This band also has been observed in Re^{185} and Re^{187}.[16] The intense 224-keV M1 transition from the 762-keV level indicates strong admixtures to the vibrational states, possibly of the (411)↓ Nilsson state.

[16] See, e.g., K. M. Bisgård, L. J. Nielsen, E. Stabell, and E. Veje, Nucl. Phys. (to be published).

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Isobaric Analog Resonances in the Scattering of Protons by Cd^{114}†

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Elastic and inelastic scattering of protons from Cd^{114} has been studied for proton energies between 7 and 10 MeV. Resonances have been identified at the following laboratory energies in MeV; numbers in parentheses give the orbital angular momentum in the elastic channel: 7.20 (0), 7.45 (2), [7.55 (2)], 7.68 (2), 7.85 (0), 8.00 (2), 8.58 (2), 9.15 (1), 9.30 (3), 9.71 (3). All are analogs of known states in Cd^{115}, from the ground state to 2.5-MeV excitation. The elastic-scattering reduced widths are compared with the stripping reduced widths to the analog levels in Cd^{115}. Agreement in relative value is found only for large reduced widths; the smaller components of the wave function seem to be different in the parent and in the analog states. The inelastic-scattering data indicate that the 0.48-MeV level has a very large overlap with the first excited state of Cd^{114} (2^+), while a level near 2.0 MeV has a very large overlap with the 3^- state of Cd^{114} at 1.95 MeV. Other states are also discussed.

I. INTRODUCTION

ISOBARIC analog resonances were discovered by Fox, Moore, and Robson.[1] These authors showed that, when a target nucleus C is bombarded with protons, resonances in the compound-nucleus pC are observed which are the isobaric analogs of the low-lying states of the parent nucleus nC. Since then, many nuclei C have been studied. It has been found that the correspondence between the resonance and the energy levels of the parent nucleus is very good; usually there is a resonance corresponding to each level of the parent nucleus. The spins, parities, and energy spacings of the resonances are the same as those of the levels of the parent nucleus.

Allan, Jones, Morrison, Taylor, and Weinberg[2] showed that the isobaric analog resonances appear not only in the elastic channel but also in several inelastic channels. The nuclear spectroscopic information about the parent nucleus which the *in*elastic-scattering experiments can in principle yield cannot be obtained from other nuclear reactions. The elastic-scattering experiment, on the other hand, should be equivalent to a stripping experiment. More precisely, the elastic-proton reduced widths of the resonances in the pC system should be related to the neutron reduced widths of the analog levels in the nC nucleus by the relation $\gamma_p^2 = \gamma_n^2/(2T_0+1)$, where T_0 is the isospin of the target C.[3] The angular momenta of the proton and neutron in question should of course be the same: $l_p=l_n$, $j_p=j_n$. The neutron reduced width γ_n^2, or equivalently the spectroscopic factor S, can be extracted from a (d,p) stripping experiment on the same target C. The inelastic proton reduced widths $\gamma_{p'}^2$, on the other hand, are related to neutron widths γ_n^{2*} which would be obtained if it were possible to do a stripping experiment with the target nucleus in an excited state C^*.

The reduced widths found experimentally in elastic-proton scattering agree roughly, but not in detail, with the stripping results.[4] It is not clear whether the discrepancies found are due mostly to an inadequate method of extracting the reduced widths from the experimental data or whether the levels in question are really not very good analogs of each other. We consider the methods of extracting reduced widths, described in Sec. III, as heuristic attempts to obtain nuclear structure information from the data, whose usefulness and accuracy can only be determined by the consistency of the information extracted.

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[1] J. D. Fox, C. F. Moore, and D. Robson, Phys. Letters **12**, 198 (1964).
[2] D. L. Allan, G. A. Jones, G. C. Morrison, P. B. Taylor, and R. B. Weinberg, Phys. Letters **17**, 56 (1965); see also D. L. Allan, Phys. Letters **14**, 311 (1965).
[3] D. Robson, Phys. Rev. **137**, 535 (1965).
[4] See, e.g., P. Richard, C. F. Moore, J. A. Becker, and J. D. Fox, Phys. Rev. **145**, 971 (1966).

The present experiment was motivated by the work of Allan *et al.*[2] We hoped to obtain new information on the structure of the $_{48}Cd^{115}$ low-lying levels by studying the analog resonances in the inelastic scattering of protons from $_{48}Cd^{114}$. In order to extract quantitative results from the inelastic-scattering data, it is necessary to know the elastic partial widths of the resonances. We therefore studied the elastic scattering first and made a comparison of the results with the recent stripping data of Rosner, of Silva and Gordon, and of Moorhead, Cohen, and Moyer,[5] for the $Cd^{114}(d,p)Cd^{115}$ reaction (see Sec. III). The inelastic scattering follows (see Sec. IV); the results, although only qualitative, are quite interesting. Several states of Cd^{115} are shown to have large overlaps with excited states of Cd^{114}. More precisely, several resonances in In^{115} (and therefore their analog states in Cd^{115}) have large spectroscopic factors for decay into an excited state of Cd^{114} and a nucleon. Such information can be used as a test for wave functions for the states of Cd^{114} and Cd^{115} derived from nuclear models, in addition to information from stripping studies and other nuclear properties. Often it will be complementary to the stripping data, in the sense that resonances which have large inelastic spectroscopic factors often are analogs of levels that have small stripping spectroscopic factors. The experimental procedure is described in Sec. II.

II. EXPERIMENTAL PROCEDURE

The proton beam was produced by the University of Pittsburgh model EN Tandem Van de Graaff accelerator, made by High Voltage Engineering Corporation. After acceleration, the beam is deflected upward by 90° in an energy-analyzing magnet and travels from the accelerator room to the scattering room where it is deflected by another 90° to a horizontal direction, and is focused by a magnetic quadrupole lens onto the target. The nuclear-magnetic-resonance fluxmeter which measures the flux in the energy-analyzing magnet was calibrated in energy by observing the $C^{13}(p,n)N^{13}$ and $Ni^{58}(p,n)Cu^{58}$ reaction thresholds.[6] Most of the data for this experiment were taken during the first few months of accelerator operation, when there was some difficulty in obtaining intense beams. The widths of the object and image slits of the analyzing magnet were consequently chosen rather large and an uncertainty (of less than 20 keV) in the relative energies was introduced. The absolute energies should be correct within better than ± 30 keV.

The scattering chamber is 61 cm in diameter and can accommodate several solid-state detectors; up to three were used. The beam entered the chamber through a circular hole, traversed the target, and was collected in a Faraday cup; it was also monitored with two small NaI scintillators set at $\pm 25°$ to the beam in the scattering plane, counting elastically scattered particles. The detectors were Oak Ridge Technical Enterprises Corp. (ORTEC) silicon semiconductor counters, depletion depth ~ 1 mm, area ~ 100 mm^2. The pulses were amplified in ORTEC model 103–203 amplifiers and preamplifiers and fed into two Nuclear Data ND160 4096 channel pulse-height analyzers. The data could be printed out photographically or dumped directly on magnetic tape for later processing in the IBM 7090 computer of the University. A monoenergetic group of scattered protons produced a peak in the spectrum of full width at half-maximum of ~ 60 keV. Later it was found that this spread could be reduced to ~ 35 keV by placing small magnets in front of the counter to deflect low-energy electrons coming from the target; however, most of the Cd^{114} data were taken without the magnets.

The targets were made by evaporating cadmium oxide enriched to 99% in Cd^{114} onto a thin (~ 20 μg/cm^2) carbon backing; they were ~ 400 μg/cm^2 thick, which corresponds to an energy loss of ~ 10 keV for 8-MeV protons. Cadmium is a difficult metal to evaporate; it will often not deposit on the carbon. The carbon foils[7] come mounted on a glass backing with a detergent substrate to facilitate removal. It was found that the evaporation was more regular if the carbon backing was floated off the original glass backing and mounted, again with a detergent base, on a polished copper backing. Some difficulty then ensued, however, in floating the target off after evaporation, and only rather small targets, approximately 8×10 mm^2, were obtained. A target made by first evaporating a thin gold layer onto a carbon foil mounted on glass[7] and then evaporating cadmium metal onto the gold was also used in the early stages of the work.[8] However, it was not suitable for elastic-scattering studies because of the intense elastic scattering from gold.

The numbers of scattered particles registered by the detectors were normalized relative to the sum of the counts registered by the two monitors at $\pm 25°$. Cross sections were obtained by assuming that the elastic cross section at 25° is given by the Rutherford formula; the contribution to the monitor counts of the elastic scattering from carbon and other contaminants was negligible. The assumption of pure Rutherford scattering at 25° should be accurate within better than 5% up to 8.5 MeV; at the higher energies the actual cross section may differ by $\lesssim 10\%$ from the Rutherford value. The absolute cross section scale is estimated to be accurate within $\pm 10\%$ up to 8.5 MeV. The reproducibility of the elastic-scattering measurements was generally within $\sim \pm 1\%$.

The accuracy of the inelastic-scattering measure-

[5] B. Rosner, Phys. Rev. **136**, B664 (1964); J. B. Moorhead, B. L. Cohen, and R. A. Moyer, Bull. Am. Phys. Soc. **12**, 18 (1967); R. J. Silva and G. E. Gordon, Phys. Rev. **136**, B618 (1964).
[6] J. B. Marion, Rev. Mod. Phys. **38**, 660 (1966).
[7] Obtained from Hebrew University, Yissum Research Development Company, Jerusalem, Israel.
[8] This target was made by Fodor Accelerator Targets, Pittsburgh.

TABLE I. Summary of results from (p,p) and (d,p) reactions.

$Cd^{114}(p,p)Cd^{114}$						$Cd^{114}(d,p)Cd^{115}$ [a]			
E_R (MeV)	$E_R-7.200$ (MeV)	l	J^π	Γ (keV)	Γ_p (keV)	E_x (MeV)	l	J^π	σ_{max} (mb/sr)
7.200	0	0		62± 6	22 ±2.5	0	0	$\frac{1}{2}^+$	2.3
						0.18	5	$\frac{11}{2}^-$	0.60
7.445	0.245	2		30±10	7 ±2	0.23	2	$\frac{3}{2}^+$	2.4
(7.55)	(0.35)	(2)				0.38	2	$\frac{5}{2}^+$	1.2
7.68	0.48	(2)		90	$\lesssim 2$	0.48	(4,2)	$(\frac{7}{2}^+,\frac{9}{2}^+)$	1.4
7.850	0.650	0		40± 8	5 ±1	0.65	0	$\frac{1}{2}^+$	0.6
8.000	0.800	2		25±10	1 ±0.5	0.77	2	$\frac{3}{2}^+$	1.2
						0.89	(4)	$(\frac{7}{2}^+)$	<0.05
						0.96	0	$\frac{1}{2}^+$	0.13
						1.10	2	$\frac{5}{2}^+$	0.40
						1.19	0	$\frac{1}{2}^+$	0.15
						1.26	2	$\frac{3}{2}^+$	0.18
8.580	1.380	2		50±15	1.3±0.4	1.37	2	$\frac{5}{2}^+$	0.72
9.15	1.95	1		50±10	2.4±0.8	1.996[b]	1		
9.30	2.10	3		70±20	1.5±0.5	2.112[b]	3[c]		
9.71	2.51	3		80±30	3 ±1.5				

[a] Data of B. Rosner, Ref. 5, except where otherwise noted.
[b] Data of J. B. Moorhead, B. L. Cohen, and R. A. Moyer, Ref. 5. Only the intense groups seen in that experiment near 2 MeV are included.
[c] R. J. Silva and G. E. Gordon, Ref. 5, assign $l=3$ to this level. Preliminary data of Moorhead *et al.* are consistent with $l=3$ or 4.

ments was limited mostly by the large background due to slit edge scattering and by the scattering from target contaminants. In the course of the experiment, it was found that the background can be reduced by a large factor by: (a) keeping the incident beam from hitting the circular entrance hole to the scattering chamber and (b) using large area detectors, so as to minimize the relative contribution from particles scattered at the edges of the diaphragm defining the sensitive area of the counter. Most of the data in the present experiment, however, were taken without observing these precautions. Recently, an experiment has been performed at this laboratory on the scattering of protons from tin isotopes[9] in which much better inelastic-scattering data were obtained.

III. RESULTS AND DISCUSSION FOR ELASTIC SCATTERING

1. Results and Analysis

Elastic-scattering measurements were made at three angles, 90°, 125°, and 170°, in the laboratory. The smaller angles were chosen because they correspond to roots of the Legendre polynomials of odd order (90°) or order 2 (125°), so that resonances of odd l or $l=2$, respectively, should show up only very weakly at these angles. At 170°, on the other hand, all resonances show up clearly because Coulomb scattering is less and the Legendre polynomials have large magnitude. The data were taken in steps of ~20 keV from 6.9 to 10.1 MeV and are shown in Figs. 1 and 2. The resonances which were identified are listed in Table I. An earlier set of data, not shown, measured with a cadmium target on a gold and carbon backing, showed the same resonances. The resonance parameters of Table I were obtained by fitting a Breit-Wigner resonance formula to the data; the fits obtained are shown in Figs. 1 and 2. The formula used is that of Brentano *et al.*,[10] who derived it from Blatt and Biedenharn's work[11] by replacing f_{pot} by $\sqrt{(\sigma_0)}e^{i\phi}$; it is valid for spin-$\frac{1}{2}$ particles incident on a spin-zero target:

$$d\sigma/d\Omega=\sigma_{Res}+\sigma_{int}+\sigma_0, \tag{1}$$

where

$$\sigma_{Res}=\frac{1}{2k^2}\left(\frac{\Gamma_p}{\Gamma}\right)^2\cos^2\beta\sum_{K=0,2}^{2J-1}Z^2(lJlJ;\tfrac{1}{2}K)P_K(\cos\theta), \tag{2a}$$

$$\sigma_{int}=\frac{(\sigma_0)}{k}(2J+1)\frac{\Gamma_p}{\Gamma}\cos\beta\sin(\alpha_l+\beta)P_l(\cos\theta), \tag{2b}$$

E is the laboratory energy, MeV, E_R is the resonance energy in the laboratory, MeV, $k=(2ME/\hbar^2)^{1/2}$, and is the wave number, M is the reduced mass of $Cd^{114}+p$ system, Γ_p is the partial elastic width of the resonance, and Γ is the total width of resonance. β is the resonance phase shift: $\tan\beta=2(E-E_R)/\Gamma$,

$$Z(lJlJ;\tfrac{1}{2}K)=(2l+1)(2J+1)(l0l0|K0)W(lJlJ;\tfrac{1}{2}K),$$

where W is the Racah coefficient,[11] and $P_K(\cos\theta)$ is the Legendre polynomial. l is the orbital angular mo-

[9] E. Schneid, Ph.D. thesis, University of Pittsburgh, 1966 (unpublished); E. Schneid and E. W. Hamburger, Proceedings of the International Conference on Nuclear Physics, Gatlinburg, 1966, paper 2, 11 (to be published).

[10] P. v. Brentano, N. Marquardt, J. P. Wurm, and S. A. A. Zaidi, Phys. Letters **17**, 124 (1965).

[11] J. M. Blatt and L. C. Biedenharn, Rev. Mod. Phys. **24**, 258 (1952).

[12] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

mentum of resonating partial wave, J is the total angular momentum of resonance, $\alpha_l=2\xi_l-\Phi$, $\Phi=\eta \times\log(\text{cosec}^2\theta/2)+\pi$, $\xi_l=\omega_l-\phi_l$ (see Ref. 12).

The potential-scattering cross section σ_{pot} is found not to be equal to the Rutherford cross section. For example, near 7.2 MeV its ratio to the Rutherford cross section is 0.94 at 90° and 0.73 at 170°. At higher energies the ratio goes down rapidly, reaching 0.4 at 170° and 0.34 at 90° at 10 MeV. However, in the energy region near each individual resonance it was found sufficient to approximate the energy dependence of σ_{pot} as

$$\sigma_{\text{pot}}=(E_R/E)^2\sigma_0, \tag{2c}$$

where the constant σ_0 is the potential-scattering cross section at the resonance energy E_R. Only for the resonances at 9.15 and 9.30 MeV was it found necessary to make an empirical fit to the observed variation of σ_{pot}. For the other resonances, the fit of the calculated curves to the data would also have been improved if σ_{pot} had been allowed to vary in a more complicated manner. However, it was felt that the improved fit would not be meaningful since it would depend on the introduction of one or more additional adjustable parameters without physical meaning.

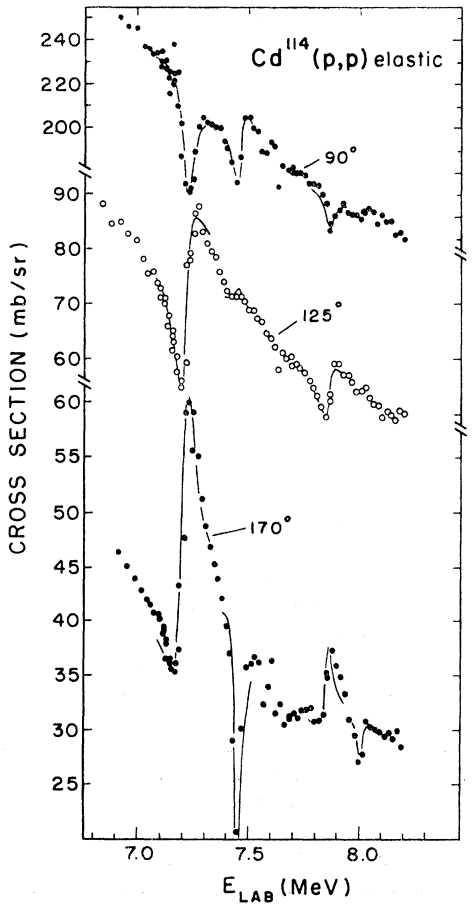

FIG. 1. Cross section for elastic scattering in the energy range E_p=6.8 to 8.2 MeV.

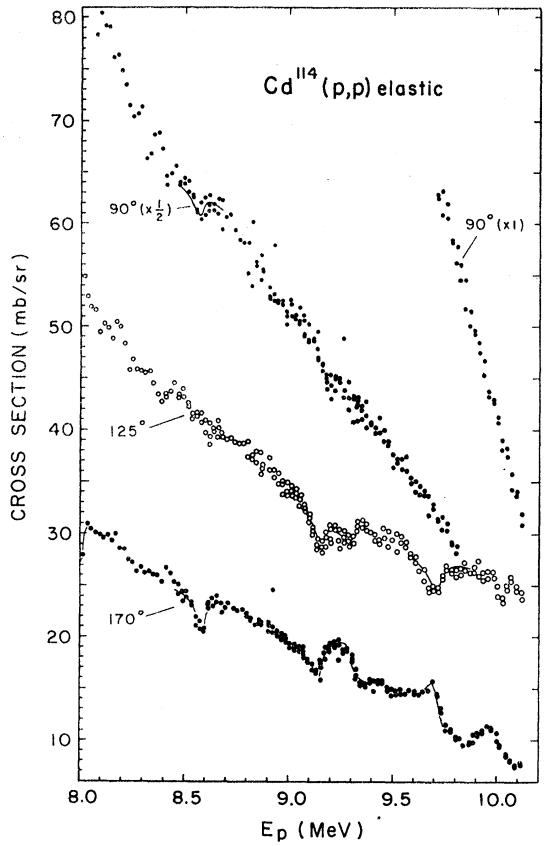

FIG. 2. Cross section for elastic scattering in the energy range 8 to 10.1 MeV.

At each angle, the parameters σ_0 and α were varied independently to achieve a good fit, while the parameters E_R, Γ_p, and Γ were chosen to give a good fit simultaneously at all three angles. The orbital angular momentum l can be determined by inspection of the resonance shapes at the three angles. In particular, at an angle where $P_l(\cos\theta)=0$ the interference cross section vanishes and the resonance appears small because for most of these resonances $\Gamma_p\ll\Gamma$ and therefore $\sigma_{\text{res}}\ll\sigma_{\text{int}}$. The total angular momentum J for the $d_{5/2}$ and $d_{3/2}$ resonances was taken from the isobaric analog levels in Cd^{115} studied by Rosner.[5]

The position of the resonance analogous to the Cd^{115} ground state allows a calculation of the Coulomb energy difference between Cd^{115} and In^{115}:

$$\Delta E_c=(114/115)E_R+B_n=13.28\text{ MeV},$$

where B_n is the binding energy of the last neutron in Cd^{115}. The value above agrees well with Coulomb energy systematics.[13]

[13] M. Harchol, S. Cochavi, A. Jaffe, and C. Drory, Nucl. Phys. 79, 165 (1966).

TABLE II. Reduced widths by various methods.

	$Cd^{114}(d,p)Cd^{115}$ [a]					$Cd^{114}(p,p)Cd^{114}$ Method I			Method II
E_x (MeV)	J^π	l	S_{nlj} [b]	$U_{nlj}^2(a_c)$ (10^{-43} cm^3)	$\gamma_n^2/(2T_0+1)$ (keV)	$\gamma_p^2/\lvert U_{cc}\rvert$ (keV)	γ_p^2 (keV)	Γ_p (keV)	S_{nlj}
0	$\frac{1}{2}^+$	0	0.35	36	13.7	22 ±2	19 ±2	22 ±2	0.23
0.23	$\frac{3}{2}^+$	2	0.53	26	7.3	13 ±4	12.5±4	7 ±2	0.19
0.65	$\frac{1}{2}^+$	0	0.085	36	3.6	3.7±0.7	3 ±0.6	5 ±1	0.030
0.77	$\frac{3}{2}^+$	2	0.13	28	3.2	1.3±0.7	1.2±0.6	1 ±0.5	0.016
1.37	$\frac{5}{2}^+$	2	0.05	29	1.0	1.3±0.4	1.1±0.3	1.3±0.4	0.012

[a] From data of B. Rosner, Ref. 5.
[b] Moorhead, *et al.*, Ref. 5.

2. Discussion of the Resonance Analysis

The resonance analysis described above is unsatisfactory from several points of view. Equations (1) and (2) are derived in the R-matrix theory by assuming that the matrix R_0L_0 is zero (see Lane and Thomas, Ref. 12). This implies that the off resonance or potential scattering is hard-sphere scattering, i.e., the phase ϕ and amplitude ρ are determined. The isobaric analog resonances lie at high-excitation energies and many other channels are open. Hard-sphere scattering is a very poor representation of the potential scattering in this case, and one actually treats σ_0 and ϕ as adjustable parameters, as explained above, which is, strictly speaking, a procedure inconsistent with the theory.

It has been suggested[14] that the error introduced by the use of Eqs. (1) and (2) can be approximately corrected for by replacing Γ_p in Eq. (2) by $e^{-2\beta}\Gamma_p$, where $e^{-2\beta}=|U_{cc}|$ is less than 1 because there is absorption of flux from the incident channel into the reaction channels. We shall see below that this "absorption correction" is not very important for our work.

On the other hand, σ_0 and α, if they are not described by hard-sphere phase shifts, should be obtainable from calculations of scattering from an optical-model potential well. It would seem preferable to derive σ_0 and α simultaneously for all angles from such a calculation, rather than choosing them arbitrarily for each angle. We made some calculations using the potential found by Perey[15] for proton scattering from silver at 9.4 MeV. The calculated cross sections generally agree with the σ_0 estimated from the data within $\sim\pm25\%$, while the calculated phase angles sometimes differ by $\sim30°$ from the α_l found by fitting the data. The experimental values of the α_l are uncertain within $\pm10°$ from uncertainties in the fitting, and are, as a matter of fact, very near the values predicted by Rutherford scattering from a point charge.

Fortunately, the uncertainties in σ_0 and α do not affect the extracted values of Γ_p and Γ too much. Γ is determined by the observed energy width of the resonance, while Γ_p/Γ depends almost only on the ratio $(\sigma_{\max}-\sigma_{\min})/\sigma_0$, where $\sigma_{\max}$ and $\sigma_{\min}$ are the maximum and minimum values of the cross section at the resonance.

Table I shows the widths Γ and Γ_p found for the Cd^{114} resonances, with estimates of upper limits of error, which are quite large.

In addition to the approximations made in Eqs. (1) and (2), described above, we have completely neglected the fact that the analog resonance is not a simple Breit-Wigner resonance at all, but rather a "giant resonance," where the mixing of the analog states with the $T_<$ states produces important effects. Robson[3] has given an expression for the scattering matrix where these effects are taken into account. However, it contains several additional parameters to be determined and we did not apply it.

3. Comparison with the Levels of Cd^{115}

The last four columns of Table I show the results[5] for the $Cd^{114}(d,p)Cd^{115}$ reaction. The three stripping experiments listed in Ref. 5 are in agreement for the purpose of the present comparison. The recent high-resolution experiment of Moorhead, Cohen, and Moyer[5] shows several weakly excited states in Cd^{115} which were not identified in the older work. However, the results of the different stripping experiments for the strongly excited states agree quite well; for example, the spectroscopic factors for the five levels listed in Table II agree within $\sim20\%$. Most of the stripping data quoted in Tables II and III are from the work of Rosner.[5]

There is very good correspondence between the analog resonances in In^{115} and the levels of Cd^{115}. The analogs of several levels observed in stripping were not identified as resonances; one of these has $l=5$ and would not be expected to appear because of the very small penetrability of h waves, while the others have very small stripping cross sections. The excitation energies in Cd^{115} agree with the distance in energy of the resonances to the ground-state analog resonance at 7.2 MeV within 20 keV, except for the small resonance at 8 MeV which is 30 keV from the expected position.

[14] S. A. A. Zaidi (private communication).
[15] F. G. Perey, Phys. Rev. **131**, 745 (1963).

TABLE III. Results from (p,p') reactions.

E_R (MeV)	J^π	Γ (keV)	Γ_p (keV)	$\frac{d\sigma}{d\Omega}$(max) (mb/sr)	$\sum\Gamma_{p'}$ (keV)	$\gamma^2(s_{1/2})$ (keV)[a]	$\gamma^2(d_{5/2})+\gamma^2(d_{3/2})$ (keV)[a]	$\gamma^2(g_{7/2})$ (keV)[a]
7.20	$\frac{1}{2}^+$	62	22	0.5	2.8	...	10	...
7.445	$\frac{3}{2}^+$	30	7	0.4	0.9	0.9	2.5	35
7.68	$(\frac{3}{2}^+,\frac{7}{2}^+)$	90	$\lesssim 2$	1.0	$\gtrsim(70,35)$	$\gtrsim 66$[b]	$\gtrsim 160$[b]	$\gtrsim 2000$[b]
7.85	$\frac{1}{2}^+$	40	5	0.7	8	...	16	...
8.00	$\frac{3}{2}^+$	25	1	0.2	2.2	1.9	4	44
8.58	$\frac{5}{2}^+$	50	1.3	0.2	5	2.6	6	55

[a] Upper limits. See text.
[b] Calculated for $J=3/2$. For $J=7/2$ the values would be half as large.

The orbital angular momenta l agree with the stripping l_n for the 5 resonances for which they were determined. The resonances at 7.55 and 7.68 MeV are too small to be identified with certainty; the data are consistent with the parameters given in the table. The 7.68-MeV resonance is very prominent in the inelastic scattering, as we shall see below, and the total width Γ quoted for it in Table I was extracted from the inelastic data.

Above 1.37-MeV excitation energy only two of the many levels identified in stripping are shown. They probably correspond to the p and f resonances at 9.15 and 9.30 MeV, respectively. No analog resonances corresponding to the other Cd^{115} energy levels in this region were identified. The occurrence of p and f states at about 2-MeV excitation energy is not surprising; the tin isotopes also exhibit them.[16]

We now make a comparison of the reduced widths of the resonances with the corresponding quantities in the analog levels of Cd^{115}. If the In^{115} resonances were perfect analogs of the Cd^{115} levels the following relation should hold between the reduced widths:

$$\gamma_p^2(\mathrm{In}^{115})=\frac{1}{2T_0+1}\gamma_n^2(\mathrm{Cd}^{115}). \tag{3}$$

We shall see below, however, that there are several reasons why this relation may not hold in actual nuclei. There are five levels for which the comparison can be made. We extracted spectroscopic factors from Rosner's[5] data by dividing the maximum measured cross section by the maximum cross section calculated from distorted-wave Born approximation (DWBA) with the code JULIE,[17] using the well parameters quoted in Ref. 5. The spectroscopic factors thus obtained are shown in column 4 of Table II; as a check it was verified that these S_{lj} yield the V_j^2 quoted in Ref. 5.

Two methods of comparison were applied. In the first, the stripping spectroscopic factors S_{lj} were multiplied by the square of the radial wave function of the captured neutron at the nuclear surface, $U_{nlj}^2(a_c)$, and by a normalizing factor, to yield a reduced width[12,18] γ_{lj}^2:

$$\gamma_{lj}^2(a_c)=\frac{\hbar^2 a_c}{2M}S_{lj}U_{nlj}^2(\alpha_c); \tag{4}$$

here a_c is the channel radius. The functions U_{nlj} are normalized so that

$$\int_0^\infty U_{nlj}^2(r)r^2dr=1$$

and were calculated by the code JULIE[17] as eigenfunctions of a real potential well of Saxon-Woods shape and depth adjusted to yield the experimentally observed binding energy of the last neutron in Cd^{115}. The function U_{nlj} is the "form factor" of DWBA calculations of stripping.

Table II shows the application of Eq. (3); the form factor is shown in the fifth column and the sixth column gives the resulting neutron reduced width, divided by $2T_0+1=19$ for comparison with the proton value. Following Robson's[19] suggestion, the radius a_c was taken smaller than the usual R-matrix value[20] as follows:

$$a_c=7.0\ \mathrm{F}\simeq 1.45A^{1/3}\ \mathrm{F}. \tag{5}$$

The next three columns of Table II give the proton resonance results: the partial width Γ_p is given in the ninth column, the reduced width

$$\gamma_p^2=\Gamma_p/(2P_l) \tag{6}$$

in the eighth column and the reduced width with the absorption correction mentioned above in the seventh column. The penetrability P_l was obtained from the tables of Thompson.[21] The scattering matrix element U_{cc} was calculated by the JULIE code for the same parameters mentioned in the discussion of σ_0 and ϕ above.

The table shows that the absorption correction is un-

[16] E. Schneid, A. Prakash, and B. L. Cohen, Phys. Rev. **156**, 1316 (1967); also O. Dietzsch and E. W. Hamburger, Bull. Am. Phys. Soc. **12**, 18 (1967).
[17] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL 3240 (unpublished).
[18] G. R. Satchler, in Lectures of Theoretical Physics Institute, Boulder, Colorado, 1965 (to be published).
[19] D. Robson, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 411.
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[21] B. Thompson, *Tables of Coulomb Penetrability Functions* (Florida State University Press, Tallahassee, Florida, 1965).

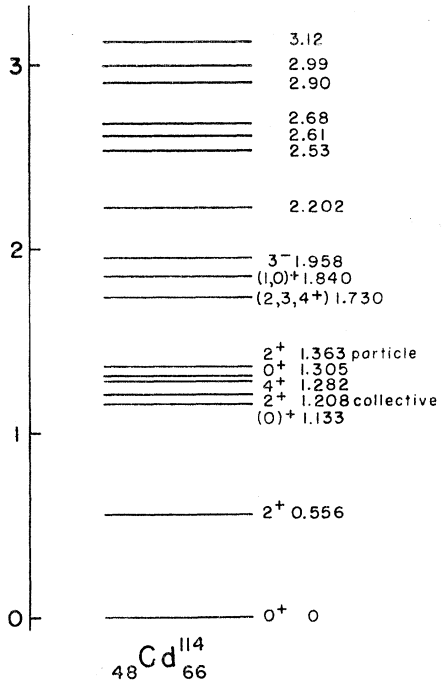

FIG. 3. Energy-level diagram for Cd^{114} following Refs. 23–25. See also Ref. 26.

important; this is because $|U_{cc}|\simeq 1$ for $l=0$ and 2. For $l=1$, on the other hand, the optical-model calculations show smaller values, $|U_{cc}|\simeq 0.4$. Moorhead, Cohen, and Moyer[5] find two $l=1$ groups in stripping, at excitation energies of 1.999 and 2.019 MeV, having spectroscopic factors 0.032 and 0.013, respectively. We observed only one broad $l=1$ resonance at 9.15 MeV, 1.95 MeV above the ground-state resonance. The spectroscopic factor extracted for this resonance, using method II, described below, is 0.011 ± 0.004 without any absorption correction. The absorption correction would more than double this value, yielding a spectroscopic factor that is in reasonable agreement with the stripping results.

We now compare column 6 in the table, $\gamma_n^2/(2T_0+1)$, with either column 7 or 8. The agreement is not good. The resonance reduced width is larger than the stripping one by a factor ~ 1.5 for the first two states and is smaller or equal to it for the three other states; not even the relative values of the reduced widths for levels with the same l value agree.

We also made calculations with the larger radius usual in resonance analysis[20];

$$a_c=1.45(A_1^{1/3}+A_2^{1/3})=8.5\text{ F}.$$

The result was to decrease the resonance reduced widths γ_p^2 by factors between 5 and 7 and to decrease γ_n^2 by factors between 2.5 and 3. The two sets of widths, however, still did not agree in absolute or in relative value.

The disagreement of the absolute values of the two sets of reduced widths is not surprising. The reduced width is proportional to the square of the wave function of the captured particle at the nuclear surface. Now it is in the surface region that the analogicity[19] of the neutron and proton wave functions is worst. In particular, one of the wave functions corresponds to a bound particle, the other to an unbound particle: the asymptotic forms are completely different. The relationship Eq. (3) which should hold for perfect analog states is therefore not expected to hold for large radius a_c. The radius which we chose, Eq. (5), is rather small but the different boundary conditions will still affect the wave function there. On the other hand, if we think in terms of a single-particle model, even inside the nucleus the neutron in Cd^{115} and the proton in In^{115} will have slightly different wavelengths because of the additional Coulomb energy of the proton.

The disagreement of the *relative* values of the reduced widths is more significant, especially for states of the same l. This discrepancy is probably not due to our method of comparison, since the penetrabilities and radial wave functions are certainly very similar for neighboring levels of the same angular momentum. On the other hand, the discrepancies are large; for example, for the two $d_{3/2}$ states the γ_n^2 are in a ratio of approximately 2.3:1, while the γ_p^2 are in a ratio of ~ 10:1. We conclude that here is evidence of a more basic type of nonanalogicity, the Cd^{115} and In^{115} analog states really contain different proportions of the states $|pC\rangle$ and $|nC\rangle$ in question.

It should be remarked that all except the first two levels of Cd^{115} are weakly excited, having spectroscopic factors $S_{nlj}\sim 0.1$. It is common in stripping studies to have the smaller spectroscopic factors disagreeing with theoretical expectations. The relative reduced widths for the two more strongly excited levels, on the other hand, agree within 20%, which seems to show that the main parts of the wave functions of the two states are analogs of each other.

One criticism which can be raised about the method of extracting a reduced width from the partial width, Eq. (6), is that the penetrability P_l is for a pure Coulomb field and does not take into account the tail of the nuclear potential which extends beyond a_c.

The second method[22] used to extract reduced widths from the resonance data for comparison with the stripping results is not open to this objection. A calculation is made of the scattering of protons from a real potential well of Saxon-Woods form and an electrostatic potential due to a uniformly charged sphere of the same radius, using the code JULIE. If now one wants, for example, to analyze the $d_{3/2}$ resonance at 7.445 MeV, one varies the depth of the well until one observes a resonance in the potential scattering d wave at this energy, i.e., until the nuclear phase shift K_L for $L=2$

[22] J. P. Schiffer, Nucl. Phys. **46**, 246 (1963); see also R. I. Andersen, J. P. Bondorf, and B. S. Madsen, Phys. Letters **22**, 651 (1966).

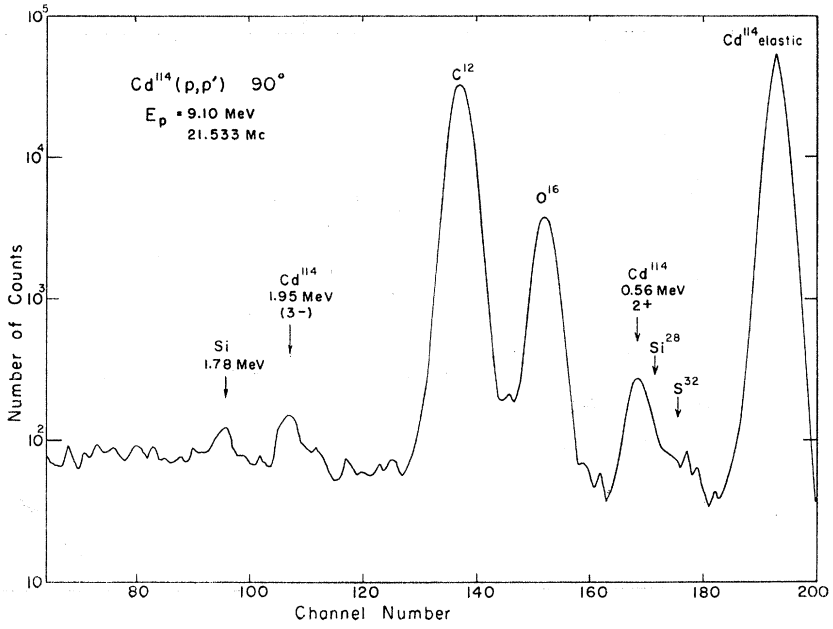

FIG. 4. Energy spectrum of protons scattered at 90° for a bombarding energy of 9.10 MeV.

is 90° at this energy. Once the well depth is fixed, the phase shift is calculated at several neighboring bombarding energies, to obtain the phase shift K_L as a function of energy near the resonance. Now the phase shift should vary according to

$$\cot K_L = (2/\Gamma_{sp})(E - E_R), \quad (7)$$

and therefore the width Γ_{sp} can be found if K_L is known as a function of E. These single-particle widths turn out to be quite large for the s and d resonances studied here so that the width actually varies appreciably across the resonance: $\cot K_L$ is not a linear function of E. We extracted the value of Γ at resonance from the slope of the curve $\cot K_L$ versus E at the resonance energy E_R. From this calculation one can actually extract the spectroscopic factor rather than the reduced width, as shown by Schiffer[22]:

$$S_{nlj} = \Gamma_{exp}/\Gamma_{sp}, \quad (8)$$

where Γ_{exp} is the partial width of the resonance, extracted from the experiment. The spectroscopic factors thus obtained are listed in the last column of Table II. Again the absolute values of the spectroscopic factors do not agree; the relative values of the two strong states do approximately agree and the relative values of the weaker states do not agree. In general, the relative values of S_{nlj} and of γ_p^2 in Table II are quite similar, i.e., they are the same within 25%: methods I and II agree as far as relative values are concerned.

We conclude that the reduced widths of the analog resonances seem to be consistent with good analogicity as far as the main components of the wave functions are concerned but show that the smaller components are different in the resonant state and in the parent state.

IV. RESULTS AND DISCUSSION FOR INELASTIC SCATTERING

1. General

The energy levels of Cd^{114} are shown in Fig. 3.[23–26] According to the vibrational model, the 2^+ state at 0.556 MeV is a single-phonon quadrupole vibrational state, while the levels at 1.133, 1.208, and 1.282 MeV comprise the two-phonon triplet. The octupole vibrational 3^- state at 1.958 MeV is strongly excited in inelastic scattering at higher energies.

A typical pulse-height spectrum from the counters is shown in Fig. 4 and shows clearly the difficulties due to large background and target contaminants, discussed in Sec. II. We were able to identify the proton groups due to the 2^+ state at 0.556 MeV (the most intense group), the 3^- state at 1.95 MeV, and the states in the two-phonon region, 1.13 to 1.36 MeV. In Fig. 4 the two-phonon region is obscured by the elastic protons from carbon, but the groups were observed at other angles. No other groups of comparable cross section were observed for excitation energies below 2 MeV; their cross sections are probably all well below 100 μb/sr at all energies where measurements were made.

[23] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Research Council—National Academy of Sciences, Washington, D. C., 1960).
[24] R. K. Smither, Phys. Rev. **124**, 183 (1961).
[25] M. Sakai, H. Ikegami, Y. Nakagima, K. Yagi, N. Ejiri, and G. R. Satchler, Phys. Letters **8**, 197 (1964).
[26] A. Bäcklin, N. E. Holmberg, and G. Bäckström, Nucl. Phys. **80**, 154 (1966).

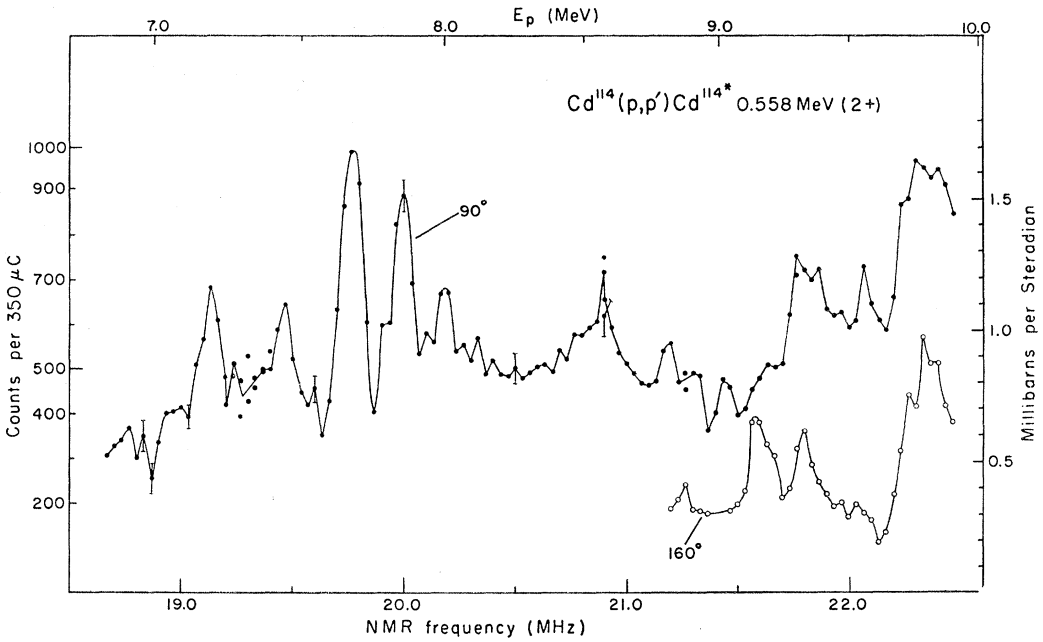

FIG. 5. Yield curve for inelastically scattered protons from Cd^{114} corresponding to the first excited state at 0.56 MeV at 90° (black dots) and 160° (empty circles). Absolute cross sections are given on the right-hand scale, energies on the upper scale.

For higher excited states we were not able to obtain consistent results. There may be some states near 2.5-MeV excitation which are strongly excited at the resonances. Their cross section is usually below 100 μb/sr and may rise to $\sim$200 μb at a few resonances. We plan to study the higher excited states in the near future using a magnetic spectrograph to obtain better resolution and lower background. It is of interest to note that in Sn^{116}, Sn^{122}, and Sn^{124} recently studied at this laboratory[9] states near 2.5-MeV excitation energy are strongly excited at the low-lying resonances, reaching cross sections as high as 500 μb/sr.

2. The First Excited State

The yield curve for the 2^+ state at 0.556 MeV is shown in Fig. 5 for 90° and 160° scattering angles. At 90° there is a slowly varying cross section of about 0.8 mb/sr on which several resonances ride. The resonances occur at the same energies as in the elastic channel within $\sim$10 keV (except the largest one at 7.68 MeV, which does not appear in the elastic) and are isobaric analogs of known levels of Cd^{115}.

There is probably an appreciable direct-reaction contribution to the cross sections even at resonance. No satisfactory theory for extracting partial and reduced widths exists in this case. However, we can make an order-of-magnitude estimate using the Breit-Wigner formula. There are no obvious effects of interference between compound nucleus formation and direct reactions visible in Fig. 5; we assume that the two mechanisms contribute incoherently to the cross section. Also for our order-of-magnitude estimate we assume the resonant cross section to be isotropic. The Breit-Wigner formula in j-j coupling is, for the total cross section to a certain final state in an even-even target nucleus:

$$\sigma_{pp'}=\frac{\pi}{2k^2}(2J+1)\frac{\Gamma_p}{(E-E_R)^2+\frac{1}{4}\Gamma^2}\sum_{j'l'}\Gamma_{p'l'j'}, \quad (9)$$

where $\Gamma_{p'l'j'}$ is the partial inelastic width to that final state with total angular momentum j' and orbital angular momentum l', and the other symbols have been defined after Eqs. (2). In general, several values of j' and l' contribute to the reaction; the coupling conditions are that l' and the proton spin couple to j', that j' and the final state spin of 2 couple to the compound angular momentum J and that l' be even (for an even parity resonance). At the resonance $E=E_R$ Eq. (9) yields

$$\sum_{j'l'}\Gamma_{p'l'j'}=\frac{2K^2\Gamma^2}{(2J+1)\Gamma_p}\left(\frac{d\sigma}{d\Omega}\right)_{\text{Res Max}}, \quad (10)$$

where $(d\sigma/d\Omega)_{\text{Res Max}}$ is the maximum differential resonance cross section, assumed isotropic and obtained from Fig. 5 by subtracting an estimated nonresonant contribution.

Table III shows the results obtained from Eq. (10). The first four columns give the resonance energy E_R, the spin and parity J^π of the analog state in the parent nucleus, the total width Γ, and the partial elastic width Γ_p from Table II. The fifth column gives $(d\sigma/d\Omega)_{\text{Res Max}}$ and the sixth gives $\sum\Gamma_{p'l'j'}$ calculated from (10). The last three columns give upper limits on the inelastic

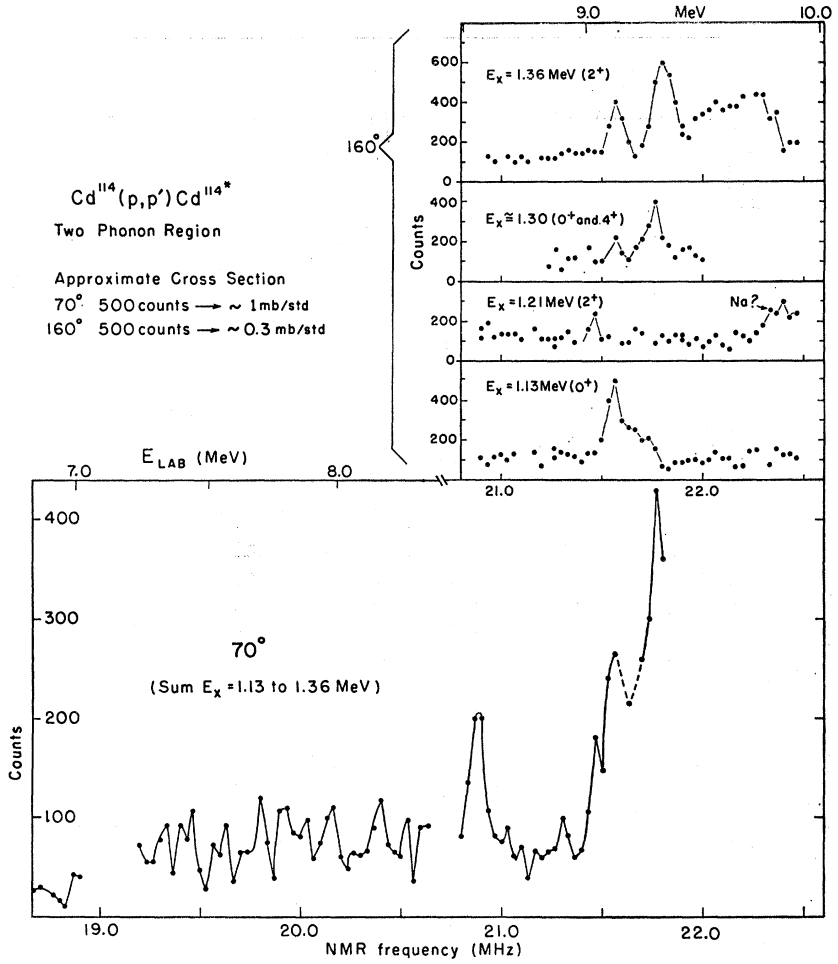

FIG. 6. Yield curve for inelastically scattered protons corresponding to the Cd^{114} levels between 1.13 and 1.37-MeV excitation energy, at 70° and 160°. Approximate conversion factors to calculate absolute cross sections are given on the upper left.

reduced widths. For example, for $J^\pi=\frac{3}{2}^+$ we have

$$\sum_{j'l'}\Gamma_{p'l'j'}=\Gamma_{p's1/2}+\Gamma_{p'd3/2}+\Gamma_{p'd5/2}+\Gamma_{p'g7/2}$$
$$=2P_0\gamma_{s1/2}{}^2+2P_2(\gamma_{d3/2}{}^2+\gamma_{d5/2}{}^2)+2P_4\gamma_{g7/2}{}^2.$$

The penetrabilities $P_{l'}$ are different for each term. The upper limits quoted in the table are obtained by assuming that only one value of l' contributes to $\sum_{j'l'}\Gamma_{p'l'j'}$ while the others have zero reduced widths. For $\frac{1}{2}^+$ resonances only $l'=2$ is allowed and the table gives directly the sum $\gamma_{d3/2}{}^2+\gamma_{d5/2}{}^2$.

Since the penetrability P_4 is quite small we will neglect the $g_{7/2}$ contribution in the following.

The two $\frac{1}{2}^+$ resonances at 7.20 and 7.85 MeV have appreciable inelastic widths to the 2^+ state; $\gamma_{d3/2}{}^2+\gamma_{d5/2}{}^2$ is half the elastic reduced width for the first resonance and about 5 times the elastic reduced width for the second. We conclude that these two resonances have sizable overlaps with an excited 2^+ core; the second resonance seems to have a larger overlap with the excited core than with the ground-state core.

The $\frac{3}{2}^+$ and $\frac{5}{2}^+$ resonances at 7.445, 8.00, and 8.58 MeV have rather small reduced widths. The resonance at 7.68 MeV, on the other hand, has a very large reduced width. The signs $\gtrsim$ for this level in Table III stem from the fact that this level was not observed in the elastic channel and an estimated upper limit of $\Gamma_p\lesssim 2$ keV (see Table I) was used to obtain $\sum\Gamma_{p'l'j'}$. Now the single-particle reduced width is ~25 keV according to method I above and ~60 keV according to method II.[27] The values given for this resonance in Table III are therefore larger than the single-particle value. We conclude that this resonance, and presumably its isobaric analog level at 0.48-MeV excitation in Cd^{115}, contains a very large fraction of excited 2^+ core; since it does not decay appreciably in other channels, it seems to be approximately a single-particle state with this core.

The conclusions in this section should be regarded as tentative, since there are large uncertainties in the values of the reduced widths. However, we believe that they are qualitatively correct.

[27] A. M. Lane (see Ref. 19) based on experiments on light nuclei, estimates the single-particle reduced width as $\sim 0.6\hbar^2/Ma_c{}^2\sim 500$ keV, which, divided by $2T_0+1=19$, yields ~25 keV.

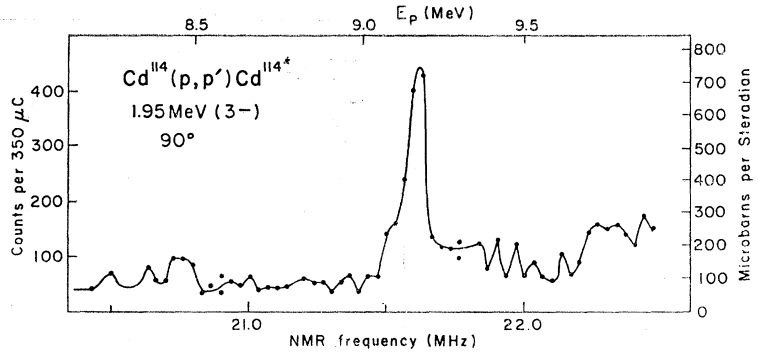

FIG. 7. Yield curve for inelastically scattered protons corresponding to the level at 1.95 MeV (3⁻) in Cd^{114}, at 90°. The cross-section scale is on the right, the energy scale is above.

3. The Two-Phonon Region and the Octupole State

Figure 6 shows yield curves for the levels with excitation energies from 1.13 to 1.36 MeV in Cd^{114} (the two-phonon region), at two scattering angles, 70° and 160°. At 70° it was not possible to separate the different levels, and the data shown refer to a sum over all; at 160° an approximate separation was possible. The cross sections are small and the data rather poor. Resonances seem to occur at 9.15, 9.3, and 8.58 MeV; the last one was observed only at 70°.

The yield curve for the 3⁻ level at 1.95 MeV is shown in Fig. 7. Only one resonance, at ~9.18 MeV, is identified, near the $p3/2$ resonance at 9.15 MeV in the elastic scattering. We assume that it is the same resonance. Then Eq. (10) gives the following value for the partial widths:

$$\sum \Gamma_{p'l'j'}=\Gamma_{p'd3/2}+\Gamma_{p'd5/2}+\Gamma_{p'g7/2}+\Gamma_{p'g9/2}=13 \text{ keV}.$$

If g waves are again neglected, we obtain

$$\gamma_{d3/2}^2+\gamma_{d5/2}^2=28 \text{ keV}$$

again of the same order of magnitude as the single-particle reduced width. This resonance also appears in the yield curves for the two-phonon states and for the first excited state; the 3⁻ contribution, however, seems to be the largest one. The isobaric analog of this resonance in Cd^{115} should be at approximately 1.95-MeV excitation energy and its wave function should have a large overlap with a core of Cd^{114} in the 3⁻ excited state.

It is interesting that we do not observe a resonance in the 3⁻ channel at the low-lying resonances, i.e., those below 9 MeV. Allan and co-workers[28,2] studying isobaric analog resonances in $Sn^{118}(p,p')$ and $Sn^{120}(p,p')$, found peaks in the 3⁻ yield already at the ground-state analog resonances. On the other hand, Schneid *et al.*[9] do not observe any low-lying resonances in the 3⁻ channel in proton scattering from Sn^{116}, Sn^{122}, and Sn^{124}. Since the structure of the low-lying levels, e.g., the $s_{1/2}$ and $d_{3/2}$ resonances, is probably quite similar in all these cases, the different behavior of the 3⁻ channel is surprising.

Finally, we remark that our data bear out the observation of Allan *et al.*[2]: that as the bombarding energy is increased, the excitation energies of the levels for which strong resonances occur in inelastic scattering also increase. For example, at the ground-state analog resonance at 7.2 MeV no strong inelastic resonance occurs. The first strong inelastic resonance occurs in the 2⁺ channel, at 7.68 MeV. The two-phonon levels first resonate at 8.58 MeV, 1.38 MeV above the ground state, and the 3⁻ only at 9.15 MeV. The table below shows the near equality of the excitation energy of the final state in the residual nucleus and the displacement of the resonance energy from the ground-state analog, for the strong resonances:

E_R−7.20 MeV	0.48 MeV	1.38 MeV	1.95 MeV
$E_x(Cd^{114})$	0.556 MeV	1.113–1.36 MeV	1.95 MeV

The approximate equality of these numbers lends support to a weak coupling picture, i.e., to the idea of single-particle states with an excited Cd^{114} core.

ACKNOWLEDGMENTS

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[28] D. L. Allan, G. A. Jones, G. C. Morrison, R. B. Taylor, and R. B. Weinberg, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 689.