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Angular Distribution of Gamma Rays from High-Spin States in Deformed Odd-A Nuclei*

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The angular distribution of γ rays emitted by deformed odd-A nuclei with high spin has been calculated. The results are applicable to emission from states populated during heavy-ion bombardment. The effect of polarization of the spin of the compound nucleus in a plane perpendicular to the heavy-ion beam is considered. Contributions from pure $E2$ and $M1$ radiations, as well as from interference between these two radiations, are included. It is shown how experimentally measured angular distributions can be used as a tool for determining both the magnitude and the relative phase of the amplitude of $E2$ and M1 transitions.

I. INTRODUCTION

A METHOD for calculating gamma emission from high-spin states in odd-A nuclei is presented. Attention is focused on highly deformed nuclei. Deformed nuclei display rotational bands, which are a manifestation of a collective degree of freedom. Therefore the calculation is based on the collective model and takes advantage of the properties of rotational states.

The present treatment shows how the anisotropy of the angular distribution of γ rays can be exploited to obtain information about nuclear structure. Since high-spin states are usually populated during heavy-ion bombardment, the spin of the compound nucleus is close to a plane perpendicular to the direction of the heavy-ion beam. This polarization is the origin of the anisotropy in the spectrum of emitted radiation has been confirmed by many workers, most recently by Diamond *et al.*[1] This alignment is affected very little by the neutron emission that precedes the γ-ray cascade.

To date, studies of angular distribution have been limited to γ emission from even-even nuclei. However, as Diamond *et al.*[1] point out, γ emission from odd-A, or odd-odd, nuclei produces interesting consequences. In particular, $E2$-$M1$ amplitude mixing is exhibited in such emission.

High-spin states have been identified in rotational bands in even-even nuclei.[2–5] Greenberg, Lark, and Diamond[4] observed excited states with spins up to 20 in the $K=0$ band. They observed that decay of the high-spin states was produced by a cascade of pure $E2$ transitions; the spin of the nucleus decreased by two units in each transition. Sperber[6] calculated the angular distribution for transitions between states with high spin within a rotational band in even-even nuclei.

The present paper shows that, for two reasons, much information about high-spin states can be obtained by studying the angular distribution of γ rays from odd-A nuclei. The two reasons are: (a) Even for the ground-states band, $K\neq 0$, so that a state with spin J decays to states with spins $J-2$ and $J-1$. (b) Three terms—$E2$, $M1$, and an interference due to the two types of transitions—contribute to the transition $J\rightarrow J-1$, whereas only $E2$ radiation contributes to the transition $J\rightarrow J-2$.

Recently, the importance of electromagnetic transitions in odd-A nuclei has been reconfirmed.[7–9] The present study explores the nuclear matrix element not only for $E2$ transitions but also for $M1$ transitions. It is shown that the angular distribution determines the relative phase of the amplitude of the two transitions.

The parity selection rules for $E2$ and $M1$ transitions are the same. Therefore, for states in which ΔJ is ± 1 or 0, the two radiations compete with one another. The significance of this competition in transitions between states with low spin has been recognized.[10–17] This competition is also important in the emission of radiation between states with high spin.[18] In highly excited states with high spin, statistical considerations are predominant. The decay probability is proportional to the density of final states, the density being spin-dependent. A state with spin J decays to states with spins

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ranging from $J+2$ to $J-2$. In such highly excited states, the detailed band structure is destroyed and mixing between bands occurs.

After neutron emission initiated by heavy-ion bombardment, however, the residual nucleus may be found in a discrete state of high spin belonging to a rotational band. In this case, the residual nucleus will be de-excited by cascading down the rotational band and emitting γ rays. The angular distribution of such γ rays due to transitions between high-spin states in rotational bands in odd-A nuclei is discussed in this paper.

The angular distribution of the first γ ray is less isotropic than the angular distribution of the following γ rays in the de-excitation cascade because each successive gamma emission causes a spread in the direction of the spin of the compound nucleus. In the present paper the angular distribution of the first γ rays, which displays maximum anisotropy, is evaluated explicitly. It is shown how the angular distribution of the other γ rays can be evaluated. Other aspects of γ emission in heavy-ion bombardment have been discussed previously.[19–25]

The theory is developed in the following section. The application of the theory to interpretation of γ-ray emission in transitions between high-spin states in odd-A nuclei is given in the last section.

II. THEORY

The probability per unit time of emitting a photon into a unit solid angle Ω is[26,27]

$$P(\Omega)d\Omega=\frac{1}{4hk^2}\left|\sum_{l,m}(-i)^{l+1}a_E(l,m)\mathbf{Y}_m^{l(l,1)}\times\mathbf{n}+a_M(l,m)\mathbf{Y}_m^{l(l,1)}\right|^2 d\Omega, \quad (1)$$

where a_E is the electric amplitude, a_M is the magnetic amplitude, $\mathbf{Y}_m^{l'(l,1)}$ is the vector spherical harmonics, and k is the wave number.

In Eq. (1), the electric and magnetic amplitudes depend on the charge and current distribution in the nucleus and are expressed in terms of the nuclear matrix elements. Because of selection rules and because of the energy dependence of the transition probability, only a few of these amplitudes contribute substantially to emitted radiation. The relation between the current and the magnetic field in rotating nuclei has been recently discussed by Grin and Zaitzev.[28]

The amplitudes a_E and a_M can be written in terms of the nuclear matrix elements as[27]

$$a_E(l,m)=-\frac{4\pi ik^{l+2}}{(2l+1)!!}\left(\frac{l+1}{l}\right)^{1/2}\langle\alpha_f|Q_m^l|\alpha_i\rangle, \quad (2a)$$

$$a_M(l,m)=\frac{4\pi ik^{l+2}}{(2l+1)!!}\left(\frac{l+1}{l}\right)^{1/2}\langle\alpha_f|M_m^l|\alpha_i\rangle, \quad (2b)$$

where $\langle\alpha_f|Q_m^l|\alpha_i\rangle$ and $\langle\alpha_f|M_m^l|\alpha_i\rangle$ are the nuclear matrices for the component of the electric and magnetic multipole tensors, respectively, for transitions from a state with quantum numbers α_i to a state with quantum numbers α_f.

For transitions within a rotational band in an odd-A nucleus, the main contributions to the radiation are due to the magnetic dipole and the electric quadrupole transitions. Considering only these transitions and using Eqs. (1) and (2), the probability per unit time per unit solid angle Ω of photon emission due to a transition from a state characterized by quantum numbers K, J, and M to a state characterized by quantum numbers K, J', and M', is

$$P(K;J,M;J',M';\Omega)=\frac{1}{2h}\frac{1}{k^3}\left|i\frac{k^4}{15}\left(\frac{3}{2}\right)^{1/2}\times\langle KJ'M'|Q_{M'-M}^2|KJM\rangle\mathbf{Y}_{M-M'}^{2(2,1)}x\mathbf{n}-\tfrac{1}{3}k^3\left(\frac{2}{1}\right)^{1/2}\times\langle KJ'M'|M_{M'-M}^1|KJM\rangle\mathbf{Y}_{M-M'}^{1(1,1)}\right|^2, \quad (3)$$

where J is the spin of the nucleus, K is the projection of the spin of the nuclear symmetry axis, and M is the projection of this spin on a space-fixed axis.

The matrix elements in Eq. (3) can be reduced by use of the Wigner-Eckert theorem[29]:

$$\langle KJ'M'|Q_{M'-M}|KJM\rangle=(-)^{J'-M'}\langle KJ'\|Q^2\|KJ\rangle\times\begin{pmatrix}J' & 2 & J\\ -M' & M'-M & M\end{pmatrix}, \quad (4a)$$

$$\langle KJ'M'|M_{M'-M^1}|KJM\rangle=(-)^{J'-M'}\langle KJ'\|M^1\|KJ\rangle\times\begin{pmatrix}J' & 1 & J\\ -M' & M'-M & M\end{pmatrix}. \quad (4b)$$

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The middle term of the right-hand side of Eq. (4) is a reduced matrix element, and the last term of the right-hand side is a Wigner 3-j coefficient.

The reduced matrix elements can be further simplified. By using (a) the transformation properties of the components of the multipole tensors between a body-fixed system and a space-fixed system,[29] (b) the appropriate wave function for the rotational states[30], and (c) Eq. (4), the reduced matrix elements can be written as

$$\langle J'K\|Q^2\|KJ\rangle=(-)^{-J'-K}[(2J+1)(2J'+1)]^{1/2}\Bigg[\langle\chi_K|Q_0{}^2|\chi_K\rangle\begin{pmatrix}J' & 2 & J\\ -K & 0 & K\end{pmatrix}+(-)^{J'+\Pi-(1/2)+2K}\langle\chi_K|Q_{-2K}{}^2|\chi_K\rangle\begin{pmatrix}J' & 2 & J\\ K & -2K & K\end{pmatrix}\Bigg], \quad (5a)$$

$$\langle J'K\|M^1\|JK\rangle=(-)^{-J'-K}[(2J+1)(2J'+1)]^{1/2}\Bigg[\langle\chi_K|M_0{}^1|\chi_K\rangle\begin{pmatrix}J' & 1 & J\\ -K & 0 & K\end{pmatrix}+(-)^{J'+\Pi-(1/2)+2K}\langle\chi_{-K}|M_{-2K}{}^1|\chi_K\rangle\begin{pmatrix}J' & 1 & J\\ K & -2K & K\end{pmatrix}\Bigg]. \quad (5b)$$

Here Π is the parity.

In Eq. (5), χ_K is the intrinsic wave function. For $K>\frac{1}{2}$ the terms $\langle\chi_{-K}|Q_{-2K}{}^2|\chi_K\rangle$ and $\langle\chi_{-K}|M_{-2K}{}^1|\chi_K\rangle$ vanish.

A state with a specified JM can decay to various substates with J'. The energy of all these γ rays is the same, but the angular distribution differs. The effective probability per unit time of emitting a photon into a unit solid angle Ω from a state characterized by quantum numbers K, J, and M to a state characterized by K, J', and all possible M' values is

$$P(K;J,M;J';\Omega)=\sum_{M'}P(K;J,M;J',M';\Omega). \quad (6)$$

For a transition from a state with spin J to a state with spin $J-2$, the only contribution comes from $E2$ transitions. Therefore the probability becomes

$$P(K;J,M;J-2,\Omega)=\frac{2\pi}{h}\frac{1}{150}k^5|\langle KJ-2\|Q^2\|KJ\rangle|^2\times\sum_{M'}\begin{pmatrix}J-2 & 2 & J\\ M' & -M'+M & -M\end{pmatrix}|\mathbf{Y}_{M'-M}{}^{2(2,1)}|^2. \quad (7)$$

For a transition from a state with spin J to a state with spin $J-1$, the probability can be written as a sum of three terms: P_1, P_2, and P_3. The first and second terms represent the contribution due to the electric quadrupole and the magnetic dipole radiations, respectively. The third term represents the contribution due to interference between these radiations.

$$P_1(K;J,M;J-1;\Omega)=\frac{4\pi}{h}\frac{1}{150}k^5|\langle KJ-1\|Q^2\|KJ\rangle|^2\times\sum_{M'}\begin{pmatrix}J-1 & 2 & J\\ M' & -M'+M & -M\end{pmatrix}^2|\mathbf{Y}_{M'-M}{}^{2(2,1)}|^2, \quad (8)$$

$$P_2(K,J,M;J-1;\Omega)=\frac{4\pi}{h}\frac{1}{9}k^3|\langle KJ-1\|M^1\|KJ\rangle|^2\times\sum_{M'}\begin{pmatrix}J-1 & 2 & J\\ M' & -M'+M & -M\end{pmatrix}^2|\mathbf{Y}_{M'-M}{}^{1(1,1)}|^2, \quad (9)$$

$$P_3(K,J,M;J-1;\Omega)=\frac{-4\pi i}{h}\left(\frac{3}{45}\right)^{1/2}k^4 2\langle KJ-1\|M^1\|KJ\rangle\langle KJ-1\|Q^2\|KJ\rangle\times\sum_{M'}\begin{pmatrix}J-1 & 2 & J\\ M' & -M'+M & -M\end{pmatrix}\begin{pmatrix}J-1 & 1 & J\\ M' & -M'+M & -M\end{pmatrix}\times[\mathbf{Y}_{M'-M}{}^{*2(2,1)}x\mathbf{n}]\cdot\mathbf{Y}_{M'-M}{}^{1(1,1)}. \quad (10)$$

The square of the vector spherical harmonics is given by Jackson.[27] The product of the vector spherical harmonics appearing in Eq. (10) can be easily derived as follows[29]:

$$i[\mathbf{Y}_{l,m}{}^{l(l,1)}x\mathbf{n}]=\left(\frac{l}{2l+1}\right)^{1/2}\mathbf{Y}_{l,m}{}^{l(l+1,1)}\times\left(\frac{l+1}{2l+1}\right)\mathbf{Y}_{l,m}{}^{l(l-1,1)}. \quad (11)$$

The expression for the probability per unit time of total photon emission due to a transition from one state to another is obtained by integrating $P(K;J,M;J',M')$ over all angles:

$$P(K;J,M;J',M')=\int P(K;J,M;J',M';\Omega)d\Omega. \quad (12)$$

By using Eq. (7), for example, the probability of total photon emission due to $E2$ transition from a state with

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spin J to a state with spin $J-2$ is

$$P(K;J,M;J-2,M')=\frac{2\pi}{h}\frac{1}{150}k^5|\langle KJ-2\|Q^2\|KJ\rangle|^2 \times\begin{pmatrix} J & 2 & J \\ M' & -M'+M & -M \end{pmatrix}^2. \quad (13)$$

Similar expressions can be obtained for other transitions.

Thus far, the expression for effective transition probability per unit time per unit solid angle and the expression for the effective integrated transition probability per unit time have been obtained for a transition from a sublevel with a given magnetic quantum number M. In nature, various sublevels are populated for a given J. When all sublevels are populated with an equal probability, the radiation is isotropic. In heavy-ion bombardment, however, sublevels with different magnetic quantum numbers are not equally populated. This is the origin of anisotropy.

Let $N(J,M)$ be the probability that a level with quantum numbers J and M is occupied. Later it is shown when and how the function $N(J,M)$ can be determined. Let $\bar{P}(K;J;J';\Omega)$ be the probability per unit time per unit solid angle of photon emission due to a transition from a state with quantum numbers K and J to a state with quantum numbers K and J'. Then

$$\bar{P}(K;J;J';\Omega)=\sum_M N(J,M)P(K;J,M;J;\Omega). \quad (14)$$

III. DISCUSSION

When a target nucleus is bombarded by heavy ions so that a compound nucleus is formed, this compound nucleus has an aligned high spin. The nucleus decays first by a neutron cascade, to form a residual nucleus, which, in turn, decays by a γ-ray cascade. If the nucleus emitting the γ-ray cascade is in the deformed region, the transitions are within a rotational band.

When the occupation function after the termination of neutron emission is known, Eq. (12) can be used for an exact determination of the occupation function in the later stages of the cascade. When either the target or the projectile has zero spin and either the projectile or fhe target has spin J_0, the function $N(J,M)$ after the tormation of the compound nucleus is

$$N(J,M)=N_0 \quad \text{for} \quad |M|<J_0, \quad (15a)$$

$$N(J,M)=0 \quad \text{for} \quad |M|>J_0. \quad (15b)$$

Since the occupation function does not change appreciably during neutron emission, the form for $N(J,M)$ suggested by Eq. (15) can be used for the first few gamma rays in the cascade. Using Eqs. (14) and (15), the function $\bar{P}(K,J;J';\Omega)$ becomes, under the above conditions

$$\bar{P}(K;J;J';\Omega)=N_0\sum_{M=-J_0}^{J_0}P(K;J,M;\Omega). \quad (16)$$

For a transition from a state with spin J to a state with spin $J-2$, Eq. (16) reduces to

$$\bar{P}(K;J;J-2;\Omega)=\frac{8\pi N_0}{h150}k^5|\langle KJ-2\|Q^2\|KJ\rangle|^2\sum_{M=-J_0}^{J_0}\sum_{M'}\begin{pmatrix} J-2 & 2 & J \\ M' & -M+M & -M \end{pmatrix}|\mathbf{Y}_{M'-M}{}^{2(2,1)}|^2. \quad (17)$$

For a transition from a state with spin J to a state with spin $J-1$, Eq. (16) reduces to

$$\bar{P}(K;J;J-1;\Omega)=\frac{8\pi N_0}{h}\sum_{M=-J_0}^{J_0}\sum_{M'}\frac{k^5}{150}|\langle KJ-1\|Q^2\|KJ\rangle^2\begin{pmatrix} J-1 & 2 & J \\ M' & -M+M' & -M \end{pmatrix}^2|\mathbf{Y}_{M-M'}{}^{2(2,1)}|^2+\frac{i(3)^{1/2}}{45}$$

$$\times k^4\langle KJ-1\|Q^2\|KJ\rangle\langle KJ-1\|M^1\|KJ\rangle\begin{pmatrix} J-1 & 2 & J \\ M' & -M'+M & -M \end{pmatrix}\begin{pmatrix} J-1 & 1 & J \\ M' & -M'+M & -M \end{pmatrix}$$

$$\times[\mathbf{Y}_{M'-M}{}^{*2(2,1)}xn]\cdot\mathbf{Y}_{M'-M}{}^{1(1,1)}+\frac{8}{9}k^3|\langle KJ-1\|M^1\|KJ\rangle|^2\begin{pmatrix} J-1 & 1 & J \\ M' & -M'+M & -M \end{pmatrix}^2|\mathbf{Y}_{M'-M}{}^{2(2,1)}|^2. \quad (18)$$

Equation (18) becomes particularly simple for $J_0=\frac{1}{2}$; for a transition from a state with spin J to a state with spin $J-2$, it reduces to

$$\bar{P}(K;J;J-2;\Omega)=\frac{N_0k^5}{480h}\frac{1}{(2J)(2J-2)(2J+1)}|\langle KJ-2\|Q^2\|KJ\rangle|^2$$

$$\times[(20J^2-32J+3)+(24J^2+48J+18)\cos^2\theta-(12J^2+48J+45)\cos^4\theta]. \quad (19)$$

Similarly, for a transition from a state with spin J to a state with spin $J-1$, the angular distribution becomes

$$\bar{P}(K;J;J-1;\Omega)=\frac{N_0}{h}\bigg\{\frac{k^5}{120}|\langle KJ-1\|Q^2\|KJ\rangle|^2\frac{1}{(2J+2)(2J+1)(2J)(2J-2)}$$

$$\times[(16J^3-22J-3)-(24J^3+36J^2-60J-90)\cos^2\theta+(24J^3+60J^2-54J-135)\cos^4\theta]+\frac{(5)^{1/2}}{30}$$

$$\times\frac{(2J+3)(J^2-1)^{1/2}}{(2J+2)(2J+1)(2J)}\langle KJ-1\|Q^2\|KJ\rangle\langle KJ-1\|M^1\|KJ\rangle(1-3\cos^2\theta)+\frac{16}{3}k^3|\langle KJ-1\|M^1\|KJ\rangle|^2$$

$$\times\frac{1}{(2J+1)(2J)}[(6J+1)-(2J+3)\cos^2\theta]\bigg\}. \quad (20)$$

When both the target and the projectile have a vanishing spin, the occupation function after the formation of the compound nucleus takes an especially simple form:

$$N(J,M)=\delta_{J,M}. \quad (21)$$

This renders the calculation even simpler.

When both the target and the projectile have appreciable spin values, the function $N(J,M)$ for a specified M depends on the possible number of ways in which this specified M value can be obtained. The calculation is a little more elaborate, but the method can be easily applied once $N(J,M)$ has been evaluated.

When $\langle KJ'\|Q^2\|KJ\rangle$ is known, Eq. (18) provides a convenient way to measure $\langle KJ'\|M^1\|KJ\rangle$. Conversely, when $\langle KJ'\|M^1\|KJ\rangle$ is known, $\langle KJ'\|Q^2\|KJ\rangle$ can be measured. In particular, $\langle KJ'\|Q^2\|KJ\rangle$ can be inferred from the total transition probabilities between the low-lying states in the same band. By using Eq. (5), the value of $\langle KJ'\|Q^2\|KJ\rangle$ for high-spin values can be calculated from its value for low-spin values.

This correlation therefore establishes a new method of evaluating $\langle KJ'\|M^1\|KJ\rangle$. The method is more efficient than the one derived from branching ratios, because measurement of lifetimes is not involved. Additional advantages are that in an angular distribution the measurement can be confirmed by measuring at a different angle and that the method provides more experimental data.

The interference term in Eq. (18) enables determination of the relative phases of the two reduced matrix elements.

Alternatively, both values of the squares of the reduced matrix elements can be obtained from transitions between low spin states, when available, by using the total transition probability, the branching ratios, and Eq. (5). This operation leaves only one free parameter in Eq. (18)—the relative phase of the two reduced matrix elements. Since measurements are performed for many angles, the predicted angular distribution can be used not only to determine this phase factor but also to check the consistency of the collective model for transitions between high spin states.

In summary, the calculated value of the angular distribution from high-spin states in odd-A nuclei populated during heavy ion bombardment can be used to determine one of the reduced nuclear matrix elements and also as a criterion for the classification of high-spin states as states of a rotational band.