to zero, so that

$$\langle I_0[\langle\sigma_{x'}^{(2)}\sigma_{z'}^{(1)}\rangle-\langle\sigma_{x'}^{(1)}\sigma_{z'}^{(2)}\rangle]\rangle_\varphi = \mathrm{Re}(\chi''^*\phi_0+\chi'\phi_1^*)\,, \quad (72)$$

or equivalently,

$$\langle I_0[(C_{PU}+C_{VK})\cos(\theta_L^{(1)}+\theta_L^{(2)}) + (C_{VU}-C_{PK})\sin(\theta_L^{(1)}+\theta_L^{(2)})]\rangle = \mathrm{Re}(\chi''^*\phi_0+\chi'\phi_1^*)\,. \quad (73)$$

Experimentally, this means that one measures all the values of

$$I_0[(C_{PU}+C_{VK})\cos(\theta_L^{(1)}+\theta_L^{(2)}) + (C_{VU}-C_{PK})\sin(\theta_L^{(1)}+\theta_L^{(2)})]$$

with their scattering planes turning around Oz as an axis of rotation. If the average of these measured quantities is equal to zero, C invariance is not violated.

VI. CONCLUSION

We have shown in this paper that all the symmetry invariances (parity, time-reversal, and charge-conjugation) can, in principle, be tested by measuring the well-known observable coefficients of the elastic scattering process $\bar{p}p\rightarrow\bar{p}p$. In these proposed tests, we have assumed that the $\bar{p}$ beams produced in large accelerators are unpolarized. Even if these $\bar{p}$ beams are polarized with an unknown degree of polarization, an unpolarized $\bar{p}$ beam still can, in principle, be produced by mixing two polarized $\bar{p}$ beams together.

Among the tests proposed in this paper, we might single out those considered as the easiest and the most recommended. For the test of time-reversal invariance, we believe that the comparison between the transverse components of the polarization and analyzing power of the proton target is the easiest. For the test of C invariance, we recommend the comparison of the transverse polarizations of the scattered $\bar{p}$ beam and the recoil proton target. As for the parity conservation test, the measurement of one component of the polarization on the scattering plane is enough.

ACKNOWLEDGMENTS

The author wishes to thank G. Cohen-Tannoudji for his critical reading of the manuscript and for some helpful discussions. Information from J.-P. Merlo on the experimental facts relating to these proposed tests is also greatly appreciated. The author is also indebted to Dr. S. P. Reddy for help in revising the manuscript and to the National Research Council of Canada for an operating grant (No. A-3962) under which part of this work was done.

PHYSICAL REVIEW VOLUME 162, NUMBER 5 25 OCTOBER 1967

K^+n Charge Exchange and the ρ' Regge Trajectory*

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(Received 14 June 1967)

Finding that the Regge exchange of ρ and A_2 fails to describe our K^+n charge-exchange data at 2.3 GeV/c (the highest energy available to date) when a simultaneous fit is attempted with higher-energy data on K^-p and π^-p charge exchange and $\pi^-p\rightarrow\eta^0n$, we introduce a second, lower-lying ρ-type trajectory (ρ'). This also provides a possible mechanism for the puzzling π^-p charge-exchange polarization. We find that we are then able simultaneously to fit all these data (including the polarization), together with related total cross-section differences up to 20 GeV/c, with a ρ' whose spin-1 mass is 1.0 GeV [perhaps the $\delta(965)$?] and whose $t=0$ intercept, 1.1 units of angular momentum below the ρ, agrees roughly with the ρ' proposed by Högaasen and Fischer to describe forward $\bar{p}p$ and np charge exchange, where the (ρ,A_2) model also fails. Our ρ and A_2 trajectories turn out essentially traditional. In the fit, we permit only small SU_3 breaking between the $K\bar{K}$ and $\pi\pi$ (or $\eta^0\pi$) couplings to the trajectories. We further constrain the fit to obey the sum rule of Igi and Matsuda. In fitting our K^+ data at 2.3 GeV/c, we include a deuteron correction, and employ exact Legendre functions rather than the high-energy asymptotic Regge forms. We offer predictions for higher-energy K^+ charge exchange.

I. INTRODUCTION

Considerable success has previously been achieved in fitting the high-energy differential cross sections for the reactions

$$\pi^-p\rightarrow\pi^0n\,, \quad (1)$$

$$K^-p\rightarrow\bar{K}^0n\,, \quad (2)$$

$$\pi^-p\rightarrow\eta^0n\,, \quad (3)$$

* Work done under auspices of the U. S. Atomic Energy Commission.

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‡ This paper was accepted by the University of California, Berkeley, as part of the Ph.D. dissertation.

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in the peripheral region $[-t\lesssim 1(\mathrm{GeV}/c)^2]$, by the t-channel Regge exchange of the $\rho(C^-)$ and $A_2(C^+)$ trajectories.[1,2] Together with

$$K^+n \to K^0 p\,, \tag{4}$$

these constitute all the isotopically independent reactions

$$P_8+N \to P_8+N \quad (P_8=\text{pseudoscalar octet},\ N=\text{nucleon}) \tag{5}$$

which require $I=1$ exchange in the t channel. For t-channel exchange, reaction (4) differs from (2) only by a relative sign change in the C^+ and C^- exchange amplitudes. Thus, if the (ρ,A_2) model is adequate, the previous fits ought to determine high-energy K^+n peripheral charge exchange.

We therefore compared K^+n charge-exchange data at 2.3 GeV/c (the highest energy available to date) of one of the authors (B.M.S.)[3] to the previous fits of the other (W.R.),[1] and attempted a simultaneous fit of data on all four reactions to the assumption of only ρ and A_2 Regge exchange in the peripheral region. The attempt was in part motivated by the correct prediction of this model that, in contrast to K^-p, the K^+n charge exchange ought to show $|\mathrm{Re}f(t=0)/\mathrm{Im}f(t=0)|\gg 1$. Despite correction for the deuteron effect in (4), and use of exact Legendre-function forms because of the relatively low energy of our K^+ data, we found that the model always gave a cross section only about half as large as that observed for K^+n charge exchange. A kinder fate would have been expected considering the relative absence of resonant activity in the K^+ nucleon system above about 1.3 GeV/c.

The model has other difficulties. It predicts, contrary to fact, zero polarization for π^-p charge exchange. Further, Högaasen and Fischer[4] find that it fails to describe the energy dependence of forward pn and $\bar{p}p$ charge exchange. There seems to be a simultaneous plausible remedy for all three difficulties, namely, the exchange of a second, lower-lying, ρ' trajectory having the same quantum numbers as the ρ. Lying well below the ρ trajectory, its relative contribution to the differential cross sections would decrease rapidly with increasing energy, thus explaining the earlier fits which ignored it.

Adding this ρ' trajectory to the C^- exchange amplitude, which we then subject to the constraint of a recent superconvergent sum rule of Igi and Matsuda,[5] we have achieved a good simultaneous fit to extensive data on processes (1) through (4), together with related total cross-section differences and π^-p charge-exchange polarization. This fit involves only small SU_3 symmetry breaking, the ρ' being assumed to belong to an octet.

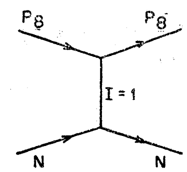

FIG. 1. Reactions (1) through (4), for which we investigate peripheral scattering data, constitute a complete, isotopically independent, set of the reactions (pseudoscalar octet + nucleon → pseudoscalar octet+nucleon) which require isovector exchange in the t (peripheral) channel.

The resulting ρ' trajectory is consistent with the intercept found by Högaasen and with the δ(965-MeV) meson at $\alpha=1$.

II. FORMALISM

All the processes (5) (see Fig. 1) require that the t-channel exchanged object satisfy $C=P=(-1)^J=G(-1)^I$. Anticipating Reggeization and SU_3, we separate the $I=1$ exchange into two parts:

$$C^+(P^+,G^-,J\text{ even}) \quad\text{and}\quad C^-(P^-,G^+,J\text{ odd}).$$

Clearly, for reaction (1) only the C^- exchange is possible, while for (3) we have only C^+. The $K^\pm N$ charge exchanges admit both. Therefore, we define the t-channel helicity-nonflip amplitudes;

$$A(K^-p\to\bar{K}^0n)={}_\kappa A^+ + {}_\kappa A^-\,, \tag{6}$$

$$A(\pi^-p\to\pi^0n)={}_\pi A^-\,, \tag{7}$$

$$A(\pi^-p\to\eta^0n)={}_\pi A^+\,, \tag{8}$$

giving

$$A(K^+n\to K^0p)={}_\kappa A^+ - {}_\kappa A^-\,, \tag{9}$$

with similar formulas for the helicity-flip amplitudes B. The superscript sign refers to both charge conjugation and J parity (or signature) $(-1)^J=C$.

Now let each amplitude be a sum over contributing Regge trajectories, e.g.,

$${}_\pi A^- = \sum_i {}_\pi A_i^- \text{ etc.} \tag{10}$$

Then we have from the factorization theorem

$${}_\pi A_i/{}_\kappa A_i = {}_\pi B_i/{}_\kappa B_i \equiv F_i(t)\,. \tag{11}$$

If SU_3 is unbroken, and all contributing trajectories are octet members, we have

$$\begin{aligned} F_i(t)=F^+&=(\tfrac{2}{3})^{1/2} \quad\text{for}\quad C_i=+1\,,\\ =F^-&=-\sqrt{2} \quad\text{for}\quad C_i=-1\,, \end{aligned} \tag{12}$$

independent of t.

The experimentally observed quantities are given in terms of the helicity amplitudes by

$$\frac{d\sigma}{dt}=\frac{1}{\pi}\left(\frac{1}{4p}\right)^2\left\{\left(1-\frac{t}{4M^2}\right)|A|^2 -\frac{t}{4M^2}\left[\frac{s+p^2}{1-(t/4M^2)}-s\right]|B|^2\right\}, \tag{13}$$

$$\sigma_T(K^\pm p)-\sigma_T(K^\pm n)=\mathrm{Im}({}_\kappa A^+\mp{}_\kappa A^-)_{t=0}/p\,, \tag{14}$$

$$\sigma_T(\pi^+p)-\sigma_T(\pi^-p)=\sqrt{2}\,\mathrm{Im}({}_\pi A^-)_{t=0}/p\,, \tag{15}$$

[1] R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965); Phys. Rev. Letters **15**, 807 (1965).
[2] F. Arbab and C. B. Chiu, Phys. Rev. **147**, 1045 (1966).
[3] Table I, footnote (a).
[4] H. Högaasen and W. Fischer, Phys. Letters **22**, 516 (1966).
[5] K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967).

$$P\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n) = \frac{1}{\pi}\left(\frac{1}{4p^2}\right)\left[\left(1-\frac{t}{4M^2}\right)\right]^{1/2} \times \left\{\frac{-t}{4M^2}\left[\frac{s+p^2}{1-(t/4M^2)}-s\right]\right\}^{1/2} {}_\pi\mathbf{A}^- \times {}_\pi\mathbf{B}^-, \quad (16)$$

where M is the nucleon mass, s and t are the invariant squares of energy and momentum transfer, p is the incoming lab momentum, and P is the $\pi^- p$ charge-exchange polarization. $\mathbf{A}$ and $\mathbf{B}$ are vectors in the complex plane.

There is only one reasonably well established trajectory for each of C^+ and C^-, namely, the A_2 and ρ. As discussed above, however, they do not seem to suffice for all the processes under consideration. Including now the speculative ρ', we parametrize the various high-energy asymptotic Regge helicity amplitudes as follows:

$${}_K A^- = {}_K A_\rho + {}_K A_{\rho'} \quad \text{and} \quad {}_K A^+ = {}_K A_{A_2} \equiv {}_K A_R, \quad (10')$$

with similar relations for the ${}_K B$, ${}_\pi A$, and ${}_\pi B$ amplitudes.

$${}_K A_i^{\pm} = -C_i(t)(\alpha_i+1)\frac{(e^{-i\pi\alpha_i}\pm 1)}{\sin\pi\alpha_i}\left(\frac{E}{E_0}\right)^{\alpha_i},$$

$${}_K B_i^{\pm} = -D(t)\alpha_i(\alpha_i+1)\frac{(e^{-i\pi\alpha_i}\pm 1)}{\sin\pi\alpha_i}\left(\frac{E}{E_0}\right)^{\alpha_i-1}, \quad (17)$$

where E/E_0 is the lab energy of the incoming meson in GeV.

We take

$$C_\rho(t) = C_\rho^0(1+C_\rho^1 t), \qquad D_\rho(t) = D_\rho^0 \exp D_\rho^1 t,$$

$$C_{\rho'}(t) = C_{\rho'}^0 \exp C_{\rho'}^1 t, \qquad D_{\rho'}(t) = D_{\rho'}^0 \exp D_{\rho'}^1 t, \quad (17')$$

$$C_R(t) = \alpha_R C_R^0 \exp C_R^1 t, \quad D_R(t) = \alpha_R D_R^0 \exp D_R^1 t,$$

$$\alpha_i^0(t) = \alpha_i^0 + \alpha_i' t, \quad (18)$$

$$F_i(t) = F_i^0 \exp F_i^1 t. \quad (11')$$

The distribution of the factors of α in Eqs. (17) implies a specific mechanism for the required vanishing of flip residue functions and the "ghost killing" for even-signature amplitudes at $\alpha=0$. We assume that the "nonsense" vertices (see Fig. 2) each provide a factor $\sqrt{\alpha}$ for all exchanged trajectories, and that in the case of even-trajectory exchange, every vertex provides an additional factor $\sqrt{\alpha}$. This is the so-called Chew ghost-killing mechanism, with all trajectories "choosing sense." Alternate mechanisms have been

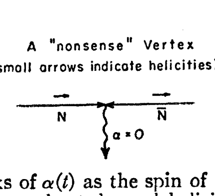

FIG. 2. If one thinks of $\alpha(t)$ as the spin of the exchanged "particle" for a given t, then the t-channel helicity flip vertex above violates angular momentum conservation when $\alpha=0$. We assume that each such "nonsense" vertex contributes a factor $\sqrt{\alpha}$ to the t-channel helicity flip amplitude.

TABLE I. Data.

Reaction	Lab momenta (GeV/c) and references
	Differential cross section
$K^+n \to K^0 p$	2.3[a]
$K^-p \to \bar{K}^0 n$	(5, 7, 9.5)[b]
$\pi^-p \to \pi^0 n$	(5.9, 9.8, 13.3, 18.2)[c]
$\pi^-p \to \eta^0 n$	(5.9, 9.8, 13.3, 18.2)/$B(\eta \to 2\gamma)$[d,e,f]
	Total cross-section differences
$\sigma_T(K^+p)-\sigma_T(K^+n)$	2.3,[g] (6, 8, 10, 12, 14, 16, 18, 20)[h]
$\sigma_T(K^-p)-\sigma_T(K^-n)$	(6, 10, 12, 14, 16, 18)[h]
$\sigma_T(\pi^-p)-\sigma_T(\pi^+p)$	(5, 5.2, 5.4, 5.6, 5.8, 6.0, 6.2, 6.4)[i] (8, 10, 12, 14, 16, 18, 20)[h]
	Polarization
$\pi^-p \to \pi^0 n$	(5.9, 11.2)[j]

[a] I. Butterworth, J. Brown, G. Goldhaber, S. Goldhaber, A. Hirata, J. Kadyk, B. Schwarzschild, and G. Trilling, Phys. Rev. Letters **15**, 734 (1965).
[b] P. Astbury, G. Brautti, G. Finocchiaro, A. Michelini, K. Terwilliger, D. Websdale, C. West, P. Zanella, W. Beuch, W. Fischer, B. Gobbi, M. Peppin, E. Polgar, C. Verkerk, and M. Pouchon, CERN Report No. 66/1057/5, 1966 (unpublished).
[c] Saclay-Orsay Collaboration: A. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, P. Borgeaud, M. Yvert, J. Guillaud, C. Caverzasio, and B. Amblard, Phys. Rev. Letters **14**, 763 (1965); Phys. Letters **20**, 75 (1966).
[d] Saclay-Orsay Collaboration (see footnote c), Phys. Letters **18**, 200 (1965).
[e] Branching ratio ($\eta \to 2\gamma/\eta \to$ all neutrals) $=0.416$, as given in G. DiGiugno, R. Querzoli, G. Triose, F. Vanoli, M. Giorgi, and P. Schiavon, Phys. Rev. Letters **16**, 767 (1966).
[f] Branching ratio ($\eta \to$ all neutrals) $=0.729$, as given in A. Rosenfeld *et al.*, Rev. Mod. Phys **39**, 1 (1967).
[g] T. Kycia, Bull. Am. Phys. Soc. **12**, 567 (1967).
[h] W. Galbraith, E. Jenkins, T. Kycia, B. Leontic, R. H. Phillips, A. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).
[i] A. Citron, W. Galbraith, T. Kycia, B. Leontic, R. H. Phillips, A. Rouset, and P. Sharp, Phys. Rev. **144**, 1101 (1966).
[j] P. Bonamy, P. Borgeaud, C. Bruneton, P. Falk-Vairant, O. Guisan, P. Sonderegger, C. Caverzasio, J. Guillaud, J. Schneider, M. Yvert, I. Mannelli, F. Sergiampietri, and L. Vincelli, Phys. Letters **23**, 501 (1966).

suggested, but there is some evidence in favor of our choice.[6]

For $C_\rho(t)$, we chose originally the form $C^0[(G+1)\times(\exp C^1 t)-G]$ to provide a possible mechanism for the crossover of the π^+p and π^-p elastic differential cross sections via a sign change in $C_\rho(t)$. The fitting program always chose the case $G \gg 1 \gg C^1$. Hence the form in (17′).

Table I shows the data to be fitted. Of the 24 parameters in Eqs. (17′) to (11′) varied to fit these data, one degree of fitting freedom is lost via the constraint of the Igi sum rule discussed below, which relates α_ρ^0, $\alpha_{\rho'}^0$, C_ρ^0, $C_{\rho'}^0$, F_ρ^0, and $F_{\rho'}^0$ in an equation of constraint. We have also constrained the six SU_3-breaking parameters F_i^0 and F_i^1 to give symmetry breaking of less than 25%. Thus we are left effectively with 17 free parameters and six restricted parameters.

III. SPECIAL TREATMENT OF LOW-ENERGY K^+n DATA

The K^+n charge-exchange data at 2.3 GeV/c require special treatment, (a) because of the relatively low energy for the applicability of the high-energy asymptotic forms (17), and (b) because the target neutron is bound in a deuteron.

[6] F. Arbab, N. Bali, and J. Dash, Phys. Rev. **158**, 1515 (1967).

A. Explicit Legendre Function Formulation

The high-energy Regge behavior ($\sim E^\alpha$) of the amplitudes comes from the asymptotic behavior of the Legendre functions;

$$\left.\begin{cases}\alpha>-\frac{1}{2}, & P_\alpha(w)\\ \alpha<+\frac{1}{2}, & M_\alpha(w)\equiv\dfrac{-\tan\pi\alpha}{\pi}Q_{-\alpha-1}(w)\end{cases}\right\}\xrightarrow[w\to\infty]{}\frac{(2w)^\alpha}{\sqrt{\pi}}\times\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)},\quad(19)$$

where P and Q are the Legendre functions of the first and second kind. Following a suggestion of Read *et al.*,[7] based on Mandelstam's extension[8] of the Regge formalism to $\alpha<-\frac{1}{2}$, we will use P_α for $\alpha\geqslant 0$ and M_α for $\alpha<0$, denoting this generically by L_α.

Retreating from asymptotically high energies, we may write the nonflip amplitudes more generally as

$$A_i^\pm=-a(t)\left(\frac{-qq'}{ME_0}\right)^\alpha(\alpha+\tfrac{1}{2})L_\alpha(w)\frac{(e^{-i\pi\alpha}\pm1)}{\sin\pi\alpha},\quad(20)$$

where q, q', and $w=-\cos\theta_t$ are the (nonphysical) momenta and scattering angle cosine in the t channel. These are given by

$$t=4(q^2+M^2)=4(q'^2+m^2),$$
$$-s=q^2+q'^2-2qq'\cos\theta_t,\quad(21)$$

giving

$$w=\frac{2(s-M^2-m^2)+t}{(4M^2-t)^{1/2}(4m^2-t)^{1/2}}.\quad(22)$$

Here $m=(E^2-p^2)^{1/2}$ is the meson mass. $(-qq'/ME_0)^\alpha=\{[(4M^2-t)(4m^2-t)]^{1/2}/4ME_0\}^\alpha$ is factored out of the residue function $a(t)$ to cancel the anomalous t-channel threshold singularities in w. From (19) we then have

$$A_i^\pm\xrightarrow[E\to\infty]{}-a(t)\left(\frac{2E}{E_0}\right)^\alpha\frac{(\alpha+\frac{1}{2})}{\sqrt{\pi}}\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\frac{(e^{-i\pi\alpha}\pm1)}{\sin\pi\alpha}.\quad(23)$$

Comparing (23) with (17) we have, finally, in terms of the fitted parameters,

$${}_KA_i^\pm=-C_i(t)(\alpha_i+1)\frac{\Gamma(\alpha_i+1)}{\Gamma(\alpha_i+\frac{1}{2})}\frac{\sqrt{\pi}}{2^{\alpha_i}}\left(\frac{-qq'}{ME_0}\right)^{\alpha_i}\times L_{\alpha_i}(w)\left(\frac{e^{-i\pi\alpha_i}\pm1}{\sin\pi\alpha_i}\right).\quad(20')$$

Similarly, we write the general form for the helicity flip amplitudes,

$$B_i^\pm=-b(t)\left(\frac{-qq'}{ME_0}\right)^{\alpha-1}(\alpha+\tfrac{1}{2})\frac{d}{dw}L_\alpha(w)\frac{(e^{-i\pi\alpha}\pm1)}{\sin\pi\alpha}.\quad(24)$$

Invoking the general property

$$d/dwL_\alpha(w)=\alpha(wL_\alpha-L_{\alpha-1})/(w^2-1),$$

we get

$$B_i^\pm\xrightarrow[w\to\infty]{}-b(t)\left(\frac{2E}{E_0}\right)^{\alpha-1}\times\frac{2\alpha(\alpha+\frac{1}{2})}{\sqrt{\pi}}\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\frac{(e^{-i\pi\alpha_i}\pm1)}{\sin\pi\alpha},\quad(25)$$

which by comparison with (17) gives

$${}_KB_i^\pm=-D_i(t)(\alpha_i+1)\frac{\Gamma(\alpha_i+1)}{\Gamma(\alpha_i+\frac{1}{2})}\frac{\sqrt{\pi}}{2^{\alpha_i}}\left(\frac{-qq'}{ME_0}\right)^{\alpha_i-1}\times\frac{d}{dw}L_{\alpha_i}(w)\left(\frac{e^{-i\pi\alpha_i}\pm1}{\sin\pi\alpha_i}\right).\quad(24')$$

The factor $[\Gamma(\alpha+1)]^{-1}$ in (19) serves to cancel the unwanted poles in the signature factors for negative α. In our formalism we have effectively replaced it by $(\alpha+1)$. So long as we have $\alpha^+>-2$ and $\alpha^->-3$ (which turns out to be the case), the empirical factors $C(t)$ and $D(t)$ can make up the difference.

We replace Eqs. (17) by (20′) and (24′) when fitting the K^+ charge-exchange and total cross-section difference data at 2.3 GeV/c. To all the rest of our data, which are above 5 GeV/c, we apply the asymptotic forms.

B. Deuteron Correction

We must now express the observed $K^+d\to K^0p(p)$ distribution $(d\sigma/dt)_d$ in terms of the free neutron K^+n charge-exchange cross section $(d\sigma/dt)_n$, and thus in terms of the Regge amplitudes. The data $(d\sigma/dt)_d$ had been determined by attributing to each K^0 the lab momentum p' it would have if its observed direction $\hat{p}'$ had resulted from a collision with a stationary free neutron.[3] Then the impulse and closure approximation gives[9]

$$\left(\frac{d\sigma}{dt}\right)_d=\left(\frac{d\sigma}{dt}\right)_n\frac{[1-H+R(1-\frac{1}{3}H)]}{(1+R)},\quad(26)$$

where

$$R(t)=\frac{(d\sigma/dt)_n{}^{\text{spin-flip}}}{(d\sigma/dt)_n{}^{\text{non-spin-flip}}},$$

and

$$H(t)=\int d^3\mathbf{r}\,|\psi_d(\mathbf{r})|^2\exp[i(\hat{p}'p'-\mathbf{p})\cdot\mathbf{r}],\quad(27)$$

where $\psi_d(\mathbf{r})$ is the Hulthén deuteron wave function.

For $t=0$ we have $H=1$ and $R=0$, causing $(d\sigma/dt)_d$ to vanish in the forward direction whether or not the two-body cross section vanishes. With increasing $-t$,

[7] A. Read, J. Orear, and H. Bethe, Nuovo Cimento **29**, 1051 (1963).

[8] S. Mandelstam, Ann. Phys. (N. Y.) **19**, 254 (1959).

[9] B. M. Schwarzschild, Lawrence Radiation Laboratory, University of California, Berkeley, thesis (UCRL-17572, Appendix B) (unpublished).

TABLE II. Fitted parameters.

	Trajectories α^0	α' $(\text{GeV})^{-2}$	KN residue parameters C^0 (mb×GeV)	D^0 (mb)	C^1 $(\text{GeV})^{-2}$	D^1 $(\text{GeV})^{-2}$	$\pi N/KN$ SU_3 breaking $F^0/F^{\pm}$	F^1 $(\text{GeV})^{-2}$
$\rho(-)$	0.58	0.92	1.30	22.7	2.92	0.26	1.10	−0.006
$\rho'(-)$	−0.48	1.44	5.02	−264	4.4	2.95	0.80	+0.20
$A_2(+)$	0.37	0.41	5.50	−116	0.42	0.66	1.01	−0.07

H approaches zero, falling to about 0.1 at $t=-0.13$ $(\text{GeV}/c)^2$, causing $(d\sigma/dt)_d$ to approach $(d\sigma/dt)_n$.

It remains to make the connection between R and the t-channel helicity amplitudes A, B. $R=f_2^2/f_1^2$, where f_1 and f_2 are, in conventional notation, given by $f(\theta)=f_1+f_2\sigma\cdot\hat{n}$. Now the s-channel helicity amplitudes g_1 and g_2 are given by $f=g_1+g_2\sigma\cdot\hat{k}_f\sigma\cdot\hat{k}_i$. Comparing the two representations and using well-known properties of the Pauli matrices σ, we get

$$R=|(g_2\sin\theta)/(g_1+g_2\cos\theta)|^2, \tag{28}$$

where θ is the center-of-mass (c.m.) scattering angle. Finally, having followed the formalism of Singh[10] for the t-channel helicity amplitudes, we have

$$g_{1,2}=\frac{(k^2+M^2)^{1/2}\pm M}{8\pi\sqrt{s}}[\pm A'+((\sqrt{s})\mp M)B],$$
$$A=A'+\frac{E+(t/4M)}{1-(t/4M^2)}B, \tag{29}$$

where k is the c.m. momentum.

In summary then, the K^+n charge-exchange data $(d\sigma/dt)_d$ are fitted to Eq. (26) in which the factor $(d\sigma/dt)_n$ is given by the Regge two-body formula (13). Note that the free-neutron cross section, especially near the forward direction, cannot be extracted from the deuterium data without the aid of a model which gives $R(t)$.

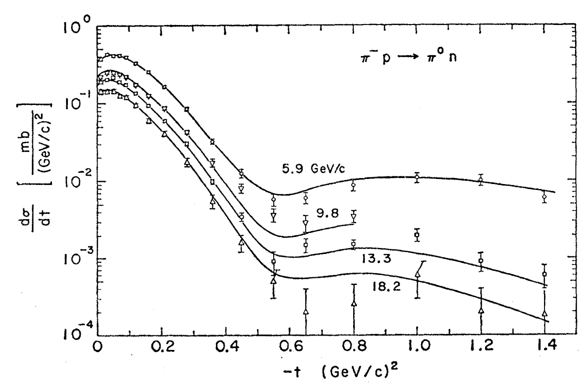

FIG. 3. π^-p charge-exchange differential cross sections, incoming lab momenta from 5.9 to 18.2 GeV/c. Data are from Saclay-Orsay Collaboration, Stirling *et al.* (Table I, footnote c). Solid curves are our Regge fits, Table II parameters.

[10] V. Singh, Phys. Rev. **129**, 1889 (1963). We have interchanged Singh's A and A'.

IV. SUPERCONVERGENT SUM RULE

To test the validity of additional Regge poles (and cuts) with the quantum numbers of the ρ, proposed to explain the π^-p charge-exchange polarization, Igi and Matsuda[5] have obtained a superconvergent sum rule. From a dispersion relation for that part of the $t=0$ amplitude which vanishes at infinity faster than E^{-1} (i.e., $\alpha<-1$), they get the sum rule

$$4\pi f^2=\frac{1}{2\pi}\int_{E=m}^{\infty}\{p[\sigma_T(\pi^+p)-\sigma_T(\pi^-p)] -\sqrt{2}\sum_i \beta_i L_{\alpha_i}{}^0(E/m)\}dE, \tag{30}$$

where $\sum\beta L$ is the sum, over contributing singularities with $\alpha^0>-1$ (in our case ρ and ρ'), of the imaginary parts of the forward Regge π^-p charge-exchange scattering amplitudes. $f^2=0.081$ is the πN coupling constant squared.

$$\beta_i L_{\alpha_i}{}^0(E/M)\xrightarrow[E\to\infty]{}\text{Im}_\pi A_i(t=0)^{\text{asymp}},$$

giving

$$\beta_i=F_i{}^0C_i{}^0(\alpha_i{}^0+1)\frac{\Gamma(\alpha_i{}^0+1)}{\Gamma(\alpha_i{}^0+\frac{1}{2})}(\sqrt{\pi})\left(\frac{m}{2E_0}\right)^{\alpha_i{}^0}. \tag{30'}$$

The integrand of (30) vanishes at energies sufficiently high that all contributions other than ρ and ρ' become negligible [cf. optical theorem, Eq. (15)]. We take the integral up to $39m$ (≈ 5.5 GeV/c), using Igi's numerical determination of $\int p\Delta\sigma_T dE$. Then Eq. (30) relates

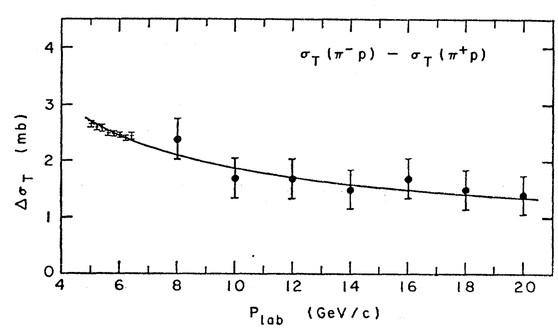

FIG. 4. πp total cross-section difference from 5 to 20 GeV/c. Data are from Galbraith *et al.*, Brookhaven (Table I, footnotes h and i). Solid curve is our Regge fit, Table II parameters.

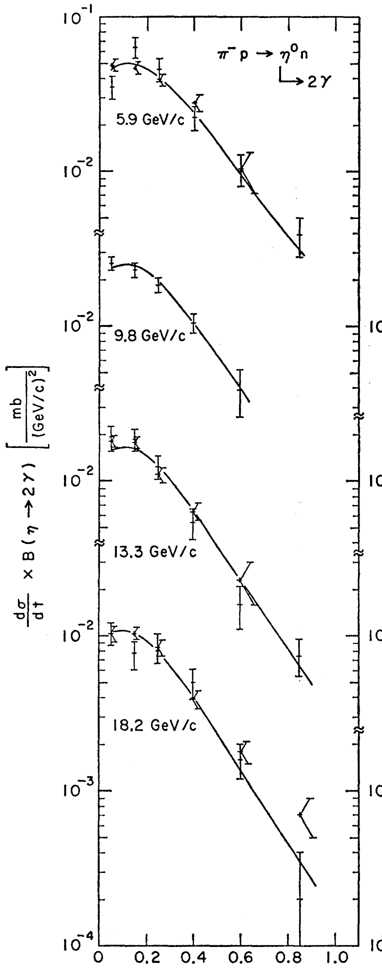

FIG. 5. $\pi^- p \rightarrow \eta^0 n$ differential cross sections reduced by branching ratio $B(\eta^0 \rightarrow 2\gamma)$, which is the only mode observed in these data, Stirling *et al.*, Saclay-Orsay Collaboration (Table I, footnote d). Solid curves are our Regge fits, Table II parameters. To make contact with the KN normalization parameters via SU_3, we use for the branching ratio 0.303 (Table I, footnotes e and f).

C_ρ^0, $C_{\rho'}^0$, α_ρ^0, $\alpha_{\rho'}^0$, F_ρ^0, and $F_{\rho'}^0$ in an equation of constraint, which we impose upon the fitting.

V. RESULTS

The fitted parameters resulting from a least-squares fit to the data are given in Table II. In Figs. 3–9 the resulting theoretical curves are shown, superimposed upon the data. For 194 data points we have a χ^2 of 191. The ρ and A_2 trajectories turn out essentially traditional, i.e., not unlike the results of the usual fits to the high-energy cross sections without a ρ'. The ρ' intercept, -0.48, is reasonably consistent with the Högaasen-Fischer determination, -0.6, from the $\bar{p}p$ and pn data.[4] The ρ' slope, $1.44/(\mathrm{GeV})^2$, gives a mass, at $\alpha=1$, of 1.01 GeV, suggesting the $\delta(965)$, about which little is known except that it is an isovector. Relative to the

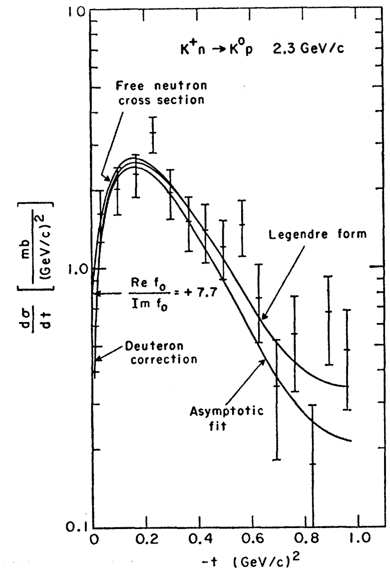

FIG. 6. K^+n charge exchange at 2.3 GeV/c, the highest-energy data available to date (Butterworth *et al.*, Berkeley, Table I, footnote a). In the forward region we show the Regge fit, with deuteron correction, to the deuterium data, as well as the free neutron cross section deduced from the fitted parameters, Table II. At larger angles one sees the correction due to the use of exact Legendre functions, as well as the fit using the high-energy asymptotic forms.

ρ, the ρ' contribution near the forward direction is given roughly by

$$\mathrm{Im}A_{\rho'}/\mathrm{Im}A_\rho \approx e^{3.7t}/E^{\Delta\alpha},$$

$$\mathrm{Im}B_{\rho'}/\mathrm{Im}B_\rho \approx 3e^{0.3t}/E^{\Delta\alpha},$$

$$\mathrm{Re}A_\rho \approx \mathrm{Im}A_\rho; \quad \mathrm{Re}A_{\rho'} \approx -\mathrm{Im}A_{\rho'},$$

where E is in GeV, and $\Delta\alpha \equiv \alpha_\rho - \alpha_{\rho'}$.

For the high-energy cross-section data we see that the ρ' plays only a small role. In Fig. 3 we have the usual dip in the $\pi^- p$ charge-exchange cross section at

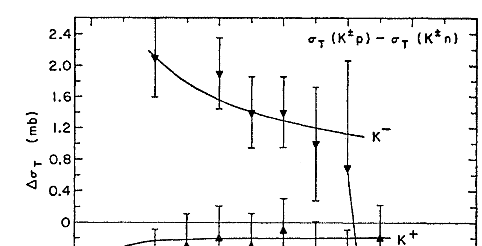

FIG. 7. K^+N and K^-N total cross-section differences, 6 to 20 GeV/c. $\sigma_T(K^-p)-\sigma_T(K^-n)>0$, and $\sigma_T(K^+p)-\sigma_T K^+n)<0$. Data of Galbraith *et al.*, Brookhaven (Table I, footnote h). We include also the K^+ datum at 2.3, for which we indicate here (and use in fitting) an uncertainty given by the amplitude of the small wiggles of $\Delta\sigma_T(K^+)$ in this region, the quoted experimental error being considerably smaller (Kycia, Brookhaven, Table I, footnote g). The solid curves are our Regge fits, Table II parameters.

$t \approx -0.6$, independent of energy, because $\alpha_\rho = 0$. Our "ghost killing and sense choosing" mechanism predicts an analogous dip in $\pi^- p \rightarrow \eta^0 n$ for $\alpha_R(t \approx -0.9) = 0$. Figure 5 shows that our data do not extend to sufficiently large momentum transfer to serve as evidence in this matter. The "nonsense choosing" mechanism of Gell-Mann would not require such a dip for even-signature trajectories.[6] Note that our η-production data measure only η's decaying to two photons. Therefore, to arrive at the SU_3-breaking parameter $F_R{}^0$, one needs to know the branching ratio for $\eta \rightarrow 2\gamma$. Using[11] $B(\eta \rightarrow 2\gamma) = 0.303$, we find $F_R{}^0$ differing from the unbroken F^+ for the A_2 by only 1%. For ρ and ρ' we have 10 and 20% SU_3-breaking in $F_i{}^0$. In each case the symmetry breaking is slightly greater for $t \neq 0$ because of the nonvanishing $F_i{}^1$. The zero of $C_\rho(t)$ occurs at $t = -0.34$ $(\mathrm{GeV}/c)^2$, roughly the first inflection point in the $\pi^- p$ charge-exchange cross sections.

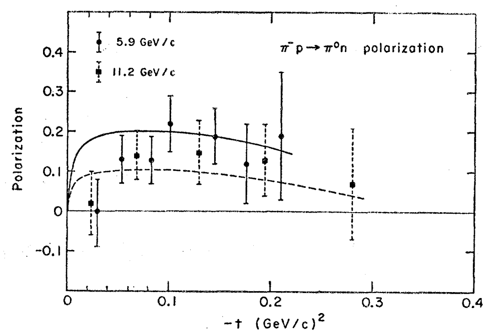

FIG. 8. $\pi^- p$ charge-exchange polarization, 5.9 GeV/c (solid error bars and fitted curve) and 11.2 GeV/c (dashed bars and curve). Data of Bonamy *et al.*, Saclay-Orsay-Pisa Collaboration (Table I, footnote j).

In the K^+N system at 2.3 GeV/c (Figs. 6 and 7), we see the ρ' asserting itself. In the forward direction, ${}_K A_\rho$ and ${}_K A_R$ are, roughly speaking, equal, and lie at $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ in the complex plane. Thus their imaginary parts subtract and real parts add for K^+ charge exchange (vice versa for K^-). The ρ' does not significantly alter the traditional result that the forward $K^+(K^-)$ amplitude is predominantly real (imaginary). We find $\mathrm{Re} f_0(K^+)/\mathrm{Im} f_0(K^+) = +7.7$. But when we include the $\Delta\sigma_T(K^+N)$ datum at 2.3 GeV/c and apply the optical theorem (Eq. 14), we see the ρ' at work. At high energies the cancellation of $\mathrm{Im}({}_K A_R - {}_K A_\rho)_{t=0}$ results in $\Delta\sigma_T(K^+N) \approx 0$ as seen in Fig. 7. This trend would continue down to 2.3 but for the ρ' term, $\sim -1/E$, which emerges at lower energies and causes $\Delta\sigma_T$ to increase negatively. This is required by the datum at 2.3. The error bar here is given by the amplitude of the small wiggles (presumably not a t-channel Regge effect) in the $\Delta\sigma_T(K^+N)$ data in this region.[12] The quoted experimental error is considerably smaller.

[11] Table I, footnotes e and f.
[12] Table I, footnote g.

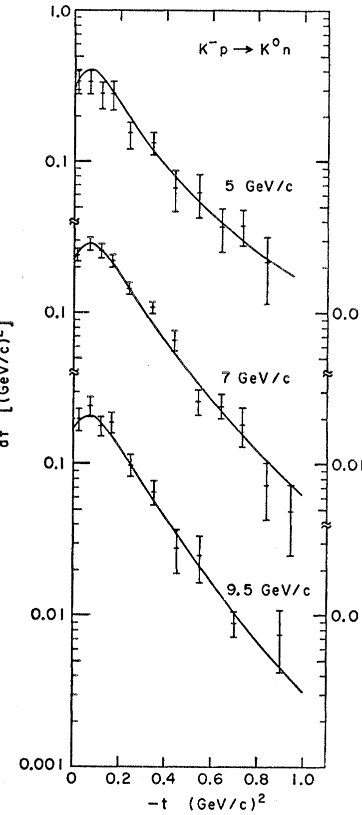

FIG. 9. $K^- p$ charge-exchange differential cross sections, incoming lab momenta 5 to 9.5 GeV/c. Data of Astbury *et al.*, CERN-ETH Zürich (Table I, footnote b). Solid curves are our Regge fits, Table II parameters.

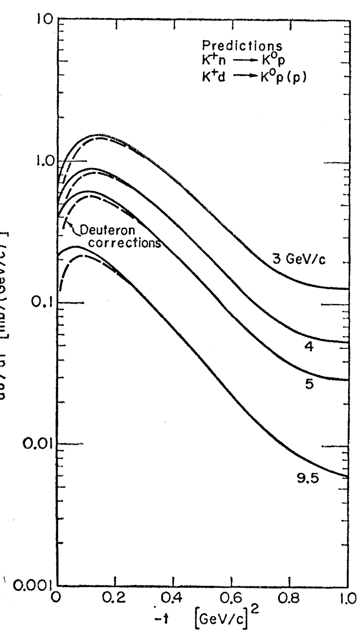

FIG. 10. Predictions of higher-energy K^+n charge exchange, from the fitted parameters, Table II. Up to 5 GeV/c we use here the exact Legendre functions. The dashed curves near $t=0$ show the predicted $K^+d \rightarrow K^0 p(p)$, i.e., the deuteron correction.

To see the contribution of the flip amplitudes to $d\sigma/dt$ (Eq. 13), we write, for small t,

$$\frac{\{(-t/4M^2)\{(s+p^2)/[1-(t/4M^2)]\}-s\}^{1/2}B_i}{(1-t/4M^2)^{1/2}A_i} \approx (-t)^{1/2}\frac{D_i^0}{C_i^0}\frac{\alpha_i^0}{2}\exp\{[D_i^1-C_i^1+(\alpha'/\alpha^0)_i]t\}.$$

For each trajectory, the t-channel helicity-flip contribution becomes comparable to the nonflip at $-t \lesssim 0.1$ (GeV)^2. This rapid rise from zero in the forward direction is responsible for the initial rise in each of the different cross sections. In the case of K^+n charge exchange at 2.3 (GeV/c), ρ' adds significantly and positively to the flip amplitude, producing a considerable initial rise and increasing the cross section in the peak region by about a factor of 2. Without the ρ', the theoretical curve in this region had stubbornly remained a factor of 2 below the data.

Figure 6 also shows the improvement in the fit at larger angles (smaller $-\cos\theta_t$) due to the use of the exact Legendre functions rather than the high-energy asymptotic expressions at this energy. Near the forward direction, where the deuteron effect is significant, the free neutron cross section is also shown. Without knowing R from the fitted Regge parameters, one could not say to what extent the forward dip in the data reflects the two-body cross section rather than the forward vanishing required by the deuteron effect.

For the π^-p charge-exchange polarization (Fig. 8), the ρ' is of course, in our model, the *conditio sine qua non*.[13] For $-t<0.34$, where our $C_\rho(t)$ goes through zero, we have

$$_\pi\mathbf{A}^-\times{}_\pi\mathbf{B}^- = \sin[\tfrac{1}{2}\pi(\alpha_\rho-\alpha_{\rho'})](|_\pi A_\rho||_\pi B_{\rho'}|-|_\pi A_{\rho'}||_\pi B_\rho|).$$

The data require, and we find, $A_\rho B_{\rho'}>A_{\rho'}B_\rho$, giving positive polarization. The approximate orthogonality of the ρ and ρ' amplitudes for small t gives roughly maximal polarization, given the magnitudes and signs of the amplitudes. If the polarization is in fact due to a ρ' trajectory lying about one unit of angular momentum below the ρ, we have perforce an $\sim 1/E$ falloff in the polarization. The data, being quite uncertain, are consistent with this, but are also consistent with an energy-independent polarization. Better polarization data would constitute a severe test of our model. The unsightly high shoulder in the theoretical curves at the smallest-t data emerged upon imposition of the sum-rule constraint. Previously, the two data around $-t=0.03$ had been better fitted. The good fit for $\Delta\sigma_T(\pi p)$, shown in Fig. 4, guarantees the convergence of the sum-rule integral (Eq. 30) above 5 GeV/c. The ρ' term turns out to make 1/15 the contribution of the ρ term to this integral.

Finally, in Fig. 10 are shown some higher-energy K^+n charge-exchange cross-section predictions from our fitted parameters. As the ρ' contribution wanes with increasing energy, the forward turnover diminishes, but, as in the case of K^-p charge exchange (Fig. 9), some turnover persists at high energies. For comparison with experiment, the deuteron effect, which causes the observed cross section to vanish at $t=0$, is also shown. Except for the 9.5 GeV/c prediction, where the difference has become negligible, exact Legendre functions were used here in place of the asymptotic forms.

VI. POSTSCRIPT

Our Regge fit gives for the real part of the forward K^+n charge-exchange scattering amplitude at 2.3 GeV/c

$$k\,\mathrm{Re}f(t=0)=-0.69.$$

Note that, unlike the imaginary part, this is not a directly observable quantity in deuterium. In a recent paper, Carter,[14] using forward K^+N dispersion relations, calculates for this quantity the value -0.70 ± 0.05.

ACKNOWLEDGMENTS

We wish to thank Dr. Charles Chiu for his helpfulness in several phases of this work, and Professor Geoffrey Chew for calling the Igi sum rule to our attention. Rarita thanks Professor Burton Moyer for the hospitality of the Physics Department, Berkeley. Schwarzschild wishes especially to thank Professor Gerson Goldhaber, who, together with his wife Sulamith, of blessed memory, gave guidance and forbearance in generous measure during his graduate studies.

[13] R. K. Logan, J. Beaupre, and L. Sertorio, Phys. Rev. Letters **18**, 259 (1967).

[14] A. A. Carter, Phys. Rev. Letters **18**, 801 (1967).