sigma\Sigma\pi}{}^2$	$-g_{\Sigma\Sigma\pi}{}^2$	$-2g_{\Sigma\Sigma\pi}{}^2$	$g_{\Sigma\Sigma\pi}{}^2$	$g_{\Sigma\Sigma\pi}{}^2$
$Y_1^*(1385)$	-6.8	3.4	5.0	-2.5	-2.5
$Y_0^*(1520)$	1.7	1.7	9.1	-9.1	9.1
$Y_1^*(1660)$	$0.23\Gamma_\Sigma$	$-0.11\Gamma_\Sigma$	$-0.36\Gamma_\Sigma$	$0.18\Gamma_\Sigma$	$0.18\Gamma_\Sigma$
$Y_0^*(1700)$	$0.06\Gamma_\Sigma$	$0.06\Gamma_\Sigma$	$0.06\Gamma_\Sigma$	$-0.06\Gamma_\Sigma$	$0.06\Gamma_\Sigma$
$Y_1^*(1770)$	-0.15	0.07	0.07	-0.03	-0.03
$Y_0^*(1820)$	0.65	0.65	-0.13	0.13	-0.13
$Y_1^*(1910)$	0.28	-0.14	0.066	-0.03	-0.03
$Y_1^*(2035)$	$-0.1\Gamma_\Sigma$	$0.05\Gamma_\Sigma$	$0.003\Gamma_\Sigma$	$-0.001\Gamma_\Sigma$	$0.001\Gamma_\Sigma$
$Y_0^*(2100)$	$0.05\Gamma_\Sigma$	$0.05\Gamma_\Sigma$	$0.007\Gamma_\Sigma$	$-0.007\Gamma_\Sigma$	$0.007\Gamma_\Sigma$
Y_2^* states	$(5/3)\Delta_t(Y_2^*)$	$\frac{1}{6}\Delta_t(Y_2^*)$	$(5/3)\Delta_u(Y_2^*)$	$\frac{5}{6}\Delta_u(Y_2^*)$	$\frac{1}{6}\Delta_u(Y_2^*)$
$f_0(1250)$	0	0	$\frac{1}{3}\Delta_u(f_0)$	$\frac{1}{3}\Delta_u(f_0)$	$\frac{1}{3}\Delta_u(f_0)$
$\rho(760)$	0	0	$-22.0\gamma_1$	$-11.0\gamma_1$	$11.0\gamma_1$
$I=2$ mesons	0	0	$(5/3)\Delta_u(I=2)$	$-\frac{5}{6}\Delta_u(I=2)$	$\frac{1}{6}\Delta_u(I=2)$

ing widths (3–35) and branching ratios to $\pi\Lambda$. Derrick *et al.*,[33] have reported a $Y_1^*(1680)$ with total width 120 MeV and undetermined spin which decays substantially into $\pi\Lambda$, while the accepted $Y_1^*(1660)$ with width 50 MeV has a branching ratio to $\pi\Lambda$ of $6\pm6\%$ according to Smart *et al.*[34] Within this uncertainty, and the error in using the narrow-width approximation for $Y_1^*(1385)$, the SCR's for $\pi\Lambda$ would be consistent with $g_{\Lambda\Sigma\pi}{}^2=10\pm3$ and $\Delta_u(f_0)\approx0$.

For $\pi\Lambda\rightarrow\pi\Sigma$ the sign of the higher resonance contributions are unknown, so that they are not included in the table. For the Σ and $Y_1^*(1385)$, the relative sign was deduced from $SU(3)$, and the coupling $g_\Lambda g_\Sigma$ is less than 8.5 from $SU(3)$ and is near this value for reasonable values of f. Thus for agreement the ρ contribution requires $\gamma_1\approx-1.0$, with large uncertainties due to neglected states, etc.

C. $\pi\Sigma\rightarrow\pi\Sigma$

The SCR's are evaluated in Table IV, where the relation for $B^{I_t=2}(t=0)$ has been written by Babu *et al.*[2] with a different normalization. The sum rule for $B^{I_t=0}(t=0)$, with acceptable values of $g_\Lambda{}^2$ and $g_\Sigma{}^2$, requires some contribution from s-channel isospin two states in order to be satisfied. Even though no Y_2^* resonances are established,[11] the contribution from nonresonant low partial waves may well be significant. Thus one may assume that if resonances are observed then they are dominant, but if none are observed the amplitude may still be large as in the KN or NN systems. Eliminating the Y_2^* contributions from the $t=0$ sum rules, one finds $4g_\Sigma{}^2-3g_\Lambda{}^2-19=0$, which, together with a value of $g_\Lambda{}^2$ of 10 ± 3, leads to $g_\Sigma{}^2=12\pm4$ when errors are estimated. The Y_2^* contribution so required has $\Delta_t(Y_2^*)\approx-21$, so that partial waves with $J=l+\frac{1}{2}$, $l\neq0$, must predominate in this system.

The $u=0$ sum rules, for which no Regge subtraction argument is needed, may also be satisfied by invoking a Y_2^* contribution and assuming fairly small contributions from f_0 and $I=2$ mesons. Using $g_\Lambda{}^2\approx g_\Sigma{}^2\approx10$, $\gamma_1\approx-2.3$, and $\Delta_u(Y_2^*)\approx-24$ one may satisfy the relations. These values are acceptable and the similarity of the Y_2^* contributions at $u=0$ to the value at $t=0$ leads one to favor P_3 waves in this state.

For $I_u=2$, a moment sum rule is expected, as pointed out above. The value of s for the ρ meson is (1515 MeV)^2 and for the f_0 it is (1143 MeV)^2, since $u=0$, so that $s=2M^2+2\mu^2-t$. Thus taking moments about $s_m=$ (1520 MeV)^2 in (2.7), the ρ and $Y_0^*(1520)$ do not contribute, and the Λ and Σ contributions will need to be balanced by the higher resonance states. As in the moment relation for πN shown in Table II, the convergence is poor so that higher resonances make considerable contributions. One concludes that this moment sum rule for $B^{I_u=2}$ could well be satisfied.

D. $\pi\Xi\rightarrow\pi\Xi$

In this case, as Table V shows, the contributions from the lowest-lying states are all much smaller than in πN, so that one should not assume that retaining only these states is a good approximation. If the $\Xi_{3/2}^*$ and higher $\Xi_{1/2}^*$ spectrum were negligible compared to the $\Xi^*(1530)$ resonance, then $g_{\Xi\Xi\pi}{}^2=3.2$. However, all that one can safely say is that it is encouraging that the s-channel resonance contribution is small, the magnetic-moment model for the ρ gives a small contribution, and $SU(3)$

TABLE V. Contribution of resonances from Ref. 10 in the narrow-resonance approximation and of other states to the sum rules for the spin-flip amplitudes for the $\Xi\pi$ system at $t=0$ with $I_t=0$ and at $u=0$ with $I_u=\frac{1}{2}$ and $\frac{3}{2}$.

	$B^{(+)}(t=0)$	$B^{I_u=\frac{1}{2}}(u=0)$	$B^{I_u=\frac{3}{2}}(u=0)$
Ξ	$g_{\Xi\Xi\pi}{}^2$	$-g_{\Xi\Xi\pi}{}^2$	$2g_{\Xi\Xi\pi}{}^2$
$\Xi_{1/2}^*(1530)$	-3.3	2.4	-4.8
$\Xi_{1/2}^*(1815)$	0.1	0.2	0.4
$\Xi_{1/2}^*(1930)$	$0.08\Gamma_\Xi$	$0.014\Gamma_\Xi$	$0.03\Gamma_\Xi$
$\Xi_{3/2}^*$ states	$\frac{2}{3}\Delta_t(\Xi_{3/2}^*)$	$\frac{4}{3}\Delta_u(\Xi_{3/2}^*)$	$\frac{1}{3}\Delta_u(\Xi_{3/2}^*)$
$\rho(760)$	0	$+6.2\gamma_1$	$-3.1\gamma_1$
$f_0(1250)$	0	$(1/\sqrt{6})\Delta_u(f_0)$	$(1/\sqrt{6})\Delta_u(f_0)$

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[35] R. Armenteros, M. Ferro-Luzzi, D. W. G. Leith, R. Levi-Setti, A. Minten, R. D. Tripp, H. Filthuth, V. Hepp, E. Kluge, H. Schneider, R. Barloutaud, P. Granet, and J. Meyer, J. P. Porte, Phys. Letters **24B**, 198 (1967).

predicts a small π coupling. The moment sum rule for $\Xi_{3/2}^*$ exchange may be computed using values of s of $(1710\text{ MeV})^2$ for ρ and $(1390\text{ MeV})^2$ for f_0 and choosing a suitable value of s_m in (2.7). This SCR is quite inconclusive because of the sparse data available.

IV. CONCLUSIONS

With the Regge-pole hypothesis for the high-energy dependence of an amplitude, together with knowledge of the trajectory intercepts, one may select those amplitudes which satisfy SCR's. For π-baryon scattering, the spin-flip amplitudes associated with the exchange of $I=2$ meson, Y_0^*, Y_1^*, Y_2^*, $\Xi_{1/2}^*$, $\Xi_{3/2}^*$, and possibly $N_{1/2}$ trajectories satisfy such conditions. Except for πN scattering below 2 GeV, where phase-shift determinations exist, no direct experimental data on these amplitudes are available and one must use the approximation of resonance saturation in order to test the SCR's. In view of the success of the model of Regge poles plus s-channel resonances in reproducing amplitudes, one might seek to add a Regge-pole contribution to these sum rules. However, this term decreases faster than s^{-1} asymptotically, so that its contribution to the SCR comes predominantly from low energies. The contribution is thus dependent on the method of fitting the correct threshold properties onto the asymptotic behavior and this is not understood.

The SCR's have been evaluated and, within the uncertainty of resonance parameters, they may be satisfied, although one slightly unexpected conclusion is that the $u=0$ sum rules for $\pi\Sigma$ suggest that the contribution from Y_2^* states, whether resonant or not, should not be negligible compared to the contributions from Y_0^* and Y_1^* states. The SCR's for the moments of the amplitudes for Y_2^* and $\Xi_{3/2}^*$ exchange processes are not saturated by the few lowest-lying states and are thus inconclusive.

If the total amplitude is constructed by adding the direct-channel resonances to a contribution which has the asymptotic behavior and analytic properties of the total amplitude, then the resonance spectrum will separately satisfy a SCR. This consideration leads us to consider SCR's for the remaining spin-flip amplitudes which have $\alpha(t=0)>0$ and $\alpha(u=0)>-\frac{1}{2}$. With the same contributions, they are found to satisfy the sum rules as well as those amplitudes for which a conventional SCR may be deduced. Taking all 14 sum rules with resonance saturation, we find that the πBB and ρBB couplings given by $SU(3)$ and ρ dominance of the form factor, are consistent with those indicated by the sum rules. In particular, $f=0.33$ for the pion couplings and a ρ coupling to nucleons such that $\gamma_1=-1.8$ have been used in the sum rules, although any claim that we have determined these numbers from the SCR's must take account of the large and rather uncertain errors involved. For the $I=0$ contribution to the t channel we find only small contributions necessary, and this is presumably due to the f_0 meson.

For the moment sum rules, the convergence is poor, as can be seen from the evaluation for πN. The relation for $\frac{1}{2}(s-u)B^{(-)}(t=0)$ is not satisfied by resonance states,[36] while for the $u=0$ moment relations, the lack of detailed knowledge on the t channel introduces an uncertainty. The relation for $sB^{I_u=1/2}(u=0)$ is possibly valid since the ρ, N, and $N^*(1236)$ contributions cancel and if the known Regge recurrences of the $N^*(1236)$ were matched by recurrences of the ρ this agreement could be sustained.

These results are in agreement with the supposition that if only the asymptotically leading term has to be removed to ensure convergence of the background, then the background should be well approximated by the resonance spectrum. However, if many asymptotic terms have to be separated and if these terms are large, then the background should be calculated explicitly by subtraction from the experimental total amplitude as Igi *et al.*, and Logunov *et al.* have done.

Our considerations suggest, for example, that sum rules at fixed t exist for amplitudes with $I_t=0$ as well as those already conjectured for $I_t=2$. If resonance saturation of the amplitude may be assumed, these sum rules may be evaluated to give useful constraints for many different amplitudes. The assumption of resonance saturation is likely to be less reliable in the former case ($I_t=0$) because of the known Regge contribution from P and P' poles but our analysis has shown that in practice this sum rule is still a very useful tool. Other extensions are clearly indicated, but one must not be too ambitious or resonance saturation will fail. It seems that one can remove all Regge trajectories above $\alpha(t=0)=0$ and $\alpha(u=0)=-\frac{1}{2}$, but if one tries to extract contributions with asymptotic behavior less than this, one begins to disturb the resonance spectrum.

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[36] C. B. Chiu, R. J. N. Phillips, and W. Rarita [Phys. Rev. **153**, 1485 (1967)] from fits to πN data with ρ, P, and P' Regge poles, deduce that the ρ contribution to $B^{(-)}$ is large, whereas the P and P' contributions to $B^{(+)}$ are small. Thus the Regge subtraction for $B^{(+)}$ should be more reliable than for $B^{(-)}$.