Study of the ρ^0 Meson Formed in $\pi^- + p \to \pi^- + \pi^+ + n$ between 1.38 and 3.00 GeV/c*†

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(Received 30 January 1967)

The final state of $\pi^- + p \to \pi^- + \pi^+ + n$ has been studied from data at laboratory pion momenta of 1.38, 1.71, 2.75, and 3.00 GeV/c. The ρ^0 resonance dominates the final state throughout the region. The ρ^0 production and decay angular distributions are compared with Gottfried and Jackson's one-pion-exchange model including absorption effects. The data are in better agreement with the model at the higher momenta, as expected. The $\pi\pi$ effective-mass distribution with $\Delta^2 \leq 4\mu^2$ and $|\cos\theta_{\pi\pi}| < 0.3$ for the combined data peaks near 730 MeV with 255 events in the plot. The effective-mass distribution without kinematic restrictions gives a ρ^0 peak at 765 MeV with 6744 events.

INTRODUCTION

PION-PROTON interaction data consisting of the incoming pion kinetic energy and the four-momentum for each of the final-state particles were obtained from Salant *et al.*[1] and Selove *et al.*[2] at the following momenta: 1.38 GeV/c, 1.71 GeV/c, and 2.75 and 3.00 GeV/c combined. The data were placed on magnetic tape and analyzed with the Ohio University IBM 360 Model 40 computer.

The production and decay of the ρ^0 is compared to the one-pion-exchange model modified by absorption effects. The model is in better agreement with experimental results as the momentum increases. The data come from an experiment at 3.00 GeV/c at the University of Pennsylvania,[3] an experiment at 2.75 GeV/c by the Saclay-Orsay-Bari-Bologna collaboration,[4] and two experiments at 1.38 and 1.71 GeV/c at Brookhaven National Laboratory.[1]

ρ PRODUCTION

The appearance of the ρ is indicated in Figs. 1(a)–1(c) at 1.38 GeV/c, 1.71 GeV/c, and the combined momenta of 2.75 and 3.00 GeV/c. The dashed curves of Figs. 1(a)–1(c) present the dipion effective-mass histograms with all events removed in which the square of the four-momentum transfer between the incident nucleon and the final nucleon is greater than $10\mu^2$. The rest mass of a charged pion is represented by μ. The square of the four-

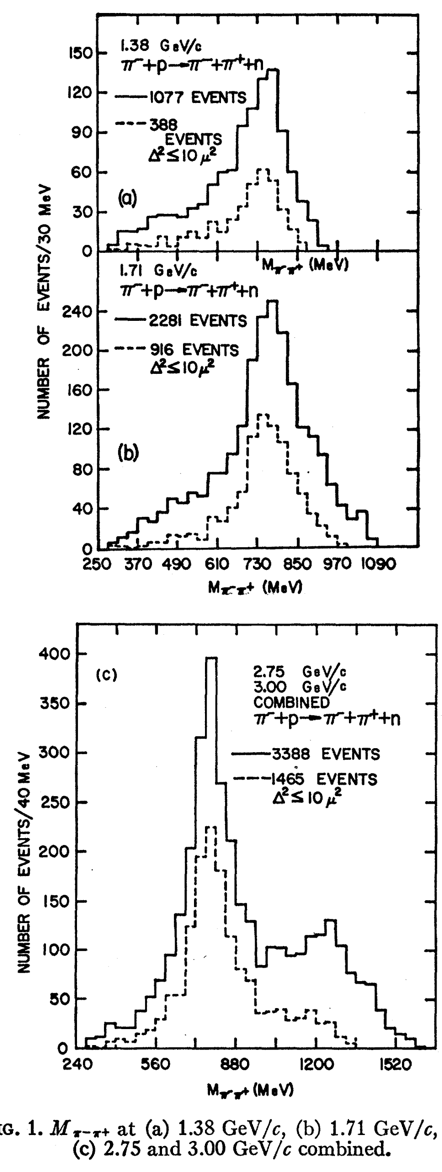

FIG. 1. $M_{\pi^-\pi^+}$ at (a) 1.38 GeV/c, (b) 1.71 GeV/c, and (c) 2.75 and 3.00 GeV/c combined.

* Supported in part by the Ohio University Fund.
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[1] E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters **9**, 170 (1962); W. J. Fickinger, D. K. Robinson, and E. O. Salant, *ibid.* **10**, 457 (1963); T. C. Bacon, W. J. Fickinger, D. G. Hill, H. W. K. Hopkins, D. K. Robinson, and E. O. Salant, in *Proceedings of the Second Topical Conference on Resonant Particles* (Ohio University, Athens, Ohio, 1965), p. 129.

[2] V. Hagopian, W. Selove, J. Alitti, J. P. Baton, and M. Neveu-Rene, Phys. Rev. **145**, 1128 (1966).

[3] V. Hagopian and W. Selove, Phys. Rev. Letters **10**, 533 (1963); V. Hagopian, Ph.D. thesis, University of Pennsylvania, 1963 (unpublished).

[4] Saclay-Orsay-Bari-Bologna collaboration, Nuovo Cimento **35**, 713 (1965).

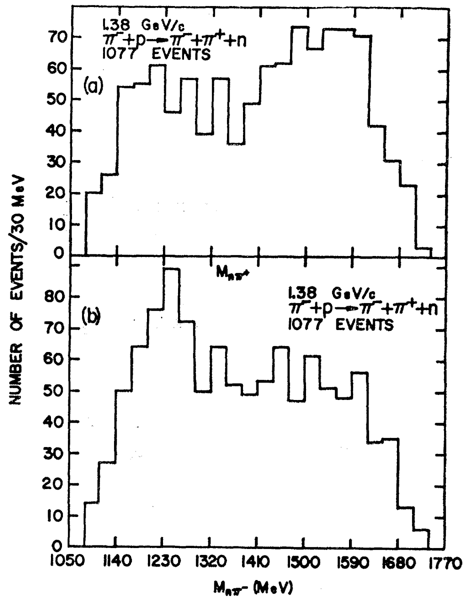

FIG. 2. Effective pion-nucleon mass distributions at 1.38 GeV/c. (a) $M_{n\pi^+}$; (b) $M_{n\pi^-}$.

momentum transfer Δ^2 is given by

$$\Delta^2 = (\mathbf{P}_n - \mathbf{P}_p)^2 - (E_n - E_p)^2, \quad (1)$$

where $(\mathbf{P}_n, E_n)$ and $(\mathbf{P}_p, E_p)$ are the four-momenta of the final nucleon and incident proton, respectively. The ρ resonance is very prominent at low Δ^2 for all sets of data, in agreement with peripheral production.

Figures 2(a) and 2(b) show the effective-mass distributions of $n\pi^+$ and $n\pi^-$, respectively, at 1.38 GeV/c; the (3,3) isobar is clearly seen in Fig. 2(b). This isobar production could not proceed through ρ exchange because a doubly charged exchange meson would be required to form the isobar in the negative charge state. Isobar production was not prominent at 1.71 GeV/c and the combined momenta of 2.75 and 3.00 GeV/c. To remove the isobar interference with the ρ, only low Δ^2 events were analyzed ($\Delta^2 \leq 10\mu^2$).

The peripherality of ρ production is clearly illustrated in the Chew-Low plots [Figs. 3(a)-3(c)]. There is a strong clustering of events at low Δ^2/μ^2 values in the ρ region, $M_{\pi\pi}{}^2/\mu^2 \sim 30$.

Figures 4(a)-4(c) are Δ^2/μ^2 distributions for $700 < M_{\pi\pi} \leq 800$ MeV at each momentum. Figs. 4(b) and 4(c) indicate the agreement of the one-pion-exchange model including absorption (OPEA) of Gottfried and Jackson[5] with the experimental differential cross sections at 1.71 GeV/c and the combined data at 2.75 and 3.00 GeV/c. The theoretical curve was omitted from Fig. 4(a) since the model was designed to describe interactions at high momentum, and the 1.38-GeV/c data stretch the limit of validity of approximations in the model.

The cross sections of Figs. 4(b) and 4(c) were not continued beyond $\Delta^2/\mu^2 = 20$ because of the large percentage of background events above this value. It was thus assumed that the proportion of ρ events for

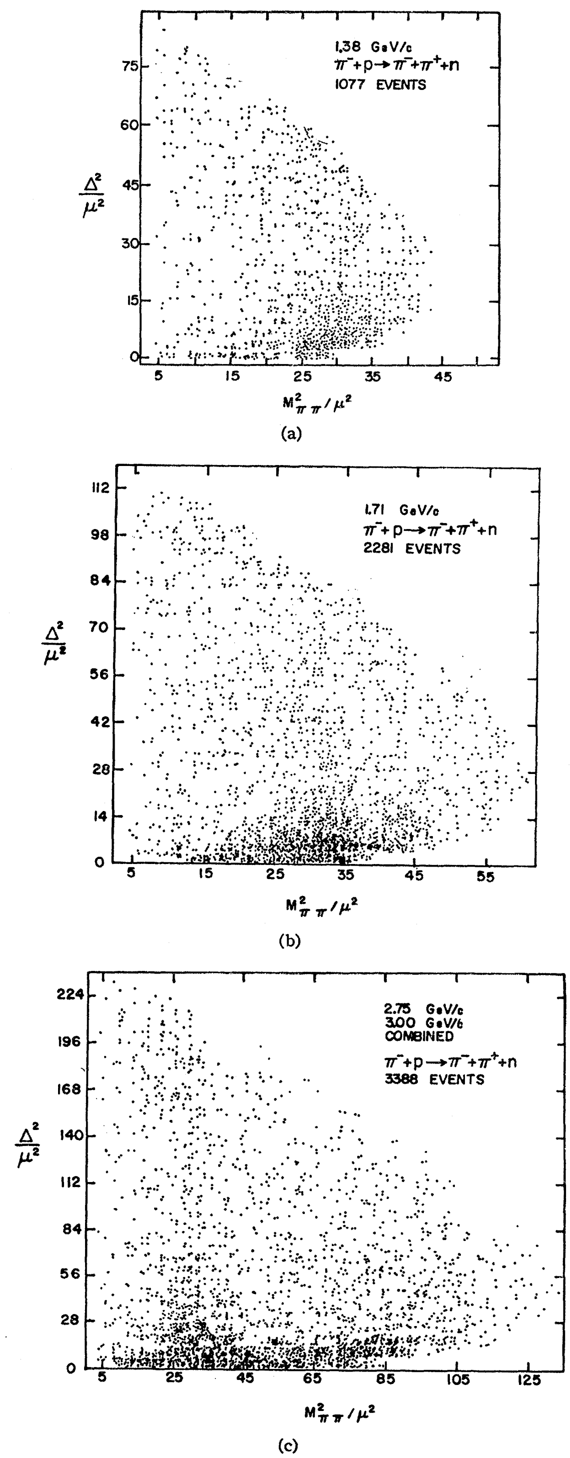

FIG. 3. Chew-Low plots of Δ^2/μ^2 versus $M_{\pi\pi}{}^2/\mu^2$ at (a) 1.38 GeV/c, (b) 1.71 GeV/c, and (c) 2.75 and 3.00 GeV/c combined.

[5] K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 735 (1964).

$\Delta^2/\mu^2 > 20$ decreased at the same rate as the theoretical cross sections in order to obtain a total experimental cross section for $\Delta^2/\mu^2 \leq 20$. The total experimental cross section for ρ production at 1.71 GeV/c was 2.48 mb, and at the combined momenta of 2.75 and 3.00 GeV/c the cross section was 1.48 mb.[6] The theoretical cross sections are 2.83 and 1.4 mb, respectively.[7]

DECAY ANGULAR DISTRIBUTIONS

The axes and angles in the dipion rest system used in the following analysis are those utilized by Jackson for OPEA.[8] $\theta_{\pi\pi}$ is the angle between the incident pion and the negative decay pion in the dipion rest system, and the azimuthal angle ϕ is equivalent to the Treiman-Yang angle.

Figures 5(a)–5(c) show $\cos\theta_{\pi\pi}$ plotted against ϕ for the four sets of data with $700 < M_{\pi\pi} \leq 820$ MeV and $\Delta^2/\mu^2 \leq 10$. On the basis of one-pion exchange, the Treiman-Yang angle should be isotropic.[9] These figures are obviously not in agreement with isotropy in ϕ. An asymmetry in $\cos\theta_{\pi\pi}$ is also very evident in these same figures. The distribution in $\cos\theta_{\pi\pi}$ should be symmetric for the decay of a pure resonance with definite quantum numbers as is true for the decay of ρ^-.[2,10]

As a convenient convention for discussion, each graph of Fig. 5 is divided into four sections numbered 1–4 starting in the upper-right quarter and counting in a counter-clockwise direction to the lower-right quarter. At 1.38 GeV/c, Fig. 5(a), the fourth quarter has the least number of events and the second quarter has the largest number. The first quarter has slightly more events than the third quarter. The same pattern is repeated at 1.71 GeV/c [Fig. 5(b)]; whereas, for ρ^0 decay of the higher momenta [Fig. 5(c)] the third quarter has the least number of events.

The angular distribution of ρ decay from the absorption model gives a definite prediction for the relation between $\theta_{\pi\pi}$ and ϕ:

$$W(\theta_{\pi\pi},\phi) = (3/4\pi)[\rho_{0,0}\cos^2\theta_{\pi\pi} + \tfrac{1}{2}(1-\rho_{0,0})\sin^2\theta_{\pi\pi} - \rho_{1,-1}\sin^2\theta_{\pi\pi}\cos 2\phi - \sqrt{2}\,\mathrm{Re}\rho_{1,0}\sin 2\theta_{\pi\pi}\cos\phi], \quad (2)$$

[6] In order to obtain the experimental cross section for ρ production of 2.48 mb at 1.71 GeV/c and 1.48 mb at 2.75–3.00 GeV/c, several approximations were required. At the lower momentum, Bacon *et al.* (Ref. 1) give 4.71 ± 0.24 mb for $\pi + N \to \rho^0 + N$. This was for combined reactions $\pi^- + p \to \pi^+ + \pi^- + n$ and $\pi^+ + n \to \pi^+ + \pi^- + p$. The estimate for $\pi^- + p \to \rho^0 + n$ was obtained by assuming that the ratio of the cross sections for ρ^0 production in the two interactions was the same as the ratio of the total cross sections given also by Bacon *et al.* (Ref. 1). The experimental cross section for $\pi^- + p \to \rho^0 + n$ for the 2.75–3.00-GeV/c data combined was obtained by assuming the ratio of ρ^0 production to total cross section was the same as for the 2.75-GeV/c data alone. Both of these cross sections were reported at 2.75 GeV by Ref. 4. The combined data of Hagopian *et al.* (Ref. 2) gave the total cross section.

[7] J. D. Jackson, J. T. Donahue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, Phys. Rev. 139, B428 (1965).

[8] J. D. Jackson, Rev. Mod. Phys. 37, 484 (1965).

[9] S. B. Treiman and C. N. Yang, Phys. Rev. Letters 8, 140 (1962).

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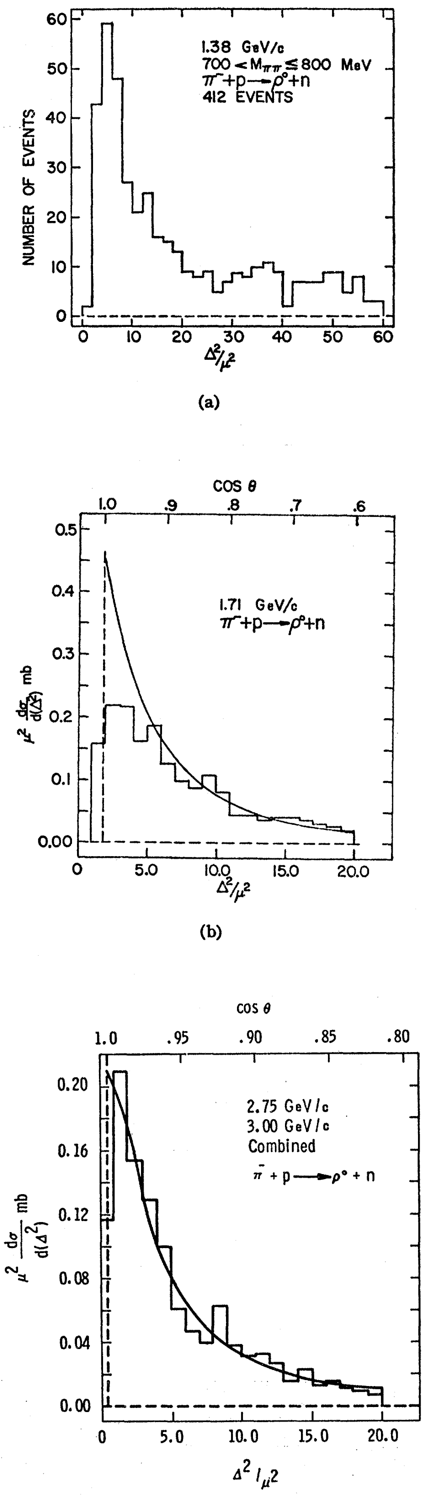

FIG. 4. (a) Δ^2/μ^2 histogram for ρ^0 production at 1.38 GeV/c. (b) Differential cross section for ρ^0 production at 1.71 GeV/c. (c) Differential cross sections for ρ^0 production at 2.75 and 3.00 GeV/c combined. Curves from the Gottfried and Jackson model for one-pion exchange.

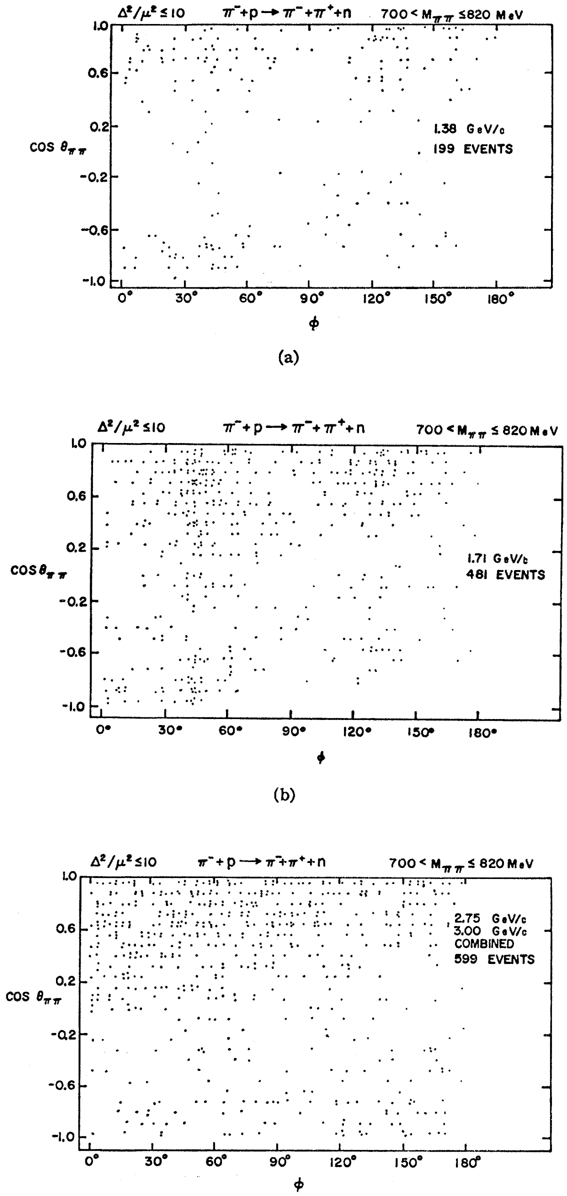

FIG. 5. Cos$\theta_{\pi\pi}$ versus Treiman-Yang angle for ρ^0 decay at (a) 1.38 GeV/c, (b) 1.71 GeV/c, and (c) 2.75 and 3.00 GeV/c combined.

where all spin-density matrix elements ($\rho_{m,m'}$) are nonzero.[5] For $\Delta^2/\mu^2 \leq 10$, the magnitude of $\sqrt{2}\,\mathrm{Re}\rho_{1,0}$ is greater than $\rho_{1,-1}$, and the last term of Eq. (2)should dominate the asymmetric part of the angular distribution.[7] Since $\mathrm{Re}\rho_{1,0}$ is negative in this Δ^2/μ^2 region and $\rho_{1,-1}$ is positive, the decay distribution should be a maximum for $\theta_{\pi\pi}=45°$, $\phi=0°$, and $\theta_{\pi\pi}=-45°$, $\phi=180°$. Thus, the second and fourth quarters of Fig. 5 should be equally populated with more events than the first and third quarters (also equally populated).

TABLE I. Numbers of events in separate regions of the ρ^0 decay angular distributions (Fig. 5). $\Delta^2/\mu^2 \leq 10$.

	Number of events			
	$700<M_{\pi\pi}\leq 760$ MeV		$760<M_{\pi\pi}\leq 820$ MeV	
Quarter	1.38 GeV/c	1.71 GeV/c	1.38 GeV/c	1.71 GeV/c
I	36	59	23	67
II	38	110	29	90
III	28	53	18	54
IV	14	27	13	21

The decay distribution of ρ^- is in good agreement with Eq. (2) at 2.75 and 3.00 GeV/c.[2,10] Figure 5(c) shows that ρ^0 decay is in disagreement with OPEA at the same momentum, with the first quarter having more events than the fourth quarter; however, Fig. 5(c) does give the predicted anisotropy in ϕ. At the lower momenta [Figs. 5(a) and 5(b)] the data no longer agree with OPEA in the anisotropic distribution of ϕ. Many more events occur in the third quarters than in the fourth quarters.

The possibility of dipion decay of ω^0 affecting the ρ^0 decay distribution at 2.75 and 3.00 GeV/c was investigated by Hagopian *et al.* and found to have a negligible effect.[2] The data at 1.71 and 1.38 GeV/c were divided into the two mass regions $700<M_{\pi\pi}\leq 760$ MeV and $760<M_{\pi\pi}\leq 820$ MeV in order to examine the possible effects of $\omega^0 \to \pi^+ + \pi^-$ in the plots of $\cos\theta_{\pi\pi}$ versus ϕ. Since ω^0 is a narrow resonance with a mass of ~ 780 MeV, only the higher mass region should be affected by $\omega^0 \to \pi^+ + \pi^-$. Table I shows the results of dividing the ρ^0 peak into the two mass regions. The decay distributions are essentially the same for both regions at each momentum, indicating that the disagreement with OPEA is not due to dipion decay of ω^0.

The asymmetry in $\cos\theta_{\pi\pi}$ for ρ^0 decay at all momenta is due to the large $T=0$, $J=0$ phase shift in the ρ^0 region. Other phase shifts have been shown to be small in this region.[11] Spark chamber experiments indicate a $T=0$, S-wave enhancement at about 700 MeV decaying into two neutral pions.[12,13] Conflicting results are seen in bubble chamber experiments where kinematic cuts of $\Delta^2 < 4\mu^2$ and $|\cos\theta_{\pi\pi}|<0.3$ have been applied to the dipion effective mass from the reaction $\pi^- + p \to \pi^- + \pi^+ + n$. Some experiments show a peak at ~ 730

[11] L. D. Jacobs and W. Selove, Phys. Rev. Letters **16**, 669 (1966); L. W. Jones, D. O. Caldwell, B. Zacharov, D. Harting, E. Bleuler, W. C. Middelkoop, and B. Elsner, Phys. Letters **21**, 590 (1966); L. J. Gutay, P. B. Johnson, F. J. Loeffler, R. L. McIlwain, D. H. Miller, R. B. Willmann, and P. L. Csonka, Phys. Rev. Letters **18**, 142 (1967).

[12] M. Feldman, W. Frati, J. Halpern, A. Kanofsky, M. Nussbaum, S. Richert, P. Yamin, A. Choudry, S. Devons, and J. Grunhaus, Phys. Rev. Letters **14**, 869 (1965).

[13] M. Wahlig, E. Shibata, D. Gordon, and D. Frisch, Phys. Rev. **147**, 941 (1966). A histogram in this paper indicates an enhancement at 700 MeV; however, the authors indicate that their spectrometer sensitivity is not sufficient at that energy to claim the peaking is a resonance.

MeV,[2,14] while others do not.[15,16] The data of this experiment were combined with those of another experiment at 2.15 GeV/c, and a peak was evident.[17] Figure 6 shows the dipion effective-mass distribution obtained here with the Δ^2 and $\cos\theta_{\pi\pi}$ restrictions. A peak exists near 730 MeV, whereas the ρ peaks at $\sim$765 MeV for the same data with no kinematic restrictions.

Because of the large number of factors which may affect Fig. 6 (for example, the measuring, analysis, and event-selection procedures may have varied in the four experiments, interference between close resonances, different bubble chambers and beam criteria), the central-limit theorem suggests that a Gaussian distribution might be consistent with the data.[18] Approximating phase space with a uniform background and excluding histogram bins with less than six events as having too few events for analysis, Eq. (3) was found consistent with Fig. 6 at better than the 55% level ($\chi^2=7.8$ with 9 degrees of freedom), using the χ^2 test.

$$dN/dM_{\pi\pi}=38.4\exp[-(M_{\pi\pi}-730)^2/2\sigma^2]+11.7, \quad (3)$$

where $\sigma=80$ MeV. Similar equations of single-level Breit-Wigner form or Gaussian distributions centered at the ρ peak of 763 MeV were inconsistent with the data, being outside the 1% level using the χ^2 test.

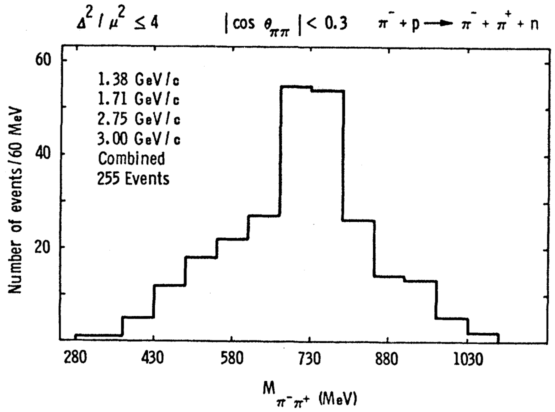

FIG. 6. $M_{\pi^-\pi^+}$ with $\Delta^2/\mu^2\leq 4$ and $|\cos\theta_{\pi\pi}|<0.3$ for all momenta combined.

Equation (4), a single-level Breit-Wigner form

$$\frac{dN}{dM_{\pi\pi}}=13.8\frac{\Gamma^2}{(M_{\pi\pi}-730)^2+\Gamma^2/2}+10, \quad (4)$$

with $\Gamma=120$ MeV, gives an even better fit to the data of Fig. 6 ($\chi^2=3.1$ with 9 degrees of freedom) although interpretation of the 730-MeV peaking as a single-level resonance appears doubtful, because of the ubiquitous ρ meson.

ACKNOWLEDGMENTS

The authors wish to thank Dr. H. L. Kraybill, Dr. D. Stonehill, Dr. E. O. Salant, Dr. D. K. Robinson, Dr. W. J. Flickinger, Dr. W. Selove, and Dr. V. Hagopian for making the experimental data available, and for discussions about the data and its possibilities. Thanks are also given to Dr. J. D. Jackson and J. Donahue for providing the predictions of the OPEA model. The services rendered by the staff of the Ohio University computer center are gratefully acknowledged.

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[17] W. Selove, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967).

[18] H. Freeman, *Introduction to Statistical Inference* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1963), pp. 181, 182.