CP-Violating Interference Effects in Radiative K^0 Decays*

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(Received 13 April 1967)

It is shown that the observation of interference between K_L and K_S decays in the partial decay rate into any nonleptonic mode is direct evidence of CP violation. The examples of $K^0 \to \gamma+\gamma$ and $K^0 \to \pi^+ + \pi^- + \gamma$ are analyzed in detail; a large interference effect may be possible in either of these cases if there is a large CP violation associated with interactions involving photon emission.

I. INTRODUCTION

THE discovery that the long-lived K^0 meson, K_L, decays into two pions[1] was clear evidence of CP violation. No other process involving CP violation has yet been uncovered. In this paper we consider the possibilities of observing CP violation in the decays $K^0 \to \gamma+\gamma$ and $K^0 \to \pi^+ + \pi^- + \gamma$. These are of particular interest because they involve electromagnetic interactions, which have been suggested as a possible source of CP violation,[2,3] and because the interference phenomena in K^0 decay provide a unique tool for identifying CP violation.

In looking for CP violation in K^0 decays other than $K^0 \to 2\pi$ or $K^0 \to 3\pi^0$, we cannot look for a simple violation of a selection rule since both K_L and K_S are allowed to decay into such final states as $\pi^+ + \pi^- + \pi^0$, $\gamma+\gamma$, and $\pi+\pi+\gamma$. In these cases, the following theorem forms the basis of identifying CP violation: *for any possible nonleptonic decay mode of the K^0 meson the observation of an interference effect between K_L and K_S decays in the partial decay rate of this mode is clear evidence of CP violation.* By a decay mode, we mean a certain set of particles, and by the partial rate we mean the rate for decay into this set of particles summed over the polarizations and momenta of the particles.[4]

The proof of this theorem is fairly trivial and undoubtedly implicit in many other papers. In order to have an interference effect after the integration over the space variables (relative momenta), the interfering final states of K_L and K_S decay must have the same parity. If CP were valid, this would mean that these states have opposite values of C. Now the general nonleptonic mode will consist of $m\gamma^0 + n\pi^0 + p(\pi^+ + \pi^-)$ where $p=0$ or 1, and $2p+n \leq 3$; for this mode, $C = (-1)^m(-1)^{pL}$ where L is the relative orbital angular momentum of π^+ and π^-. Therefore states with opposite values of C have different values of L so that integration over the relative momentum of the $\pi^+\pi^-$ pair destroys the interference effect. This argument cannot be applied to final states involving neutrinos, since these are not C eigenstates.

In Sec. II we review the general formalism governing interference experiments.[5] This allows us to separate the CP-violating term into that due to the CP impurity of the K_L and K_S states and that due to the CP violation in the specific decay amplitude. The division depends to some extent, of course, on the Wu-Yang phase convention[6] which we employ.

II. GENERAL FORMALISM

In the most general interference experiment, if one starts with a coherent mixture of $|K_S\rangle$ and $|K_L\rangle$ given by

$$N(|K_L\rangle + R|K_S\rangle), \tag{1}$$

the partial decay rate into a set a of final states is given by[7]

$$I_a(t) = N^2[\gamma_{La}e^{-\gamma_L t} + |R|^2\gamma_{Sa}e^{-\gamma_S t} + 2(\gamma_{La}\gamma_{Sa})^{1/2}\,\mathrm{Re}(RV_a e^{-i\delta t})e^{-(\gamma_L+\gamma_S)t/2}], \tag{2}$$

where $\delta = m_S - m_L$, and γ_i and m_i are the total widths and masses, respectively. The four real parameters γ_{La}, γ_{Sa}, $\mathrm{Re}V_a$, $\mathrm{Im}V_a$ are related to the decay amplitudes $\langle\alpha|T|K_L\rangle$ and $\langle\alpha|T|K_S\rangle$ by

$$\gamma_{La} = \sum_\alpha |\langle\alpha|T|K_L\rangle|^2, \tag{3a}$$

$$\gamma_{Sa} = \sum_\alpha |\langle\alpha|T|K_S\rangle|^2, \tag{3b}$$

$$V_a = \sum_\alpha \langle\alpha|T|K_L\rangle^* \langle\alpha|T|K_S\rangle / (\gamma_{La}\gamma_{Sa})^{1/2}, = \frac{\sum_\alpha \langle\alpha|T|K_L\rangle^* \langle\alpha|T|K_S\rangle}{(\sum_\alpha |\langle\alpha|T|K_L\rangle|^2)^{1/2}(\sum_\alpha |\langle\alpha|T|K_S\rangle|^2)^{1/2}}. \tag{3c}$$

Here $|\alpha\rangle$ denotes a final state which is completely

* This work was supported by the U. S. Atomic Energy Commission.

[1] J. H. Christenson *et al.*, Phys. Rev. Letters **13**, 138 (1964).

[2] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. **139**, B1650 (1965); S. Barshay, Phys. Letters **17**, 78 (1965).

[3] F. Salzmann and G. Salzmann, Phys. Letters **15**, 91 (1965).

[4] As will be seen by the proof below the theorem only requires a sum (integral) over the internal *angle* variables. For example, for the cases of $\pi^+ + \pi^- + \pi^0$ or $\pi^+ + \pi^- + \gamma$ the theorem holds as long as the direction of the relative momentum of the $\pi^+\pi^-$ pair is integrated over, independent of the range of effective dipion masses included.

[5] This formalism has been summarized in a number of places; see, for example, J. S. Bell and J. Steinberger, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).

[6] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964).

[7] For a pure $K^0(\bar{K}^0)$ beam we have $R=1(-1)$. It has been noted by P. K. Kabir (private communication) that CP invariance implies that it is impossible to distinguish in an absolute sense K^0 from $\bar{K}^0$. Thus if the set of states a transforms into itself under CP it follows that the linear term in R must vanish and so $V_a = 0$. This is the content of the theorem discussed in Sec. I.

specified, perhaps by the polarizations of all the particles and all the independent momenta, and the sum is over all states in the set a. The parameters γ_{La} and γ_{Sa} are the partial decay widths for K_L and K_S decay.

The complex parameter V_a defines the possible interference effect in the sense that if $|V_a|=1$ complete interference is possible and can be obtained at $t=0$ by using a beam with $|R|^2=\gamma_{La}/\gamma_{Sa}$. Such a beam can be obtained by regeneration from a K_L beam provided the requisite $|R|$ is much less than unity; that is, $\gamma_{Sa}\gg\gamma_{La}$. As long as the requisite $|R|$ is less than unity, such a beam can be obtained from the decay of a beam which is initially pure K^0 or $\bar{K}^0$. From Eq. (1) it is seen that the phase of V_a determines the phase of the interference effect. The theorem proved in Sec. 1 states that if CP were conserved V_a would equal zero provided the set a includes a sum over the momenta and polarizations of the particles.[4] The unitarity condition states

$$\sum_a (\gamma_{La}\gamma_{Sa})^{1/2}V_a=\left(\frac{\gamma_L+\gamma_S}{2}+i\delta\right)\langle K_L|K_S\rangle, \tag{4}$$

where the sum is over all possible sets (nonintersecting) of final states.

We now assume CPT invariance and see how V_a is related to CP violation in the mass matrix and the CP violation in the decay amplitude. For this purpose we use as a basis the CP eigenstates

$$|K_+\rangle=(|K\rangle+|\bar{K}\rangle)/\sqrt{2},$$
$$|K_-\rangle=(|K\rangle-|\bar{K}\rangle)/\sqrt{2},$$

in terms of which[8]

$$K_S=n\begin{pmatrix}1\\ \epsilon\end{pmatrix},\quad K_L=n\begin{pmatrix}\epsilon\\ 1\end{pmatrix}, \tag{5}$$
$$n=(1+|\epsilon|^2)^{-1/2}.$$

The decay amplitudes may be written

$$\begin{aligned}\langle\alpha|T|K_+\rangle&=(i)^{p_\alpha}c_\alpha e^{i\rho_\alpha},\\ \langle\alpha|T|K_-\rangle&=(i)^{p_\alpha-1}d_\alpha e^{i\mu_\alpha},\end{aligned} \tag{6}$$

where $p_\alpha=0$ for CP-even modes and $p_\alpha=1$ for CP-odd modes. If there were no final state interactions ρ_α and μ_α (the "unitarity phases") would be zero, since it is understood that c_α and d_α are real; in this case, the CP-violating amplitude is the pure imaginary one. If the state α is an eigenstate of the strong-plus-electromagnetic interactions, $\rho_\alpha=\mu_\alpha$.

[8] Our definition of ϵ agrees with that of T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965). In the notation of Ref. 6 our ϵ would be called $(\epsilon/2)$. Note also the usual phase convention in which $|K_-\rangle$ and all other CP-odd stationary states are odd under CPT.

Neglecting $|\epsilon|^2$ compared to unity,

$$\gamma_{La}=|\epsilon|^2\gamma_{+a}+\gamma_{-a}+2\,\mathrm{Re}(\epsilon b_a), \tag{7a}$$

$$\gamma_{Sa}=\gamma_{+a}+|\epsilon|^2\gamma_{-a}+2\,\mathrm{Re}(\epsilon^* b_a), \tag{7b}$$

$$V_a=(\epsilon^*\gamma_{+a}+\epsilon\gamma_{-a}+b_a)/(\gamma_{La}\gamma_{Sa})^{1/2}, \tag{7c}$$

where

$$\gamma_{+a}=\sum_\alpha c_\alpha^2, \tag{8a}$$

$$\gamma_{-a}=\sum_\alpha d_\alpha^2, \tag{8b}$$

$$b_a=\sum_\alpha ic_\alpha d_\alpha e^{i(\rho_\alpha-\mu_\alpha)}. \tag{8c}$$

Two limiting cases are of interest: (1) If the only CP violation for this decay mode comes from the mass matrix, $b_a=0$ and the interference parameter V_a can be written

$$V_a=\frac{\epsilon^*(\gamma_{Sa}-|\epsilon|^2\gamma_{La})+\epsilon(\gamma_{La}-|\epsilon|^2\gamma_{Sa})}{(\gamma_{La}\gamma_{Sa})^{1/2}}. \tag{9}$$

Since for practical purposes, interference experiments probably require $\gamma_{Sa}\gg\gamma_{La}$, we have for this case

$$V_a\simeq\epsilon^*(\gamma_{Sa}/\gamma_{La})^{1/2}. \tag{10}$$

(2) If $|\epsilon|\simeq0$, or if the CP-violating effect in the mass matrix is too small to be of importance, then

$$\begin{aligned}V_a&=b_a/(\gamma_{La}\gamma_{Sa})^{1/2}\\ &=\frac{\sum_\alpha ic_\alpha d_\alpha e^{i(\rho_\alpha-\mu_\alpha)}}{(\sum_\alpha c_\alpha^2\sum_\alpha d_\alpha^2)^{1/2}}.\end{aligned} \tag{11}$$

III. DECAY $K^0\rightarrow\gamma+\gamma$

There are two possible states of the two-photon system,

$$\begin{aligned}|X_e\rangle&=(|LL\rangle+|RR\rangle)/\sqrt{2},\\ |X_o\rangle&=(|LL\rangle-|RR\rangle)/\sqrt{2},\end{aligned} \tag{12}$$

of which $|X_e\rangle$ is even under CP and $|X_o\rangle$ is odd. Following the notation of Eq. (6), the transition amplitudes to these states are shown in Table I. The interference parameter V_γ is obtained from Eqs. (7c) and (8c).[9]

$$V_\gamma=\frac{\epsilon^*\gamma_{+\gamma}+\epsilon\gamma_{-\gamma}+ic_ed_e\eta_e+ic_od_o\eta_o}{(\gamma_{S\gamma}\gamma_{L\gamma})^{1/2}}, \tag{13}$$

[9] The subscript γ here replaces a and refers to the final two-photon states summed over polarizations. Interference effects occur when the polarization of the photons is observed even if CP is not violated, J. Dreitlein and H. Primakoff, Phys. Rev. **124**, 268 (1961). For the CP-violating case this is discussed in Appendix A.

where the partial decay widths are given by

$$\gamma_{L\gamma}=|\epsilon|^2\gamma_{+\gamma}+\gamma_{-\gamma}-2c_ed_e\,\mathrm{Im}(\epsilon\eta_e)-2c_od_o\,\mathrm{Im}(\epsilon\eta_o),$$
$$\gamma_{S\gamma}=\gamma_{+\gamma}+|\epsilon|^2\gamma_{-\gamma}-2c_ed_e\,\mathrm{Im}(\epsilon^*\eta_e)-2c_od_o\,\mathrm{Im}(\epsilon^*\eta_o), \quad (14)$$

and

$$\eta_i=e^{i(\rho_i-\mu_i)},$$
$$\gamma_{+\gamma}=c_e^2+c_o^2, \quad (15)$$
$$\gamma_{-\gamma}=d_e^2+d_o^2.$$

The decay rate of $K_L\rightarrow\gamma+\gamma$ has recently been measured and two values have been quoted:

$$\gamma_{L\gamma}=(2.6\pm1.2)\times10^3\ \mathrm{sec}^{-1}\quad(\text{Ref. }10),$$
$$\gamma_{L\gamma}=(14.8\pm3.2)\times10^3\ \mathrm{sec}^{-1}\quad(\text{Ref. }11).$$

The rate for $K_S\rightarrow\gamma+\gamma$ can be estimated by assuming that it proceeds predominantly through the two-pion intermediate state with the quantum numbers $J=0$, $I=0$. (For the purpose of estimating the rate, we can ignore CP violation.) Such an estimate was made by Barger[12] and the result expressed in terms of the $\pi\pi$ $I=0$ s-wave phase shifts. If the interaction is neglected, one obtains the perturbation-theory value $\gamma_S=3\times10^4$ sec^{-1}. For small positive scattering lengths such as those indicated by recent analyses of $\pi\pi$ scattering there is some enhancement. It therefore seems quite possible that $\gamma_{S\gamma}$ is considerably larger than $\gamma_{L\gamma}$ so that interference experiments may be considered. We consider as a possible range of values $(\gamma_{S\gamma}/\gamma_{L\gamma})$ between 10 and 1000; a lower ratio would probably make the interference experiments impossible. With these assumptions, and using $|\epsilon|\lesssim2\times10^{-3}$, Eqs. (13) and (14) may be approximated (to at least 10% accuracy)

$$\frac{\gamma_{L\gamma}}{\gamma_{S\gamma}}\simeq\frac{\gamma_{-\gamma}}{\gamma_{+\gamma}}\simeq D_e^2+D_o^2 \quad (16)$$

$$V_\gamma\simeq(\epsilon^*+iC_eD_e\eta_e+iC_oD_o\eta_o)(\gamma_{S\gamma}/\gamma_{L\gamma})^{1/2} \quad (17)$$

where

$$D_e=d_e/(c_e^2+c_o^2)^{1/2},\quad D_o=d_o/(c_e^2+c_o^2)^{1/2},$$
$$C_o=c_o/(c_e^2+c_o^2)^{1/2},\quad C_e=c_e/(c_e^2+c_o^2)^{1/2}.$$

The phase factors η_e and η_o are nonreal since the "effective Hamiltonian" responsible for $K^0\rightarrow\gamma+\gamma$ is, in general, not Hermitian because of the possibility of real intermediate states. To say the same thing another way, we may note that the states $|X_e\rangle$ and $|X_o\rangle$ are not eigenstates of the strong plus electromagnetic interaction to order e^2; there is more than one eigenstate of which $|X_e\rangle$ (or $|X_o\rangle$) is a component, and $|K_+\rangle$ and $|K_-\rangle$ decay to different combinations of these eigenstates. We now give rough estimates of ρ_e and μ_o. It seems reasonable to assume that the only real intermediate state that need be considered is the 2π state; to the extent that 3π intermediate states contribute to $K^0\rightarrow\gamma+\gamma$, the virtual states should be much more important than the real states, because of the small phase space for three pions at the kaon mass. With this assumption, since the CP-conserving decay $K_-\rightarrow X_o$ certainly does not involve a 2π intermediate state, we have $\mu_o=0$. The phase ρ_e depends on the dynamical calculation of the CP-conserving process $K_+\rightarrow2\pi\rightarrow2\gamma$; ignoring the $\pi\pi$ interaction, Barger[12] finds $\rho_e\simeq0.4\pi$. On the other hand, the phases μ_e and ρ_o for the CP-violating amplitudes depend on the model of CP violation.

TABLE I. Definition of amplitudes for $K^0\rightarrow\gamma+\gamma$.

	K_+	K_-
X_e	$c_ee^{i\rho_e}$	$-id_ee^{i\mu_e}$
X_o	$+ic_oe^{i\rho_o}$	$d_oe^{i\mu_o}$

We now consider possible numerical values for the CP-violating parameter V_γ on the basis of various models of CP violation.

(A) There are no CP-violating contributions to the amplitude $K^0\rightarrow\gamma+\gamma$ so that the only CP violation is in the mass matrix. In this case, Eq. (17) reduces to Eq. (10):

$$V_\gamma\simeq\epsilon^*(\gamma_{S\gamma}/\gamma_{L\gamma})^{1/2}.$$

(B) A more reasonable assumption would be that the only CP violation was *either* in the mass matrix *or* in the virtual process $K^0\rightarrow\pi^++\pi^-$ which contributes to $K^0\rightarrow\gamma+\gamma$.[13] This means assuming that the only CP violation contributing to $K^0\rightarrow\gamma+\gamma$ is that which contributes directly to $K^0\rightarrow2\pi$, and thus we may hope to relate the CP violation in the two cases. In the standard analysis[6] of $K^0\rightarrow2\pi$ the CP violation is given by[8]

$$\eta_{+-}=\frac{\langle\pi^+\pi^-|T|K_L\rangle}{\langle\pi^+\pi^-|T|K_S\rangle}=\epsilon+\frac{\langle\pi^+\pi^-|T|K_-\rangle}{\langle\pi^+\pi^-|T|K_+\rangle}. \quad (18)$$

The second term in the notation of Ref. 6 is $\frac{1}{2}\sqrt{2}i\,\exp i(\delta_2-\delta_0)\,\mathrm{Im}A_2/A_0$. If we now make the approximation that the virtual process $\pi^++\pi^-\rightarrow\gamma+\gamma$ is the same for K_- decay as for K_+ decay

$$-i\frac{D_e\eta_e^*}{C_e}\equiv\frac{\langle X_e|T|K_-\rangle}{\langle X_e|T|K_+\rangle}=\frac{\langle\pi^+\pi^-|T|K_-\rangle}{\langle\pi^+\pi^-|T|K_+\rangle}. \quad (19)$$

It should be emphasized that the approximation is not too reasonable because the K_- decay involves an inter-

[10] L. Criegee, J. D. Fox, H. Frauenfelder, A. O. Hanson, G. Moscati, C. F. Perdrissat, and J. Todoroff, Phys. Rev. Letters **17**, 150 (1966).

[11] J. W. Cronin, P. F. Kunz, W. S. Risk, and P. C. Wheeler, Phys. Rev. Letters **18**, 25 (1967).

[12] V. Barger, Nuovo Cimento **32**, 127 (1964).

[13] The model of T. Truong, Phys. Rev. Letters **13**, 358a (1964), in which CP violation occurs only for $\Delta I>\frac{1}{2}$ decays would effectively be an example of this case, since virtual $I>1$ states other than 2π, such as 3π, would be expected to make a very small contribution to the CP violation in $K^0\rightarrow\gamma+\gamma$.

mediate dipion state that has a large $I=2$ component, whereas the K_+ decay goes almost completely to the $I=0$ state; therefore, when the pion-pion interaction is included the "virtual rate" for $\pi^+ + \pi^- \to \gamma+\gamma$ may be quite different for K_+ and K_-. If we substitute Eq. (19) into Eq. (17), note that $c_o=0$ in this model, and make use of Eq. (18)

$$V_\gamma = (\eta_{+-})^* (\gamma_{S\gamma}/\gamma_{L\gamma})^{1/2}. \quad (20)$$

This result amusingly depends only on η_{+-} and not on the separate values of ϵ and $\mathrm{Im}(A_2/A_0)$; this follows almost directly from the assumptions and approximations made. On this assumption $|V_\gamma|$ ranges from 6×10^{-3} to 6×10^{-2} as $(\gamma_{S\gamma}/\gamma_{L\gamma})$ ranges from 10 to 1000.

(C) Continuing to consider CP violation in the weak interactions, we assume CP violation in the parity-conserving nonleptonic Hamiltonian which contributes to $|X_o\rangle$ final states. As a simple model[14] we consider that the parity-conserving and parity-violating parts of the nonleptonic Hamiltonian have a relative phase factor $e^{i\phi}$. It then follows for the final state $|X_o\rangle$, as for any CP-odd final state, that

$$\frac{\langle X_o|T|K_+\rangle}{\langle X_o|T|K_-\rangle} \equiv i\frac{C_o}{D_o}\eta_o = i\tan\phi.$$

If we neglect ϵ^* and D_e in Eqs. (16) and (17) we then have

$$V_\gamma = i\tan\phi(\gamma_{L\gamma}/\gamma_{S\gamma})^{1/2}. \quad (21)$$

Alternatively we can assume that both this source of CP violation and that assumed in B are operative, in which case we find[15]

$$V_\gamma \simeq \left(\frac{\gamma_{S\gamma}}{\gamma_{L\gamma}}\right)^{1/2}\left[\eta_{+-}^* + i\left(\frac{\gamma_{L\gamma}}{\gamma_{S\gamma}}\right)\tan\phi\right]. \quad (22)$$

In Appendix B, an argument is given indicating that ϕ is probably quite small; however, it should be noted that direct experimental evidence from $K^0\to 3\pi$ does not put much restriction on the value of ϕ. Assuming $\phi\leq 0.1$, Eq. (22) gives an upper limit on V_γ of 0.04, 0.03, and 0.07 for values of $(\gamma_{S\gamma}/\gamma_{L\gamma})$ of 10, 100, and 1000, respectively.

(D) Finally there are suggestions that CP violation may be associated with photon emission, either as a C-violating part of the electromagnetic interaction[2] or a CP-violating part of the effective weak-plus-electromagnetic interaction.[3] In either case one might expect large CP-violating effects in the electromagnetic process $K^0\to\gamma+\gamma$. The only real intermediate states that can enter are for the case of C violation in electromagnetism in which case there might be a contribution from $K^0\to$ ($S=0$ state with $C=-1$) $\to\gamma+\gamma$. The only possibility is the 3π state with $C=-1$, which corresponds to the very unlikely $I=0$ or $I=2$ final states in the CP-conserving three-pion decay of K_+. Thus we may assume $\mu_e=\rho_o=0$. The phase of V_γ then is pure imaginary if the CP violation is only in the K_+ decay but is given by $ie^{i\rho_e}$ if the CP violation is only in the K_- decay.

The fact that the most obvious intermediate states, those with one and two pions or three pions in the $I=1$ state all have $C=+1$ may suggest that a C-violating electromagnetic interaction is not of importance in $K\to\gamma+\gamma$ decay. However, many diagrams involving intermediate vector-meson states or the emission of one of the photons before the weak interaction allow CP-violating photon emission.

Given the small CP-violating rates predicted if CP violation were only in the weak interactions (case A to C above), a large magnitude for V_γ would be significant evidence in favor of CP violation associated with a photon emission process.

IV. DECAY $K^0\to\pi^+ + \pi^- + \gamma$

The states of the $\pi^+\pi^-\gamma$ system may be written $|\lambda\mathbf{kp}\rangle$ where λ is the photon helicity, $\mathbf{k}$ the photon momentum vector in the dipion center-of-mass frame, and $\mathbf{p}$ the momentum of the π^+ in this frame. From energy conservation

$$p^2 = \tfrac{1}{4}(E_0-k)^2 - m^2 \quad (p\equiv|\mathbf{p}|), \quad (23)$$

where $E_0=(M^2+k^2)^{1/2}$ is the energy of the K meson in this frame. M and m are the kaon and pion masses respectively. In place of helicity states we define the states $|E\mathbf{kp}\rangle$ and $|M\mathbf{kp}\rangle$, which are even and odd under CP, respectively.[16] If we choose the positive z-axis along the direction $\mathbf{k}$ and consider only odd order multipoles, these states may be written explicitly as

$$|E\mathbf{kp}\rangle = \frac{1}{\sqrt{2}}(e^{-i\varphi}|R\mathbf{kp}\rangle - e^{i\varphi}|L\mathbf{kp}\rangle)$$
$$|M\mathbf{kp}\rangle = \frac{1}{\sqrt{2}}(e^{-i\varphi}|R\mathbf{kp}\rangle + e^{i\varphi}|L\mathbf{kp}\rangle) \quad (24)$$

where φ is the azimuthal angle of $\mathbf{p}$. The transition amplitudes to these states are defined in Table II. It follows from the fact that the K meson has zero spin that the amplitudes depend only on k and θ where

[14] An example of this model has been given by W. Alles, Phys. Letters 14, 348 (1965) and discussed by L. Wolfenstein, Nuovo Cimento 42, 17 (1966). See also Appendix B.

[15] If the only CP violation is due to the phase factor $e^{i\phi}$, then for the process $K^0\to 2\pi$ the only CP violation is in the mass matrix. (Reference 14.) Recent experiments on $K^0\to 2\pi^0$ have shown that this is not the case, so that there must be some CP violation in the effective parity-violating nonleptonic Hamiltonian H_v by itself. It is possible to assume, however, that most of H_v giving rise to $K^0\to(\pi\pi)$ in an $I=0$ state is CP-conserving and that CP violation shows up both as a small part of H_v and in the relative phase ϕ between H_c and the main part of H_v; it is this assumption that leads to Eq. (22).

[16] The labels E and M are used for even and odd CP states, respectively. For the odd-order multipoles which we consider, these correspond to electric and magnetic radiation.

TABLE II. Definition of amplitudes for $K^0 \to \pi^+ + \pi^- + \gamma$.

	K_+	K_-
$\vert E\mathbf{kp}\rangle$	$c_E(\mathbf{kp})e^{i\rho_E}$	$-id_E(\mathbf{kp})e^{i\mu_E}$
$\vert M\mathbf{kp}\rangle$	$ic_M(\mathbf{kp})e^{i\rho_M}$	$d_M(\mathbf{kp})e^{i\mu_M}$

$\cos\theta = \hat{k}\cdot\hat{p}$. The phase factors in Table II also depend on k and θ but we omit writing this explicitly.

For an interference experiment that measures the decay rate into all $\pi^+\pi^-\gamma$ states for which the photon energy lies between $k_{\max}$ and $k_{\min}$, we obtain from Eqs. (7c) and (8c)[17]

$$V_\pi = \frac{\epsilon^*\gamma_{+\pi} + \epsilon\gamma_{-\pi} + b_\pi}{(\gamma_{L\pi}\gamma_{S\pi})^{1/2}}, \tag{25}$$

where the partial decay widths are given by

$$\begin{aligned} \gamma_{L\pi} &= |\epsilon|^2\gamma_{+\pi} + \gamma_{-\pi} + 2\,\mathrm{Re}(\epsilon b_\pi), \\ \gamma_{S\pi} &= \gamma_{+\pi} + |\epsilon|^2\gamma_{-\pi} + 2\,\mathrm{Re}(\epsilon^* b_\pi), \end{aligned} \tag{26}$$

and

$$\gamma_{+\pi} = M\int_{k_{\min}}^{k_{\max}} dk\,\phi(k)\int_{-1}^{+1} d\cos\theta(c_E^2 + c_M^2), \tag{27a}$$

$$\gamma_{-\pi} = M\int_{k_{\min}}^{k_{\max}} dk\,\phi(k)\int_{-1}^{+1} d\cos\theta(d_E^2 + d_M^2), \tag{27b}$$

$$b_\pi = M\int_{k_{\min}}^{k_{\max}} dk\,\phi(k)\int_{-1}^{+1} d\cos\theta\, i\{c_E d_E e^{i(\rho_E-\mu_E)} + c_M d_M e^{i(\rho_M-\mu_M)}\}. \tag{27c}$$

$\phi(k)$ is the phase-space factor

$$\phi(k) = \frac{1}{(2\pi)^3}\frac{1}{8M^2}k\left[1 - \frac{4m^2}{(E_0-k)^2}\right]^{1/2}. \tag{27d}$$

The decay $K_S \to \pi^+ + \pi^- + \gamma$ is known to occur at a rate of the order of 10^7 sec^{-1}, for $k > 50$ MeV.[18] It seems likely that it can be described by the CP-conserving inner-bremsstrahlung process. We shall assume therefore that c_M is negligible, while c_E is given by the internal bremsstrahlung amplitude

$$eg\left[\frac{\epsilon\cdot p_+}{k\cdot p_+} - \frac{\epsilon\cdot p_-}{k\cdot p_-}\right]. \tag{28}$$

Here ϵ, k, p_+, and p_- are four-vectors denoting photon polarization, photon momentum, π^+ momentum, and π^- momentum, respectively. The coupling constant g is defined by

$$\mathrm{Rate}(K_S \to \pi^+ + \pi^-) = \frac{g^2}{16\pi M}\left(1 - \frac{4m^2}{M^2}\right)^{1/2}. \tag{29}$$

In the dipion center-of-mass frame, the amplitude (28) becomes

$$c_E(\mathbf{k},\mathbf{p}) = eg\frac{2}{k}\frac{x}{1 - x^2\cos^2\theta}\sin\theta, \tag{30}$$

where $x = (2p/E_0 - k)$. Clearly the amplitude c_E contains only odd electric multipoles. The phase ρ_E, at least for low-energy photons is approximately the phase of the $\pi\pi$ $I=J=0$ scattering amplitude at the kaon mass: $\rho_E \simeq \delta_0$.

The decay rate $K_L \to \pi^+ + \pi^- + \gamma$ has not been measured[19] but it is not improbable that it is of the order of 10^4 sec^{-1}. In this case, it clearly cannot be described as internal bremsstrahlung from $K_L \to \pi^+ + \pi^-$ but must occur as a direct emission process. Thus d_o and d_e are the CP-conserving and CP-violating parts of the direct emission amplitude for $K_- \to \pi^+ + \pi^- + \gamma$. If we restrict the decay to dipole radiation, we may write[20]

$$\begin{aligned} d_M(\mathbf{k},\mathbf{p}) &= h_M M^{-2} pk\sin\theta, \\ d_E(\mathbf{k},\mathbf{p}) &= h_E M^{-2} pk\sin\theta. \end{aligned} \tag{31}$$

h_M and h_E are real form factors which we shall assume to vary slowly enough to be treated as constants. The phases μ_E and μ_M are probably not very different from the $\pi\pi$ p-wave shift at the energy of the dipion; this phase is known to be small at the energies involved in this decay. We shall therefore take $\mu_E = \mu_M = 0$. Using Eqs. (28) and (29) together with $c_M = 0$ and our approximations for the phases, we obtain for the angular integrals in Eq. (27)

$$F_1(k) \equiv \int_{-1}^{+1} d(\cos\theta)c_E^2 = \left(\frac{2}{k}\right)^2(eg)^2 \times\left\{-1 + \frac{1}{2}\left(x + \frac{1}{x}\right)\ln\frac{1+x}{1-x}\right\}, \tag{32a}$$

$$F_2(k) \equiv \int_{-1}^{1} d(\cos\theta)(d_E^2 + d_M^2) = \tfrac{4}{3}(h_M^2 + h_E^2)M^{-4}p^2k^2, \tag{32b}$$

[17] The subscript π here replaces a and refers to the indicated set of states. Interference effects occur when the γ-ray polarization is observed even if CP is not violated; S. Barshay and C. Iso, Phys. Rev. **125**, 2168 (1962).

[18] Some observations on $K_s \to \pi^+ + \pi^- + \gamma$ have been made by P. Franzini *et al.*, Phys. Rev. **140**, B127 (1965).

[19] The upper limit for $K_L \to \pi^+ + \pi^- + \gamma$ is quoted as 5×10^4 sec^{-1} by G. Trilling, in Proceedings International Conference on Weak Interactions, Argonne National Laboratory, 1965, Argonne National Laboratory Report No. ANL-7130 (unpublished). The direct emission process $K_L \to \pi^+ + \pi^- + \gamma$ is calculated as giving a rate of the order of 10^4 sec^{-1} by S. V. Pepper and Y. Ueda, Nuovo Cimento **33**, 1614 (1964); their calculations are not inconsistent with observations on $K^+ \to \pi^+ + \pi^0 + \gamma$.

[20] CP violation in the process $K_L \to \pi^+ + \pi^- + \gamma$ may be observed directly as an asymmetry between the spectra of π^+ and π^-. This requires going beyond the dipole approximation which we consider here. See T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **16**, 567 (1966).

$$F_{12}(k)\equiv\int_{-1}^{+1}d(\cos\theta)c_E d_E$$
$$=\left(\frac{4}{x}\right)egh_E M^{-2}p\left\{1+\frac{1}{2}\left(x-\frac{1}{x}\right)\ln\frac{1+x}{1-x}\right\}. \quad (32c)$$

The interference parameter V_π given in Eq. (25) thus becomes, in the limit in which CP violation in the mass matrix is ignored (i.e., $\epsilon\simeq0$),

$$V_\pi=ie^{i\delta_0}\int_{k_{\min}}^{k_{\max}}dk\phi(k)F_{12}(k)\Big/\left(\int_{k_{\min}}^{k_{\max}}dk\phi(k)F_1(k)\right)^{1/2}\left(\int_{k_{\min}}^{k_{\max}}dk\phi(k)F_2(k)\right)^{1/2} \quad (33)$$
$$\equiv ie^{i\delta_0}\frac{h_E}{(h_E^2+h_M^2)^{1/2}}X(k_{\max},k_{\min}).$$

The parameter X clearly represents the maximum possible interference effect, limited only by essentially kinematical considerations. Some numerical values of X are given in Table III for the case where the interference experiment detects all photons whose energy exceeds a certain minimum value. We observe that the interference parameter V_π stays large and fairly constant as long as the interference experiment excludes the very low-frequency photons. This is a reflection of the fact that in the low-frequency region, the amplitude c_E becomes very much larger than d_E owing to the infrared divergence and rapidly decreases the over-all interference effect. Therefore, if the CP-violating direct emission measured by h_E is comparable to the CP-conserving measured by h_M, as might be expected on models with electromagnetic CP violation,[2,3] a large interference effect may be observed as long as the low-energy photons are excluded.

TABLE III. Maximum possible interference effect X as a function of photon energy threshold. [$k_{\max}$(lab)$\simeq$168 MeV.]

$k_{\min}$(lab) in MeV	X
160	0.99
150	0.98
140	0.96
130	0.94
120	0.93
110	0.91
100	0.88
90	0.85
80	0.82
70	0.77
60	0.72
50	0.67
40	0.61
30	0.55
20	0.47
10	0.37

APPENDIX A

The photons resulting from the two-photon decay of a coherent mixture of K_S and K_L have a net circular polarization. The intensity of left- (right-) circularly polarized photons is obtained from Eq. (2), with the subscript $l(r)$ replacing a. The parameters $\gamma_{Ll}, \gamma_{Sl}, V_l(\gamma_{Lr},\gamma_{Sr},V_r)$ are obtained by the same substitution in Eqs. (7) and (8), where the sum $\sum_\alpha$ is restricted to states of left- (right-) circular polarization. Since the intensities $I_l(t)$ and $I_r(t)$ are not equal, there will be a time-dependent asymmetry between the number of left- and right-circularly polarized photons. Such an effect is present even when there is no CP violation[9] (i.e., when $d_e=c_o=0$), the effect of CP violation being to modify the asymmetry. There is, however, one purely CP-violating effect, viz., the net circular polarization of photons in the decay of a pure K_S or a pure K_L beam. For the latter case, $R=0$, and we get

$$I_l(t)=\gamma_{Ll}e^{-\gamma_L t},\quad I_r(t)=\gamma_{Lr}e^{-\gamma_L t}, \quad (A1)$$

where

$$\gamma_{Ll}=|\langle LL|T|K_L\rangle|^2,\quad \gamma_{Lr}=|\langle RR|T|K_L\rangle|^2. \quad (A2)$$

So the net circular polarization, using Eq. (12) and Table I, is

$$P=\frac{I_l-I_r}{I_l+I_r}=\frac{2\,\mathrm{Re}(\langle X_e|T|K_L\rangle^*\langle X_o|T|K_L\rangle)}{|\langle X_e|T|K_L\rangle|^2+|\langle X_o|T|K_L\rangle|^2}, \quad (A3)$$

or

$$P=\frac{2c_e d_o}{\gamma_L}\mathrm{Re}\left[\left(\epsilon e^{i\rho_e}-i\frac{d_c}{c_e}e^{i\mu_e}\right)^*\left(i\epsilon\frac{c_o}{d_o}+e^{i\mu_o}\right)\right]. \quad (A4)$$

The expression in parentheses is purely CP-violating.

APPENDIX B

We consider an effective nonleptonic decay Hamiltonian of the following form[14]

$$H=H_v+e^{i\phi}H_c, \quad (B1)$$

where H_c is parity-conserving, H_v is parity-violating, and H is CP invariant if $\phi=0$. We attempt to find a possible limit on ϕ from information on CP violation in $K^0\to2\pi$.[21] The phase ϕ affects $K^0\to2\pi$ only via the mass matrix which contains intermediate states from both H_c and H_v. The CP-violating parameter ϵ in the mass matrix may be written[6]

$$\epsilon=\frac{-M_i+iy}{(\gamma_S-\gamma_L)+2i\delta}, \quad (B2)$$

where M_i is the dispersive part and y the absorptive part of the CP-violating portion of the self-energy matrix. Here we focus our attention on M_i. Neglecting

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terms in ϕ^2 and letting δ_c equal the magnitude of the contribution to the mass difference from intermediate odd-parity states we have

$$M_i/\delta_c \simeq \tan\phi. \tag{B3}$$

In order to estimate δ_c we may use the pole model. It has been suggested[22] that the entire mass difference δ may be explained by δ_c, but we find this argument unconvincing. However, arguments given below indicate that it seems very reasonable that

$$\delta_c \geq 0.1\delta. \tag{B4}$$

From (B2), (B3), and (B4), and the empirical fact that $2\delta \simeq (\gamma_S - \gamma_L)$, we find

$$|\epsilon| \geq \frac{\delta_c \tan\phi}{2\sqrt{2}\delta} \geq 0.035 \tan\phi. \tag{B5}$$

Values of $|\epsilon|$ may be deduced from experiments[1,11] on $K^0 \rightarrow 2\pi$. Actually there are two solutions[23] for $|\epsilon|$, but either gives $|\epsilon| \lesssim 3\times10^{-3}$, whence we reach as a conservative conclusion $\tan\phi < 0.1$.

To derive Eq. (B4) we may use the pole model to compare δ_c with the rate for $K^0 \rightarrow 2\gamma$. One finds considering only the π^0 pole[24]

$$\Gamma(K_L \rightarrow \gamma+\gamma) = 2\Gamma(\pi^0 \rightarrow \gamma+\gamma) \times \left(\frac{m_K}{m_\pi}\right)^3 \frac{m_K^2}{(m_K - m_\pi)^2} \frac{\delta_c}{m_K}. \tag{B6}$$

If we use[25] $\Gamma(\pi^0 \rightarrow \gamma+\gamma) = 1.8\times10^{16}$ sec^{-1} and let $\Gamma(K_L^0 \rightarrow \gamma+\gamma) = (6\pm3)\times10^3$ sec^{-1} we find $(\delta_c/\delta) = 0.4 \pm 0.2$. If we combine the π^0 and η^0 poles using physical masses and SU_3 coupling constants we find $(\delta_c/\delta) = 0.9 \pm 0.45$. The errors are not statistical, but represent a reasonable range on the value of $K_L^0 \rightarrow \gamma+\gamma$ from two different experiments. We conclude that Eq. (B4) represents a reasonable inequality.

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PHYSICAL REVIEW VOLUME 162, NUMBER 5 25 OCTOBER 1967

Tests of Symmetry Invariance in Strong-Interaction Physics by Using the Elastic Scattering Process $\bar{p}p \rightarrow \bar{p}p$†

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(Received 4 August 1966; revised manuscript received 13 February 1967)

Various types of symmetry-invariance tests through the elastic scattering process $\bar{p}p \rightarrow \bar{p}p$ are proposed. The tests of charge-conjugation invariance are discussed in particular. Polarized proton targets make some of the experiments proposed for these tests easier. By these experiments, time-reversal invariance and charge-conjugation invariance can be tested independently. One can therefore make a direct test for *CPT* invariance.

I. INTRODUCTION

SINCE the discovery of the decay process $K_2^0 \rightarrow \pi^+ + \pi^-$, the problem of *CP* invariance has been a controversial one.[1,2] Recently, Cohen-Tannoudji and Messiah[3] proposed a test of *C* invariance using the inelastic collision process $\bar{p}p \rightarrow \bar{Y}Y$. It is known that some transition matrix elements are zero if *C* conservation is not violated. According to the test,[3] it is possible to find whether some matrix elements are zero. Since the density matrix elements of the final spin states can be derived directly from the angular-correlation measurements of the decay pions from $\bar{Y}Y$ pairs,[4] the *C* test, as proposed, can in principle be realized. But unfortunately the statistics of this *C* test are rather poor.[5]

In this paper, it will be shown that the *C* test can in principle be realized with the elastic scattering process $\bar{p}p \rightarrow \bar{p}p$, where the collision cross section is much greater. Here one must replace the angular-correlation measurements of the decay particles in the inelastic

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