This result can be compared with the results of others[1,3] in 3.2 and 4.0 GeV/c π^-p four-prong events. At 3.2 and 4.0 GeV/c approximately 75% of the Δ^{++} (1238) production in $\pi^-p\rightarrow\pi^-\pi^-p\pi^+$ occurs at $|t|$ to the Δ^{++} of less than 0.8 GeV^2. The ω^0 in $\pi^-p\rightarrow\pi^-\pi^-p\pi^+\pi^0$ at 4.0 GeV/c is strongly peaked forward.

CONCLUSIONS

Strong resonance production of Δ^{++} (1238) has been observed in the final state $3\pi^-p2\pi^+$, ω^0 in the final state $3\pi^-p2\pi^+\pi^0$, and Δ^- (1238) in the final state $3\pi^-3\pi^+n$. They each occurred in approximately one third of the corresponding events. The nucleons from all events were peaked backward in the center-of-mass system, but less strongly than in experiments at lower energy with fewer outgoing particles. However, the neutrons were peaked backward more strongly than the protons. The pions from all events were distributed approximately isotropically in the center-of-mass system. The distribution of ω^0 and Δ^{++} (1238) in four-momentum transfer was quite similar to the distribution of background.

[3] S. Chung, O. Dahl. L. Harky, R. Hess, G. Kalbfleisch, J. Kirz, D. Miller, and G. Smith, Phys. Rev. Letters **12**, 621 (1964). S. Chung, O. Dahl, L. Hardy, R. Hess, L. Jacobs, J. Kirz, and D. Miller, *ibid.* **15**, 325 (1965).

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Application of $SU(6)_W$ in a Model for Vector-Meson Production at High Energies*

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A double-Regge-pole model incorporating $SU(3)$ symmetry, previously applied to pseudoscalar meson-baryon exchange reactions, is extended to include certain vector-meson production reactions with $SU(6)_W$ symmetry imposed at the forward direction. A large number of predictions for forward vector-meson production cross sections are obtained in terms of pseudoscalar-meson cross sections. These are compared where possible with the meager data available above 3 BeV/c, with generally favorable results.

It has been observed by Jackson[1] that comparison of some simple $SU(6)_W$ symmetry predictions for forward scattering amplitudes with data yields generally bad agreement. However, at the same time it was pointed out[1] that these simplest relations (which are model-independent) are just the ones expected to be disturbed violently by symmetry-breaking effects, such as mass differences among the pseudoscalar mesons. In fact, similar tests of $SU(3)$ in high-energy reaction amplitudes do not fare well either.[2]

If symmetries such as $SU(3)$ and $SU(6)$ are preserved well in the couplings of states, but the masses of the states are not degenerate within a multiplet, it seems logical to incorporate the empirical mass splitting in some way in the construction of S-matrix elements.[3] For high-energy two-body inelastic (exchange) reactions, it is appropriate in a Regge-pole approach to include empirical (or linearly extrapolated) trajectories $\alpha(t)$, but retain symmetry predictions for pole residues. This procedure has met with considerable success in the case of $SU(3)$ in the charge- and hypercharge-exchange reactions at forward angles involving pseudoscalar mesons.[4] In this paper we employ $SU(6)_W$ in a similar fashion to obtain predictions for forward cross sections for certain vector meson production reactions.

We consider the classes of reactions $PB\rightarrow PB$, $PB\rightarrow PB^*$, $PB\rightarrow VB$, and $PB\rightarrow VB^*$ where pseudoscalar P and vector V belong to the same $SU(6)_W$ representation (**35**), while B and B^* also belong to the same (**56**) representation. A model for the first two of these classes has been developed in a previous paper[4]; the model involves a pair of exchange-degenerate octets of Regge trajectories which include ρ, A_2, K_{890}^*, and K_{1400}^* exchanges. This double-pole model appears to give at least the gross features of the first two classes of reactions correctly and incorporates exact $SU(3)$ symmetry in the couplings.

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[1] J. D. Jackson, Phys. Rev. Letters **15**, 990 (1965).

[2] S. Meshkov, 1966 Boulder (Colorado) Summer School Proceedings (unpublished); H. Harari, in *High Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965).

[3] This philosophy has been employed at low energies; e.g., K. C. Wali and R. L. Warnock, Phys. Rev. **135**, B1358 (1964).

[4] R. C. Arnold, Phys. Rev. **153**, 1506 (1967).

Extending this model to include vector-meson production requires additional justification, since the selection rules forbidding pseudoscalar exchanges are not present in the last two classes above. We discuss each reaction separately below in order to see the range of applicability of the generalized model; then we apply $SU(6)_W$ and obtain many predictions.

The exchange degeneracy assumption[4,5] will play cnly a minor role in the present work. It is employed essentially only to determine that even-signature exchanges dominate over odd-signature in the hypercharge exchange reaction near the forward direction.

The exchange of 1^- vector (V) and 2^+ tensor (T) contributions, which are associated with parity $(-1)^J$ in the t channel, is consistent with W-spin symmetry as regards the population of helicities of the produced vector meson; such exchanges populate[6] only helicities ± 1, and contribute to production of pseudoscalar mesons. This means only $W=1$ states are produced, but not $W=0$ (helicity-zero components of vector mesons).

As will be explained below, the $SU(6)_W$ symmetry in our model implies only spin-singlet and spin-triplet exchanges in the forward direction. The helicity amplitudes that are nonzero in any model incorporating only spin singlet and triplet exchanges are defined in Table I. In terms of these amplitudes, the differential cross sections may be written as follows:

$$\frac{d\sigma}{dt}(PB)=|G_+|^2+|G_-|^2,$$

$$\frac{d\sigma}{dt}(VB)=|H_+|^2+|H_-|^2,$$

$$\frac{d\sigma}{dt}(PB^*)=|\tilde{G}_+|^2+|\tilde{G}_0|^2+|\tilde{G}_-|^2,$$

$$\frac{d\sigma}{dt}(VB^*)=|\tilde{H}_+|^2+|\tilde{H}_-|^2.$$

At $t=0$, there is no orbital-angular-momentum transfer, and the amplitudes with subscript $(-)$ [as well as $\tilde{G}_+$] vanish. Writing σ for the forward differential cross section, we obtain

$$\sigma(PB)=|G_+(0)|^2,$$
$$\sigma(VB)=|H_+(0)|^2,$$
$$\sigma(PB^*)=|\tilde{G}_0(0)|^2,$$
$$\sigma(VB^*)=|\tilde{H}_+(0)|^2.$$

To exploit the assumed $SU(6)_W$ symmetry in the exchange mechanism, a decomposition of these amplitudes in the t channel is appropriate in terms of $SU(6)_W$

TABLE I. Amplitudes relevant to model. Amplitudes preceded by an asterisk are not forbidden by angular-momentum conservation to be nonzero at $t=0$. Blank entries are identically zero with only spin triplet and singlet exchanges.

Outgoing baryon Spin	Change in helicity	Outgoing meson ($W=1$) $W_z=+1$	0	-1
$\frac{1}{2}$	$+1$		G_-	$*H_+$
	0	H_-	$*G_+$	H_-
	-1	$*H_+$	G_-	
$\frac{3}{2}$	$+2$			
	$+1$		$\tilde{G}_+$	$*\tilde{H}_+$
	0	$\tilde{H}_-$	$*\tilde{G}_0$	$\tilde{H}_-$
	-1	$*\tilde{H}_+$	$\tilde{G}_-$	
	-2			

state labels. We assume the external mesons (baryons) are states of a **35** (**56**) representation of $SU(6)_W$, that the reaction mechanism is $SU(6)_W$ symmetric (in the forward direction), and that no $SU(3)$ representations higher than **8** can occur in the exchange. These assumptions limit the intermediate states in the t channel to singlet and **35** representations of $SU(6)_W$. Since elastic scattering is not considered, singlet exchange is irrelevant, and we consider only **35** states.

In the decomposition of the direct product $\mathbf{56}\times\mathbf{56}^*$ only one **35** occurs, while in $\mathbf{35}\times\mathbf{35}$ there are two such representations which may be selected to have opposite permutation symmetry. Thus, there are two independent exchange amplitudes, $\mathbf{35}_F$ and $\mathbf{35}_D$. Only the former (antisymmetric) couples two identical π mesons at the meson vertex; it is associated in this model with the exchange of $C=-1$ states, and in this case the latter (symmetric) ($C=+1$) is absent because of charge conjugation invariance (exchange degeneracy implies the baryon couplings are the same for both exchanges, e.g., F). In certain other cases, such as $\pi^- p\rightarrow \eta n$, only $\mathbf{35}_D$ is present. In general both contribute. Exchange degeneracy gives a relation between the two amplitudes, since in the double pole model the "vector" (odd-signature) trajectories will be associated with $\mathbf{35}_F$ states, while the "tensor" (even-signature) trajectories are associated with $\mathbf{35}_D$ states.

Let these two exchange amplitudes be represented by matrix elements, F_V (vector) and F_T (tensor), reduced with respect to $SU(6)_W$ indices. Their S dependence and phase at $t=0$ will be given in the double-pole model[4] by

$$F_V(S,0)=\beta_V[1-e^{-i\pi\alpha_V(0)}](S/S_0)^{\alpha_V(0)}/\sin\pi\alpha_V(0),$$
$$F_T(S,0)=\beta_T[1+e^{-i\pi\alpha_V(0)}](S/S_0)^{\alpha_T(0)}/\sin\pi\alpha_T(0),$$

where β_V, β_T are real, and independent of S.

With exchange degeneracy, $\alpha_V(0)\cong\alpha_T(0)$, while β_V and β_T are connected, e.g., by the condition that the ρ and R couple equally in $\bar{K}N$ charge exchange.[4]

The assignments of vector and tensor mesons to $\mathbf{35}_F$ and $\mathbf{35}_D$ exchanges proposed here coincides with the assignments of meson states to l-excitation repre-

[5] A. Ahmadzadeh, Phys. Rev. Letters **16**, 952 (1966); Physics Letters **22**, 669 (1966).
[6] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

TABLE II. Forward hypercharge-exchange reaction-amplitude ratios in the double-pole model with $SU(6)_W$ symmetry. Entries in the table are magnitudes of ratios of amplitude at left of table to amplitude in column heading. Entries preceded by an asterisk are direct tests of $SU(6)_W$ symmetry with $\mathbf{35}_D$ exchange.

	$K^-p \to \pi^0\Lambda$	$\pi^-\Sigma^+$
$K^-p \to \rho^0\Lambda$	*1	
$\rho^-\Sigma^+$		$*(\frac{2}{3})^{1/2}$
$\rho^0\Sigma^0$		$1/6^{1/2}$
$\omega\Sigma^0$		$1/6^{1/2}$
$\omega\Lambda$	1	
$\phi\Lambda$	$\sqrt{2}$	
$\phi\Sigma^0$		$1/3^{1/2}$
$\rho^0 Y_1^{*0}$		$5^{1/2}/3$
$\rho^- Y_1^{*+}$		$*(\frac{2}{3})5^{1/2}$
ωY_1^{*0}		$5^{1/2}/3$
ϕY_1^{*0}		$10^{1/2}/3$
$\pi^-p \to K^{*0}\Lambda$	$\sqrt{2}$	
$K^{*0}\Sigma^0$		$1/3^{1/2}$
$K^{*0}Y_1^{*0}$		$10^{1/2}/3$
$\pi^+p \to K^+\Sigma^+$		1
$K^{*+}Y_1^{*+}$		$5^{1/2}/3$
$K^{*+}\Sigma^+$		$1/6^{1/2}$

sentations of $U(6)\times O(3)$ given by various other authors.[7] These assignments are interpretable in a quark-model framework by assigning the 2^+ mesons to a p-wave quark-antiquark system, while the 1^- mesons (and 0^-) fit into an s-wave $q\bar{q}$ system.

Now the required amplitudes $G_+(0)$, $H_+(0)$, $\tilde{H}_+(0)$, and $\tilde{G}_0(0)$ can be expressed in terms of $SU(6)_W$ Clebsch-Gordan coefficients multiplying F_V and F_T. One simple observation is the vanishing of $\tilde{G}_0(0)$ in this model; this means the differential cross section for $PB \to PB^*$ must vanish in the exact forward direction, as was noted in previous work.[4] For the nonvanishing coefficients we refer to the tables of Cook and Murtaza.[8] It will be sufficient to give explicitly ratios $H_+(0)/G_+(0)$ and $\tilde{H}_+(0)/G_+(0)$ for the reactions appropriate to the double-pole model, since the G_+ factors have been given elsewhere.[4]

In $PB \to PB$ hypercharge-exchange reactions, with the exchange degeneracy assumption, it was found that the F_T contributions dominate numerically[4] over the F_V contributions. The predictions will be simpler for vector-meson production if $|F_V|^2$ is ignored relative to $|F_T|^2$ when hypercharge exchange occurs, and we consistently make this assumption below. The reactions without hypercharge exchange will be discussed separately.

Consider first the pion reactions. We consider explicitly only π^-; π^+ reactions can be related to these by isospin coefficients in the double-pole model. The reactions $\pi^-p \to \rho^0 n$, $\rho^0\Delta^0$ allow pion exchange which apparently dominates, and cannot be included in the model; the former reaction may in fact be completely accounted for up to 8 BeV/c by pion exchange with absorptive corrections. Similarly $\pi^-p \to \rho^-p$, $\rho^-\Delta^+$ may be entirely consistent in a similarly wide energy region with only pion exchange. Although ω exchange is allowed in the former case, and A_2 exchange in both cases, the long-range pion exchange appears to be more important than V or T amplitudes. At sufficiently high energies the V and T exchanges must, however, become more important since their S dependence is less sharply dropping than that of pion exchange. We assume it is the small mass of the π which causes its exchange to be disproportionately emphasized at intermediate energies.

The reactions $\pi^-p \to \omega n$ and $\omega\Delta^0$ are, however, expected to be described by the double-pole model since pion exchange is not allowed. In these cases ρ exchange (F_V) is present. In order to explain the ω density matrix with only ρ exchange (as in the double-pole model) it is necessary to invoke absorptive corrections, which should in fact be present in all the cases considered here, in comparable amounts.[9] We will compare only forward differential cross sections among reactions involving similar exchanges, and the damping from absorptive corrections will presumably be the same in all; thus, we do not explicitly consider such corrections.

In principle, $\pi^-p \to \phi n$ (and $\phi\Delta^0$) is also to be considered. However, to treat ϕ and ω we will assume the $SU(3)$ singlet-octet mixing and couplings as suggested by the $SU(4)_S$ decomposition[10] of $SU(6)$ (or alternatively, the quark model[11] or the Okubo *Ansatz*[12] for couplings); ϕ is not coupled to a $\rho\pi$ vertex. [Similarly, as shown by Levinson, Wall, and Lipkin,[13] an exchanged ϕ is assumed to be not coupled to nucleons; only ω is exchanged with $SU(3)$ mixing equal to the mass-shell value.] Thus $\sigma(\pi^-p \to \phi n) = 0$ and $\sigma(\pi^-p \to \phi\Delta^0) = 0$.

The other π^- reactions to be considered involve hypercharge exchange. These final states are $K^*\Lambda$, $K^*\Sigma$, and $K^*Y_1^*$. Although K exchange is allowed we assume it (because its range is relatively short and $V+T$ exchanges can compete favorably) to be negligible; and as explained above, we ignore $|F_V|^2$ relative to $|F_T|^2$ for simplicity. This is certainly justified with the present degree of accuracy desired.

Turning to the KN reactions, we similarly examine only K^-p explicitly. The vector production reactions not involving hypercharge exchange, with baryons in the final state, are $K^-p \to K^{*-}p$ and $K^-p \to K^{*0}n$. The latter involves π, ρ, and A_2 exchange. Estimates based on $SU(6)$-coupling-constant ratios between π and ρ indicate[14] that ρ exchange should be small compared to π exchange, for momenta below 6 BeV/c. Since A_2 and ρ are comparable if exchange degeneracy is assumed,

[7] M. Gell-Mann, Phys. Rev. Letters **14**, 77 (1965); R. Gatto, L. Maiani, and G. Preparata, Nuovo Cimento **39**, 1192 (1965); P. G. O. Freund, A. N. Maheshwari, and E. Schonberg, Phys. Rev. **159**, 1232 (1967).

[8] C. L. Cook and G. Murtaza, Nuovo Cimento **39**, 531 (1965).

[9] For a discussion of "absorptive corrections" in a Regge-pole formalism, see R. C. Arnold, Phys. Rev. **153**, 1523 (1967).

[10] M. A. Baqi Bég and V. Singh, Phys. Rev. Letters **13**, 418 (1964).

[11] H. J. Lipkin, Phys. Rev. Letters **13**, 590 (1964); H. J. Lipkin, F. Scheck, and H. Stern, Phys. Rev. **152**, 1375 (1966).

[12] S. Okubo, Phys. Letters **5**, 165 (1963).

[13] C. A. Levinson, N. S. Wall, and H. J. Lipkin, Phys. Rev. Letters **17**, 1122 (1966).

we expect this reaction will in fact be dominated by π exchange at such energies, as in $\pi^- p \to \rho^0 n$, and is outside the double-pole-model framework except at momenta higher than presently available. However, for $K^- p \to K^{*-} p$, similar estimates (at energies above 3 BeV) indicate that ω exchange is more important than π or ρ; so this reaction may fit into the double-pole ($\omega + f^0$) framework. In the VB^* reactions $K^- p \to K^{*-}\Delta^+$, $K^{*0}\Delta^0$ the ω and f^0 exchanges are not allowed; presumably ρ will be dominated by π exchange in these cases, and the double-pole model inapplicable (at least below 8 BeV/c) in these cases.

Finally in the $K^- p$ reactions involving hypercharge exchange we again assume K exchange is negligible compared to V and T terms (above 3 BeV/c), and apply the double-pole model to reactions with final states $\rho\Lambda$, $\rho\Sigma$, $\phi\Lambda$, $\phi\Sigma$, $\omega\Lambda$, $\omega\Sigma$, ρY_1^*, ϕY_1^*, and ωY_1^*. Again $|F_V|^2$ is neglected compared to $|F_T|^2$, to obtain the given ratios.

It should be restated for emphasis at this point that a combination of V and T exchanges, with kinematically equal baryonic couplings as suggested by exchange degeneracy, will yield the Stodolsky-Sakurai ($M1$ coupling) density matrix for N^* and Y^* final states when a pseudoscalar meson is in the final state (at the other vertex). This is a consequence of the Gottfried-Jackson[6] analysis and was remarked upon previously.[4] Thus, the isobar density matrices are valuable checks (whenever π exchange is forbidden) on these $PB \to PB^*$ models.

Carrying out the $SU(6)_W$ Clebsch-Gordan calculations, augmented by $SU(3)$ factors from the tables of McNamee and Chilton,[15] we obtain for hypercharge exchange the forward-reaction-amplitude ratios given in Table II. Some subsets of these relations do not depend on $SU(6)_W$ symmetry directly (e.g., are consequences of octet exchange), but those with asterisk involve definite tests of W-spin symmetry in the context of the double-pole model.

If σ denotes the cross section (square of the amplitude) in the forward direction, we find

$$\sigma(\pi^- p \to \omega n):\sigma(\pi^- p \to \omega\Delta^{++}):\sigma(\pi^- p \to \pi^0 n) = 1:(6/5):1.$$

The other new results found in Table II following from $SU(6)_W$ relations may be stated as follows:

$$\sigma(K^- p \to \rho^0\Lambda)/\sigma(K^- p \to \pi^0\Lambda) = 1,$$
$$\sigma(K^- p \to \rho^-\Sigma^+)/\sigma(K^- p \to \pi^-\Sigma^+) = 2/3,$$
$$\sigma(K^- p \to \rho^- Y_1^{*+})/\sigma(K^- p \to \pi^-\Sigma^+) = 20/9.$$

The other ratios obtained follow from these, and $SU(3)$, in a double-octet exchange model. Since we do not have a prediction for the ratio between Pomeranchuk trajectory exchange and the other exchanges, we do not make any statements about ratios such as $\sigma(K^- p \to K^{*-} p)/\sigma(K^- p \to K^- p)$.

Of all these predicted ratios, only a few have any data to compare with at present, at momenta at or above 3 BeV/c; and that data which exists is difficult to extrapolate to the forward direction. However, we can examine the orders of magnitude involved. As before, σ will denote the forward value of the differential cross section.

At 3.25 BeV/c, which is the highest-energy published data available,[16] $\sigma(\pi^+ n \to \omega p)/\sigma(\pi^- p \to \pi^0 n) = 0.33 \pm 0.09$, while the prediction is 1. This is difficult to interpret, however, since at this energy the ω-π mass difference presumably is important. (If we were to compare these cross sections at comparable Q values, the agreement would be worse.)

At 3.5 BeV/c (again the highest-energy data published[17] at the present), $\sigma(K^- p \to \omega\Lambda)/\sigma(K^- p \to \pi^0\Lambda) = (0.180 \pm 0.05)/(0.250 \pm 0.100)$ while the prediction is 1; this is consistent. At the same energy

$$\sigma(K^- p \to \rho^-\Sigma^+)/\sigma(K^- p \to \pi^-\Sigma^+) = (0.50 \pm 0.25)/(0.45 \pm 0.20)$$

which should be compared to the predicted value of $(\frac{2}{3})$; again we find consistency within the large estimated errors.

It is clear that accurate data on the reactions treated here would be very desirable, at energies above 4 BeV/c, in order to test the relevance of $SU(6)_W$ in strong interaction reaction mechanisms.

I would like to acknowledge illuminating conversations with H. J. Lipkin, B. Sakita, K. C. Wali, and P. G. O. Freund concerning the matters discussed in this paper.

[14] R. Jabbur (private communication). Jabbur and Griffiths have done an absorptive-model calculation for $K^- p \to K^* p$ involving elementary vector (ρ,ω) and pseudoscalar (π) exchanges, with $K^*K\pi$ coupling constant determined by $K^* \to K\pi$ decay widths, using $SU(6)$ relations for the ρ and ω couplings to the mesons and baryons. It is found that a reasonably good absolute fit to differential cross section and K^* density-matrix data in the range 4–5 GeV/c can be obtained, although such a model is not adequate for higher energies because of the elementary-vector-exchange energy dependence. They find, as discussed in the text, that π is much more important than ρ exchange in this energy region for K^{*-}, while for K^{*0}, ω dominates over π.

[15] P. McNamee and F. Chilton, Rev. Mod. Phys. **36**, 1005 (1964).

[16] Date quoted by M. Barmawi, Phys. Rev. Letters **16**, 595 (1966).

[17] Birmingham-Glasgow-London (I.C.)-Oxford-Rutherford Collaboration, Phys. Rev. **152**, 1148 (1966).