PHYSICAL REVIEW VOLUME 162, NUMBER 4 20 OCTOBER 1967

Investigation of Excited States of P^{31} with the $Si^{30}(p,\gamma)P^{31}$ Reaction

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(Received 13 April 1967)

Some previously unknown properties of excited states of P^{31} were investigated by studying the γ rays from the reaction $Si^{30}(p,\gamma)P^{31}$. A coincidence spectrum taken at the resonance proton energy $E_p=1177$ keV reduced the maximum possible branching ratio of the decay of the 2.23-MeV level via the 1.27-MeV level to 0.8%. Spectra taken at the $E_p=1694$-keV resonance yielded decay schemes for the unbound level at 8.93 MeV and the bound levels at 4.78, 5.12, and 5.66 MeV. Angular-correlation measurements at the same resonance led to the assignment of $J^\pi=5/2^+$ to the 4.78-MeV level. γ-ray linear-polarization measurements at the resonances at $E_p=1204$, 1322, 1509, and 2187 keV verified experimentally the even-parity assignments for the levels at 3.29, 3.41, 3.51, and 4.26 MeV and the virtual levels at 8.45 and 8.75 MeV. By comparison of the polarization results with results of earlier triple-correlation measurements, the unique values $\delta=-0.41\pm0.03$ and $\delta=-0.32\pm0.04$ were obtained for the $M1$-$E2$ mixing ratios of the $3.51\rightarrow0$ and $4.26\rightarrow0$ transitions, respectively.

I. INTRODUCTION

THE excited states of P^{31} have been the subject of numerous experimental and theoretical studies. For work done before 1962, the reader is referred to the review of Endt and van der Leun.[1] References to subsequent work can be found in the paper by Harris and Breitenbecher,[2] which gives a summary of measured properties of the bound levels and a comparison of these properties with some recent model calculations.

Except for the 4.78-MeV level, the spins of all levels up to 5.01-MeV excitation energy have been determined. In the present work, angular-correlation measurements were performed on the γ rays from the $Si^{30}(p,\gamma)P^{31}$ reaction at the $E_p=1694$-keV resonance in order to determine the spin of the 4.78-MeV level. The analysis of the data also yielded multipolarity mixings for several transitions. The need for more data on dynamic properties was pointed out by Harris and Breitenbecher.[2] Spectra were taken at the 1694-keV resonance, therefore, in order to obtain branching ratios for the decays of the 4.78-, 5.12-, 5.66-, and 8.93-MeV levels. A re-examination of the decay of the 2.23-MeV level was also conducted at the $E_p=1177$-keV resonance.

A number of even-parity assignments had been made for levels below 5-MeV excitation by arguing that significant amounts of $E1$-$M2$ mixing are unlikely. Even parities are also favored by most model calculations.[2] Recently, however, there have been demonstrated some clear cases of mixed $E1$-$M2$ transitions for $2s$-$1d$ shell nuclei, including decays of at least three virtual levels of P^{31} itself.[3,4] Betigeri *et al.*[5] recently verified experimentally the even parity of the 3.29-, 3.51-, and 4.26-MeV levels in a $Si^{30}(He^3,d)P^{31}$ stripping experiment. However, Table 4 of Ref. 5 has parentheses around the $l=2$ entries for the 3.29- and 3.51-MeV levels, indicating that the determinations were not conclusive in these two cases.[6] In the present work, the even parities of the 3.29-, 3.41-, 3.51-, and 4.26-MeV levels were verified by measuring the linear polarizations of γ rays at the $E_p=1204$-, 1322-, 1509-, and 2187-keV resonances.

II. EXPERIMENTAL PROCEDURE

The experiments were performed with the 2-MeV Van de Graaff accelerator of the Aerospace Research Laboratories. The proton beam was deflected onto the target with a 30° analyzing magnet. The targets, which averaged about 2-keV thickness for 1.5-MeV protons, were prepared by evaporation of elemental silicon onto thin tantalum backings. The targets were enriched to 78.4% Si^{30}. Typical beam currents of 5 μA were used. Additional details of the beam transport system may be found in Ref. 2.

Singles spectra of γ rays were recorded with an 8 in. long by 8-in. diam. NaI(Tl) crystal, and with a 2 cm^3 Ge(Li) detector. In each case, the detector was placed at 55° relative to the beam direction in order to reduce the effect of angular distributions on the relative intensities of various γ rays. The Ge(Li) detector spectra were limited to 20-keV resolution by the channel width required to cover the entire spectrum in a 512-channel analyzer. This resolution still resulted in sufficiently accurate values for the transition energies to greatly facilitate the determination of the complex decay scheme of the 8.93-MeV resonance level ($E_p=1694$ keV).

Triple-correlation measurements were obtained for two geometries by recording coincidence spectra in the 8 by 8 in. NaI(Tl) detector at angles $\theta_2=0°$, 30°, 45°, 60°, and 90° relative to the beam direction. In geometry I, the spectra were required to be in coincidence with a primary γ ray observed in a 5 by 5 in. NaI(Tl) detector at $\theta_1=90°$ to the beam direction and $\phi=90°$ to the

* An element of the Office of Aerospace Research, U. S. Air Force.
[1] P. M. Endt and C. van der Leun, Nucl. Phys. 34, 1 (1962).
[2] G. I. Harris and D. V. Breitenbecher, Phys. Rev. 145, 866 (1966).
[3] H. Van Rinsvelt and P. B. Smith, Physica 30, 59 (1964).
[4] H. Van Rinsvelt and P. M. Endt, Physica 32, 513 (1966).
[5] M. Betigeri, R. Bock, H. H. Duhm, S. Martin, and R. Stock, Z. Naturforsch, 21a, 980 (1966).
[6] The spin of the 3.29-MeV level is erroneously tabulated as $\frac{3}{2}$ in Ref. 5.

plane of motion of the 8 by 8 in. detector. In geometry II, a similar coincidence gate was established with a second 5 by 5 in. detector at $\theta_1=135°$ and $\phi=180°$. In the notation of Ref. 2, these geometries are described, respectively, as $(\theta_1,\theta_2,\phi)=(90,V,90)$ and $(135,V,180)$, where V denotes the variable angle of the set.

Linear polarizations of γ rays were determined by means of a Compton-scattering polarimeter, which is described in greater detail elsewhere.[7] The scatterer was a 2 by 2 in. NaI(Tl) crystal, mounted at 90° relative to the proton beam direction, i.e., directly above the Si^{30} target. Scattered γ rays were observed in two 5 by 5 in. NaI(Tl) crystals. Whenever a coincidence occurred between pulses in the scattering crystal and one of the final detectors, the sum of the pulses was recorded in one of the halves of the memory of a 512-channel analyzer, according to which final detector produced a pulse.

The final detectors were mounted so as to subtend a mean scattering angle of 60° from the initial γ-ray direction at the center of the 2 by 2 in. crystal. The final detectors were separated by an azimuthal angle of 90°. Thus, one detector would observe γ rays scattered in the (p,γ) reaction plane, and the other one those scattered normal to this plane. At regular intervals during the experiments, the whole polarimeter was rotated through 90° about a vertical axis in order to interchange the roles of the two final detectors. This removed a major contribution of spurious asymmetries due to small errors in alignment, beam walk, or differences in detector efficiency.

In all three types of measurements, γ-ray spectra were analyzed (stripped) using the computer program of Graber and Watson.[8] Standard line shapes available from previous work were used for all spectra recorded in the 8 by 8 in. crystal. For the sum spectra obtained with the polarimeter, new standard shapes were generated using some well-known (p,γ) reactions yielding monoenergetic γ rays, or a few γ rays of well-separated energies. The reaction $C^{12}(p,\gamma)N^{13}$ at the resonance energy $E_p=459$ keV was used not only to obtain a standard shape, but also to check for any inherent spurious asymmetry of the apparatus. The 2.367-MeV γ ray corresponds to the transition from the $J^\pi=\frac{1}{2}^+$ first excited state of N^{13} to the $J^\pi=\frac{1}{2}^-$ ground state, and is therefore not polarized. The observed ratio $N(0°)/N(90°)$ between scattering intensities in the reaction plane and normal to it was 0.991 ± 0.018. A typical polarimeter spectrum and the results of the stripping computation are shown in Fig. 8.

III. ANALYSIS OF THE DATA

The angular-correlation data were analyzed by the same techniques that were used in Ref. 2, which are based on the formalism of Harris, Hennecke, and Watson.[9] It is also convenient to use this formalism in the analysis of polarization data. Formulas for the polarization in terms of the coefficients of Ref. 9 are developed in a paper by Watson and Harris, which also gives tables of numerical values of the coefficients.[10]

The linear-polarization distribution of a γ ray emitted at an angle θ to the proton-beam direction (z axis) can be expressed as

$$W(\theta,\phi)=W(\theta)\pm\sum_m P_m\sum_\nu \cos2\phi B_{\nu m}P_\nu{}^2(\cos\theta)\,, \quad (1)$$

where ϕ is defined as the angle between the electric vector of the emitted γ ray and the plane defined by the z axis and the γ-ray direction. The plus sign is used if no parity change occurs in the transition and the minus sign is used if parity change does occur. The angular distribution is given by

$$W(\theta)=\sum_m P_m\sum_\nu A_{\nu m}P_\nu(\cos\theta)\,, \quad (2)$$

where for a primary γ ray

$$A_{Km}=\sum_{L_1L_1'}\frac{\delta_1{}^{p_1}}{1+\delta_1{}^2}(2K+1)^{1/2}E_{K0}{}^0(J_1L_1L_1'J_2m)\,, \quad (3)$$

while for a secondary γ ray

$$A_{Mm}=\sum_{L_1L_1'\cdots L_iL_i'\cdots L_eL_e'}\frac{\delta_1{}^{p_1}\cdots\delta_i{}^{p_i}\cdots\delta_e{}^{p_e}}{(1+\delta_1{}^2)\cdots(1+\delta_i{}^2)\cdots(1+\delta_e{}^2)}\times(2M+1)^{1/2}E_{0M}{}^0(J_1L_1L_1'J_2m)\cdots\times u_M(J_iL_iJ_j)\cdots h_M(J_eL_eL_e'J_f)\,. \quad (4)$$

In Eqs. (3) and (4), $p_j=0$, 1, or 2 for pure L_j, mixed L_j-L_j', or pure L_j' radiation, respectively.

The expression for $B_{\nu m}$ in Eq. (1) for a primary γ ray is

$$B_{Km}=\sum_{L_1L_1'}\frac{\delta_1{}^{p_1}}{1+\delta_1{}^2}\eta_K(L_1L_1')E_{K0}{}^0(J_1L_1L_1'J_2m)\,, \quad (5)$$

and for a secondary γ ray

$$B_{Mm}=\sum_{L_1L_1'\cdots L_iL_i'\cdots L_eL_e'}\frac{\delta_1{}^{p_1}\cdots\delta_i{}^{p_i}\cdots\delta_e{}^{p_e}}{(1+\delta_1{}^2)\cdots(1+\delta_i{}^2)\cdots(1+\delta_e{}^2)}\times E_{0M}{}^0(J_1L_1L_1'J_2m)\cdots u_M(J_iL_iJ_j)\cdots\times\eta_M(L_eL_e')h_M(J_eL_eL_e'J_f)\,. \quad (6)$$

The coefficients $E_{K0}{}^0$, $E_{0M}{}^0$, u_M, h_M, and η_ν in Eqs. (3) to (6) are tabulated in Ref. 10. The P_m and δ_i are the population parameters of the magnetic substates and the multipolarity mixing ratios, respectively. The $P_\nu(\cos\theta)$ and $P_\nu{}^2(\cos\theta)$ are Legendre polynomials.

[7] F. D. Lee and D. D. Watson (to be published).

[8] H. D. Graber and D. D. Watson, Nucl. Instr. Methods 43, 355 (1966).

[9] G. I. Harris, H. J. Hennecke, and D. D. Watson, Phys. Rev. 139, B1113 (1965).

[10] D. D. Watson and G. I. Harris, Nuclear Data (to be published).

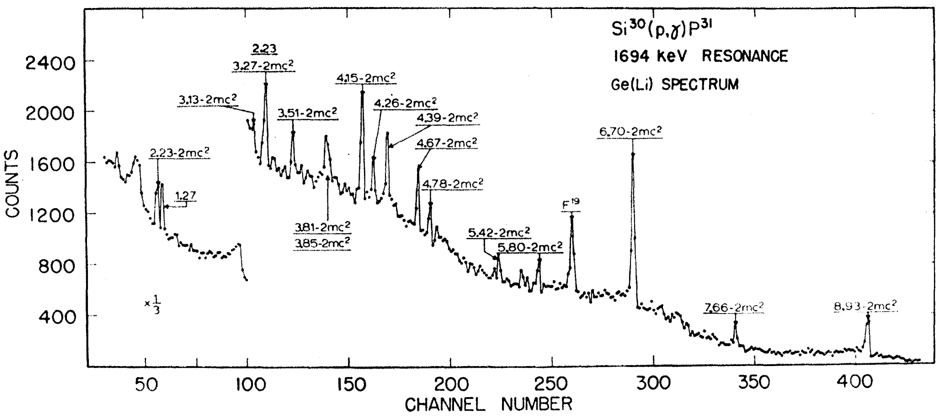

FIG. 1. Ge(Li) spectrum taken at the Si^{30}-$(p,\gamma)P^{31}$ resonance at $E_p=1694$ keV. The energies are labeled in MeV.

The degree of linear polarization is defined as

$$P(\theta)=\frac{W(\theta,90°)-W(\theta,0°)}{W(\theta,90°)+W(\theta,0°)}$$
$$=\pm\frac{\sum_m P_m \sum_\nu B_{\nu m} P_\nu{}^2(\cos\theta)}{\sum_m P_m \sum_\nu A_{\nu m} P_\nu(\cos\theta)}, \quad (7)$$

where the sign convention is the same as in Eq. (1). $P(\theta)$ is related to the numbers of scattered quanta $N_{\parallel}$ and $N_{\perp}$ observed with a Compton polarimeter by a detector located in the plane defined by the emitted γ ray and the z axis and a detector perpendicular to this plane, respectively. The relationship is

$$\frac{N_{\perp}-N_{\parallel}}{N_{\perp}+N_{\parallel}}=pP(\theta). \quad (8)$$

The polarization analyzing power p for "point" detectors and scatterer is given by[11]

$$p=\frac{\sin^2\beta}{k_0/k+k/k_0-\sin^2\beta}, \quad (9)$$

where k_0 and k are the wave numbers of the incoming and scattered radiation, respectively, and β is the Compton scattering angle. In the present work, effective values of p, taking into account finite size effects, were obtained by numerical integrations over detector and scatterer volumes. The results of these calculations are shown in Table I. Effective values of p at other energies were obtained by interpolation.

TABLE I. Polarimeter analyzing power as a function of γ-ray energy.

E_γ (MeV)	p
1.0	0.178
1.5	0.176
2.0	0.173
2.5	0.169
3.0	0.164
3.5	0.158
4.0	0.146
4.5	0.131
5.0	0.112

IV. RESULTS

1. Decay Schemes

A coincidence spectrum was taken at the 1177-keV resonance, gated by the primary γ ray feeding the 2.23-MeV level. A small peak appeared in the spectrum at 1.27 MeV, presumably due to a weak primary transition to the first excited state. There was no indication of a peak at 0.96 MeV, the energy difference between the first two excited states. The upper limit of the branching ratio of the decay of the 2.23-MeV level to the 1.27-MeV level was reduced from its previous value of 3%[12] to 0.8%.

The 8.93-MeV virtual level, corresponding to the 1694-keV resonance, was assigned $J^\pi=\frac{3}{2}^{(+)}$ by Van Rinsvelt and Endt.[4] Branching ratios for the decays of this level and of some levels fed by this resonance were obtained from a number of spectra taken with the 8 by 8 in. crystal. The transition energies were accurately determined by a Ge(Li) spectrum, shown in Fig. 1, to aid in stripping the NaI(Tl) spectra. The resulting branching ratios for the 8.93-MeV level are shown in Fig. 2, and are compared with those of Van Rinsvelt and Endt[4] in Table II. The discrepancies concerning transitions to the 3.51-, 4.26-, 5.01-, and 5.66-MeV levels are probably due primarily to the limited accuracy of energy determinations in a NaI(Tl) spectrum and the problems in stripping this spectrum which contains five pairs of unresolved γ rays.

The recently discovered 5.12- and 5.66-MeV

[11] M. Suffert, P. M. Endt, and A. M. Hoogenboom, Physica **25**, 659 (1959).

[12] A. E. Litherland, E. B. Paul, G. A. Bartholomew, and H. E. Gove, Can. J. Phys. **37**, 53 (1959).

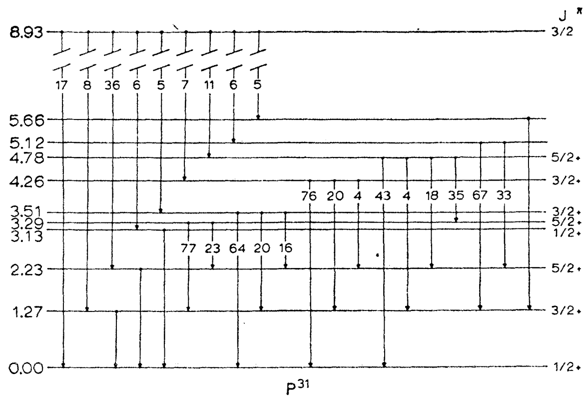

FIG. 2. Decay scheme of the 8.93-MeV level of P^{31}, corresponding to the $E_p=1694$-MeV resonance of the $Si^{30}(p,\gamma)P^{31}$ reaction. The level energies are shown in MeV.

levels[4,13–15] are fed by the 1694-keV resonance. The 5.12-MeV level decays to the 1.27- and 2.23-MeV levels with branching ratios of $(67\pm9)\%$ and $(33\pm9)\%$, respectively. The large uncertainties stem from the complexity of the spectrum at the 1694-keV resonance. The 5.66-MeV level decays strongly to the 1.27-MeV level. No other decay modes were observed.

The 4.78-MeV level has been known since the inelastic-proton-scattering work of Endt and Paris in 1957.[16] A later $P^{31}(p,p')P^{31}$ experiment indicated that it might decay strongly to the 3.13-MeV level.[17] The level is fed by a number of $Si^{30}(p,\gamma)P^{31}$ resonances, but usually rather weakly.[4,18] Its decay scheme was determined in the present work from the coincidence spectra of triple-correlation measurements at the 1694-keV resonance. The transition energies involved were found from the Ge(Li) spectrum. Contributions from the tails of higher energy γ rays included in the coincidence gate had to be taken into account. These corrections were obtained by calculating the fraction of a standard-shape spectrum for each energy falling in the window, and multiplying this number by known branching ratios of expected coincident γ rays. Branching ratios obtained in the analysis of the singles spectra at the 1694-keV resonance as well as branching ratios obtained for various bound states in previous work were used.

A small peak seen in the coincidence spectra, corresponding to a 3.51-MeV transition, could only partially be accounted for by coincidences with the tail of the 5.42-MeV transition which feeds the 3.51-MeV level.

[13] P. Kossanyi-Demay, R. M. Lombard, and G. R. Bishop, Nucl. Phys. **62**, 615 (1965).
[14] G. M. Crawley and G. T. Garvey, Bull. Am. Phys. Soc. **10**, 526 (1965); G. M. Crawley, thesis, Princeton University, 1965 (unpublished).
[15] B. Cujec, W. G. Davies, W. K. Dawson, T. B. Grandy, G. C. Neilson, and K. Ramavataram, Phys. Letters **15**, 266 (1965).
[16] P. M. Endt and C. H. Paris, Phys. Rev. **106**, 764 (1957).
[17] T. Wakatsuki *et al.*, in *Proceedings of the Kingston Conference* (University of Toronto Press, Toronto, Canada, 1960), p. 971.
[18] G. I. Harris and L. W. Seagondollar, Phys. Rev. **128**, 338 (1962).

TABLE II. Branching ratios of the decays of the 8.93-MeV level.

Transition energy (MeV)	Final-state energy (MeV)	Branching ratio (%) Van Rinsvelt and Endt[a]	Present work
8.93	0.00	22	17 ± 1
7.66	1.27	6	8 ± 1
6.70	2.23	33	36 ± 1
5.80	3.13	7	6 ± 1
5.42	3.51	<2	5 ± 1
4.67	4.26		7 ± 1
4.15	4.78	20	11 ± 2
3.92	5.01	12	
3.81	5.12		6 ± 1
3.27	5.66		5 ± 1

[a] Reference 4.

The remainder is presumably due to a weak decay of the 4.78-MeV level to the first excited level. A strong 1.49-MeV transition to the 3.29-MeV level probably corresponds to the γ-ray peak observed in the proton-scattering work, which was thought to indicate a decay to the 3.13-MeV level.[17] Other decay modes are to the ground and second excited states. The branching ratios to the indicated levels are as follows: ground state, $(43\pm2)\%$; 1.27-MeV level, $(4\pm2)\%$; 2.23-MeV level, $(18\pm2)\%$; 3.29-MeV level, $(35\pm2)\%$.

2. Angular-Correlation Measurements

Angular-correlation measurements were performed at the 1694-keV resonance in order to determine the spin of the 4.78-MeV level. As a result of these measurements, the spins of all levels in P^{31} up to 5.01-MeV excitation energy are now known. Triple-correlation analyses were carried out on the following cascades: $r\rightarrow4.78\rightarrow3.29$, $r\rightarrow4.78\rightarrow2.23$, and $r\rightarrow4.78\rightarrow3.29\rightarrow1.27$. An attempted analysis on the cascade $r\rightarrow4.78\rightarrow0$ failed because of difficulties encountered in separating the contributions of the 4.67-MeV primary to the 4.26-MeV level from the 4.78-MeV peak. Similarly, the cascade $r\rightarrow4.78\rightarrow1.27$ could not be analyzed successfully, since the 3.51-MeV peak was rather weak, and the ground-state decay of the 3.51-MeV level accounted for about half of it.

Searches for best fits were made for the triple-correlation data on the $r\rightarrow4.78\rightarrow3.29$ and $r\rightarrow4.78\rightarrow2.23$ cascades for $J(4.78)=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$. The goodness of fit was given by the parameter Q^2, defined in Ref. 2 as

$$Q^2=\frac{1}{A-P-q}\sum_a(W_a-W_a^*)^2\omega_a^2\,, \qquad (10)$$

where W_a and W_a^* are experimental and calculated values of the correlations, ω_a is the inverse of the standard deviation of W_a, A is the number of observation points, P is the number of population parameters, and q is the number of mixing ratios varied. The mixing ratios resulting in the lowest Q^2 values (designated as χ^2) for each case are shown in Table III. Projections

TABLE III. Summary of the angular-correlation analysis at the 1694-keV resonance. The abbreviations T.C. and A.D. identify triple correlations and angular distributions, respectively.

Cascade	$J(4.78)$	χ^2	δ_1	δ_2	δ_3
$r \to 4.78 \to 3.29$ T.C.	$\frac{1}{2}$	8.95	-0.10 ± 0.10		
			$2.0_{-0.5}^{+1.5}$		
	$\frac{3}{2}$	6.07	$4.6_{-1.0}^{+4.0}$	-0.13 ± 0.15	
				$16.0_{-11.0}^{+\infty}$	
	$\frac{5}{2}$	1.41	-0.02 ± 0.04	0.05 ± 0.06	
				$-1.70_{-0.40}^{+0.20}$	
	$\frac{7}{2}$	1.35	-0.47 ± 0.05	-0.47 ± 0.06	
$r \to 4.78 \to 2.23$ T.C.	$\frac{1}{2}$	1.39	-0.18 ± 0.10		
			$2.5_{-0.5}^{+1.0}$		
	$\frac{3}{2}$	1.11	$11.0_{-6.0}^{+\infty}$	-0.05 ± 0.15	
				$7.0_{-4.0}^{+\infty}$	
	$\frac{3}{2}$	1.39	$0.40_{-0.15}^{+0.30}$	0.12 ± 0.20	
				$2.8_{-1.0}^{+3.0}$	
	$\frac{5}{2}$	0.87	-0.02 ± 0.08	0.18 ± 0.11	
				$-2.5_{-2.0}^{+0.5}$	
	$\frac{7}{2}$	0.93	-0.35 ± 0.14	-0.30 ± 0.12	
$r \to 4.78 \to 3.29 \to 1.27$ T.C.	$\frac{5}{2}$	1.64	0.16 ± 0.08	0.05	-0.32 ± 0.08
	$\frac{5}{2}$	1.62	0.15 ± 0.07	-1.80	$-0.55_{-0.60}^{+0.25}$
	$\frac{7}{2}$	1.65	-0.17 ± 0.08	-0.49	∞
	$\frac{7}{2}$	1.83	-0.10 ± 0.10	-0.49	-0.28 ± 0.10
$r \to 4.78 \to 0.00$ A.D.	$\frac{5}{2}$	0.75	0.03 ± 0.05	-0.01 ± 0.12	
	$\frac{7}{2}$	0.38	-0.19 ± 0.03	$0.4_{-0.3}^{+2.3}$	
$r \to 4.78$ mean	$\frac{5}{2}$	1.39	0.03 ± 0.05		
	$\frac{7}{2}$	2.18	-0.25 ± 0.05		

showing the minimum Q^2 values obtainable for all possible values of the primary and secondary mixing ratios are shown in Figs. 3 and 4 for the case $J(4.78)=\frac{5}{2}$. In Fig. 7, the theoretical predictions resulting with mixing ratios which correspond to χ^2 in the spin-$\frac{5}{2}$ case are compared with the experimental correlations.

For the cascade to the 3.29-MeV level, spins $\frac{1}{2}$ and $\frac{3}{2}$ are seen to result in unacceptably large values for χ^2. In the cascade to the 2.23-MeV level, reasonable fits can be obtained for these spins, but significantly lower χ^2 values are again obtained with spins $\frac{5}{2}$ and $\frac{7}{2}$. The cascade $r \to 4.78 \to 3.29 \to 1.27$, with the transition $4.78 \to 3.29$ unobserved, was analyzed with $J(4.78)=\frac{5}{2}$ and $\frac{7}{2}$. Projections of the Q^2 surface are shown in Figs. 5 and 6 for the two mixing ratios obtained for the $4.78 \to 3.29$ transition in the above analysis. Theoretical and experimental correlations are compared in Fig. 7. From the figures and Table III it is seen that acceptable fits result for both spin $\frac{5}{2}$ and $\frac{7}{2}$. Q^2 minima at very large values of δ_3 can be ignored, since the mixing ratio for the $3.29 \to 1.27$ transition is known to be $\delta_3=-0.44 \pm0.02$.[2] The arrival of a 40 cm^3 Ge(Li) detector made it possible to measure the angular distributions of the 4.15- and 4.78-MeV γ rays in a reasonable amount of time. The presence of the 4.26- and 4.69-MeV γ rays make these measurements subject to large errors if NaI(Tl) detectors are used. An improvement in the target cooling technique permitted use of beam currents up to 40 μA, further facilitating the measurements.

Angular distributions were measured at 0°, 45°, and

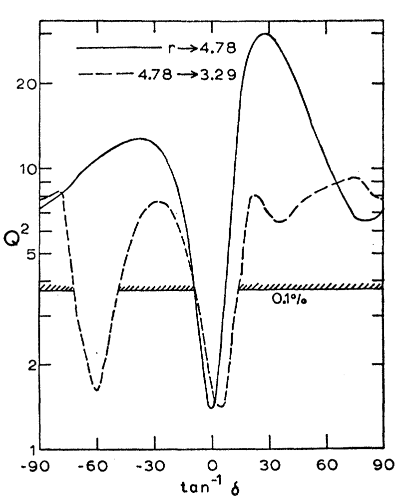

FIG. 3. Projections of the Q^2 surface for the cascade $r \to 4.78 \to 3.29$ at the 1694-keV resonance for $J(4.78)=\frac{5}{2}$.

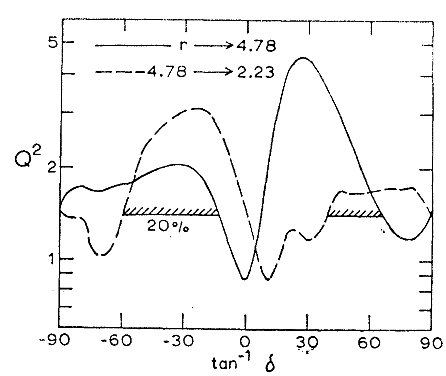

FIG. 4. Projections of the Q^2 surface for the cascade $r \to 4.78 \to 2.23$ at the 1694-keV resonance for $J(4.78)=\frac{5}{2}$.

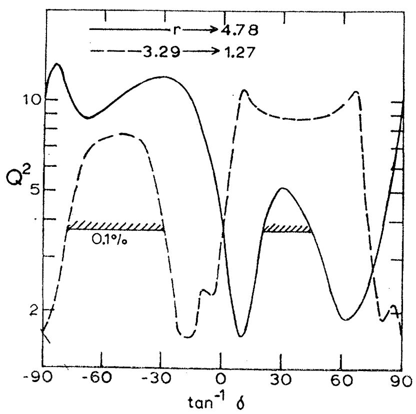

FIG. 5. Projections of the Q^2 surface for the cascade $r \to 4.78 \to 3.29 \to 1.27$ at the 1694-keV resonance for $J(4.78)=\frac{5}{2}$ and $\delta(4.78 \to 3.29)=0.05$.

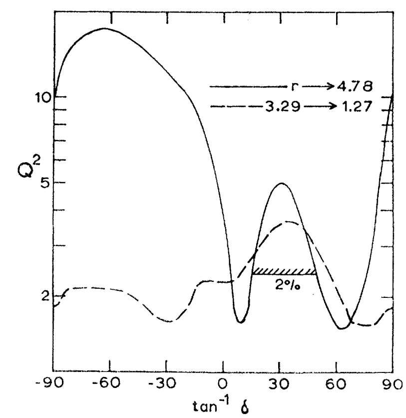

FIG. 6. Projections of the Q^2 surface for the cascade $r \to 4.78 \to 3.29 \to 1.27$ at the 1694-keV resonance for $J(4.78)=\frac{5}{2}$ and $\delta(4.78 \to 3.29)=-1.80$.

90°, and the data were analyzed with $J(4.78)=\frac{5}{2}$ and $\frac{7}{2}$. The results are shown in Table III. For spin $\frac{7}{2}$, an unlikely admixture of 2^4-pole radiation in the 4.78-MeV γ ray is implied by the range of values of δ_2 which give a good fit to the data.

The last entries in Table III show the Q^2 minima obtained for the mixing ratio of the 4.15-MeV γ ray when the three triple-correlation analyses and the angular-distribution analysis were folded together. Before folding, the individual Q^2 projections were normalized to $\chi^2=1.00$. Each projection was weighted by the number of degrees of freedom in the corresponding analysis. Spin $\frac{5}{2}$ is seen to result in a significantly lower value of χ^2.

The resonance strength of Ref. 4 and the branching ratio of the 4.15-MeV transition determined in the present work imply a width $\Gamma_\gamma=(0.047\pm0.014)$ eV for this transition. In the angular-distribution measurements, a Doppler shift of about 8 keV was observed between the spectra at 0° and 90° for both the 4.15- and the 4.78-MeV transition. It can be concluded from this evidence that the 4.78-MeV transition has a width $\Gamma_\gamma>0.02$ eV.

Let $\Gamma_w(EL)$ and $\Gamma_w(ML)$ be the extreme single-particle model estimates of the width assuming EL and ML transitions. For the 4.78-MeV transition we have then

$$\Gamma_w(E2)=1.2\times10^{-2}\text{ eV},$$
$$\Gamma_w(M2)=3.7\times10^{-4}\text{ eV},$$
$$\Gamma_w(E3)=1.3\times10^{-6}\text{ eV},$$
$$\Gamma_w(M3)=3.8\times10^{-8}\text{ eV}.$$

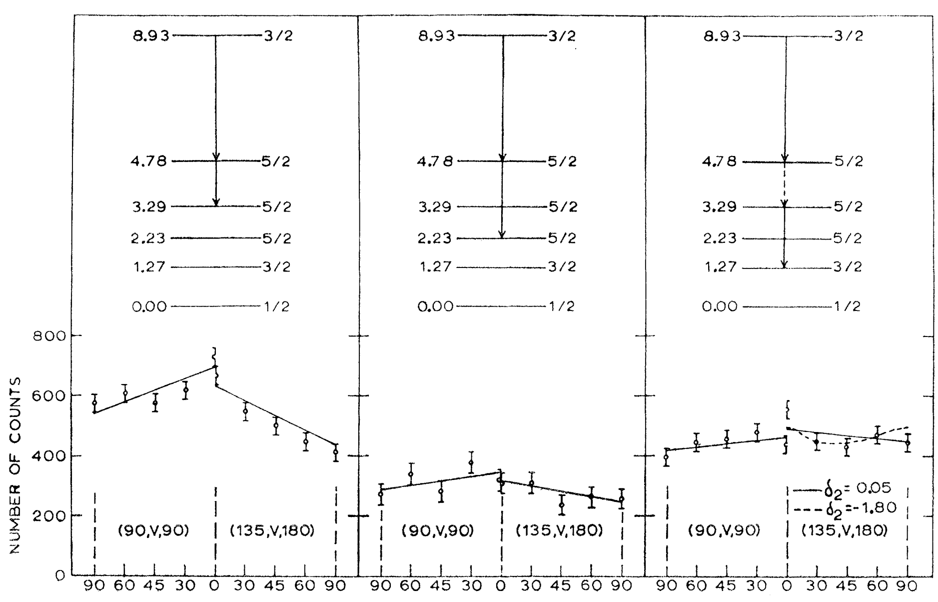

FIG. 7. Comparison between theoretical and experimental triple-correlations at the 1694-keV resonance for the best values of the mixing ratios with $J(4.78)=\frac{5}{2}$. Level energies are in MeV.

TABLE IV. Properties of the resonance levels used in the polarization measurements. Column 1 lists the level energies in MeV, column 2 the energy levels (in MeV) to which the resonance decays, and column 3 the branching ratios of the decays in percent

E_p (keV)	J^π	1	2	3	References
1204	$\frac{5}{2}+$	8.45	1.27	7	a, b
			2.23	27	
			3.29	32	
			3.51	10	
			4.26	24	
1322	$\frac{5}{2}+$	8.57	0.00	5	a, b, c
			1.27	32	
			2.23	21	
			3.51	30	
			4.26	12	
1509	$\frac{5}{2}+$	8.75	1.27	56	d, b
			2.23	10	
			3.41	28	
			4.43	4	
			6.4	2	
2187	$\frac{7}{2}-$	9.40	4.43	>95	e

[a] Reference 3.
[b] Reference 19.
[c] J. Walinga, H. A. Van Rinsvelt, and P. M. Endt, Physica 32, 954 (1966).
[d] Reference 2.
[e] G. I. Harris, H. J. Hennecke, and F. W. Prosser, Jr., Phys. Letters 9, 324 (1964).

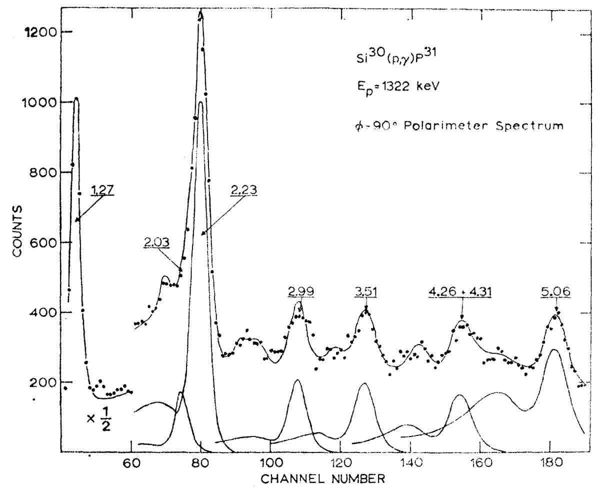

FIG. 8. Polarimeter spectrum accumulated at the 1322-keV resonance with the final detector at $\phi=90°$. The total fitted spectrum includes the contributions from some relatively weak γ-rays, whose partial spectra are not shown in order to avoid cluttering the drawing. The energies are labeled in MeV.

Therefore $J(4.78)=\frac{7}{2}$ is also ruled out on the basis of the width of the 4.78-MeV transition. Furthermore, $J^\pi=\frac{5}{2}-$ for the 4.78-MeV level would require an $M2$ transition, and thus an enhancement by at least a factor of 50 over the Weisskopf estimate. Thus the 4.78-MeV level must have $J^\pi=\frac{5}{2}+$.

3. Polarization Measurements

The properties of the resonances used for polarization measurements are shown in Table IV, and the results of the polarization analysis in Table V. Table VI summarizes the known properties of the bound levels of P^{31}. The theoretical values P_{th} were calculated using the previously known properties of the P^{31} levels and their decays as well as the properties determined in this work. The uncertainties in the values of P_{th} reflect the uncertainties in the known mixing ratios. The uncertainties indicated for the experimental values P_{ex} are based on the uncertainties of the γ-ray intensities given by the stripping program.

From Eq. (7) and the selection rules for electromagnetic transitions it is seen that changing the parity of one of the levels between which a transition occurs changes the sign of the polarization of the corresponding γ ray, but does not change the absolute value of the polarization. As a result it was only necessary for most transitions of interest to accumulate sufficient statistics to determine unambiguously the sign of the polarization.

The experimental polarizations of the 2.02-MeV γ rays observed at the 1204- and 2187-keV resonances show that the 3.29-MeV level must have even parity. The somewhat large uncertainty at the 2187-keV reso-

TABLE V. Summary of the linear polarization analyses. Levels are identified by their energies in MeV. Column (a) lists the energy in MeV of the γ ray whose polarization is considered. Column (b) gives the parameters varied in the analyses and their resultant values. Properties not listed in column (b) had the values shown in Table VI.

Cascade	a	P_{th}	P_{ex}	b
8.45 → 3.29 → 1.27	5.16	$\pm(0.89\pm0.01)$	$+0.68\pm0.39$	$\pi(3.29)=+$
	2.02	$\pm(0.65\pm0.02)$	-0.54 ± 0.11	$\pi(3.29)=+$
8.45 → 3.51	4.94	$\pm(0.73\pm0.06)$	-0.79 ± 0.41	$\pi(8.45)=+$
8.45 → 4.26 → 0.00	4.19	$\pm(0.57\pm0.03)$		
	4.26	$\pm(0.72\pm0.02)$		
	$\begin{Bmatrix}4.19\\4.26\end{Bmatrix}$	$\begin{Bmatrix}\pm(0.63\pm0.03)\\\pm(0.07\pm0.04)\end{Bmatrix}$	-0.42 ± 0.24	$\begin{Bmatrix}\pi(8.45)=+\\\delta_2=-0.32\end{Bmatrix}$
8.45 → 4.26 → 1.27	2.99	$\pm(0.43\pm0.04)$	$+0.46\pm0.22$	$\pi(4.26)=+$
8.57 → 3.51 → 0.00	5.06	$\pm(0.72\pm0.02)$	-0.49 ± 0.21	$\pi(3.51)=+$
	3.51	$\pm(0.72\pm0.02)$	-0.88 ± 0.25	$\delta_2=-0.41$
8.75 → 3.41 → 1.27	2.14	$\pm(0.75\pm0.02)$	$+0.65\pm0.23$	$\pi(3.41)=+$
	5.34	$\pm(0.32\pm0.02)$	-0.51 ± 0.29	$\pi(8.75)=+$
9.40 → 4.43 → 3.29 → 1.27	2.02	$\pm(0.69\pm0.01)$	-0.22 ± 0.35	$\pi(3.29)=+$

TABLE VI. The properties of the bound levels of P^{31}. Under γ_0, γ_1, γ_2, and γ_4 are listed the branching ratios of decays to the ground state, 1.27-, 2.23-, and 3.29-MeV levels (in percent) and the mixing ratios of these transitions. Additional levels have been observed at 5.53, 5.77, 5.89, 6.05, 6.18, and 6.24 MeV. The uncertainties in the level energies refer to the last decimal place. An asterisk in the reference column indicates that some of the properties listed were determined in the present work. [*Note added in proof*: The results of very recent high-resolution Ge(Li) measurements at Utrecht show that the properties listed for the 4.633-MeV level (taken from Ref. 2) belong, instead, to the 4.590-MeV level. The 4.633-MeV level has most probable spin values of $\frac{5}{2}$ or $\frac{7}{2}$. (P. M. Endt, private communication)].

Energy (MeV)	J^π	γ_0	γ_1	γ_2	γ_4	References
0	$\frac{1}{2}^+$					a
1.265±3	$\frac{3}{2}^+$	100 0.28±0.02				a, b
2.232±4	$\frac{5}{2}^+$	100 0.0	≤0.8			a, *
3.133±4	$\frac{1}{2}^+$	100 0.0				a, c, d
3.292±4	$\frac{5}{2}^+$	0.8±1.2	77±2 −0.44±0.02	23±2 −0.41±0.06		a, e, f, g, *
3.413±5	$\frac{7}{2}^+$		100 0.0			a, h, b, *
3.505±5	$\frac{3}{2}^+$	64±3 −0.41±0.03	20±14 0.06±0.19	16±14 $-0.05_{-0.11}^{+1.05}$		a, e, f, *
4.188±5	$\frac{5}{2}^+$		65 0.25±0.03 or 1.8±0.4	35		a
4.257±5	$\frac{3}{2}^+$	76±3 −0.32±0.04	20±3 −0.25±0.05	4±3		a, e, f, *
4.430±5	$\frac{7}{2}^-$			55 −0.04±0.04	45 −0.012±0.014	a, i
4.590±5	$\frac{5}{2}$				100	a
4.633±5	$\frac{3}{2}$	27±4 0.07±0.04 or 1.48±0.12	52±4 −0.02±0.05 or −3.9±1.0	21±6		a, e
4.784±5	$\frac{5}{2}^+$	43±2 −0.01±0.12	4±2	18±2 0.18±0.11 or $-2.5_{-2.0}^{+0.5}$	35±2 0.05±0.06 or $-1.70_{-0.40}^{+0.20}$	a, *
5.012±5	$\frac{3}{2}^-$	70	30			a, b, j
5.12±10			67±9	33±9		c, *
5.25±10	$\frac{1}{2}^+$					c
5.66±10			100			c, *
6.38±10	$\frac{3}{2}^+$					c, j
6.46±10	$\frac{1}{2}^-$					c, b, j
6.61±10	$(\frac{1}{2}^-,\frac{3}{2}^-)$					c, j
7.15±10	$\frac{1}{2}^+$	100				c, j, k

[a] Reference 1.
[b] Reference 19.
[c] Reference 15.
[d] H. Van Rinsvelt and P. M. Endt, Phys. Letters 9, 266 (1964).
[e] Reference 2.
[f] Reference 5.
[g] H. Van Rinsvelt and P. M. Endt, Physica 32, 529 (1966).
[h] Reference 3.
[i] G. I. Harris, H. J. Hennecke, and F. W. Prosser, Jr., Phys. Letters 9, 324 (1964).
[j] W. G. Davies, W. K. Dawson, and G. C. Neilson, Phys. Letters 19, 576 (1965).
[k] P. F. Hinrichsen and C. P. Swann, Phys. Rev. 140, B549 (1965).

nance resulted from poor counting statistics. Operation of the accelerator 10% above its rated voltage gradually led to beam instabilities which made it necessary to stop the experiment. Additional verification of the even parity of the 3.29-MeV level resulted from the 5.16-MeV transition at the 1204-keV resonance, after the resonance parity had been determined through the study of the 3.51-MeV level. The presence of a 4.94-MeV transition of comparable intensity made the determination of the intensity of the 5.16-MeV γ ray subject to a larger error than would otherwise be the case. This is reflected in the large uncertainties of P_{ex} in Table V for the 4.94- and 5.16-MeV transitions at the 1204-keV resonance.

At the 1509-keV resonance, the polarization of the 2.14-MeV γ ray emitted in the transition from the 3.41-MeV level to the first excited state shows that the parity of the 3.41-MeV level is even. The 5.34-MeV primary γ ray was then used to confirm the tentative even-parity assignment to the resonance made in Ref. 19.

The spectrum at $\phi=90°$ for the 1322-keV resonance is shown in Fig. 8 as an example of a typical polarimeter spectrum. The curves represent the stripping results for the more intense γ rays and the total fitted spectrum. The 5.06-MeV primary γ ray at the 1322-keV resonance has negative polarization. The 3.51-MeV level must therefore have even parity. The 3.51-MeV secondary

[19] G. I. Harris and L. W. Seagondollar, Phys. Rev. 131, 787 (1963).

TABLE VII. Mixing ratios of various transitions in P^{31} determined in the present work.

Transition	δ
$3.51 \rightarrow 0.00$	-0.41 ± 0.03
$4.26 \rightarrow 0.00$	-0.32 ± 0.04
$4.78 \rightarrow 0.00$	-0.01 ± 0.12
$4.78 \rightarrow 2.23$	0.18 ± 0.11 or $-2.5_{-2.0}^{+0.5}$
$4.78 \rightarrow 3.29$	0.05 ± 0.06 or $-1.70_{-0.40}^{+0.20}$
$8.93 \rightarrow 4.78$	0.03 ± 0.05

γ ray to the ground state was shown to have a mixing ratio $\delta = -0.41$ or $\delta = 7.1$ in the angular-correlation work of Harris and Breitenbecher.[2] The measured polarization of this γ ray combined with the knowledge that the 3.51-MeV level has even parity, shows that the smaller mixing ratio is the correct one. The large mixing ratio would result in the value $P_{\text{th}} = +0.72$.

Knowing that the 3.51-MeV level has even parity, we were also able to confirm the tentative even parity assignment to the 1204-keV resonance[19] by measuring the polarization of the 4.94-MeV primary γ ray from the resonance to the 3.51-MeV level.

The polarization of the 2.99-MeV γ ray shows that the 4.26-MeV level has even parity, in agreement with Ref. 5. With this knowledge we could calculate the polarization expected for the 4.19-MeV γ ray, $P_{\text{th}} = -0.57 \pm 0.03$. The polarization expected for the 4.26-MeV γ ray is $P_{\text{th}} = -0.72 \pm 0.02$ if $\delta = -0.32$ and $P_{\text{th}} = +0.72 \pm 0.02$ if $\delta = 4.7$. The mean of the polarizations of the 4.19- and 4.26-MeV γ rays then becomes $P_{\text{th}} = -0.63 \pm 0.03$ or $P_{\text{th}} = -0.07 \pm 0.04$, depending on whether the small or large mixing ratio for the 4.26-MeV transition is correct. In determining the mean, the polarizations were weighted by the relative intensities of the two γ rays and by the values of their angular-distribution functions $W(\theta)$ evaluated at 90°. Our measurements show that $\delta = -0.32$.

The 4.19- and 4.26-MeV transitions also give additional evidence that the 1204-keV resonance parity is even. The assumption of odd-resonance parity would result in a weighted mean polarization of $P_{\text{th}} = +0.07 \pm 0.04$ or $P_{\text{th}} = +0.63 \pm 0.03$ when the 4.26-MeV transition has a mixing ratio $\delta = -0.32$ or $\delta = 4.7$, respectively.

In summary, our polarization measurements have shown that the 3.29-, 3.41-, 3.51-, 4.26-, 8.45-, and 8.75-MeV levels all have even parities. The last two levels are the virtual levels corresponding to the proton resonance energies of 1204 and 1509 keV. The mixing ratios determined in the present work are summarized in Table VII.

V. CONCLUSIONS

In their comparison of experimental results with model calculations, Harris and Breitenbecher pointed out that existing P^{31} data agreed best with weak-coupling unified models. The present work lends further support to this conclusion. The description of P^{31} states as $2s_{1/2}$ hole configurations weakly coupled to S^{32} core excitations by Crawley,[14] for example, predicts even parities for the 3.41- and 3.51-MeV levels, and $J^\pi = \frac{5}{2}+$ for the 4.78-MeV level. An alternative coupling scheme of Crawley's, which predicts $J^\pi = \frac{9}{2}+$ for the 4.78-MeV level, is contradicted by our results.

Similarly, Thankappan's weak-coupling calculations based on a Si^{30} core[20] predict even-parity states of spins $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ with excitation energies near 3 to 4 MeV. These states can be identified with the 3.29-, 3.41-, 3.51-, and 4.26-MeV levels. However, the large mixing ratio predicted for the transition from the 3.51-MeV level to the ground state is not borne out by our polarization measurements, which select the smaller of the two possible mixing ratios of Harris and Breitenbecher.[2] These weak-coupling calculations also fail, for reasonable values of the model parameter k,[20] to yield the very small upper limit of the decay of the 2.23-MeV level ($\frac{5}{2}+$) to the 1.27-MeV level ($\frac{3}{2}+$) relative to its decay to the ground state ($\frac{1}{2}+$). On the other hand, this small value is consistent with interpretations of the 2.23-MeV level as a $1d_{5/2}$ hole state.

ACKNOWLEDGMENTS

We wish to express our gratitude to D. D. Watson and A. K. Hyder, Jr. for their many helpful comments concerning the use of the polarimeter and of the computer programs. We also wish to express our appreciation of the invaluable assistance of D. V. Breitenbecher and W. A. Anderson in the maintenance and operation of the accelerator.

[20] V. K. Thankappan, Phys. Letters 2, 122 (1962).