channels do not contribute much (less than few percent) to the proton reaction cross section since the energies involved are considerably below the respective Coulomb barrier. We included the (p,n) channels leading to the ground state $(\frac{9}{2})$, 595-keV state $(\frac{1}{2})$, 1095, 1465, and 1645-keV states.

The differential cross section for a given channel is coupled to the rest of the open channels through Eq. (2). Therefore, in general, $\sigma(\theta)$ for a particular channel depended on the assumed I^{π} 's of the relevant open channels. However, we found, by actual calculations assuming different spin-parities for the unknown states, that, insofar as the shapes and branching ratios of the $\sigma(\theta)$'s to the three low-lying states were concerned, the assumed values of the spin-parities did not matter. On the other hand, the absolute values of $\sigma(\theta)$ did change noticeably with particular choice of the I^{π} . The differential crosssections shown in Figs. 5 and 6 are calculated assuming $I^{\pi} = \frac{3}{2}^{+}$ for the 1465-keV state and $I^{\pi} = \frac{5}{2}^{+}$ for the 1645keV state.

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Nuclear Structure of Na²². II. Some Gamma-Ray Doppler-Shift and Correlation Measurements*

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Levels of Na²² below an excitation energy of 4.4 MeV have been investigated through the $F^{19}(\alpha, n\gamma)$ Na²² and $Ne^{20}(He^3, p\gamma)Na^{22}$ reactions. Using the first reaction, initiated in a CCl_2F_2 gas target, the Doppler shifts of γ -ray transitions from the 0.891-, 2.211-, and 2.572-MeV levels were measured using α energies between 4 and 7 MeV. From these results upper limits were placed on the lifetimes of the three initial states. Combining these upper limits with previously obtained lower limits yields the following restrictions on the mean lifetimes (in psec) for these three levels: $8 < \tau < 52$, $11 < \tau < 42$, and $10 < \tau < 42$, respectively. Proton- γ angular correlations were measured at $E_{\text{He}^3} = 6.56$ MeV using the Ne²⁰(He³, $\rho\gamma$)Na²² reaction. These results gave information on γ -ray branching and multipole-mixing ratios and served to limit the possible spin-parity assignments of some of the levels.

I. INTRODUCTION

'N this report we present further results in a con-L tinuing experimental investigation into the properties of the energy levels of the nucleus Na²². Information available on the quantum numbers of the first 18 levels of Na²² and the first 3 levels of Ne²² is shown in Fig. 1. The figure is taken from a previous report¹ on the levels of Na²² with some additions. Recent results from γ -ray linear polarization measurements² are included. These fixed the 1.528-MeV level as $J^{\pi} = 5^+$ and determined the parity of the 2.211-MeV level to be odd. The results of the present work are also included.

In our previous report¹ we presented results bearing on the levels below an excitation energy of 3.1 MeV. The emphasis in this report is on the levels between 2.5 and 4.4 MeV. These levels were studied by proton- γ angular-correlation measurements via the $Ne^{20}(He^3, p\gamma)$ Na²² reaction. The interpretation of these results was aided by γ - γ coincidence measurements obtained with both NaI(Tl)-NaI(Tl) and NaI(Tl)-Ge(Li) γ -ray detector combinations.

In addition to these correlation measurements, we also present the results of Doppler-shift measurements on the most intense γ rays originating from each of the three levels at 0.89, 2.21, and 2.57 MeV. These measurements were analyzed to give upper limits on the lifetimes of these three levels. Combining them with previously obtained¹ lower limits results in fairly limited ranges of allowed values for the lifetimes of the three states.

II. LIFETIME MEASUREMENTS

A. Experimental Procedure and Results

Levels of Na²² were populated via the $F^{19}(\alpha, n)$ Na²² reaction. The target consisted of CCl₂F₂ gas at a pressure of 1 atm, which was confined by a 0.1-mil Ni foil. The α beam passed through this foil, through 2 cm of gas, and was stopped in tantalum. Gamma rays were detected in an 8-cc Ge(Li) detector placed 10 cm from the gas target and at angles θ_{γ} to the beam axis of 0° and 90°. Gamma-ray spectra were recorded using a 4096-channel digitally stabilized analog-to-digital converter in conjunction with a Technical Measurements Corporation (TMC) 16384 pulse-height analyzer.

The three γ -ray transitions studied were: 0.891 \rightarrow 0, $2.211 \rightarrow 0.657$, and $2.572 \rightarrow 0$. For each level studied

^{*} Work performed under the auspices of the U. S. Atomic

Energy Commission. ¹ E. K. Warburton, J. W. Olness, and A. R. Poletti, Phys. Rev. **160**, 938 (1967).

²A. R. Poletti, E. K. Warburton, and J. W. Olness, Phys. Rev. (to be published).

the beam energy was chosen to be the lowest commensurate with a reasonable peak-to-background ratio for the full-energy-loss peak of the main γ -ray branches given above.¹ The difference, ΔE_{γ} , between the γ -ray energy corresponding to $\theta_{\gamma} = 0^{\circ}$ and that corresponding to 90° was determined from computer analysis of the 0° and 90° spectra. This energy difference was then divided by the full kinematical shift (including the uncertainty associated with the unknown angular distribution of the outgoing neutrons) to obtain the Doppler-shift attenuation factor $F(\tau)$. Except for the use of a gas target instead of a solid target, all procedures were identical to those used in previous measurements¹ of the Doppler shifts of these transitions.

The values of $F(\tau)$ obtained are listed in column 2 of Table I. The results of our measurements are all consistent with $F(\tau) = 1$. Thus we conclude that these three levels have lifetimes short compared to the stopping time of the target, and so these measurements yield only upper limits for the lifetimes of the three levels in question. Lower limits on $F(\tau)$ are listed in column 3 of Table I; they correspond to 2 standard deviations from the measured values of $F(\tau)$. Upper limits on the mean lifetime (τ) corresponding to these limits on $F(\tau)$ are given in column 4. The procedure used to obtain these limits has been fully explained previously.¹ The lower limits on τ given in column 5 are from the previous investigation¹ of the Doppler shifts of these transitions. In that work a solid CaF₂ target was used and no Doppler shifts were discernible. Finally, the upper and lower limits on the mean lifetime are combined to give the most probable values of τ listed in the last column of Table I. In obtaining these most probable values we have used electronic stopping times, α , of 0.69 and 252 psec, respectively, for Na²² ions recoiling in CaF_2 and CCl_2F_2 (1 atm). The uncertainties assigned to these most probable values are such that two standard deviations brings τ to the upper or lower limit. The most probable values are given for purposes of discussion; the limits on τ are actually the pertinent experimental results.

B. Discussion

The Na²² 0.891-MeV level has J=4 and decays to the $J^{\pi}=3^{+}$ ground state by a mixed quadrupole/dipole transition with a mixing amplitude, x, of $-(2.6\pm0.5)$.¹ Combining this mixing ratio with the most probable

TABLE I. Results of observations of the Doppler shifts of three γ -ray transitions following the $F^{19}(\alpha, n\gamma)Na^{22}$ reaction.

Transition (MeV)	$F(\tau)$	$F(au)_{\min}$	τ_{\max} (psec)	$ au_{\min}^{a}$ (psec)	$\tau_{\text{most probable}}$ (psec)
$\begin{array}{c} 0.891 \rightarrow 0 \\ 2.211 \rightarrow 0.66 \\ 2.572 \rightarrow 0 \end{array}$	0.96 ± 0.07	0.82	52	8	30 ± 11
	0.99 ± 0.07	0.85	42	11	26 ± 8
	0.95 ± 0.05	0.85	42	10	26 ± 8

^a From Ref. 1.

	4.325 4.363 1.2
<u>3.357 4⁺,⊺≖I</u>	<u></u>
	3.711 ≥2
	3.527 ≥2
	3.059 (2) 2.969 (3)
	2.572 I ⁽⁺⁾ ,2
	2.211 1-
1.275 2+,1=1	$\frac{1.984}{1.952} = \frac{2^{+},3^{+}}{2^{-},3^{+}}$
	1.937 I ⁺
	1.528 5 ⁺ ;T=0
	0.89I 4 ⁺ ;T⁼0
0 0 ⁺ ,T=1	0.657 0 ⁺ ;T≖I
Ne ²²	0.583 I ⁺ ;T⁼O
	<u>03⁺;⊺≈0</u> Na ²²

FIG. 1. The low-lying levels of Na^{22} and Ne^{22} . The figure is from Ref. 1 with some additions. The sources of the data are cited in Ref. 1. Additions include results from Ref. 2 and from the present work.

mean lifetime (Table I) gives E2 and M2 transition strengths of 11.1_{-3}^{+7} and 290_{-80}^{+170} Weisskopf units (W.u.),³ respectively. The M2 strength is unreasonably large so M2/E1 mixing can be rejected. Thus the parity of the Na²² 0.891-MeV level is found to be even in agreement with a previous conclusion.^{1,4} The M1strength of the 0.891 \rightarrow 0 transition is $\sim 2 \times 10^{-4}$ W.u. which is reasonable for an isotopic-spin-forbidden $M1.^5$ The E2 strength also appears to be quite reasonable.

The $E1^2$ 2.211 \rightarrow 0.657 transition is allowed by the isotopic-spin selection rule. It has a strength of $(1.3_{-0.34}^{+0.50})\times10^{-5}$ W.u. and thus we conclude that the transition is highly retarded by a nuclear-structure effect (see Refs. 3 and 5). The 2.211-MeV level has a ground state branch of $(1\pm1)\%^{.1}$ This transition, if indeed present, would be a M2/E3 mixture. A 1% branch corresponds to an M2 strength of 0.04 W.u., which is a representative value for an isotopic-spin-

⁸ D. H. Wilkinson in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), part B, p. 862 ff.

<sup>beig-conv. (1998)
w. F. Vogelsang and J. N. McGruer, Phys. Rev. 109, 1663 (1958).</sup>

⁶ E. K. Warburton, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, (1966), pp. 90-112.

(MeV)	3.53	3.71	3.95	4.07ª	4.32	4.36
$\begin{array}{c} 0\\ 0.58\\ 0.66\\ 0.89\\ 1.53\\ 1.94\\ 1.95\\ 1.98\\ 2.21\\ 2.57\\ 2.97\\ 3.06\\ 3.53\\ 3.71 \end{array}$	$ \begin{array}{c} 14\pm 6 \\ <5 \\ <5 \\ \leq(13\pm 5)^{\flat} \\ <5 \\ <25 \\ <25 \\ <35^{\flat} \\ <14 \\ <10 \\ \dots \\ \dots\end{array} $	$<7 < 7 < 7 < 7 <7 65\pm1035\pm10<10<10<10<10<10<10<15 $		}100ª		$ \begin{array}{c} 4\pm4 \\ 24\pm4 \\ <5 \\ <7 \\ <20 \\ 71\pm4^{\circ} \\ <7 \\ <5 \\ <7 \\ <5 \\ <8 \\ \end{array} $

TABLE II. Summary of branching ratios (or limits) determined for γ -ray transitions in Na²² from the listed initial states (E_i) to various final states (E_f) . This summary is an average of the present work with that given in Ref. 8.

Level weakly excited. Other possible decay modes could have been missed.
^b The measurements do not distinguish between the two alternate decay modes 3.53 → 0.89 → 0 and 3.53 → 2.57 → 0.
^c The decay to the 0.66-MeV level is preferred by the energy measurement of the γ-ray transition.
^d The major decay mode is probably to the 1.98-MeV level.
^e The decay to the 1.95-MeV level is preferred by the energy measurement of the γ-ray transition.

forbidden M2 transition in this region of atomic number.5

The Na²² 2.572-MeV level decays to the ground state and to the 0.583-MeV level with branches of $(81\pm3)\%$ and $(19\pm3)\%$, respectively. Both transitions can be at most quadrupole; i.e., octupole (and higher order) radiation is forbidden by the lifetime limit of Table I. Thus, the 2.572-MeV level has J=1, 2, or 3. If J=1, then the ground-state transition has, depending on its parity, an E2 strength of $(6.0_{-1.4}^{+2.6}) \times 10^{-2}$ W.u. or an M2 strength of $(1.6_{-0.4}^{+0.6})$ W.u. The latter is stronger than any known $\Delta T = 0$ M2 transition in a self-conjugate nucleus⁵ and thus if the 2.572-MeV level has J=1 it is most likely to have even parity. Whatever the spin-parity of the 2.572-MeV level, the possible dipole transitions, M1 or E1, to the ground state and 0.583-

TABLE III. Results of least-squares Legendre polynomial fits to proton-gamma angular correlations measured in the Ne²⁰-(He³, $p\gamma$)Na²² reaction. The coefficients are not corrected for the finite solid angle subtended by the γ -ray detector.

Initial level (MeV)	Transition (MeV)	<i>a</i> 2	<i>a</i> 4	x ²
2.57	$2.57 \rightarrow 0$	$+(0.01\pm0.03)$	$+(0.01\pm0.03)$	2.5
	$2.57 \rightarrow 0.58$	$-(0.33\pm0.06)$	$+(0.02\pm0.06)$	0.3
2.97	$2.97 \rightarrow 1.95$	$-(0.32\pm0.03)$	•••	0.03
	$1.95 \rightarrow 0.58$	$-(0.30\pm0.04)$	•••	0.1
3.06	$3.06 \rightarrow 1.95$	$+(0.26\pm0.03)$	•••	1.9
	$1.95 \rightarrow 0.58$	$-(0.02\pm0.04)$	•••	0.2
3.53	(1.56 ± 0.02)	$-(0.39\pm0.05)$	$-(0.01\pm0.06)$	0.2
	(1.36±0.02)	$-(0.06\pm0.06)$	$-(0.22\pm0.06)$	a
3.71	$3.71 \rightarrow 0.89$	$+(0.31\pm0.14)$	$-(0.17\pm0.15)$	0.2
	$0.89 \rightarrow 0$	$+(0.43\pm0.07)$	$+(0.20\pm0.08)$	0.03
3.95	$3.95 \rightarrow 0.66$	$-(0.75\pm0.03)$	•••	1.7
4.36	$4.36 \rightarrow (0.66 + 0.58)$	$-(0.42\pm0.04)$	$-(0.03\pm0.05)$	0.3
	$4.36 \rightarrow (1.95)$	$+(0.44\pm0.06)$	$-(0.01\pm0.06)$	0.1
	$(1.95) \rightarrow 0.58$	$-(0.16\pm0.08)$	$+(0.00\pm0.09)$	1.0

* The coefficients for this transition were obtained from an average of three angular-correlation measurements consisting of 3, 3, and 7 data points, respectively,

MeV level are retarded by factors of at least $\sim 10^4$. This factor is large even for isotopic-spin-forbidden transitions and again indicates the presence of nuclearstructure effects.

III. PROTON-GAMMA ANGULAR CORRELATIONS

A. Experimental Procedures

The experimental method which we used has been described in detail previously¹; consequently we will merely give a brief summary of it. The reaction Ne²⁰- $(He^3, p)Na^{22}$ was used at a bombarding energy of 6.56 MeV. The details of the gas target, target chamber, and detectors were the same as those previously described.¹ The detection of the reaction protons in an annular detector at 180° limits the possible magnetic substates which can be populated in the residual excited Na²² nucleus to $\alpha=0, \pm 1$. The angular distribution of the resultant γ rays is then characteristic of the spin J of the excited level, the multipolarity of the γ ray de-

TABLE IV. Some results of χ^2 analyses of $p-\gamma$ angular correlations in the Ne²⁰ (He³, ρ)Na²² reaction. Restrictions for x, the (L+1)/L mixing ratio, are given for the possible spins of the initial state.

Initial level (MeV)	Final level (MeV)	J_i	${J}_{f^{\pi}}$	Restrictions on x
2.57 3.71	0 0.58 0 0.58 0.89 0.89 0.89 0.89 0.89 0.89	2 2 1 1 2 3 4 5 6	3+ 1+ 3+ 1+ 4+	$\begin{array}{c} + (0.13 \pm 0.10), \text{ or } + (3.5 \pm 1.0) \\ + (0.03 \pm 0.15), \text{ or } + (2.8 \pm 0.7) \\ \text{no restriction} \\ x \neq + (0.14 \pm 0.11), x \le 4 \\ - 0.3 < x < + 0.7 \\ - 0.04 < x < 3.0 \\ - 2.2 < x < + 0.7 \\ - 0.6 < x < - 0.1 \\ - 0.2 < x < + 0.3 \end{array}$

TABLE V. Restrictions on the quadrupole/dipole mixing ratios of the first (x_1) and second (x_2) members of the Na²² 2.97 \rightarrow 1.95 \rightarrow 0.58 and 3.06 \rightarrow 1.95 \rightarrow 0.58 cascades. The restrictions result from lifetime measurements (Ref. 1) and the present angularcorrelation measurements. The restrictions obtained for the mixing ratios of the 3.06 \rightarrow 1.95, 2.97 \rightarrow 1.95 and 1.95 \rightarrow 0.58 transitions from the two cascades considered are interdependent.

J_i	J_f	Mixing-ratio restrictions
		$2.97 \rightarrow 1.95 \rightarrow 0.58$
1	1	Not Allowed
1	2	$-0.30 < x_1 < -0.19; -0.08 < x_2 < +0.56$
2	1	$-0.12 < x_1 < +0.30; +0.34 < x_2 < +0.56$
2	2	Not Allowed
3	2	$-0.07 < x_1 < +0.07; -0.14 < x_2 < +0.14$
		$3.06 \rightarrow 1.95 \rightarrow 0.58$
1	1	$-0.32 < x_1 < 0.00; +0.34 < x_2 < +0.56$
1	2	$-0.32 < x_1 < -0.19; -0.14 < x_2 < -0.02$
2	1	Not Allowed
2	2	$-0.18 < x_1 < +0.21; -0.14 < x_2 < +0.56$
3	2	$-0.30 < x_1 < -0.24; -0.14 < x_2 < -0.08$

exciting it, the spin of the final level and the (unknown) population ratio P(0)/P(1), where $P(\alpha)$ describes the relative population of the α substate.^{6.7} Often the spin of the final level is known and in favorable cases the measured angular distribution can determine the spin of the excited level uniquely. In the next subsection we shall discuss the results for each level in turn.

Information upon decay modes was obtained for all of the levels to be discussed but because of low yields, analysis of the p- γ angular correlations was only carried out for the more strongly excited of the levels. The branching ratios determined from the present work were combined with previous results⁸ to yield the average values summarized in Table II. The decay modes of the 2.57-, 2.97-, and 3.06-MeV levels are not included in Table II since they were given previously.¹ Table III summarizes the correlation data which were obtained. The angular-correlation data for each of the listed transitions were fitted with an expansion $I_{\gamma}[1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta)]$ where $P_k(\cos\theta)$ is a kth-order Legendre polynomial. The coefficients a_2 and a_4 are listed together with a normalized χ^2 value. Tables IV and V list the mixing ratios which were obtained from our analysis of the correlation data. The mixing ratio, which is the amplitude ratio of the next-to-lowest (L+1) to the lowest (L) allowed multipolarity (for example quadrupole and dipole radiation), is as defined by Poletti and Warburton.⁴ For natural mixtures (e.g. E2/M1), the phase is identical with that recommended by Brink and Rose.9

Figure 2 shows the spectrum observed in the annular detector in coincidence with all detected γ rays with energies greater than 200 keV. Gamma rays were seen from all the known levels of Na²² between 2.2- and $\overline{^{6}A}$. E. Litherland and A. J. Ferguson, Can. J. Phys. **39**, 788



FIG. 2. Partial results of a two-parameter analysis of $p \cdot \gamma$ coincidences from the Ne³⁰ (He³, $p\gamma$)Na²² reaction at a bombarding energy of 6.56 MeV. The plot shows the proton spectrum measured in coincidence with all γ -ray pulses of energy $E_{\gamma} > 200$ keV. The data is the sum of spectra recorded at 5 different angles θ_{γ} . The proton peaks are identified by the excitation energies (in MeV) of the Na²² levels to which they correspond and by their level sequence. The two N¹⁴ peaks both correspond to the first-excited state of N¹⁴ from C¹² (He³, $p\gamma$)N¹⁴ arising from carbon contamination of the exit and entrance Ni foils of the gas target.

4.4-MeV excitation, although the levels at 3.71 and 4.07 MeV were only weakly excited. The peak at channel 7 corresponds to excitation of one or more of the three levels near an excitation energy of 4.5 MeV in Na²².

B. Results

1. The 2.57-MeV Level

For this level the decay modes $(19\pm4\%)$ to the 0.58 MeV level, $81\pm4\%$ to the ground state) inferred from the summed γ spectrum (Fig. 3) are in agreement with those of two previous investigations.^{1,8} The p- γ angular correlations for the 2.57- and 1.99-MeV transitions were both obtained; that to the ground state was found to be isotropic within the experimental errors (see Table III). The only limitation on the spin of the 2.57-MeV level from this source was that $J \leq 4$. For the 1.99-MeV transition, however, $a_2 = -(0.33 \pm 0.06)$. Fitting this angular distribution for various choices of the spin of the 2.57-MeV level gave the χ^2 curves shown in Fig. 4 and eliminated all spins except J = 1 and 2. The limitations on the mixing ratios for the transitions to the ground state and first-excited state for these spin choices are given in Table IV. For the ground-state transition, they are in good agreement with those determined by Maier et al.8

2. The 2.97- and 3.06-MeV Levels

The annular detector did not have sufficiently good resolution to completely resolve the particle groups leading to the 2.97- and 3.06-MeV levels; however, the decay modes and angular correlations for each state could be obtained by a suitable decomposition of the two-dimensional spectra.¹ Figure 5 gives the γ -ray spectra which represent the decay modes of the 2.97-

<sup>(1961).
&</sup>lt;sup>7</sup> A. R. Poletti and E. K. Warburton, Phys. Rev. 137, B595 (1965).
⁸ H. I. Maier, D. Pelte, I. G. Propho, and C. Rolfs, Nucl. Phys.

⁸ H. J. Maier, D. Pelte, J. G. Pronko, and C. Rolfs, Nucl. Phys. 84, 1 (1966).
⁹ H. J. Rose and D. Brink, Rev. Mod. Phys. 39, 306 (1967).



FIG. 3. Spectra of γ rays from the Ne²⁰(He³, $\rho\gamma$)Na²² reaction measured in coincidence with proton groups ρ_0 and ρ_{14} populating, respectively, the 2.572- and 3.95-MeV levels of Na²². The spectra are the sum of spectra measured at 5 different angles θ_{γ} . For each spectrum the γ -ray peaks are identified by the energies (in MeV) of the initial and final states between which the transition takes place and/or the γ -ray energy (in MeV). The expected positions of some unobserved transitions are indicated. The dashed curves indicate some of the details of the analyses of the spectra. The inserts indicate the decay modes inferred from these spectra.

and 3.06-MeV levels, respectively. Both states were observed to decay $\sim 100\%$ to the 1.95-MeV level. The 3.06-MeV level has a weak branch $(3\pm1\%)$ to the 0.583-MeV level. Limits on possible decays to all other lower-lying levels are given in Table IV of Ref. 1. The angular correlations for the 1.02- and 1.37-MeV γ rays from the 2.97-MeV level are shown in the upper part of Fig. 6 while the angular correlations for the 1.11- and 1.37-MeV γ rays from the 3.06-MeV level are shown in the upper part of Fig. 7.

For both the 2.97- and 3.06-MeV levels $J \leq 3$ was given previously¹ and J=0 is ruled out by the observed anisotropies of the γ rays from these levels. For the 1.95-MeV level, the spin was previously¹ limited to J=1 or 2. The 0.583-MeV level has J=1. The 2.97 \rightarrow 1.95, 3.06 \rightarrow 1.95, and 1.95 \rightarrow 0.583 transitions must be predominantly dipole with the quadrupole/dipole mixing ratio restricted to |x| < 0.30, < 0.32, and < 0.56, respectively.¹ In our present analysis of the correlation data as presented below, we have incorporated these previously imposed restrictions on the spins of the levels involved and on the mixing ratios of the three transitions. In the analysis of results for the 2.97-MeV level (and also similarly for the 3.06-MeV level) the experimental angular correlations of *both* members of the cascade were compared simultaneously to theory; thus χ^2 was obtained as a function of the mixing ratios of the first (x_1) and second (x_2) members of the cascade, $J_i \rightarrow J_f \rightarrow 1$, for each of the five possible combinations J_i and J_f . Unfavored solutions were excluded at the 0.1% limit. The results of this procedure are given in Table V which presents the allowed regions of x_1 and x_2 for each J_i , J_f combination. It is seen that not all J_i , J_f combinations are allowed. Table V also summarizes the results obtained, in an identical fashion, for the 3.06-MeV level.

Although the spin of the 1.95-MeV level is not definitely known, it is the only state of Na²² whose properties are consistent with its being the T=1, $J^{\pi}=2^+$ analog of the first-excited state of Ne²². If so, the $1.95 \rightarrow 0.58$ transition would be expected to be an allowed *M*1 transition with a negligible unenhanced *E*2 component, i.e., $|M(E2)|^2 \lesssim 1.0$ W.u. To illustrate the most probable explanations of the angular correlations



FIG. 4. Proton-gamma angular correlations for the Na²² 2.57 \rightarrow 0.58 transition resulting from population of the 2.572-MeV level in the Ne³⁰(He³, p)Na²² reaction. The points with error bars (upper plot) show the experimental correlation data. The results of a χ^2 analysis of these data are shown in the lower part of the figure. Here we have plotted χ^2 , representing the goodness-of-fit to the experimental data, as a function of arctan x, where x is the (L+1)/L mixing ratio in the 2.57 \rightarrow 0.58 transition. For a correct solution the expectation value of χ^2 is unity and the probability of χ^2 exceeding the 0.1% limit is 0.1%. Plots are shown for assumed values of J=0 through 4 for the 2.57-MeV level. The effect of the departure of the proton scattering angle from 180° (called the FSE) is unimportant for these plots. It is seen that both J=1 and 2 give acceptable solutions. The best fit for these two values of J is shown in the upper part of the figure.

of the γ decays from the 2.97- and 3.06-MeV levels, the mixing ratio of the $1.95 \rightarrow 0.58$ transition was constrained to the region $|x_2| < 0.05$. This region corresponds to $|M(E2)|^2 < 1.0$ W.u. The chi-squared versus $\arctan x_1$ curves corresponding to this constraint are shown in the lower parts of Figs. 6 and 7. In these figures the region of x_1 corresponding to $|M(E2)|^2 < 10$ W.u. is indicated. Since the $3.06 \rightarrow 1.95$ and $2.97 \rightarrow$ 1.95 transitions are most probably $\Delta T = 1,^{1}$ this region is heavily favored if these transitions are M1, E2. If either is E1, M2 then this region corresponds to $|M(M2)|^2 < 260$ W.u. and clearly is the only region allowed. In both Figs. 6 and 7 only one value of J_i gives a χ^2 curve which drops below the 0.1% limit in the preferred region of x_1 . Thus we conclude that the 2.97-MeV level is probably J=3 and the 3.06-MeV level is probably J=2. These conclusions are qualified because of our assumption of $J^{\pi} = 2^+$ for the 1.95-MeV level and the nonrigorous restrictions on x_1 and x_2 .

3. The 3.53-MeV Level

The γ decay of this level is complex. There are clearly decays to the ground state and to the 0.89-MeV level and/or the 2.57-MeV level (see Table II); these together account for only 27% of all decays from the level. The levels through which the remaining 73% of the decays occur are less certain. From an analysis of Fig. 2 of Maier *et al.*⁸ decays through the 1.94- and 2.21-MeV levels *combined* must be less than 25%. The



FIG. 5. Spectra of γ rays from the Ne²⁰(He³,p)Na²² reaction measured in coincidence with proton groups p_{10} and p_{11} populating, respectively, the 2.97- and 3.06-MeV levels of Na²². For details see the caption of Fig. 3.



FIG. 6. Proton-gamma angular correlations for the Na²² 2.97 \rightarrow 1.95 \rightarrow 0.58 cascade. The upper plot shows the experimental correlation data and the least-squares fits to these data (Table II). The results of a χ^2 -analysis of these data are shown in the lower part of the figure. The solid curves are for $x_2=0$. For details see the caption of Fig. 4 and the text.

limits obtained from the spectrum of Fig. 8 for decays to other levels are given in Table II. From γ - γ coincidence measurements the decay to the 2.57-MeV level is <35%. The width of the peak at ~1.36 MeV in Fig. 8 compared to the width of the 1.54-MeV peak gives definite evidence that the 3.53-MeV level decays to at least two of the four states between 1.9 and 2.3 MeV. The angular distribution of the photopeak at 1.36±0.02 MeV contained a significant term in P_4 (cos θ), $a_4 = +(0.22\pm0.06)$. This implies that $J \ge 2$ for the 3.53-MeV level, and that the $3.53 \rightarrow 1.95 \rightarrow 0.58$ cascade is not the only decay mode constituting the 73% of decays in question.

4. The 3.71-MeV Level

The γ -ray spectrum corresponding to the decay of this level is shown in Fig. 8. The spin of the 3.71-MeV level is restricted to $J \ge 2$ by the presence of a term in $P_4(\cos\theta)$ in the correlation of the second member of the cascade: $3.71 \rightarrow 0.89 \rightarrow 0$. This restriction is also given by the transition to the $J^{\pi} = 5^+$ 1.53-MeV level (Fig. 8) since this transition would be of multipole order $L \ge 4$ for J < 2 and this is not allowed by the mean lifetime limit, $\tau < 10^{-7}$ sec, imposed by the coincidence conditions pertaining in the correlation measurements.

The angular correlations for the two transitions arising from the major decay mode were simultaneously



FIG. 7. Proton-gamma angular correlations for the Na²² 3.06 \rightarrow 1.95 \rightarrow 0.58 cascade. The upper plot shows the experimental correlation data and the least-squares fits to these data (Table II). The results of a x²-analysis of these data are shown in the lower part of the figure. For details see the caption of Fig. 4 and the text.

fitted for assumed spin values for the 3.71-MeV level of 2 to 6. All of these spin values were allowed for certain regions of x which are listed in Table IV. The decay modes of this level have not previously been observed.

5. The 3.95-MeV Level

The major decay of the level was confirmed as being to the 0.66-MeV level by the p- γ results (Fig. 3) and by γ - γ coincidence measurements. It was also observed that there was a branch of $(8\pm3)\%$ to the 1.95-MeV level. The γ -ray spectrum representing the decay of this state is shown in Fig. 3 while the decay modes and limits are given in Table I. Displayed in Fig. 9 is the angular distribution which was obtained for the decay to the 0.66-MeV level. It is consistent only with a J=1assignment to the 3.95-MeV state, J=2 and 3 being rejected by wide margins. This spin assignment is in agreement with that made by Maier *et al.*⁸

6. The 4.07-MeV Level

This level was very weakly excited (c.f. Fig. 2). From the γ -ray spectrum observed in coincidence with that portion of the proton spectrum where protons leading to the 4.07-MeV level were expected to occur, three γ rays of energy (1.40 \pm 0.02) MeV, (2.07 \pm 0.03) MeV, and (0.58 \pm 0.01) MeV were observed. We ascribe these to the decay of this level. The most likely major decay mode is therefore $4.07 \rightarrow 1.98 \rightarrow 0.58 \rightarrow 0$. This is in agreement with the observations of Arnell and Wernbom-Selin¹⁰ at the 813- and 874-keV resonances in Ne²¹(p,γ)Na²². From a comparison with the energy levels of Ne²² (Fig. 1) we see that the 4.07-MeV level is the most likely candidate for the 4⁺, T=1 analog of the 3.35-MeV level of Ne²². If this hypothesis is correct, the observed γ decay indicates a preference for the 3⁺ alternative for the 1.98-MeV level which has $J^{\pi}=2^+$ or 3⁺.¹

7. The 4.32- and 4.36-MeV Levels

Again, as for the doublet levels at 3 MeV, the resolution of the particle detector was not sufficient to resolve the individual proton groups. It was possible, however, by careful decomposition of the two-parameter spectrum to obtain the decay modes of these two levels, as is illustrated in Fig. 10. Our results are summarized in Table II. Of the two levels, the one at 4.36 MeV was the more strongly excited. We were thus able to extract the



FIG. 8. Spectra of γ rays from the Ne²⁰(He³, $p\gamma$)Na²² reaction measured in coincidence with proton groups p_{12} and p_{13} populating respectively the 3.53- and 3.71-MeV levels of Na²². For details see the caption of Fig. 3. In the upper plot the mean energies of two unresolved peaks are given (in MeV) with errors and the expected positions of possible contributors to these peaks are indicated.

¹⁰ S. E. Arnell and E. Wernbom-Selin, Arkiv Fysik 27, 1 (1964).



FIG. 9. Proton-gamma angular-correlation results for the Na²² $3.95 \rightarrow 0.66$ transition. The points show the experimental data while the curves show the best fits to the data obtained, as indicated in the insert, for assumed spins of J=1, 2, and 3 for the 3.95-MeV level. For the J=2 and 3 assumptions the minimum value of χ^2 is indicated. The uncertainties have been adjusted so that $\chi^2=1$ for J=1. The assignments J=2 and 3 are excluded by these results.

angular distributions for the more prominent γ rays those with energies of 1.37, 2.41, and 3.73 MeV. The 3.73-MeV γ ray feeds the 0.66- and 0.58-MeV levels with $J^{\pi} = 0^+$ and 1⁺, respectively. A chi-squared analysis was made for the angular distribution of this γ ray with both the magnetic substate ratio P(1)/P(0) of the 4.36-MeV level and the intensity ratio of the 4.36 \rightarrow 0.66 and 4.36 \rightarrow 0.58 branches as unknowns. The results definitely ruled out all spin assignments for the 4.36-MeV level except J=1 and 2.

The NaI(Tl)-NaI(Tl) γ - γ measurements showed clearly that one or both of the 4.32- and 4.36-MeV levels decays to the 0.66-MeV level. The NaI(Tl)-Ge(Li) γ - γ measurements gave some indication that the 4.32-MeV level decays in this manner. These results are in agreement with those of Maier *et al.*⁸ From the observed decay modes, a J=1 assignment appears to be favored for the 4.32-MeV level.

IV. CONCLUSIONS

Conclusions regarding spin-parity assignments resulting from the present work have been incorporated into Fig. 1. Other final results are summarized in Tables I, II, IV, and V. These results are in agreement with previous experimental work^{1,2,4,8,10} which has also been incorporated into Fig. 1 and Tables I and II.

To date, the most comprehensive theoretical study of Na^{22} has been the effective-interaction shell-model



FIG. 10. Spectra of γ rays from the Ne²⁰(He³, $p\gamma$)Na²² reaction measured in coincidence with proton groups p_{16} and p_{17} populating, respectively, the 4.32- and 4.36-MeV levels of Na²². For details see the caption of Fig. 3.

calculation of Wildenthal, et al.11 This calculation predicts seven T=0 even parity Na²² states below an excitation energy of ~ 3 MeV. We see from Fig. 1 that six such states are known and three more possible. If two of the three states at 2.572, 2.969, and 3.059 MeV had odd parity, and if the 1.984-MeV level and either the 2.969- or 3.059-MeV level were $J^{\pi} = 3^+$ and 2^+ (or 2⁺ and 3⁺), respectively, then the known evenparity levels would be in one-to-one correspondence with the theoretically predicted ones for $E_x \lesssim 3$ MeV. It may be that the 2.211-MeV level starts an odd-parity rotational band, and that the 2.572- and 2.969-MeV levels are the next two members of this band. If so, the three levels of Na²² at excitation energies of 3.527, 3.711, and 3.947 MeV could be the next three theoretically predicted even-parity states. These are predicted to have $J^{\pi} = 6^+, 4^+$, and 1^+ (not necessarily in that order).¹¹ The decay modes of the Na²² 3.527- and 3.711-MeV levels are consistent with those expected for 4^+ and 6^+ states, respectively. It is clear that the remarks in this paragraph are highly speculative. They are given to stress the need for further experimental work on the nuclear spectroscopy of Na²², and to indicate that such work can be of immediate theoretical interest.

¹¹ B. H. Wildenthal, P. W. M. Glaudemans, E. C. Halbert, and J. B. McGrory, Bull. Am. Phys. Soc. **12**, 48 (1967); and (private communication).