Interference of Fine-Structure Levels of He II⁺

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Passage of a beam of foil-excited He⁺ ions through a small electric field induced quasiperiodic fluctuations in the intensity of the light emitted from levels n=7, 8, 9, and 10. Both He³ and He⁴ beams were used. Only one foil was needed, in contrast to the double-foil arrangement reported necessary for hydrogen. The observations are discussed in terms of Stark-induced interference of the fine-structure levels of He π , but discrepancies between the data and the calculations are large.

I. INTRODUCTION

 \mathbf{I} T has been shown experimentally¹ that the applica-tion of an electric field of a few tens of V/cm to fast beams of excited hydrogen atoms can cause a quasiperiodic oscillation in the intensity of the light emitted from certain levels. That experiment, which used the beam-foil technique² to excite the atoms, gave a one-to-one correspondence between the instant at which an excited atom radiated light, the point in space at which the radiation occurred, and a point on the photographic images, formed in dispersed light, of the slit of a stigmatic spectrograph. The line images which appear on the photographic plate show a decline in blackening from one end to the other because of the finite mean lives of the excited levels which give rise to the observed spectral lines. As a result, the intensity of a spectral line declines with increasing distance of the emitting particles from the foil. In other words, temporal variations in intensity are linearly transformed into spatial variations in plate blackening. Thus the timedependent oscillations were transformed into spatially distinct variations in the intensities of certain spectral lines. The previous specific observation¹ was that the Balmer lines H_{γ} , H_{δ} , and H_{ϵ} , exhibited rippled appear-



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¹ S. Bashkin, W. S. Bickel, D. Fink, and R. K. Wangsness, Phys. Rev. Letters 15, 284 (1965).
² S. Bashkin, D. Fink, P. R. Malmberg, A. B. Meinel, and S. G. Tilford, J. Opt. Soc. Am. 56, 1064 (1966).

ances, with the spacing between ripples differing from line to line. This modulation of the line intensities was interpreted^{1,3} in terms of the Stark effect which caused interference between nearly degenerate levels of opposite parity.

Use of the beam-foil technique to excite electronic levels of He I and He II showed qualitatively⁴ that some of the light from levels in He II exhibited a similar timedependent variation in intensity. We have confirmed this result by means of a more quantitative experiment, but the problem of interpretation has been accentuated by the new data. We also corroborated an interesting difference between the hydrogen and helium cases.

II. THE EXPERIMENT

A beam of mass four He⁺ ions, with an energy of 400 keV ($v=4.4\times10^8$ cm/sec), passed through a carbon foil (thickness $\sim 10 \,\mu {
m g/cm^2}$) and then entered a uniform transverse electric field. (The energy loss on going through the foil has been neglected.) Optical radiation from the beam was analyzed with a stigmatic Meinel spectrograph (f/0.8). In the wavelength range between λ 3100 and λ 6700 Å, we detected 13 spectral lines from He I, 11 from He II, and two others, each of which is probably a blend of lines from He I and He II. These lines are listed in Table I, with the designation B or BB following the wavelengths of the blended lines.

Four of the He II lines, indicated by an asterisk in Table I, showed the oscillations. Figure 1 shows a portion of the spectrum including three of the lines on which the oscillations appeared. Reference 4 showed the effect for λ 5411, λ 4859, and λ 4541 Å. None of the other lines exhibited the oscillations, but the He II lines from n > 10 were so faint that grain interfered with determining whether the phenomenon occurred or not.

The patterns on the He II lines were less distinct than those obtained for hydrogen,¹ but were nonetheless easy to recognize. Figure 2 is a microphotometer tracing along the length of λ 4859 Å; the applied field was 60 V/cm. There is more "noise" on the helium plates than on those for hydrogen because the helium exposures were made with a very narrow (10μ) slit width, whereas the slit was 2-mm wide in the earlier work. A wide slit

⁸ R. K. Wangsness, Phys. Rev. 149, 60 (1966).

⁴ S. Bashkin and G. Beauchemin, Can. J. Phys. 44, 1603 (1966).

TABLE I. Spectral lines detected in He I and He II. In column 2 for He II, the entries indicate the principal quantum numbers of the levels involved in the transitions with listed wavelengths. B and BB indicate blended lines. The asterisk designates a line exhibiting the time-dependent intensity modulation.

	λ (Å)	Transitions		
He I	5016 3965 6678 5047 4922 4437 4388 4143 3889 BB 3187 4713 4471 4121 4026 B	$\begin{array}{c} 2 {}^{1}S - 3 {}^{1}P^{0} \\ 2 {}^{1}S - 4 {}^{1}P^{0} \\ 2 {}^{1}P^{0} - 3 {}^{1}D \\ 2 {}^{1}P^{0} - 4 {}^{1}D \\ 2 {}^{1}P^{0} - 5 {}^{1}S \\ 2 {}^{1}P^{0} - 5 {}^{1}S \\ 2 {}^{1}P^{0} - 5 {}^{1}D \\ 2 {}^{3}S - 3 {}^{3}P^{0} \\ 2 {}^{3}S - 4 {}^{3}P^{0} \\ 2 {}^{3}S - 4 {}^{3}D \\ 2 {}^{3}P^{0} - 4 {}^{3}S \\ 2 {}^{2}P^{0} - 4 {}^{3}S \\ 2 {}^{2}P^{0} - 5 {}^{3}S \\ 2 {}^{2}P^{0} - 5 {}^{3}D \end{array}$		
Неп	3819 4686 3203 (2nd order) 6560 5411* 4859* 4541* 4338* 4199 4100 4026 B 3968 3923 3887 BB	$ \begin{array}{r} 2 \ {}^{3}P_{0-6} \ {}^{3}D\\ 3-4\\ 3-5\\ 4-6\\ 4-7\\ 4-8\\ 4-9\\ 4-10\\ 4-11\\ 4-12\\ 4-13\\ 4-14\\ 4-15\\ 4-16\\ \end{array} $		

gives better signal averaging than a narrow slit, and the best signal-to-noise ratio is obtained with the widest slit that is consistent with the desired resolution. Despite the "noise" on the plate, seven prominent peaks, approximately equally spaced in time, are seen. The data are summarized in columns 2, 3, and 4 of Table II. The uncertainty in each frequency is estimated as $\pm 10\%$. Similar measurements, made with He³ particles at the same velocity as used for the He⁴ particles, gave similar patterns.

Reference 1 states that the intensity modulation was seen when the incident particles were HH⁺ or HHH⁺, but not protons. However, we have since shown that the effect does appear with protons. The patterns are less distinct than when HH⁺ or HHH⁺ particles are used, but they are present nonetheless. We have also found that the patterns appear when deuterons, DD⁺, and DDD⁺ are employed as incident particles. It is possible that the difference in modulation amplitude for the different beams results from a preferential excitation of levels of different *l* value for each type of incident particle.

III. DISCUSSION

The interpretation of the experiment on hydrogen involved two assumptions:

1. The oscillations arose because of Stark mixing of the fine-structure levels.

TABLE II. Experimentally determined oscillation frequencies for lines in He π . The listed values are based on the average observed spacing of the intensity maxima.

E		Oscillation frequency (sec ⁻¹) measured		
Transition	λ (Å)	40 V/cm	60 V/cm	
4-7 4-8 4-9 4-10	5411 4859 4541 4338	5.6×10^8 6.1×10^8	7.7×10^8 8.2×10^8 8.8×10^8 9.5×10^8	

2. Weak-field Stark calculations were adequate.

Despite the interpretative difficulties to be described below, we still ascribe the oscillations to the Stark effect on the fine-structure levels because

1. The intensity patterns are strongly influenced by a transverse electric field which is either applied directly by sending the beam between a pair of charged parallel plates, or, as was shown in the earlier experiment, results from passage of the rapidly moving particles through a transverse magnetic field. (It is perhaps significant that the intensity patterns reported in Ref. 4 were less distinct than those described herein, but they were detected without the use of an external field. This finding, reminiscent of the (unpublished) circumstances which led to the work of Ref. 1, presumably means that there is an intrinsic field associated with the beam itself. This field might be due to the proximity of electrons and positive ions in the beam or simply from the presence of nearby grounded conductors. Measurements made in the course of the work of Ref. 1 showed that the local magnetic field was at least an order of magnitude too small to produce the patterns via a Lorentz field.)



FIG. 2. Densitometer tracing along the central part of the line λ 4859 Å. The beam velocity was 4.4×10^8 cm/sec and the applied field was 60 V/cm. The first five of the seven observed intensity maxima are indicated by the numbers 1 to 5.

TABLE III. Calculated Stark-mixing frequencies, in units of $10^{9} \sec^{-1}$, for level pairs conforming to $\Delta l = \pm 1$, $\Delta j = \pm 1$, 0, $\Delta m = 0$ for a field of 60 V/cm and n = 7 in He II. The combining levels are described by j and j'. Entries with an asterisk approximate the observed frequency.

j	j'	$\frac{1}{2}$	32	5/2	$\frac{7}{2}$	<u>9</u> 2	11/2
$\frac{1}{2}$	1/2	1.86					
	32	4.32					
<u>3</u> 2	32	0.36	1.08				
-	52	1.90	3.52				
52	52	0.15	0.44	0.73*			
-	7/2	1.42	3.50	4.50			
72	7	0.74	0.22	0.37	0.52		
2	9	1.22	3.32	4.85	5.12		
8	92	0.04	0.12	0.20	0.28	0.36	
	11/2	1.02	2.86	4.43	5.34	5.10	
11/2	11/2	0.02	0.06	0.10	0.14	0.18	0.22
	13/2	0.75*	2.13	3.35	4.28	4.72	4.27

2. The only transitions which exhibit the patterns in the present work are in He II. We repeat that the failure to detect the oscillations in the lines from n > 10 does not necessarily mean that the oscillations are absent. Relatively speaking, the patterns become increasingly clear as *n* goes from 7 to 10, but for n > 10, the absolute line intensity is too low to permit us to see what is happening. In the earlier work on hydrogen,¹ the patterns also became clearer with increasing n up to the highest n value detected. None of the He I lines, irrespective of exposure field strength or n, gives any indication of oscillations. It is the case that some of the He II lines (3-4, 3-5, 4-6) fail to exhibit the oscillations. Neither did H_{α} (2-3) in hydrogen, and the pattern was present but difficult to identify in H_{β} (2–4). Concerning H_{α} , it was argued¹ that the pattern frequencies were either too rapid or too slow for the plates to respond to the variations. While that may be the correct explanation for the nonoscillating lines in H and He II, our present inability (see below) to calculate the observed frequencies in He II makes us reluctant to insist on that reason. Cascade effects may play a part in causing the patterns from levels of small n to disappear, but again, the argument is neither quantitative nor applicable to levels of large n.

Concerning the second assumption, it was realized that the perturbation introduced by the external field was not small compared to the zero-field energy spacing $\Delta \sigma$ of the fine-structure levels, so that a weak-field Stark theory could only approximate the actual situation. In hydrogen, for example, a field of 60 V/cm causes the $7P_{3/2}$ - $7S_{1/2}$ separation to change from (+) 8.5×10^{-3} cm⁻¹ (the fine-structure splitting) to (-) 37×10^{-3} cm⁻¹ (for the $m = \frac{1}{2}$ substates). However, the weak-field approximation seemed adequate, given the experimental uncertainty in the oscillation frequencies. In the present case, the fine-structure splitting is about 16 times larger, and the perturbation significantly

smaller, than for the equivalent levels in hydrogen, so that the weak-field approximation should be better here than it was for hydrogen. Thus, the $7P_{3/2}$ - $7S_{1/2}$ fine-structure separation is (+) 136 cm⁻¹, while a field of 60 V/cm reduces the separation to (+) 80 cm⁻¹ (for the $m=\frac{1}{2}$ substates). Relatively speaking, this is a markedly smaller level displacement than occurs for hydrogen.

The calculations discussed below take account of the fine-structure splitting of the hydrogen levels. This represents an improvement in the theory over what was done in the work on hydrogen, since the expression including the fine-structure splitting was not available until after the paper on hydrogen was written.

In the hydrogen work, it appeared that the observed effects could be described in terms of the $P_{1/2}$ and $S_{1/2}$ levels. This was not unreasonable in view of the evidence⁵ that the small-l levels are preferentially populated in hydrogen. However, the contrary seems to be true⁶ for heavy elements, and the situation for helium is ambiguous at this time. Consequently, we have carried out the calculations for all values of l for n=7, 8, 9, and 10. One can show that the Einstein A coefficients⁷ for the appropriate level pairs (i.e., those connected by the selection rules, $\Delta l = \pm 1$, $\Delta j = \pm 1$, 0, $\Delta m=0$), combined as (A_1-A_2) in the frequency formula, are at least a factor of 10 smaller than other additive terms. With this condition, the two-level formula obtained by Wangsness³ reduces to

$$|c_{2}(t)|^{2} = |c_{10}|^{2} \frac{v^{2}}{v^{2} + \frac{1}{4}\delta^{2}} \exp\left[-\left(\frac{A_{1} + A_{2}}{2}\right)t\right] \\ \times \sin^{2}\left[(v^{2} + \frac{1}{4}\delta^{2})^{1/2}t\right] \quad (1)$$

for the simplified special case, where $|c_{10}|^2$ is the initial population of level 1 and $|c_{20}|^2$ has been taken to be zero, $\delta = 2\pi c \Delta \sigma$, and hv, the matrix element connecting states of the same m, is given by^{3,8,9}

$$hv = -\frac{3eFan}{4j(j+1)Z} |m| [n^2 - (j+\frac{1}{2})^2]^{1/2} \\ \times \{j[(j+1)^2 - m^2]^{1/2}\}, \quad (2)$$

where e is the electronic charge (taken to be positive), F is the electric field strength, a is the Bohr radius for hydrogen, n is the principal quantum number, m is the magnetic quantum number, j is the total angular momentum quantum number, and Z is the atomic number.

Note that the { } term is used for pairs of levels like $S_{1/2}$ and $P_{3/2}$ or $P_{3/2}$ and $D_{5/2}$, and is omitted for pairs of levels like $S_{1/2}$ and $P_{1/2}$ or $P_{3/2}$ and $D_{3/2}$.

- ⁵ A. S. Goodman and D. J. Donahue, Phys. Rev. **141**, 1 (1966). ⁶ S. Bashkin, L. Heroux, and R. K. Wangsness, Phys. Rev. **151**, 87 (1966).

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 ⁷ E. R. Caproitti, Astrophys. J. 139, 225 (1964).
 ⁸ V. Rojansky, Phys. Rev. 33, 1 (1929).
 ⁹ R. Schlapp, Proc. Roy. Soc. (London) A119, 313 (1928).

Since the measurements yield $|c_2(t)|^2$, we compare the observed frequency with $(1/\pi)(v^2+\frac{1}{4}\delta^2)^{1/2}$ rather than with $(1/2\pi)(v^2+\frac{1}{4}\delta^2)^{1/2}$.

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Although terms with l>4 cannot be involved in the transitions which were detected, it is in principle possible for higher-l levels to populate the lower-l levels via the Stark mixing for each value of n. Hence the decays from the lower-l values could be modulated by Stark-induced transitions having the frequencies characteristic of higher-l level pairs. We have, therefore, calculated all the theoretical frequencies; Table III lists the results for n=7.

Two conclusions are immediately obvious from the entries in Table III:

1. Of the many frequencies, two, identified by an asterisk in the table, match the observed value to within experimental error. There is similar agreement in one case for n=8 and 9, and two cases for n=10. However, there does not appear to be any particular physical significance to these occasional points of agreement.

2. The calculations yield such a variety of frequencies that excitation of even a few of the available l levels could give an incoherent addition of terms of different frequencies which would obscure the patterns.

The same results obtained for levels n=8, 9, and 10. Comparison of the theory and experiment for all four of the observed levels gives the following:

1. With exceptions as noted above, the theoretical frequencies fail to match the observed frequencies, even after allowance is made for a $\pm 10\%$ uncertainty in the measured repetition rates.

2. Equation (1) predicts a general decline in $|c_2(t)|^2$ by virtue of the factor $\exp[-\frac{1}{2}(A_1+A_2)t]$. The densitometer tracing (see Fig. 2) shows that the actual decline in intensity is far faster than predicted. We point out that the data from the photographic plate from which the tracing was made were not corrected for the HD curve; correction for the HD curve would emphasize further the discrepancy between theory and experiment.

3. From Fig. 2, it appears that the general decline is not monotonic. Rather, the peak heights seem to go in pairs, two being of comparable heights, followed by another set of two of lower, but again equal, heights.

4. The dependence of frequency on n is strongly dependent on Δj and m. Thus $\Delta j=0$, all m and $\Delta j=1$, $m\neq \frac{1}{2}$ lead to an increase in frequency by a factor of 2 to 3 as n goes from 7 to 10; on the other hand, $\Delta j=1$, $m=\frac{1}{2}$ yields a decrease in frequency with rising n. Since the observations show a 20% increase in frequency between n=7 and 10, it appears that the calculations do not reflect the proper variation of frequency with n.

5. Another point is that the matrix element hv is proportional to the electric field. When $v \gg \delta$ (as occurs when $\Delta j=0$ or $\Delta j=1$ and *m* is large), the frequency is expected to be proportional to the field. Table II shows

this to be the case, so that one might infer that the $\Delta j=0$ and/or $\Delta j=1$, *m* large levels are the main contributors to the data. Unfortunately, these conditions also give far too rapid a dependence of frequency on *n*.

6. As can be seen in Fig. 2, the frequency appears to increase as the radiating ions proceed through the field, i.e., the spacing of the intensity maxima is about 20% closer at the low-intensity end of each line than at the start. (The average of the periods is listed in Table I.) Moreover, the modulation becomes less pronounced, and even disappears in some cases, towards the low-intensity end of the line, although the emitters are still in the field. There is nothing in the theory to account for this behavior.

In concluding this summary of the conflicts between theory and experiment, we call attention to an interesting difference between the work on hydrogen and the present experiment. Reference 1 emphasized that the intensity modulation could be seen only after the beam had passed through *two* foils in succession. This requirement was stringent, as was shown, for example, in the work with a magnetic field.¹ Although that field permeated the region between the two foils, the oscillations appeared only downstream from the second foil. The experiments on helium, on the other hand, have been done with a single foil. It appears that the interference phenomenon depends on a close, but still unknown, relationship between relative level populations and level decay characteristics.

IV. CONCLUSIONS

We find that

1. The experimental patterns are unmistakable and the measured frequencies are well enough determined to warrant a quantitative comparison with theory.

2. The Stark effect is the basic one with which we must deal.

3. The two-level, weak-field approximation is not adequate.

It is our tentative suggestion that the disparity between theory and experiment can be removed by using a multilevel theory. The multilevel approach is being studied by Wangsness.¹⁰ Even this approach, however successful, would leave untouched the one-foil and two-foil requirements for producing the patterns.

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¹⁰ R. K. Wangsness (private communication).



FIG. 1. Enlargement of a partial spectrogram showing several spectral lines from He I and He II. The spectral lines with underlined wavelengths show the oscillatory intensity pattern.