Spectrum of High-Energy Gamma Rays Following Thermal-Neutron Capture by Er^{166}†

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The thermal-capture γ-ray spectrum of the $Er^{166}(n,\gamma)Er^{167}$ reaction in a sample enriched in Er^{166}(99.97%) has been studied with a Ge(Li) pulse-height spectrometer. Some of the transitions are interpreted as populating states having excitation energies in good agreement with those of known states, and several additional levels with excitation energies less than 2 MeV are tentatively assigned on the basis of the data. The distribution of the radiative strength over final states previously assigned to a specific nuclear motion is compared with that expected for an initial state consisting of an *s*-wave neutron weakly coupled to the deformed target nucleus.

1. INTRODUCTION

THE spectrum of γ rays resulting from the capture of *s*-wave neutrons is particularly informative when even-even targets are used. An initial state of spin and parity $J^\pi=\frac{1}{2}^+$ is formed; and the emission of primary electromagnetic radiation, which has been observed to be predominantly dipole in nature,[1] leads to the excitation of final states of spin $\frac{1}{2}$ or $\frac{3}{2}$. The reaction is, therefore, extremely useful in locating the positions of the states of the odd-neutron product nucleus having the two lowest possible spin values.

In the mass region for which nuclei are found to have large static deformations, the dynamical origin of the observed levels having excitation energies less than 1 MeV is rather well understood within the framework of individual-particle motion strongly coupled to collective motion.[2] The particular states populated in this reaction can be described as the first two members of a $K=\frac{1}{2}$ rotational band, or the first member of a $k=\frac{3}{2}$ rotational band. The intrinsic states upon which the rotational bands are built have been described in detail as single-particle eigenstates of a deformed potential.[3] In addition, more complex configurations corresponding to single-particle states coupled to collective vibrational modes have been identified.[4]

The low-energy region of the capture γ-ray spectrum of $Er^{166}(n,\gamma)Er^{167}$ has been studied by Koch,[5] who used a crystal diffraction spectrometer. Harlan,[6] who studied the $Er^{166}(d,p)Er^{167}$ reaction, has observed Er^{167} excited states extending to approximately 2.7 MeV. Ivanov *et al.*[7] have studied the spectrum of primary transitions following thermal neutron capture in a natural erbium sample. These authors have assigned two transitions of energy 6224±5 keV and 6168±6 keV to Er^{167}. Finally, information on the excited states has been obtained[8] from the decay of 9.5-day Tm^{167} and 3-*h* Ho^{167}.

2. EXPERIMENTAL PROCEDURE AND RESULTS

A sample consisting of 400 mg of Er_2O_3 enriched in Er^{166} was enclosed in a high-purity graphite holder and placed in the high-flux position of the through-hole facility at the Cp-5 reactor.[9] The isotopic composition of the sample is shown in Table I, together with the expected integral contributions based on the known[10] thermal cross sections. In addition, an upper limit for the intensity of contributions from likely rare-earth contaminants is indicated in the same units as for the erbium isotopes.

The resultant γ radiation was collimated to a beam approximately 0.13 in. in diameter, and detected with a cooled Ge(Li) counter having an active volume of 4 cc. The pulse spectrum was amplified in a low-noise, single-differentiating, linear system and the distribution was accumulated in a 4096-channel analyzer with a channel width corresponding to 1.74 keV. The gain of the system was digitally stabilized by sensing the centroid of the 6.23-MeV peak, the most intense peak in the spectrum. In order to obtain adequate resolution, a differentiating time constant of 1.6 μsec was used and the total counting rate was limited to 2500 counts/sec with polyethylene absorbers. The resolution width for a counting period of 72 h was 6.7 keV full width at half maximum (FWHM) at 6.2 MeV.

In a separate experiment using the same sample, the spectrum of events corresponding to simultaneous escape of two annihilation quanta was recorded with a

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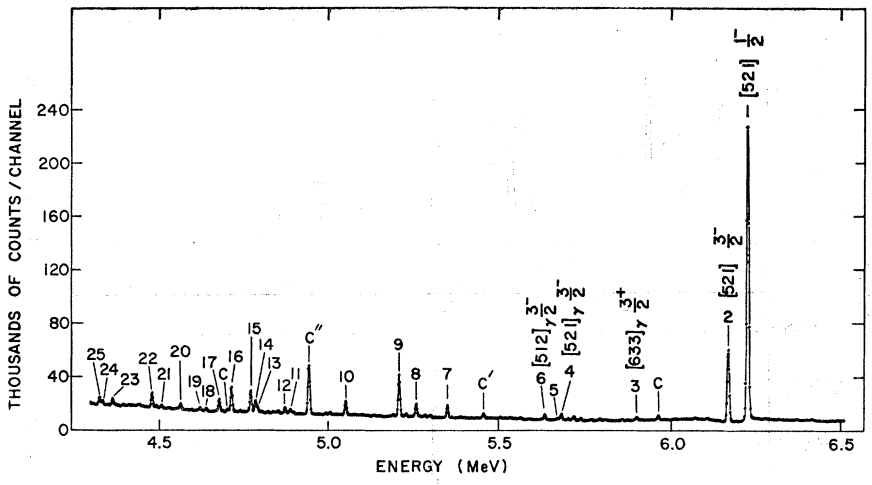

FIG. 1. The singles spectrum for the $Er^{166}(n,\gamma)Er^{167}$ reaction observed with a Ge(Li) detector. Each peak indicated by a number is a double-escape peak corresponding to a primary transition in Er^{167}. The peaks labeled *C*, *C'*, and *C''* are the full-energy, first-escape, and double-escape peaks of the 4946-keV transition in carbon.

standard three-detector pair spectrometer. The annihilation quanta were detected in NaI scintillation detectors 4 in. in diameter and 4-in. high. In order to obtain good statistics, the singles counting rate was increased to 6000 counts/sec. As a result the resolution was significantly impaired. This spectrum was used to confirm the identification of peaks observed in the singles spectrum as double-escape, single-escape, or full-energy peaks.

The spectrum obtained in the region corresponding to 4.3 to 6.5 MeV is shown in Fig. 1. The energy scale has been adjusted so that the position of a double-escape peak corresponds to the full energy of the associated γ ray. The positions of all double-escape peaks identified as belonging to Er^{167} are indicated as well as all those corresponding to capture in the C^{12} of the graphite sample holder. The double-escape peak associated with a given transition is identified with the number of that transition, while the double-escape, single-escape, and full-energy peaks from the carbon ground-state transition are designated *C''*, *C'*, and *C*, respectively.

As can be seen from the figure, the spectrum is dominated by a transition at approximately 6.23 MeV. Below the energy region shown, the spectrum is characterized by a smooth continuum rising slowly to a plateau at 600 keV, where the height is 25% of the height of the 6.23-MeV double-escape peak. The most intense peak in this region, approximately 5% that of the 6.23-MeV peak, is the double-escape peak of the 3.68-MeV transition in C^{13}.

The nonlinearity of the system was determined with a precision pulser as described elsewhere,[11] and the gain was deduced from the spacing between double-escape, single-escape, and full-energy peaks—which are assumed to be separated by 511.006 keV.[12] The position and area of each peak was determined from a least-squares fit to the data. It was assumed that over a limited region, approximately 20 keV, the data could be represented by a Gaussian peak superimposed on a linearly varying background. The absolute energy scale was obtained from the energy[13] of the C^{13} ground-state transition at 4945.5 keV. The energies (quoted to the nearest keV) and intensities of the transitions assigned to Er^{167} are shown in Table II. The relative intensity was determined from the area of the double-escape peak, corrected for the energy dependence of the Ge(Li) detector efficiency. The absolute intensity was estimated from the relative intensity of the 531-keV line, reported by Koch[5] to have an intensity of $6.8\pm1.0\%$. The uncertainty in the absolute energies is estimated to be 3 keV and that in the relative intensities is thought to be 10%. Because of the large correction for the difference between the efficiency at the full-energy peak at 531 keV and that at the double-escape peak at 6.23 MeV, the absolute intensity may be in error by as much as 50%.

The observed energies are in good agreement with the only previously published study of the erbium spectrum in this energy region.[7] In Fig. 2, the spectrum obtained with the enriched sample and that for a natural sample are compared in the energy region near 6 MeV.

TABLE I. Isotopic composition of the Er^{166} sample.

Isotope	Abundance, *F* (%)	σ (b)	F_σ (10^{-2}b)
162	0.005	2	0.01
164	0.005	1.7	0.009
166	99.97	45	4500
167	0.03	620	19
168	0.01	2	0.02
170	0.005	9	0.05
Sm	<0.05	5500	<300
Gd	<0.02	46000	<1000
Dy	<0.2	1100	<200

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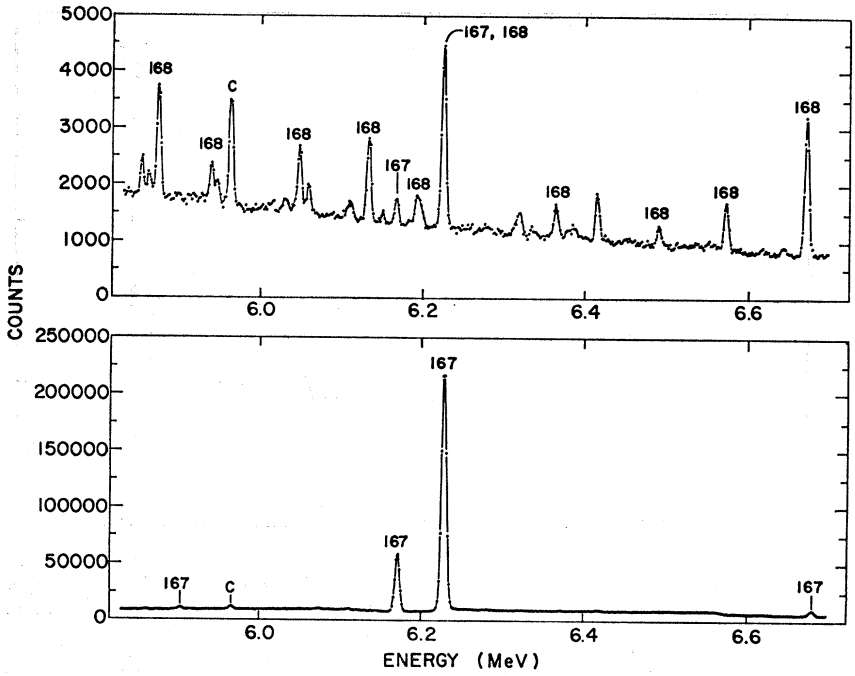

FIG. 2. Comparison of the spectra obtained for natural (upper curve) and enriched (lower curve) samples in the 6.0–6.6-MeV region. All transitions are labeled by the mass number of the compound nucleus, regardless of their escape nature. Note the lack of transitions from the $Er^{167}(n,\gamma)Er^{168}$ reaction in the spectrum obtained with the enriched sample.

It is obvious that the contribution from $Er^{167}(n,\gamma)Er^{168}$ transitions, identified by comparison with the work of Ivanov *et al.*,[7] is negligible in the enriched sample. An estimate of the intensity of the strong Er^{168} transition at 6140 keV is given in Table III. In their work on natural erbium, Ivanov *et al.*[7] point out that the 6228-keV transition is also consistent with an assignment to Er^{168}. In the present work, this is quantitatively confirmed on the basis of the relative intensities of the 6171-keV and 6228-keV γ rays as observed in both samples. In the natural sample, the intensity of the 6171-keV γ ray was observed to be 13.5% that of the 6228-keV transition, in good agreement with the value of 14% determined by the above authors. In the enriched sample, this value increases to 23%. If it is assumed that the transition at 6171 keV is entirely from the decay of Er^{167}, then in the natural sample 59% of the observed intensity at 6228 keV is from the decay of Er^{167} and the remainder from the decay of Er^{168}.

TABLE II. γ rays from the $Er^{166}(n,\gamma)Er^{167}$ reaction.

Peak No.	γ-ray energy Present[a] (keV)	γ-ray energy Previous[b] (keV)	Intensity (Photons per 1000 captures)
1	6228	6244±5	100
2	6171	6186±6	23
3	5904		0.87
4	5683		1.7
5	5670		0.58
6	5634		0.82
7	5351		3.3
8	5259		3.5
9	5210		12.1
10	5053		3.7
11	4891		0.95
12	4872		1.6
13	4795		1.3
14	4789		3.3
15	4775		6.2
16	4719		7.4
17	4682		5.1
18	4644		0.70
19	4625		0.83
20	4569		1.5
21	4513		0.74
22	4486		3.9
23	4369		1.9
24	4339		1.2
25	4330		2.1

[a] The assignment to the decay of Er^{167} of all transitions with intensity less than 1 photon/1000 captures is tentative.
[b] Reference 7.

The data were examined for the possibility of contributions from likely contaminants—in particular, Sm, Gd, and Dy. The region of the spectrum corresponding to the strongest transition[14] in Sm at 7210±4 keV showed no indication of this transition. An estimate of the net intensity for this transition is given in Table III, as well as that for the similarly strong transition[15] in Dy. An upper limit for this contribution is given by the statistical error. Also included in the table is an estimate for the total contribution from each of the elements considered. It was obtained from the relative intensity of the γ ray shown, with a correction for the known branching ratios. It is obvious that the contribution from Sm and Dy is negligible. Groshev *et al.*[16] report transitions at 5901±4, 5672±4, and 4872±4 keV in

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[15] R. K. Sheline, W. N. Shelton, H. T. Motz, and R. E. Carter, Phys. Rev. **136**, B351 (1964); O. W. B. Schult, B. P. Maier, and U. Gruber, Z. Physik **182**, 171 (1964).
[16] L. V. Groshev, A. M. Demidov, A. V. Ivanov, V. N. Lutsenko, and V. I. Pelekhov, Izv. Akad. Nauk SSR, Ser. Fiz. **26**, 1119 (1962) [English transl.: Columbia Tech. transl. **26**, 1127 (1962)].

Gd. Transitions with energies similar to these have been assigned to Er^{167} in this work. The strongest transition reported for Gd has an energy of 6746±4 keV. A transition at this energy would be masked by the single-escape peak of the strong 6228-keV transition in Er^{167}. An estimate of the intensity of this transition was obtained from the pair-spectrometer distribution, which does not show the single-escape peak, and is given in the table. On the basis of this result and the relative intensity values given by Groshev *et al.*,[16] this contribution could at most account for only 16% of the 5904-keV transition, 10% of the 5670-keV transition, and 4% of the 4872-keV transition. In addition, there is no indication of several other Gd transitions of comparable or greater intensities in this energy region. Since the available data are inadequate to exhaust all possibilities of contamination, it is felt that the assignment of transitions with intensities less than 0.1% must be considered tentative. In addition, four transitions in the energy region below 5 MeV have been observed, with intensities less than 0.05% and have not been assigned.

3. DISCUSSION

The mass spectrometric data[17] yield a neutron separation energy for the $Er^{166}(n,\gamma)Er^{167}$ reaction of 6444±5 keV, which is in good agreement with the (d,p) value[6] of 6434±10 keV. On this basis, the final state corresponding to the emission of the 6228-keV γ ray must lie between 206 and 216 keV above the ground state. It is most natural to identify this level as the $\frac{1}{2}^-$ level[5] at 207.80 keV characterized by the Nilsson assignment of [521], $K^\pi=\frac{1}{2}^-$. For this state, primary electric-dipole radiation from the initial capture state with $J^\pi=\frac{1}{2}^+$ is to be expected. The energy separation between the two highest-energy transitions observed is 57.09 ±0.10 keV, in excellent agreement with the value of 57.07 keV obtained by Koch[5] for the separation between the $\frac{3}{2}^-$ and $\frac{1}{2}^-$ members of the [521], $K^\pi=\frac{1}{2}^-$ rotational band. We therefore assign a neutron separation energy of 6436±3 keV to the reaction.

Table IV compares the levels observed in this work with the previous measurements.[5,6] The errors quoted are based on the statistical uncertainty in peak positions and the uncertainty in gain. The discrepancy between the crystal measurement of the 530-keV level and the value obtained in this work lies outside the expected error. As discussed previously, the transition at 5904 keV may contain a 16% contribution from Gd. It is not clear whether this might explain the observed discrepancy, or whether other systematic effects occur so that the estimate of the uncertainty in the measurement is low. The agreement between the energies E_i of the levels observed in this work and the energies $\mathcal{E}_i$ of the corresponding (d,p) levels may be characterized by the standard deviation

$$\sigma=\frac{[\sum_{i=1}^{N}(E_i-\mathcal{E}_i)^2]^{1/2}}{N-1}=2.4 \text{ keV}.$$

The largest single difference is 7 keV, which occurs for the level at 2113 keV. This agreement is sufficient to assume that the correspondence between levels is relatively unambiguous, although there could be unresolved doublets in either spectrum.

The identification of transitions to the levels at 532, 752, and 801 keV is in agreement with the previous $J=\frac{3}{2}$ assignment for these states.[5] On the basis of a comparison with the level structure of the isotone Dy^{165},

[17] H. E. Duckworth (private communication).

TABLE III. Estimated contributions from probable impurities.

Element	E_γ (keV)	I_γ (Photons per 1000 captures)	Total contribution (%)
Sm	7210	<0.01	<0.2
Gd	6746	<0.25	<1.3
Dy	5607	<0.05	<0.1
Fe	7646	0.02 ±0.002	0.006
Cl	6112	0.03 ±0.001	0.02
Ti	6759	0.02 ±0.001	0.005
N	6321	0.02 ±0.01	0.01
Er^{168}	6140	0.008±0.04	

TABLE IV. Excitation energies of levels in Er^{167}. The neutron separation energy used is $S_n=6436\pm3$ keV.

Transition No.	Excitation energy (keV) S_n-E_γ	Crystal spectrom[a]	(d,p)[b]
1	207.80	207.80[c]	208
2	264.89±0.1	264.87	262
(3)[d]	532.05±0.3	531.54	538
4	752.46±0.5	752.75	750
(5)[d]	762.88±0.5		
(6)[d]	801.39±0.6	801.62	803
7	1085.6 ±0.8		
8	1177.2 ±0.8		1173
9	1226.4 ±0.9		1227
10	1383.4 ±1.1		1385
(11)[d]	1545.4 ±1.3		1546
12	1564.2 ±1.3		
13	1641.1 ±2		
14	1647.5 ±2		
15	1661.2 ±1.4		
16	1717.6 ±1.5		1720
17	1753.9 ±1.6		1749
(18)[d]	1692.1 ±0.7		
(19)[d]	1810.8 ±1.8		1813
20	1867.6 ±1.8		1867
(21)[d]	1923.0 ±2		
22	1950.6 ±2		
23	2067.6 ±2		2067
24	2097.5 ±2		
25	2105.8 ±2		2113

[a] Reference 5.
[b] Reference 6.
[c] This excitation energy has been used as the reference level in constructing the levels given in the second column.
[d] Transitions corresponding to these excitation energies have intensities <1 photon/1000 captures and are considered tentative.

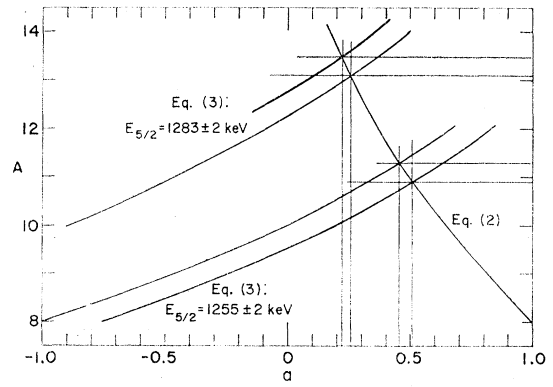

FIG. 3. Graphical determination of the $K=\frac{1}{2}$ rotational-band parameters from Eqs. (2) and (3) for the alternative sequences 1177, 1226, 1255 keV and 1177, 1226, 1283 keV corresponding to the $J=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ members, respectively.

the 801-keV level has been identified as the $J=\frac{3}{2}$ member of a $K^\pi=\frac{1}{2}^-$ band corresponding to a γ vibration coupled to the [512] Nilsson orbital.[15] The level observed at 763 keV in this work may be identified as the $J=\frac{1}{2}$ member of the same band. A similar situation obtains in Dy^{165}, for which the $[512]_\gamma$ $J^\pi=\frac{1}{2}^-$ level lies approximately 4 keV below the $[521]_\gamma J^\pi=\frac{3}{2}^-$ state. All the transitions identified as populating excited states below 801 keV have electric-dipole character except for the 5904-keV γ ray, which terminates on the positive parity $[633]_\gamma$ $\frac{3}{2}^+$ state and therefore would be $M1$ in character. From the level scheme given by Koch,[5] there is a possibility of a 5862-keV $E2$ transition to the $[633]_\gamma$ $\frac{5}{2}^+$ final state. Such a transition is not apparent in the spectrum and the observed intensity at this energy is $(6.6\pm6.4)\times10^{-3}\%$. The sensitivity for this transition is limited by the presence in the singles spectrum of the single-escape peak of the 5351-keV γ ray. The intensity limit quoted was obtained from analysis of the pair-coincidence data, for which the statistical accuracy is reduced. It may be concluded, therefore, that primary $E2$ radiation to this state has an intensity of less than $1.3\times10^{-2}\%$.

Another feature of the level structure is that the levels at 1085, 1177, and 1226 keV in Er^{167} may correspond to a similar isolated set at 919, 1104, and 1167 keV in Dy^{165}. Transitions from the capture state to the 1104- and 1167-keV states of Dy^{165} have been reported.[15] This indicates that these states have low spin, just as do the 1177- and 1226-keV states in Er^{167}. However, no such transition to the 919-keV state of Dy^{165} has been observed, while a γ ray to the 1085-keV level in Er^{167} is observed.

A possible interpretation for the 1177- and 1226-keV levels is that they are the lowest spin members of a $K=\frac{1}{2}$ rotational band. In the (d,p) reaction, for which orbital-angular-momentum transfers extending to $l_n=3$ are common, the first four states ($J=\frac{1}{2}$ to $J=\frac{7}{2}$) can be populated. Two levels, at 1255 and 1283 keV, observed in the (d,p) reaction[6] are possible candidates for the $J=\frac{5}{2}$ member of such a band. The sequence of level energies for a $K=\frac{1}{2}$ band is given by[2]

$$E_J-E_{1/2}=A\{[J(J+1)-\tfrac{3}{4}]+a[(-1)^{J+1/2}(J+\tfrac{1}{2})+1]\}. \quad (1)$$

This equation leads to the relations

$$E_{3/2}-E_{1/2}=3A(1+a), \quad (2)$$

$$E_{5/2}-E_{1/2}=2A(4-a). \quad (3)$$

The range of values of the parameters satisfying the experimental spacings calculated from Eqs. (2) and (3) are plotted in Fig. 3. On the basis of analysis of the variance between corresponding (n,γ) and (d,p) excitation energies as discussed above, an uncertainty of 2 keV has been assumed for the spacing in Eq. (3). The values obtained for the band parameters when the 1255-keV level is used are $A=11.1\pm0.2$ keV and $a=0.485\pm0.025$, while for the 1283-keV level they are 13.6 ± 0.35 and 0.24 ± 0.02, respectively. A decision between these alternatives can be made on the basis of systematics. The values of the inertial parameter A observed[5,15] for five negative-parity rotational bands in Du^{165} and Er^{167} vary from approximately 10.6 to 11.2 keV. The parameters obtained for the 1255-keV level are therefore more consistent with the systematics.

A serious criticism of this interpretation is the fact that the $J=\frac{7}{2}$ member of such a band, which would be expected at 1370 ± 5 keV, was not observed in the (d,p) reaction. The same situation applies to the alternative band parameters. If this interpretation is nevertheless correct, then a possible explanation for the dynamical origin of such a band is that it corresponds to a $K=0$ β vibration coupled to the $[521]$ $\frac{1}{2}^-$ particle state. Such a state would be expected to have inertial and decoupling parameters similar in value to those for the particle state, observed to be $A=11.2206$ and $a=70033$ in this nucleus.[5]

The most striking feature of the spectrum is the large relative radiation widths of the 6228-keV γ ray. This transition has a reduced width which is sixteen times the observed average reduced width. Qualitatively, this result is not inconsistent with the statistical model. In the random-matrix approximation, the compound radiation state formed by the capture of a neutron is expected to have a complex configuration. The decay of this state should exhibit little sensitivity to the structure of the final state, so the average radiation widths should be approximately equal for different final states. For a given capture state, however, the observed widths represent a sampling from a frequency distribution function; and this allows large fluctuations from the average. The detailed nature of the distribution function has been calculated in the random-matrix

approximation,[18] and this result has been confirmed experimentally.[19]

Because of the manner in which the radiative widths are distributed, the capture γ-ray spectrum will be featured by a majority of transitions that have smaller than average widths and very few transitions with larger than average widths, just as is observed for the $Er^{166}(n,\gamma)Er^{167}$ reaction. According to the statistical model, however, the probability of a particular width being sixteen times the average is only $\sim 10^{-7}$.

It is interesting to note that this large width, for which statistical considerations predict such a low probability, occurs for the 6228-keV transition leading to the final state at 207 keV, identified as the $[521]\frac{1}{2}^-$ Nilsson orbital. This state is rather accurately described as corresponding to single-particle motion of the odd neutron in the deformed potential of the even-even core. In contrast, the average reduced width of transitions to the negative-parity states described as vibrational in character is only 22% as large as the over-all average, and only 1.4% of the reduced width for the 6228-keV transition. In addition, the ratio $\Gamma_\gamma([521]\frac{1}{2}^-, J=\frac{3}{2}^-)/\Gamma_\gamma([521]\frac{1}{2}^-, J=\frac{1}{2}^-)=0.24$ observed in this work is similar to that for corresponding transitions in the thermal spectra of W^{184}(0.35), Hf^{179}(0.17), and Hf^{181} (0.29) observed previously[20,21] for the $[510]\frac{1}{2}^-$ Nilsson orbital.

Because of these qualitative regularities, it was decided to make a more quantitative investigation of the possibility that the initial radiating state might be simpler in structure than is generally assumed. Lynn and Lane[22] have proposed a mechanism of direct capture in which radiation occurs from an initial state described as an s-wave neutron coupled to the target nucleus. The wave function for an initial state described as an s-wave neutron coupled to the deformed even-even target is[23]

$$\psi_i = X_0(Rx)\{D_{M_1\,1/2}^{(1/2)}(R)S_{1/2}(Rx) - D_{M_1\,1/2}^{(1/2)}(R)S_{-1/2}(Rx)\},$$

where $X_0(Rx)$ is the wave function of the even-even core in the intrinsic coordinate system (Rx), $D_{M_1\,1/2}^{(1/2)}(R)$ is the symmetric-top wave function for spin $\frac{1}{2}$, and R refers to the Eulerian angles. This wave function is of exactly the same form as that for the final state, described by strong coupling, except that the intrinsic functions $S_{\pm 1/2}(Rx)$ are replaced by the Nilsson wave function $\chi_{\pm\kappa}(Rx)$. Because of this result, which is true only for $S_{1/2}$ states, the reduced transition probability

$$B(E1; i\to f)=(2I_i+1)^{-1}\sum_{M_i,M_f}(\psi_f|E_\mu^{(1)}|\psi_i)^2$$

can be calculated from the standard formula for strong coupling wave functions. This gives the result

$$B(E1)=\text{const}\times(p\|E^{(1)}\|s)^2[(2J+1)C_{pJ}{}^2],$$

where J is the total angular momentum of the final state, $(p\|E^{(1)}\|s)$ is the reduced matrix element between single-particle s and p states, and $C_{pJ}(N,n_z,\Lambda,\Omega)$ is a factor which depends upon the intrinsic structure of the final-state Nilsson orbit. This factor is given by

$$C_{pJ}=\sum_\Lambda(1-\tfrac{1}{2}\Lambda\Sigma/K\Omega)d_{p\Lambda}(N,n_z,\Omega),$$

where the $d_{p\Lambda}$ are the $l=1$ Nilsson coefficients.[3] For this extreme model, therefore, the relative radiative strength for the first two members of an $\Omega=\frac{1}{2}$ rotational band built upon a single-particle Nilsson orbit is uniquely defined by the structure coefficient.

The coefficients $d_{p\Lambda}$ are themselves a function of the three parameters δ, μ, K corresponding to the deformation, angular-momentum dependence, and spin-orbit interaction for the Nilsson potential.[3] For the rare-earth region, it has been found that $\mu=0.4$ and $K=0.5$ are the parameters most consistent with the systematics,[3] and that the deformation parameter $\delta=0.3$ is appropriate[24] for Er^{167}. The predicted ratio of the population of the $\frac{3}{2}^-$ state to that of the $\frac{1}{2}^-$ state in the $[521]\frac{1}{2}^-$ band is

$$\frac{B(E1;[521]\frac{1}{2}^-, J=\frac{3}{2}^-)}{B(E1;[521]\frac{1}{2}^-, J=\frac{1}{2}^-)}=0.18\pm 0.03(\delta=0.30\pm 0.015)$$

if the deformation parameter is taken[25] to be $\delta=0.30\pm 0.015$. In view of the sensitivity of the transition matrix elements to the details of the level structure, this value is not inconsistent with the observed value 0.24. In this context, it should be noted that the agreement is no worse than that between the observed and predicted decoupling constants for this band. The value calculated for the set of parameters used in this calculation is $a=0.881\pm 0.02$, approximately 25% larger than the observed value 0.70033. In fact, the predicted value is significantly larger than the observed value over the range $0.1\leqslant\delta\leqslant 0.33$. One must conclude that the uncertainty in the final-state wave function alone might account for the slight difference between the observed and calculated relative radiative widths.

In extreme application of this model, if one assumes the collective γ vibrations to be normal modes of the even-even core, then single-particle transitions to the

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states thought to be of this character would be strictly forbidden. As can be seen from Table II, transitions to these states appear to be inhibited but the radiative widths are finite and significant. It is impossible to say whether this violation of implied selection rules results from inaccuracies in the description of the initial or final states or both.

Because of the apparent regularities observed in corresponding transitions in W and Hf isotopes,[20,21] the predicted ratios for the $[510]\frac{1}{2}^-$ band were also calculated for this model. The calculated results indicate that the intensity of the transition to the $J=\frac{3}{2}^-$ state should be approximately two orders of magnitude larger than that of the transition to the $J=\frac{1}{2}^-$ state. This is at complete variance with the experimental results, which exhibit ratios similar to that observed for Er^{167}. Also, in Dy^{165} the radiative width to the $J^\pi=\frac{3}{2}^-$ member of the $[521]\frac{1}{2}^-$ band is approximately 70% of that of the γ ray to the $J^\pi=\frac{1}{2}^-$ level.[15] This value would require a much smaller deformation (≈ 0.2) for Dy^{165} than is consistent with the systematics. In addition, transitions to vibrational states in this nucleus occur with radiative strengths up to 40% of that of the $[521]\frac{1}{2}^-$ level. It must be concluded, therefore, that although the concept of a direct radiative process occurring from the channel region of the reaction phase space provides a rather detailed description of the Er^{176} spectrum, it fails to explain the observations in the ratio $\Gamma_\gamma([510]\frac{3}{2}^-/\Gamma_\gamma([510]\frac{1}{2}^-)$ and is not consistent with the data observed in Dy^{165}.

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Conversion-Electron Particle Parameters in the Decay of Eu^{152} and Eu^{154}†

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The internal conversion-electron particle parameters of several fast low-energy $E2$ transitions in the deformed nuclei Sm^{152} and Gd^{154} were determined by comparing the e^--γ directional correlations and γ-γ directional correlations displayed by the same sources. Special attention was paid to eliminating the effects of scattering in the source, and the corrections caused by the presence of extranuclear perturbations of the directional correlations were carefully studied. The results are: 0.122-MeV transition in Sm^{152}: $b_2(E2,K)=1.60\pm0.05$, $b_2(E2,L^-)=1.03\pm0.05$; 0.245-MeV transition in Sm^{152}: $b_2(E2,K)=1.72\pm0.07$; 0.344-MeV transition in Gd^{154}: $b_2(E2,K)=1.57\pm0.13$; 0.123-MeV transition in Gd^{154}: $b_2(E2,K)=1.37\pm0.07$. The two particle parameters of the 0.122-MeV ground-state transitions in Sm^{152} are about 15% lower and that of the 0.123-MeV ground state-transition in Gd^{154} is 25% lower, than the theoretical predictions for finite-size nuclei. The particle parameters of the 0.245-MeV and of the 0.344-MeV transitions agree with the theoretical values within experimental errors.

I. INTRODUCTION

In recent years experimental conversion-electron particle parameters for pure $E2$ transitions have been reported which show significant deviations from the theoretical values.[1] These discrepancies from a well-established theory are difficult to explain. Recently, more accurate calculations[2] of conversion-electron particle parameters for finite-size nuclei including higher-order effects indicated that even large nuclear deformations cannot explain the observed discrepancies. In view of these difficulties, it seemed desirable to re-examine the experimental results and to investigate the "anomalous" conversion-electron particle parameters, taking into account all possible effects which could cause systematic errors.

The observed "anomalous" conversion particle parameters are all smaller than the theoretical predictions. Thus the effects of scattering of the conversion electrons in the source and the attenuation of the directional correlation by extranuclear perturbations could be offered as possible explanation of the observed discrepancies.

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