Multiple Pion Production in 5.5-GeV/c $\pi^- p$ Interactions*

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About 2000 six-prong events and 130 eight-prong events from an exposure in the Brookhaven 80-in. bubble chamber were analyzed. Production of $\Delta^{++}(1238)$, $\Delta^-(1238)$, and ω^0 resonances was observed. The distribution of Δ^{++} and ω^0 as a function of four-momentum transfer was similar to that of background. Center-of-mass angular distributions of the nucleons and pions were examined. The nucleons were peaked backward and the pions were essentially isotropic with a slight forward peaking in the π^+ case.

INTRODUCTION

THIS paper is a report on multiple pion production in $\pi^- p$ interactions at an incident pion momentum of 5.48 GeV/c. The particular final states analyzed were those leading to topologies of six or eight-charged-outgoing prongs and are listed in Table I.

This study utilized photographs obtained in an exposure of the BNL 80-in. hydrogen bubble chamber to an unseparated π^- beam at the Brookhaven Alternating Gradient Synchrotron. The 80-in. chamber was operated at a hydrogen vapor pressure of 63 psia, which corresponds to a temperature of about 26.2°K and a mass density of 62.1±0.1 grams per liter. This yields an interaction length of 884±2 feet per millibarn. The results quoted in this paper are based on an analysis of approximately 2000 measurable six-prong events found in 25 000 photographs.

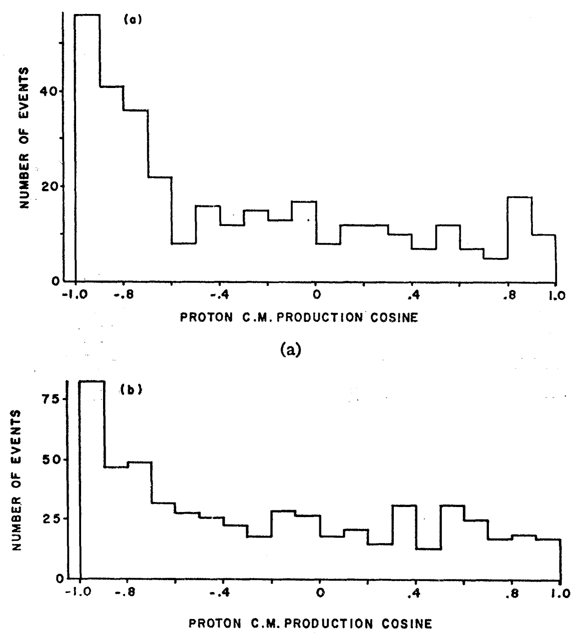

FIG. 1. Distribution of proton center-of-mass production cosine. (a) For events of the type $\pi^- p \to \pi^-\pi^-\pi^-\pi^+\pi^+ p$; 337 events. (b) For events of the type $\pi^- p \to \pi^-\pi^-\pi^-\pi^+\pi^+ p\pi^0$; 519 events.

IDENTIFICATION OF EVENTS AND CROSS SECTIONS

A. Six-Prong Events

Six-prong events that were found in a scan of approximately 25 000 photographs and which satisfied fiducial volume and measurability requirements were measured on precision engines with a least count of one micron. Each track was measured in two views with an average of ten points per view. The events were then processed

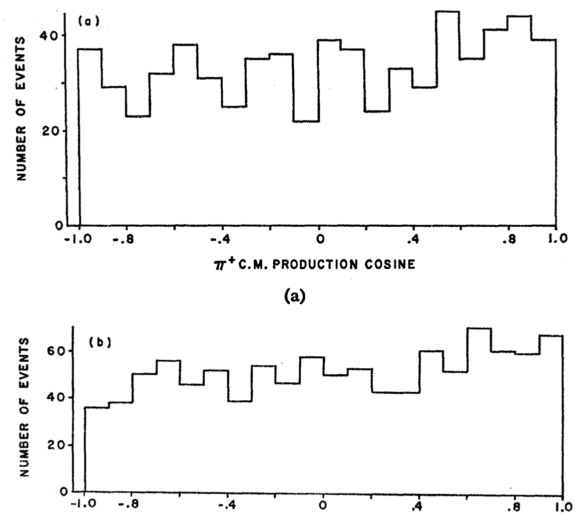

FIG. 2. Distribution of π^+ center-of-mass production cosine. (a) For events of the type $\pi^- p \to \pi^-\pi^-\pi^-\pi^+\pi^+ p$; 2 pions each from 337 events. (b) For events of the type $\pi^- p \to \pi^-\pi^-\pi^-\pi^+\pi^+ p\pi^0$; 2 pions each from 519 events.

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TABLE I. $\pi^- p$ interactions into 6 and 8 charged prongs at 5.5 GeV/c.

Reaction	Final state	Number of events	Cross section (mb)
1	$3\pi^- p 2\pi^+$	337	0.21 ± 0.02
2	$3\pi^- p 2\pi^+\pi^0$	519	0.34 ± 0.03
3	$3\pi^- 3\pi^+ n$	195	0.12 ± 0.01
4	$3\pi^- p 2\pi^+ N\pi^0 (N\geq 2)$	791	0.49 ± 0.04
5	$3\pi^- 3\pi^+ n N\pi^0 (N\geq 1)$		
	all eight-prong events	131	0.064 ± 0.007

through the Brookhaven FILCH-KICK, geometry and kinematics programs. FILCH is a FORTRAN version of the Berkeley program, FOG.

Events were classified into the various categories shown in Table I on the basis of kinematic fits and ionization. If the calculated track ionizations for a particular fit were not consistent with the observed track ionizations for all tracks, the fit was immediately discarded. Events were classified as type 1, ($\pi^- p \rightarrow 3\pi^- p 2\pi^+$), if they fit with $\chi^2 \leq 25$. The remaining events were classified as type 2 ($\pi^- p \rightarrow 3\pi^- 2\pi^+\pi^0$), or type 3 ($\pi^- p \rightarrow 3\pi^- 3\pi^+ n$) if they fit the type with a $\chi^2 \leq 10$. Events ambiguous between type 2 and type 3

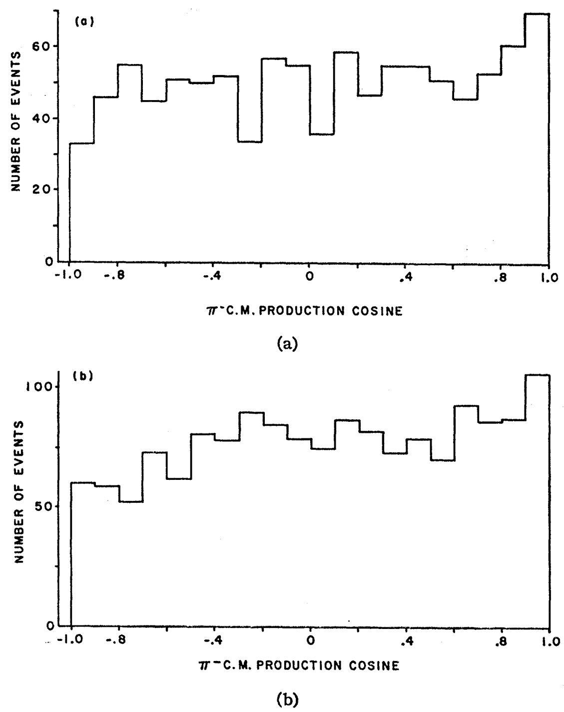

FIG. 3. Distribution of π^- center-of-mass production cosine. (a) For events of the type $\pi^- p \rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+ p$; 3 pions each from 337 events. (b) For events of the type $\pi^- p \rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+ p\pi^0$; 3 pions each from 519 events.

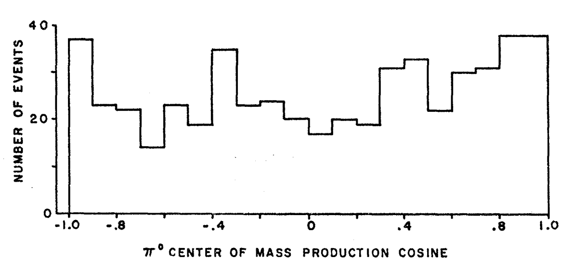

FIG. 4. Distribution of π^0 center-of-mass production cosine for events of the type $\pi^- p \rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+ p\pi^0$; 519 events.

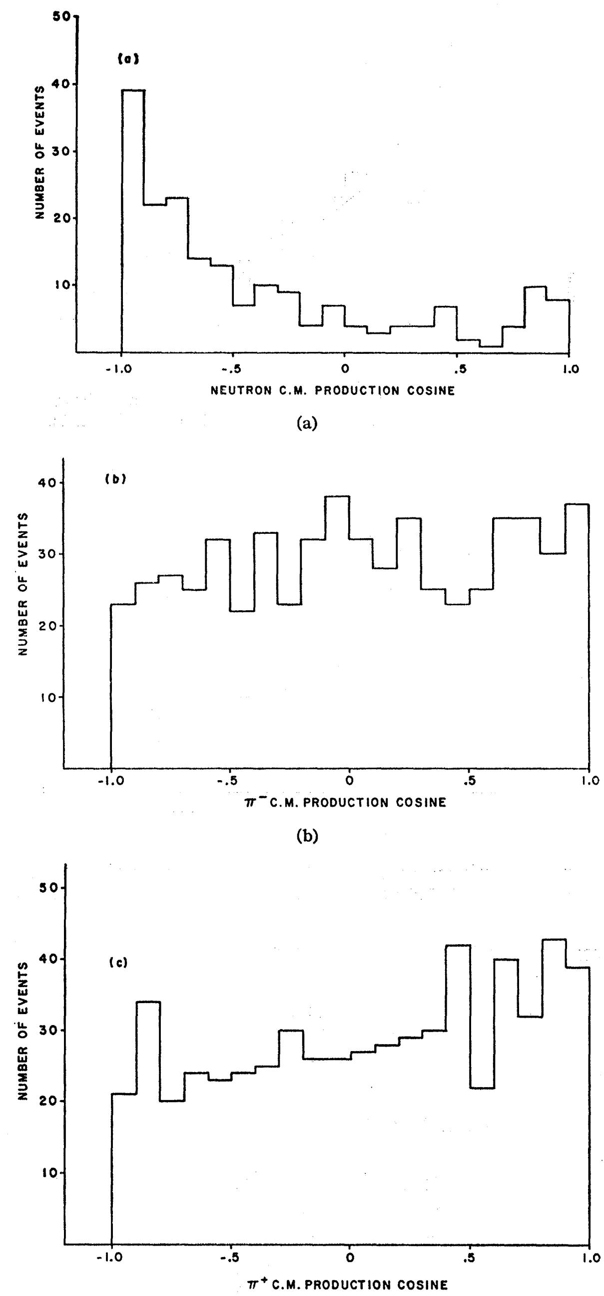

FIG. 5. Distribution of center-of-mass production cosine for 195 events of the type $\pi^- p \rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+\pi^+ n$. (a) Neutron distribution. (b) π^- distribution; 3 pions per event. (c) π^+ distribution; 3 pions per event.

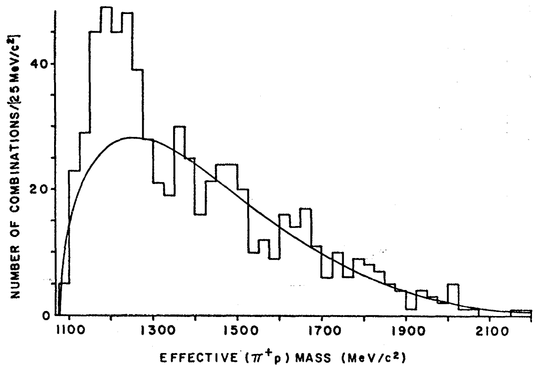

FIG. 6. Effective mass of the $p\pi^+$ system from events of the type $\pi^-p\rightarrow\pi^-\pi^-\pi^-\pi^+\pi^+p$; 2 combinations each from 337 events.

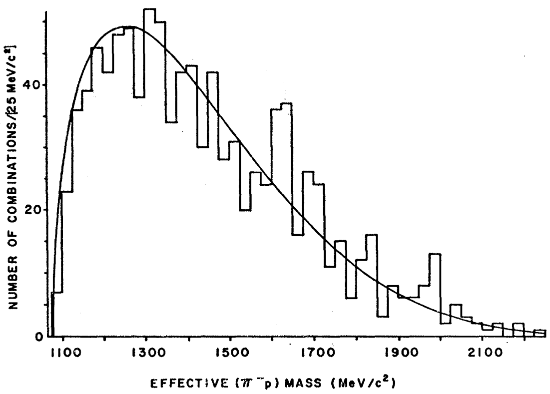

FIG. 7. Effective mass of the $p\pi^-$ system from events of the type $\pi^-p\rightarrow\pi^-\pi^-\pi^-\pi^+\pi^+p$; 3 combinations each from 337 events.

were used if the χ^2 for one type was at least a factor of 10 lower than the other. The remaining ambiguous events were not used in the following analysis. Events of type 4 ($\pi^-p\rightarrow 3\pi^-p2\pi^+N\pi^0$) were not separated from those of type 5 ($\pi^-p\rightarrow 3\pi^-3\pi^+nN\pi^0$) since in approximately one half of all the events, it was not possible using ionization to determine which track, if any, was a proton.

The cross sections shown in Table I were calculated from the number of events assigned to each type after applying corrections for (a) scan efficiency as determined from a partial second scan, (b) ambiguous events, (c) nonmeasurable events, and (d) effective shortening of beam track length due to interactions.

B. Eight-Prong Events

There were not enough eight-prong events to permit a reasonable analysis of this topology. The only result quoted for these events is the total cross section for the topology, which was found to be 0.064 ± 0.007 mb.

ANGULAR DISTRIBUTIONS

Figures 1 through 4 show the center-of-mass angular distributions for the protons and pions from reactions 1 and 2. It may be noted that the proton angular distributions in our 6- and 7-particle final states are less sharply peaked than the distributions in 4- and 5-particle final states[1] at 4.0 GeV/c. The pion distributions are consistent with isotropy with a suggestion of a small forward peaking in the π distributions. This is similar to the 6- and 7-particle final states distributions in π^+p at 5.0 GeV/c^2 where the π^- distributions were peaked forward.

Figure 5 shows the center-of-mass angular distributions for the neutrons and pions from reaction 3. The neutrons from this reaction are peaked backward much more strongly than the protons from the other reactions. The positive pions are peaked slightly forward.

Table 2 is a list of center-of-mass forward/backward

TABLE II. Forward (F) to backward (B) ratios, $(F-B)/(F+B)$.

Reaction	p	n	π^+	π^-	π^0
1	-0.40 ± 0.05	...	0.09 ± 0.04	0.05 ± 0.03	...
2	-0.32 ± 0.04	...	0.08 ± 0.03	0.08 ± 0.03	0.08 ± 0.04
3	...	-0.52 ± 0.07	0.14 ± 0.04	0.04 ± 0.04	...

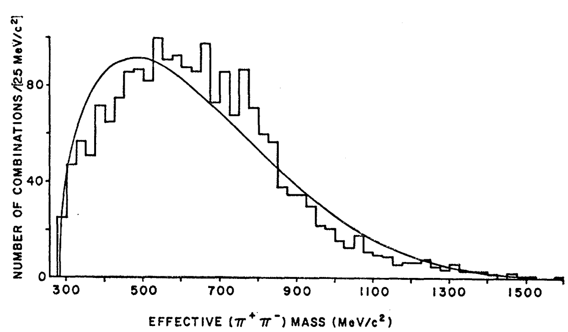

FIG. 8. Effective mass of the $\pi^+\pi^-$ system from events of the type $\pi^-p\rightarrow\pi^-\pi^-\pi^-\pi^+\pi^+p$; 6 combinations each from 337 events.

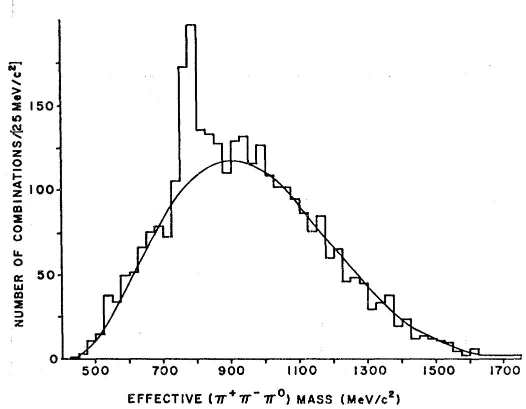

FIG. 9. Effective mass of the $\pi^+\pi^-\pi^0$ system from events of the type $\pi^-p\rightarrow\pi^-\pi^-\pi^-\pi^+\pi^+p\pi^0$; 6 combinations each from 519 events.

[1] Aachen-Birmingham-Bonn-Hamburg-London (I.C.)-Munich Collaboration, Nuovo Cimento **31**, 485 (1964).

[2] Bockmann *et al.*, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).

ratios for the nucleons and pions from reactions 1, 2, and 3. Forward is in the direction of the incident π^- in the center-of-mass system. The error limits shown are statistical, with no correction for possible biases.

INVARIANT-MASS DISTRIBUTIONS

To search for resonance production in reactions 1, 2, and 3, all possible invariant mass histograms were examined. Significant production was observed for three resonances, the Δ^{++} (1238) in reaction 1, the ω^0 in reaction 2, and the Δ^- (1238) in reaction 3.

The only invariant-mass distributions for events of type 1 that are significantly different from either a statistical-model phase space or a smooth curve are the $p\pi^+$, $p\pi^-$, and $\pi^+\pi^-$ distributions. These are shown in Figs. 6 through 8. The $p\pi^+$ distribution shows a large Δ^{++} (1238) peaked at 1200 MeV/c^2 corresponding to a cross section of 65 ± 15 μb. The $p\pi^-$ distributions shows a small bump peaked at 1650 MeV/c^2. If this bump were interpreted as evidence of $N^0(1688)$ production, it would correspond to a partial production cross section

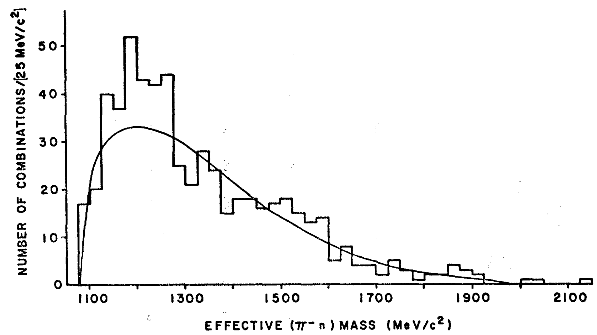

FIG. 11. Effective mass of the π^-n system from events of the type $\pi^-p\rightarrow\pi^-\pi^-\pi^-\pi^+\pi^+\pi^+n$; 3 combinations each from 195 events. The smooth curve is phase-space normalized to 521 combinations.

of 28 ± 10 μb for N^0's decaying into $p\pi^-$. The $\pi^+\pi^-$ distribution is skewed toward high mass values, showing a depletion of events relative to phase space in the mass interval from 300 to 525 MeV/c^2 and an approximately constant excess from 525 to 850 MeV/c^2. This region does not correspond to any known resonance and the excess of events is not yet explained. The apparent lack of ρ production might have been expected from 4.0 GeV/$c\pi^-p$ interactions, where the ρ production drops from 33% for events with 4 particles in the final state to 8% for events with 5 particles in the final state.[1]

Figures 9 and 10 are invariant mass distributions for events of type 2. The $\pi^+\pi^-\pi^0$ mass distribution showed very strong ω^0 production with the center of the ω^0 at about 780 MeV/c^2. The cross section corresponding to this peak is 116 ± 24 μb. The $\omega^0\pi^+$ and $\omega^0\pi^-$ distributions show no evidence for B^+ or B^- production. In the $2\pi^+2\pi^-\pi^0$ mass plot, not shown, there are no combinations below 1100 MeV/c^2. Thus there is no production of η^0 (960) in these events.

Figure 11 is an invariant-mass distribution for events of type 3. The $n\pi^-$ distribution has a large Δ^- (1238), peaked at about 1200 MeV/c^2, corresponding to a cross section of 39 ± 16 μb.

The production of Δ^{++} (1238) and ω^0 in reaction types 1 and 2, respectively, is sufficiently strong to permit further study. In particular we looked for but did not find significant correlations between resonance production and four-momentum transfer to either the proton or the resonant system.

Figure 12 is a scatter plot of the mass of the $p\pi^+$ system versus the magnitude of the four-momentum transfer from the target proton to the outgoing proton. The fraction of combinations in the Δ^{++} band (1.12–1.27 GeV/c^2) is seen to be approximately independent of t. The number of combinations in the Δ^{++} band peaks at a t of about 1.1 GeV with a width of about 0.5 GeV. This peaking, however, is also displayed by the region outside the Δ^{++} band and thus does not indicate peripheral production.

Figure 13 is a scatter plot of the mass of the $p\pi^+$

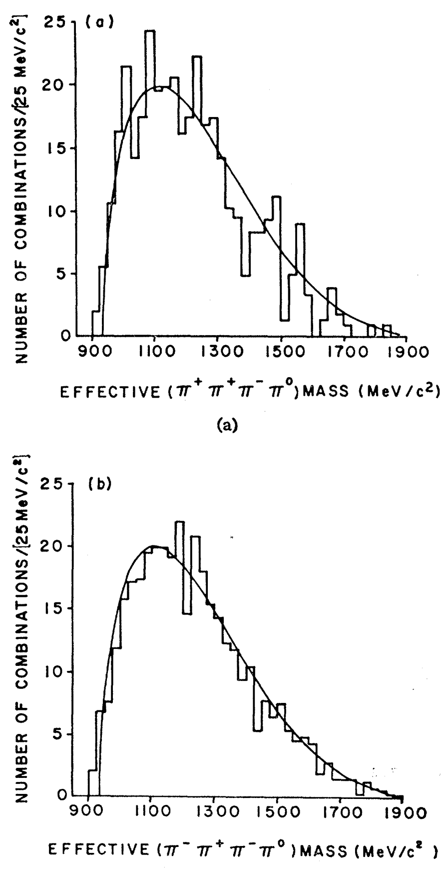

FIG. 10. Four-pion effective mass distributions from 366 events of the type $\pi^-p\rightarrow\pi^-\pi^-\pi^-\pi^+\pi^+p\pi^0$, when the $\pi^+\pi^-\pi^0$ effective mass is required to lie in the ω band ($750\leq M_\omega\leq850$ MeV/c^2). (a) $\pi^+\pi^+\pi^-\pi^0$ effective mass; (b) $\pi^+\pi^-\pi^-\pi^0$ effective mass.

MASS $(p\pi^+)$ GeV/c^2: 2.17, 2.02, 1.87, 1.72, 1.57, 1.42, 1.27, 1.12

			1						
		1							
	1	1					1		
	2	2	2	1					2
		4	5	1			1		
		1	5	4	3	3	3	1	
	3	3	6	3	1	5	1		1
	4	1	5	8	7	3	5	4	
	2	5	7	6	6	8	2	2	1
		3	9	6	13	5	4	2	2
	1	7	16	13	12	12	4	1	
	7	10	17	13	11	6	4	6	
	2	5	17	13	19	9	7	2	1
Δ^+	3	19	37	19	20	16	12	4	2
	4	8	21	21	29	15	13	4	1
	1	4	1	3	7	2	3	2	

0 .5 1.0 1.5 2.0 2.5

| t | (target → proton) (GeV)

FIG. 12. Effective mass of the $p\pi^+$ versus the magnitude of the four-momentum transfer from the target to the proton for events of the type $\pi^- p \to \pi^-\pi^-\pi^-\pi^+\pi^+ p$.

MASS SQUARED $(p\pi^+)$ (GeV^2/c^4): 4.66, 4.26, 3.86, 3.46, 3.06, 2.66, 2.26, 1.86, 1.46, 1.06

				1				
				1				
		2	2					
		3	2	1	1			
		1	5					
		2	3	1				
		2	9	3	1	1		
	1	3	6	4	3	1		
	2	7	5	2	5		1	
	3	11	5	11	1			
	2	11	11	6	2	1		
	1	16	11	2	5	1	1	
	9	18	17	7	9	1		
	11	24	13	4	8	2	1	
	17	24	12	5	8	1		1
Δ^{++}	46	36	19	17	9	7	2	
	35	39	22	19	13	8	3	1
	5	6	7	2	2		2	

.1 1.0 1.9 2.8 3.7 4.6 5.5 6.4 7.3

$|t|^2$ (target → $p\pi^+$) (GeV^2)

FIG. 13. Effective mass squared of the $p\pi^+$ system versus the magnitude of the four-momentum transfer squared from the target to the $p\pi^+$ system for events of the type $\pi^- p \to \pi^-\pi^-\pi^-\pi^+\pi^+ p$.

system versus the magnitude of the 4-momentum transfer squared from the target proton to the outgoing $p\pi^+$ system. The fraction of combinations in the Δ^{++} band (1.26–1.66 GeV^2/c^4) again seems to be independent of t^2 although there may be a slight excess below 1 GeV^2.

The same analysis for ω^0 production gives like results; that is, ω's are produced with a t distribution similar to that of the background. The t distributions checked were t from the target to the proton, Fig. 14, and $|t|^2$ from the incident π^- to the $\pi^+\pi^-\pi^0$ system, Fig. 15.

FIG. 14. Effective mass of the $\pi^+\pi^-\pi^0$ system versus the magnitude of the four-momentum transfer from the target to the proton for events of the type $\pi^- p \rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+ p\pi^0$.

MASS $(\pi^+\pi^-\pi^\circ)$ (GeV/c^2)

1.800				1	1						
				3	3						
1.650			1	5	4						
			4	3	8	4					
1.500		2	8	5	6	9					
		6	22	18	18	12	4				
1.350		6	17	26	30	21	6	1			
		12	32	37	37	28	10	3			
1.200	2	14	41	43	51	48	17	7	1		
	4	24	64	65	50	47	18	10	3		
1.050	3	37	70	64	64	60	27	14			
	3	39	86	57	74	60	40	12	5		
.900	6	37	78	86	62	47	32	18	5		
	14	58	119	81	85	71	44	32	8		ω°
.750	7	31	57	42	41	35	19	14	6	2	
	9	23	54	34	16	32	11	8	3	2	
.600	5	16	26	22	12	17	11	5	5	1	
		1	11	8	2	1	1	2		1	
.450	1										

0 .25 .50 .75 1.00 1.25 1.50 1.75 2.00 2.25 2.50 2.75

$|t|$ (target $\rightarrow$ proton) (GeV)

FIG. 15. Effective mass squared of the $\pi^+\pi^-\pi^0$ system versus the magnitude of the four-momentum transfer squared from the incident π^- to the $\pi^+\pi^-\pi^0$ system for events of the type $\pi^- p \rightarrow \pi^-\pi^-\pi^-\pi^+\pi^+ p\pi^0$.

MASS SQUARED $(\pi^+\pi^-\pi^\circ)$ (GeV^2/c^4)

3.11		1		1				
			3	1				
2.81		1	3					
		1	5	2				
2.51		2		4				
		5	7	4	1			
2.21	2	3	10	7				
		12	15	9	1	1		
1.91	3	25	29	7	8	2		
	2	31	28	16	8	1		
1.61	7	45	46	21	13	2		
	7	57	59	27	18	4		
1.31	19	81	77	40	15	6	1	
	35	90	85	49	32	9		
1.01	43	116	115	69	32	9		
	51	158	105	59	36	16	1	
.71	112	214	143	84	37	12	5	ω°
	59	115	89	44	11	16	4	
.41	53	65	61	28	17	6		
	3	7	6	1	3	1		
.11								

.1 1.0 1.9 2.8 3.7 4.6 5.5 6.4

$|t|^2$ (incident $\pi^- \rightarrow \pi^+\pi^-\pi^\circ$) (GeV^2)

This result can be compared with the results of others[1,3] in 3.2 and 4.0 GeV/c $\pi^- p$ four-prong events. At 3.2 and 4.0 GeV/c approximately 75% of the Δ^{++} (1238) production in $\pi^- p \to \pi^-\pi^- p\pi^+$ occurs at $|t|$ to the Δ^{++} of less than 0.8 GeV^2. The ω^0 in $\pi^- p \to \pi^-\pi^- p\pi^+\pi^0$ at 4.0 GeV/c is strongly peaked forward.

CONCLUSIONS

Strong resonance production of Δ^{++} (1238) has been observed in the final state $3\pi^- p 2\pi^+$, ω^0 in the final state $3\pi^- p 2\pi^+\pi^0$, and Δ^- (1238) in the final state $3\pi^- 3\pi^+ n$. They each occurred in approximately one third of the corresponding events. The nucleons from all events were peaked backward in the center-of-mass system, but less strongly than in experiments at lower energy with fewer outgoing particles. However, the neutrons were peaked backward more strongly than the protons. The pions from all events were distributed approximately isotropically in the center-of-mass system. The distribution of ω^0 and Δ^{++} (1238) in four-momentum transfer was quite similar to the distribution of background.

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[3] S. Chung, O. Dahl. L. Harky, R. Hess, G. Kalbfleisch, J. Kirz, D. Miller, and G. Smith, Phys. Rev. Letters 12, 621 (1964). S. Chung, O. Dahl, L. Hardy, R. Hess, L. Jacobs, J. Kirz, and D. Miller, *ibid.* 15, 325 (1965).

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Application of $SU(6)_W$ in a Model for Vector-Meson Production at High Energies*

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A double-Regge-pole model incorporating $SU(3)$ symmetry, previously applied to pseudoscalar meson-baryon exchange reactions, is extended to include certain vector-meson production reactions with $SU(6)_W$ symmetry imposed at the forward direction. A large number of predictions for forward vector-meson production cross sections are obtained in terms of pseudoscalar-meson cross sections. These are compared where possible with the meager data available above 3 BeV/c, with generally favorable results.

IT has been observed by Jackson[1] that comparison of some simple $SU(6)_W$ symmetry predictions for forward scattering amplitudes with data yields generally bad agreement. However, at the same time it was pointed out[1] that these simplest relations (which are model-independent) are just the ones expected to be disturbed violently by symmetry-breaking effects, such as mass differences among the pseudoscalar mesons. In fact, similar tests of $SU(3)$ in high-energy reaction amplitudes do not fare well either.[2]

If symmetries such as $SU(3)$ and $SU(6)$ are preserved well in the couplings of states, but the masses of the states are not degenerate within a multiplet, it seems logical to incorporate the empirical mass splitting in some way in the construction of S-matrix elements.[3] For high-energy two-body inelastic (exchange) reactions, it is appropriate in a Regge-pole approach to include empirical (or linearly extrapolated) trajectories $\alpha(t)$, but retain symmetry predictions for pole residues. This procedure has met with considerable success in the case of $SU(3)$ in the charge- and hypercharge-exchange reactions at forward angles involving pseudoscalar mesons.[4] In this paper we employ $SU(6)_W$ in a similar fashion to obtain predictions for forward cross sections for certain vector meson production reactions.

We consider the classes of reactions $PB \to PB$, $PB \to PB^*$, $PB \to VB$, and $PB \to VB^*$ where pseudoscalar P and vector V belong to the same $SU(6)_W$ representation (**35**), while B and B^* also belong to the same (**56**) representation. A model for the first two of these classes has been developed in a previous paper[4]; the model involves a pair of exchange-degenerate octets of Regge trajectories which include ρ, A_2, K_{890}^*, and K_{1400}^* exchanges. This double-pole model appears to give at least the gross features of the first two classes of reactions correctly and incorporates exact $SU(3)$ symmetry in the couplings.

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] J. D. Jackson, Phys. Rev. Letters 15, 990 (1965).

[2] S. Meshkov, 1966 Boulder (Colorado) Summer School Proceedings (unpublished); H. Harari, in *High Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965).

[3] This philosophy has been employed at low energies; e.g., K. C. Wali and R. L. Warnock, Phys. Rev. 135, B1358 (1964).

[4] R. C. Arnold, Phys. Rev. 153, 1506 (1967).