Comprehensive Formalism for Nuclear-Reaction Problems. II. Applications—Giant Resonances and Doorway States

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The formal theory of nuclear reactions introduced in a preceding paper is applied to several phenomena of current interest. Particular attention is given to the derivation of a single-level form for the transition amplitude, the interpretation of this form being dependent on the physical situation involved. Under certain conditions the single-level form is appropriate for the deseription of an isolated resonance; under other conditions it corresponds to a giant resonance. The width of the giant resonance is discussed in detail, in particular the possibility of observing the so-called "spreading" width. The theory is used to describe "doorway" states and their connection with giant resonances. Some clarification of the relationship between earlier alternative approaches to this problem is obtained. The occurrence of statistical fluctuations near a giant resonance is considered and leads to the suggestion that fine structure and gross structure are intimately connected, the actual connection being simple in some cases. Several of these results are illustrated by considering the phenomena of isobaric analog resonances within the framework of the present formalism.

I. INTRODUCTION

THE general theory of nuclear reactions[1] developed in the previous paper (hereinafter referred to as I) is applied here to several phenomena of current interest. Because of the flexibility of the theory the particular type of phenomena to be described is obtained by choosing appropriate operators for H^0 and $\mathfrak{L}^0$ and then arranging the various terms into the most suitable form. The various expressions obtained in this way are all equivalent forms of the same basic formula and are only arranged into a particular form in order to highlight the interesting features of immediate concern.

First we obtain an exact "single-level" form for the transition amplitude in Sec. II without having to be too specific about the choice of H^0 or $\mathfrak{L}^0$. This particular form of the transition amplitude is most appropriate for describing isolated resonance phenomena. The resonant amplitude is found to reduce to the well-known Breit-Wigner form when there are no nearby thresholds or levels. The "potential" scattering is not described by a hard sphere but by an appropriate operator such as the optical-model potential.

The single-level formula may also be used to represent a size or giant resonance.[2] The present formalism allows the mixing or sharing of strength to be written exactly and no recourse to the use of a random-phase approximation or the use of projection operators is made. The concept of a "spreading" width (such as, for example, characterized in some situations by the imaginary part of the optical-model potential) for the giant resonance is shown to require careful consideration because the conventional expression (see, for example, Ref. 3) does not correspond to an observable quantity. The theory of giant resonances given in Sec. III suggests an alternative definition.

In recent years the concept of intermediate structure has become of increasing interest because it represents the early stages of compound-nucleus formation. This attempt to bridge the gap between simple direct interactions and the complex compound nucleus has been investigated by Block and Feshbach[4] and others[5] via the language of "doorway" states and projection-operator formalism.[6,7] Such investigations suggest that the giant resonance will break down into smaller substructures. Unfortunately, Block and Feshbach[4] were unable to relate their results to the more conventional intermediate model of Lane, Thomas, and Wigner.[2] Indeed, there appears to be some contradiction between the two approaches as to the expected width of a giant resonance. In Sec. IV the important results of both approaches are obtained from the same initial assumptions by use of different forms for the transition amplitude, and consequently no contradictions occur.

The existence of fluctuations or fine structure in nuclear reaction cross sections is usually explained by statistical arguments. The results of such assumptions are to a large extent independent of the particular formalism applied and it is therefore straightforward to

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[3] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

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[5] C. M. Shakin, Ann. Phys. (N. Y.) **22**, 47 (1963); A. K. Kerman, L. S. Rodberg, and J. E. Young, Phys. Rev. Letters **11**, 244 (1963); H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. (N. Y.) **41**, 230 (1967).

[6] H. Feshbach, Ann. Phys. (N. Y.) **5**, 357 (1958); **19**, 287 (1962).

[7] L. Fonda and R. G. Newton, Ann. Phys. (N. Y.) **10**, 490 (1960); A. Agodi and E. Eberle, Nuovo Cimento **18**, 718 (1960).

obtain the standard results[8,9] from the present theory. However, the possibility of a nonrandom contribution is not easily treated without resorting to a particular formalism. The present theory is useful in such situations, and in Sec. V it is applied in order to investigate the relationship between fine structure and gross structure, the latter being attributed to a nonrandom component of the compound system.

Several of the results obtained in Secs. II–V are very well illustrated by the phenomenon of isobaric analog resonance.[10] The four operators introduced in I are particularly convenient for describing the two similar nuclei involved in isobaric analog studies and yields a rigorous theory of such phenomena. Most of the results given here in Sec. VI have been obtained earlier,[11] but the present approach is more rigorous and indicates the relationship between several of the various approaches[11,12] to the problem of calculating isobaric spin impurities in heavy nuclei.

II. SINGLE-LEVEL FORMULA

One interesting application of nuclear-resonance theory is to the situation where one particular "compound-nucleus" state is singled out as being important. This leads to a particular separation of the collision matrix usually referred to as the single-level formula which under certain conditions reduces to the Breit-Wigner formula. Unfortunately, the definition of a compound-nucleus state is not unique because the operator $(H^0+\mathfrak{L}^0)$ introduced in I is arbitrary, e.g., it may be chosen to provide the conventional R-matrix[3,13] set of eigenstates or it could be chosen to provide the set of states considered by Feshbach.[6] Each approach leads to an exact single-level formula but with different interpretations as to the nature of the compound nucleus state actually responsible for the resonance phenomenon.

To derive an *exact* single level formula we need not be too specific about $(H^0+\mathfrak{L}^0)$ but simply require that the corresponding Green's operator $\mathcal{G}^0$ may be written in the form

$$\mathcal{G}^0=\mathcal{G}_\infty{}^0+|\lambda\rangle\langle\lambda|/(E_\lambda-E), \tag{2.1}$$

where $|\lambda\rangle$ is an eigenstate of $(H^0+\mathfrak{L}^0)$, i.e.,

$$(H^0+\mathfrak{L}^0)|\lambda\rangle=E_\lambda|\lambda\rangle. \tag{2.2}$$

For simplicity we assume that $(H^0+\mathfrak{L}^0)$ is chosen such that $|\lambda\rangle$ and E_λ are real. Non-Hermitian operators $(H^0+\mathfrak{L}^0)$ can be treated also, but a more cumbersome notation is required. Straightforward algebra yields the single-level result

$$\mathcal{G}=\mathcal{G}_\infty+|g_\lambda\rangle\langle\tilde{g}_\lambda|/(E_\lambda-E+\Phi_\lambda), \tag{2.3}$$

in which

$$\mathcal{G}_\infty=(1+\mathcal{G}_\infty{}^0h)^{-1}\mathcal{G}_\infty{}^0 \tag{2.4}$$

yields the scattering via all levels other than $|\lambda\rangle$. The Green's operator $\mathcal{G}_\infty$ involves a corresponding Hamiltonian H_∞ and a set of eigenstates $|\mu_\infty\rangle$ which are obtained by diagonalizing H in the subspace spanned by all states except $|\lambda\rangle$. The *resonance* states $|g_\lambda\rangle$ are not eigenstates of any Hamiltonian[14–17] but are defined by the relations

$$|g_\lambda\rangle=(1-\mathcal{G}_\infty h)|\lambda\rangle, \tag{2.5a}$$

$$\langle\tilde{g}_\lambda|=\langle\lambda|(1-h\mathcal{G}_\infty), \tag{2.5b}$$

and the quantity Φ_λ by either of the two equivalent expressions

$$\Phi_\lambda=\langle\lambda|h-h\mathcal{G}_\infty h|\lambda\rangle \tag{2.6a}$$

or

$$\Phi_\lambda=\langle g_\lambda|h+h^\dagger(\mathcal{G}_\infty{}^0)^\dagger h|g_\lambda\rangle, \tag{2.6b}$$

the dagger indicating Hermitian conjugate.

Transition amplitudes are obtained by substituting the above expression for $\mathcal{G}$ into either Eq. (47) or Eq. (51) of I, which leads to the two respective results

$$\begin{aligned}\mathcal{T}_{ba}=&\langle\chi_b{}^{(-)}|\mathfrak{L}\chi_a{}^{(+)}\rangle-\langle\mathfrak{L}^*\chi_b{}^{(-)}|\mathcal{G}_\infty|\mathfrak{L}\chi_a{}^{(+)}\rangle\\&-\langle\mathfrak{L}^*\chi_b{}^{(-)}|g_\lambda\rangle\langle\tilde{g}_\lambda|\mathfrak{L}\chi_a{}^{(+)}\rangle/\\&\times(E_\lambda-E+\Phi_\lambda), \qquad (2.7a)\\=&\mathcal{T}_{ba}{}^{\rm pot}+\mathcal{T}_{ba}{}^{(\lambda)}, \qquad (2.7b)\end{aligned}$$

and

$$\begin{aligned}\mathcal{T}_{ba}=&\langle\chi_b{}^{(-)}|V_b|\chi_a{}^{(+)}\rangle-\langle\chi_b{}^{(-)}|V_b\mathcal{G}_\infty V_a|\chi_a{}^{(+)}\rangle\\&-\langle\chi_b{}^{(-)}|V_b|g_\lambda\rangle\langle\tilde{g}_\lambda|V_a|\chi_a{}^{(+)}\rangle/\\&\times(\text{cf eq. 2.7a})(E_\lambda-E+\Phi_\lambda), \qquad (2.8a)\\=&\mathcal{T}_{ba}{}'^{(\rm pot)}+\mathcal{T}_{ba}{}'^{(\lambda)}, \qquad (2.8b)\end{aligned}$$

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[9] P. A. Moldauer, Phys. Rev. **135**, B642 (1964).

[10] J. D. Fox, C. F. Moore, and D. Robson, Phys. Rev. Letters **12**, 198 (1964); C. F. Moore, P. Richard, C. E. Watson, D. Robson, and J. D. Fox, Phys. Rev. **141**, 1166 (1966); G. Vourvopoulos and J. D. Fox, *ibid.* **141**, 1180 (1966).

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[12] A. M. Lane and J. M. Soper, Phys. Rev. Letters **7**, 420 (1961); Nucl. Phys. **37**, 663 (1962); C. Bloch and J. P. Schiffer. Phys. Letters **12**, 22 (1964); L. A. Sliv and Yu. I. Kharitonov, *ibid*, **16**, 176 (1965).

[13] E. P. Wigner and L. Eisenbud, Phys. Rev. **74**, 29 (1947).

[14] The "resonance states" $|g_\lambda(E)\rangle$ discussed here are, however, intimately connected with the complex eigenvalue states ("radioactive" states) introduced by Siegert (Ref. 15) and Humblet (Ref. 16). The "states" $|g_\lambda(E)\rangle$ used here satisfy the equation

$$[H+\mathfrak{L}(E)-E]|g_\lambda(E)\rangle=f(E)|\lambda\rangle,$$

with

$$f(E)=E_\lambda-E+\Phi_\lambda(E).$$

Now at the energy $E=\epsilon_\lambda$, where $f(E)=0$, we have the eigenvalue equation

$$[H+\mathfrak{L}(\epsilon_\lambda)-\epsilon_\lambda]|g_\lambda(\epsilon_\lambda)\rangle=0.$$

Clearly ϵ_λ is a pole of the S matrix and $\mathfrak{L}(\epsilon_\lambda)$ is the boundary condition operator appropriate for a radioactive state at the pole energy $E=\epsilon_\lambda$. For E sufficiently close to ϵ_λ, one easily obtains [using $\mathcal{G}_\infty(E)\approx\mathcal{G}_\infty(\epsilon_\lambda)$]

$$\mathcal{G}=\mathcal{G}_\infty(\epsilon_\lambda)+|g_\lambda(\epsilon_\lambda)\rangle\langle\tilde{g}_\lambda(\epsilon_\lambda)|/(\epsilon_\lambda-E),$$

which produces an S matrix of the form suggested by Rosenfeld and Humblet (Ref. 17).

[15] A. J. F. Seigert, Phys. Rev. **56**, 750 (1939).

[16] J. Humblet, thesis, Roy. Soc. Sci. Liege. Ser. 4, **7**, No. 4 (1952).

[17] J. Humblet and L. Rosenfeld, Nucl. Phys. **26**, 529 (1961).

where the matrix elements are understood to be limited to the "internal" region $\gamma_c \leqslant a_c$, this region being arbitrarily large if desired. The notation $\chi_c^{(\pm)}$ is an arbitrary representation for the $\mathcal{T}$ operator which can be chosen to be distorted waves with plane-wave normalization (i.e., $\pi^{(c\pm)}$ of Ref. 1) or partial waves of the type $|\alpha sl\rangle$ used in Eq. (2.9) below.

The usual R-matrix form[3] of $\mathcal{T}_{ba}$ may be obtained by evaluating Eqs. (2.7) using partial waves suitably norlized, e.g., if we choose

$$\chi_c^{(+)} = (i\hbar v_c)^{-1/2}(I_c/r_c - e^{2i\omega_c}O_c/r_c)\varphi_c, \qquad (2.9)$$

where I_c and O_c are ingoing and outgoing Coulomb radial functions, ω_c is the relative Coulomb phase, and φ_c represents the orthonormal set of surface functions used in I. The relative velocity v_c is as defined in Ref. 3.

The first term of Eq. (2.7a) gives the hard-sphere term, e.g.,

$$\langle \chi_b^{(-)} | \mathfrak{L} \chi_a^{(+)} \rangle = e^{2i\omega_a}(1 - e^{-2i\phi_a})\delta_{ba}, \qquad (2.10)$$

in which ϕ_a is the hard-sphere phase,[3] and the last term has the usual single-level form

$$\mathcal{T}_{ba}^{(\lambda)} = -2ie^{i(\omega_b-\phi_b)}P_b^{1/2}\alpha_{\lambda b}\alpha_{\lambda a}P_a^{1/2}e^{i(\omega_a-\phi_a)}/ \times (E_\lambda - E + \Phi_\lambda), \qquad (2.11)$$

where P_c is the penetrability[3] and the integral

$$\alpha_{\lambda c} = (\hbar^2 a_c/2m_c)^{1/2}\langle (1/r_c^2)\delta(r_c - a_c)\varphi_c | g_\lambda \rangle \qquad (2.12)$$

yields a complex and energy-dependent reduced width. This resonant contribution may be regarded as an *isolated* resonance when $\mathcal{G}_\infty$, h are independent of energy in the energy region $E \approx E_\lambda$. In this case $\alpha_{\lambda c}$ is independent of energy for $E \approx E_\lambda$ and it is useful to use the Breit-Wigner form, i.e.,

$$\mathcal{T}_{ba}^{(\lambda)} = -\frac{ie^{i(\omega_b-\phi_b)}e^{i\delta_b}\Gamma_b^{1/2}\Gamma_a^{1/2}e^{i\delta_a}e^{i(\omega_a-\phi_a)}}{E_\lambda + \mathrm{Re}\Phi_\lambda - E - \frac{1}{2}i\Gamma}, \qquad (2.13)$$

with partial widths given by

$$\Gamma_c = 2P_c|\alpha_{\lambda c}|^2, \qquad (2.14)$$

and the phase factors by

$$e^{i\delta_c} = \alpha_{\lambda c}(|\alpha_{\lambda c}|)^{-1}. \qquad (2.15)$$

For real energies $(h^\dagger(\mathcal{G}_\infty^0)^\dagger h)$ is real and it may be verified using Eqs. (2.6b), (2.12), and (2.14) that the total width satisfies the sum rule

$$\Gamma = -2\ \mathrm{Im}\Phi_\lambda = \textstyle\sum_c \Gamma_c, \qquad (2.16)$$

as expected for an isolated level. Finally, in this section we note that "potential" scattering is *not* given by hard-sphere scattering but rather by the first two terms of Eq. (2.7a), which in the present situation may be parametrized as

$$\mathcal{T}_{ba}^{(\mathrm{pot})} = \delta_{ba}e^{2i\omega_a} - e^{i\lambda_b}t_{ba}e^{i\lambda_a}, \qquad (2.17)$$

with λ_a, λ_b, and t_{ba} being real, so that $|t_{ba}| \leqslant 1$ from unitarity requirements. In the single-channel case there is an important simplification, i.e., $t_{ba} = \delta_{ba}$ and $\lambda_a \equiv \delta_a$ (see Ref. 3). The only difference between the results in this section and those of the conventional R-matrix theory is that the basis states $|\lambda\rangle$ need not be eigenstates of H. This important difference will now be made use of in the following sections.

III. GIANT-RESONANCE THEORY

In a fashion similar to Lane, Thomas, and Wigner,[2] we consider the origin of giant resonances to be due to the existence of a "simple" Hamiltonian H^0 which involves a particular eigenstate $|\lambda\rangle$. We might consider $|\lambda\rangle$ to be a state having a large reduced width in only one open channel c, in which case we refer to $|\lambda\rangle$ as a single-particle state in channel c. Alternatively, $|\lambda\rangle$ may have large reduced widths in several open channels, in which instance $|\lambda\rangle$ corresponds to a collective state. In either event, $|\lambda\rangle$ will be assumed to have a large reduced width in at least one open channel which in practice will usually be the entrance channel.

Because of the open-channel nature of the basic state it is not easily constructed in the Feshbach formalism,[6] or similar approaches.[7] However, its construction affords no problem if we adopt the R-matrix type of formalism. Consequently, we choose $\mathfrak{L}^0$ to be real with constant parameters b_c such that

$$\mathcal{G}_\infty^0 = \sum_{\mu \neq \lambda} \frac{|\mu\rangle\langle\mu|}{E_\mu - E}, \qquad (3.1)$$

in which $|\mu\rangle$ are all the remaining eigenstates in the spectrum of H^0. When the particular state $|\lambda\rangle$ is included we have a complete set which is discrete, real, and orthonormal over the hypervolume bounded by the hypersurface $r_c = a_c$ provided, of course, that H^0 is Hermitian and suitably behaved for $r_c \leqslant a_c$.

With these assumptions for H^0 and $\mathfrak{L}^0$, we note that the single-level formula (Eq. 2.3) above may also be regarded as an *exact* expression of giant-resonance phenomena. Contrary to the work of Bloch[18] and others,[3,4] we do not need to invoke approximate procedures (e.g., second-order perturbation theory or random-phase arguments) in order to describe the way in which the simple state $|\lambda\rangle$ is dissolved and spread out amongst all other states.

According to Eq. (2.6a), we may write down a giant-resonance width

$$\Gamma = -2\ \mathrm{Im}[\langle\lambda|h|\lambda\rangle - \langle\lambda|h\mathcal{G}_\infty h|\lambda\rangle], \qquad (3.2)$$

where the first term is the natural width arising via the coupling to the continuum, i.e.,

$$\Gamma_\lambda = 2\langle\lambda|\mathcal{P}|\lambda\rangle. \qquad (3.3)$$

The "extra" width W_t given by the second term in Eq.

[18] C. Bloch, Nucl. Phys. 4, 503 (1957).

(3.2) arises via the coupling to all states in the spectrum of $\mathcal{G}_\infty$.

Similarly, there is a level shift Δ given by the real part of Φ_λ in Eq. (2.6a) which has a natural shift

$$\Delta_\lambda = -\langle\lambda|\mathcal{S}|\lambda\rangle, \tag{3.4}$$

in which

$$h = -(\mathcal{S}+i\mathcal{P}), \tag{3.5}$$

with

$$\mathcal{S} = -H' + \sum_c |c\rangle \frac{\hbar^2 a_c}{2m_c} \delta(r_c - a_c)(S_c - b_c)\langle c|, \tag{3.6}$$

and

$$\mathcal{P} = \sum_c |c\rangle \frac{\hbar^2 a_c}{2m_c} \delta(r_c - a_c) P_c \langle c|. \tag{3.7}$$

The last two equations define the shift and penetrability operators, respectively. Note that Δ_λ has a contribution from $H' = H - H^0$ in addition to the usual Thomas-Ehrmann[19] shift.

If we expand $\mathcal{G}_\infty$ into a set of states which diagonalizes $\mathcal{G}_\infty$ according to a Kapur-Peierls prescription,[20] then the second term of Eq. (3.2) may be written

$$W_t = 2\ \mathrm{Im} \sum_\mu \frac{\langle\lambda|h|\mu_\infty\rangle\langle\tilde{\mu}_\infty|h|\lambda\rangle}{E_\mu(E) - E}, \tag{3.8}$$

which for reasons discussed below we refer to as a *theoretical* spreading width. The states $|\mu_\infty\rangle$ are linear combinations of the original states $|\mu\rangle$, the appropriate coefficients being complex and energy dependent (see Sec. VI). It is interesting to compare this result to that obtained using the random-phase approximation[3,18] which yields

$$W_t \approx 2\ \mathrm{Im} \sum_{\mu\neq\lambda} \frac{\langle\lambda|h|\mu\rangle\langle\mu|h|\lambda\rangle}{E_\mu - E}, \tag{3.9}$$

corresponding to the approximation $\mathcal{G}_\infty \approx \mathcal{G}_\infty{}^0$ being used. In practice this random type of coupling may not occur and it seems likely that the important terms in the sum over μ in Eq. (3.8) will be those involving *coherent* superpositions of the original states $|\mu\rangle$. In particular, if the basic operator $(H^0 + \mathcal{L}^0)$ has a spectrum involving single-particle states, two-particle–one-hole states, three-particle–two-hole states, and so on, then a single-particle state $|\lambda\rangle$ is coupled to $|\mu_\infty\rangle$ via essentially the particle-hole interaction[5] [for the moment we neglect the continuum coupling $(h - H')$]. As pointed out by several workers,[4,5] the particle-hole interaction only couples to two-particle–one-hole states. Nevertheless, the exact expression Eq. (3.8) tells us that we should use the *diagonalized* set of two-particle–one-hole states, which allows for the possibility of coherent or collective states in the spectrum of $\mathcal{G}_\infty$.

The importance of including the continuum coupling via $(\mathcal{L} - \mathcal{L}^0)$ is non-negligible if on the average

$$|\langle\lambda|\mathcal{P}|\mu\rangle| \gg |\langle\lambda|\mathcal{S}|\mu\rangle|$$

because in this case W_t is actually *negative* although Γ remains positive, i.e., $\mathcal{P}$ produces a *narrowing* of the giant resonance. In the usual case where

$$|\langle\lambda|\mathcal{P}|\mu\rangle| \ll |\langle\lambda|\mathcal{S}|\mu\rangle|,$$

such as considered by Lane, Thomas, and Wigner, W_t is positive corresponding to the usual interpretations of W_t as a spreading width.

The statement above that W_t is a theoretical spreading width is based on the fact that W_t is not a measurable width. To see this we note that the sum of the partial widths implicitly contained in the numerator of Eq. (2.7a) above is found to be given by

$$\Gamma_N = 2\langle g_\lambda|\mathcal{P}|g_\lambda\rangle, \tag{3.10}$$

which is not equal to the natural width Γ_λ. For this reason we introduce an *observable* spreading width

$$W_0 = \Gamma - \Gamma_N, \tag{3.11}$$

which is easily evaluated from Eqs. (2.6b) and (3.10) to yield

$$W_0 = 0, \quad E = \text{real}, \tag{3.12a}$$

$$W_0 = -2\ \mathrm{Im} \sum_{\mu\neq\lambda} \frac{|\langle g_\lambda|h^\dagger|\mu\rangle|^2}{E_\mu - E^*}, \quad E = \text{complex}. \tag{3.12b}$$

We see that W_0 is zero for a physical system and the usual sum rule between partial widths and total width is satisfied. However, a giant resonance is not equivalent to an isolated level. The point is that many relatively narrow $|\mu\rangle$ states may occur within the width of the giant resonance. These states give rise to rapid energy variations within the width of the giant resonance. An average or smoothed *amplitude* can be obtained[21] by the use of complex energies $E + i|F|$ (cf. Wigner's statistical R matrix[22]) where $|F|$ is made large enough so that the rapid energy dependence is damped out. Consequently, the averaged amplitude obeys a single-level formula but with $W_0 > 0$ arising from the positive contribution given by Eq. (3.12b) when E is complex.

Alternative expressions for W_0 are

$$W_0 = -2\ \mathrm{Im}\langle\lambda|h^\dagger \mathcal{G}_\infty{}^\dagger h - h^\dagger \mathcal{G}_\infty h^\dagger \mathcal{G}_\infty h|\lambda\rangle, \tag{3.13}$$

or

$$W_0 = -2\ \mathrm{Im} \sum_{\mu\neq\lambda} |\langle\lambda|h^\dagger \mathcal{G}_\infty{}^\dagger|\mu\rangle|^2 (E_\mu - E), \tag{3.14}$$

showing that W_0 is not singular at $E = E_\mu$ as naively suggested by Eq. (3.12b).

Finally, we note the occurrence of $|g_\lambda\rangle$ in the numera-

[19] J. B. Ehrman, Phys. Rev. **81**, 412 (1951); R. G. Thomas, *ibid.* **88**, 1109 (1952).

[20] P. L. Kapur and R. E. Peierls, Proc. Roy. Soc. (London) **A116**, 277 (1938).

[21] G. E. Brown, Rev. Mod. Phys. **31**, 893 (1959).

[22] E. P. Wigner, Ann. Math. **53**, 36 (1951).

tor of the giant-resonance amplitude allows a single-particle-type giant resonance to be observed in other channels as well as the channel containing $|\lambda\rangle$ itself (see Sec. VI).It therefore follows that a size resonance may be observed in many open channels, the observed magnitudes depending of course on the effective strength of the coupling interaction.

IV. DOORWAY STATES

We now consider the way in which giant resonances are broken down into "subgiant" resonances. Such a concept was qualitatively described by Weisskopf[23] and then more quantitatively by Block and Feshbach.[4] Due to the considerable effort being made[24] to identify the doorway type of structure, it is important to have a rigorous theory of such phenomena. The difficulty with the Block-Feshbach theory is that there seems to be no connection between the doorway states and the conventional giant-resonance pciture of Lane, Thomas, and Wigner. This is also pointed out by Block and Feshbach and stems from an intrinsic difficulty with the Feshbach formalism, i.e., the open- and closed-channel resonances are treated in different ways. For this reason we adopt a real discrete eigenvalue expansion which has the advantage of giving a close connection between doorway states and giant-resonance theory.

For completeness we must consider three types of states, all being solutions of the unperturbed or basic Hamiltonian H^0 with R-matrix boundary conditions. These three types depend on the choice of H^0 which we assume here to have a spectrum defined by:

(i) $|p\rangle$ *states* are entrance-channel states corresponding to the situation where the target and projectile are not excited. The complete set gives rise to potential scattering including single-particle resonances of the entrance-channel character.

(ii) $|d\rangle$ *states* are exit-channel (open or closed) states which are *directly* coupled to the entrance channel via the residual interaction h, i.e., for at least one $|p\rangle$ state, $\langle p|h|d\rangle\neq 0$. In order that $|d\rangle$ correspond to the concept of a doorway, we require that H^0 be chosen so that all $|d\rangle$ states have zero reduced widths for decay into the entrance channel.

(iii) $|b\rangle$ *states* are all other states and which, therefore, correspond to more complicated target or projectile excitations than those of the doorway type. These must satisfy the requirement $\langle p|h|b\rangle=0$, and we again assume all $|b\rangle$ states have zero reduced widths for decay into the entrance channel.

Firstly, we shall obtain the type of result emphasized by Block and Feshbach. The appropriate Green's operator $\mathcal{G}^0$ in the present case is given by

$$\mathcal{G}^0=\sum_p \frac{|p\rangle\langle p|}{E_p-E}+\sum_d \frac{|d\rangle\langle d|}{E_d-E}+\sum_b \frac{|b\rangle\langle b|}{E_b-E}, \tag{4.1}$$

or abbreviating,

$$\mathcal{G}^0=\mathcal{G}_p{}^0+\mathcal{G}_d{}^0+\mathcal{G}_b{}^0. \tag{4.2}$$

Using the fundamental relation from I,

$$\mathcal{G}=(1+\mathcal{G}^0h)^{-1}\mathcal{G}^0, \tag{4.3}$$

we may separate out the doorway contributions in a straightforward manner,

$$\begin{aligned}\mathcal{G}=&[1+(\mathcal{G}_p{}^0+\mathcal{G}_b{}^0)h]^{-1}(\mathcal{G}_p{}^0+\mathcal{G}_b{}^0)\\&+[1+(\mathcal{G}_p{}^0+\mathcal{G}_b{}^0)h]^{-1}\\&\times\{1+\mathcal{G}_d{}^0h[1+(\mathcal{G}_p{}^0+\mathcal{G}_b{}^0)h]^{-1}\}^{-1}\mathcal{G}_d{}^0\\&\times[1+h(\mathcal{G}_p{}^0+\mathcal{G}_b{}^0)]^{-1}.\end{aligned} \tag{4.4}$$

To simplify this relation we define

$$\mathcal{G}_p=(1+\mathcal{G}_p{}^0h)^{-1}\mathcal{G}_p{}^0,\quad \mathcal{G}_b=(1+\mathcal{G}_b{}^0h)^{-1}\mathcal{G}_b{}^0 \tag{4.5}$$

and utilize the identity

$$\mathcal{G}_p{}^0h\mathcal{G}_b{}^0=0 \tag{4.6}$$

to obtain the result

$$\begin{aligned}\mathcal{G}=&\mathcal{G}_p+\mathcal{G}_b+(1-\mathcal{G}_ph-\mathcal{G}_bh)\\&\times[1+\mathcal{G}_d{}^0h(1-\mathcal{G}_ph-\mathcal{G}_bh)]^{-1}\\&\times\mathcal{G}_d{}^0(1-h\mathcal{G}_p-h\mathcal{G}_b).\end{aligned} \tag{4.7}$$

Considering the entrance channel to involve the limited set of channels c, then the surface integrations in these channels yields matrix elements $\mathcal{G}_{cc'}$. Within this subspace we have $(\mathcal{G}_b)_{cc'}=0$ and

$$\mathcal{G}\equiv\mathcal{G}_p+\sum_{d,d'}|g_d\rangle A_{dd'}\langle\tilde{g}_{d'}|, \tag{4.8}$$

where

$$|g_d\rangle=\mathcal{G}_ph|d\rangle, \tag{4.9a}$$

$$\langle\tilde{g}_{d'}|=\langle d'|h\mathcal{G}_p, \tag{4.9b}$$

and A is the inverse of a matrix with elements

$$\begin{aligned}(E_d-E)\delta_{dd'}+&\langle d|h-h\mathcal{G}_ph-h\mathcal{G}_bh|d'\rangle\\&=(E_d-E)\delta_{dd'}+\xi_{dd'}{}^{\uparrow}+\xi_{dd'}{}^{\downarrow}.\end{aligned} \tag{4.10}$$

The last equation conforms to the idea proposed by Feshbach that the doorway states have an upward coupling to the "continuum," i.e.,

$$\xi_{dd'}{}^{\uparrow}=\langle d|h-h\mathcal{G}_ph|d'\rangle \tag{4.11}$$

and a downward coupling to the compound nucleus via

$$\xi_{dd'}{}^{\downarrow}=-\langle d|h\mathcal{G}_bh|d'\rangle. \tag{4.12}$$

In the weak-coupling case we may assume that A is diagonal, which yields a doorway-state contribution to

[23] V. F. Weisskopf, Phys. Today **14**, 18 (1961).
[24] See, for example, J. A. Farrell, G. C. Kyker, Jr., E. G. Bilpuch, and H. W. Newson, Phys. Letters **17**, 286 (1965).

the transition amplitude

$$\mathcal{T}_{cc'}{}^{(D)}=-\sum_d \frac{\langle \mathcal{L}^*\chi_c{}^{(-)}|g_d\rangle\langle \tilde{g}_d|\mathcal{L}\chi_{c'}{}^{(+)}\rangle}{E_d+\mathrm{Re}(\xi_{dd}{}^{\downarrow}+\xi_{dd}{}^{\uparrow})-E-\frac{1}{2}i(\Gamma_d{}^{\downarrow}+\Gamma_d{}^{\uparrow})}, \tag{4.13}$$

where the widths

$$\Gamma_d{}^{\uparrow}=-2\ \mathrm{Im}\xi_{dd}{}^{\uparrow},\quad \Gamma_d{}^{\downarrow}=-2\ \mathrm{Im}\xi_{dd}{}^{\downarrow}, \tag{4.14}$$

can be directly compared to those used by the Feshbach group only when $|d\rangle$ is a closed-channel resonance. In this case $\mathrm{Im}\langle d|h|d\rangle$ is zero and width only arises via coupling to $|p\rangle$ or $|b\rangle$ states. Note, however, that if $|d\rangle$ is an open-channel resonance, then it has a natural width typical of perhaps a single-particle state so that in this case the doorway state may be very broad and a giant resonance in its own right. Such doorway states are not easily discussed in the Feshbach theory for reasons already discussed. Of course, there is no requirement to call these latter states doorways; one could simply include them in the set of states $|p\rangle$. The present definition seems simpler, however, since the concept of a doorway state should not depend on whether a particular channel is open or closed. Certainly the available nuclear models involve no such considerations of the boundary conditions imposed on a physical system.

To complete the picture we must connect the foregoing approach to that of Lane, Thomas, and Wigner. This is easily achieved in the present formulation by use of an alternative separation. Instead of separating off $\mathcal{G}_d{}^0$, we now separate off $\mathcal{G}_p{}^0$. After some algebra and restricting ourselves to the entrance-channel subspace we find

$$\mathcal{G}\equiv[1+\mathcal{G}_p{}^0h(1+\mathcal{G}_d{}^0h(1-\mathcal{G}_bh))^{-1}]^{-1}\mathcal{G}_p{}^0. \tag{4.15}$$

Expanding $\mathcal{G}_p{}^0$ into its eigenstates we obtain the exact result

$$\mathcal{G}=\sum_{p,p'}|p\rangle A_{pp'}\langle p'|, \tag{4.16}$$

in which A is the inverse of a matrix with elements

$$(E_p-E)\delta_{pp'}+\langle p|h-h(1+\mathcal{G}_d{}^0h_b)^{-1}\mathcal{G}_d{}^0h|p'\rangle =(E_p-E)\delta_{pp'}+\Phi_{pp'}, \tag{4.17}$$

with

$$h_b=h-h\mathcal{G}_bh. \tag{4.18}$$

To be consistent with the giant-resonance theory of Lane *et al.*, we must keep only diagonal terms, i.e.,

$$\mathcal{G}\approx\sum_p \frac{|p\rangle\langle p|}{E_p-E+\Phi_{pp}}. \tag{4.19}$$

In the present case Φ_{pp} may be approximately evaluated by assuming a similar expansion for $(1+\mathcal{G}_d{}^0h_b)^{-1}\mathcal{G}_d{}^0$, i.e.,

$$(1+\mathcal{G}_d{}^0h_b)^{-1}\mathcal{G}_d{}^0\approx\sum_d \frac{|d\rangle\langle d|}{E_d-E+\Phi_{dd}}, \tag{4.20}$$

where

$$\Phi_{dd}=\langle d|h-h\mathcal{G}_bh|d\rangle, \tag{4.21}$$

which yields

$$\Phi_{pp}\approx\langle p|h|p\rangle-\sum_d \frac{\langle p|h|d\rangle\langle d|h|p\rangle}{E_d-E+\Phi_{dd}}. \tag{4.22}$$

Consequently, we see that the giant resonance may break up into a series of subresonances of the doorway type whose width depends in most cases on how the doorway states are in turn broken down into the next state of complication via the term involving $\mathcal{G}_b$ in Eq. (4.21) for Φ_{dd}. Although the intermediate model of Lane, Thomas, and Wigner may be valid for $|p\rangle$ states, it is not obvious that the same assumptions apply to the doorway states. For this reason the unperturbed states $|d\rangle$ appearing in Eqs. (4.21) and (4.22) should be replaced by a suitably diagonalized set of states.

The results obtained here show the equality between the two treatments of giant resonances given in Refs. 3 and 4. This equality is exact in the simple situation wherein one only considers one giant resonance and one doorway state because the diagonal assumptions are automatically fulfilled. In this case Eqs. (4.8) and (4.19) can easily be shown to be simply different ways of writing the same expression. The conclusion arrived at by Block and Feshbach concerning the widths of giant resonances appears to be invalid and presumably arises from their inadequate treatment of the open-channel resonances.

V. CROSS SECTIONS FOR A SPECIAL STATE WITH FINE STRUCTURE

Up until now no explicit attempt has been made to directly relate the phenomena of the previous sections to experimental data. It is appropriate, therefore, to consider the cross section within some finite energy interval wherein there occurs an isolated broad structure with fine structure superimposed upon it in some way. In such situations it appears reasonable to associate the origin of the broad structure with one special type of eigenstate corresponding to an ideal operator $H^0+\mathcal{L}^0$ as in Sec. II. Usually there will also be some nearby eigenstates of the same operator $H^0+\mathcal{L}^0$ which will give rise to fine structure in the broad-resonance region.

It is convenient to discuss the situation of a special state plus its nearby states in terms of a nonrandom state plus a set of random states in the sense that the fluctuations arising from the nearby states can be evaluated in terms of statistical theory. The remaining eigenstates of $H^0+\mathcal{L}^0$ will lie outside the energy region being considered and need not be treated via statistical arguments since they are easily included in the formal theory as a slowly varying contribution. This model of one nonrandom state imbedded in a sea of random states is no doubt an oversimplification of the true situation. As has already been pointed out in Sec. IV, a simple

resonance structure may in fact be made up of several substructures corresponding to a further breakdown in the statistical assumptions normally invoked. The problem treated here does illustrate, however, some of the possible effects which may arise when a nonrandom component is included.

It is convenient to separate the discussion of the problem into two distinct cases, each one corresponding to a definite experimental situation. In the first part we consider the situation wherein the experimental resolution is insufficient to observe the fine structure but sufficient to observe the broad resonance and in the second part the situation wherein the resolution is sufficient to observe the fine structure itself. In the former situation the interesting results are contained in energy-averaged cross-section expressions, whereas in the latter situation we are more concerned with the interplay between the fine structure and the gross structure. We now consider each situation in turn.

A. Average Cross Sections

From the above discussion we assume that the experimental resolution characterized by δ (energy units) lies somewhere between the mean fluctuation width Γ_{fs} of the fine structure and the total width Γ of the broad structure. Ideally, then, we require

$$\Gamma_{fs}\ll\delta\ll\Gamma,$$

although larger values of δ up to the order of Γ itself can be considered in practice.

In formulating the expression for a typical cross section we only need a slight extension of the usual optical-model approach. The scattering matrix is separated into two parts, e.g.,

$$S_{ba}=\bar{S}_{ba}+S_{ba}{}^{\mathrm{fl}}, \tag{5.1}$$

where the first term is the appropriate energy-averaged amplitude and the second term describes all the fluctuations away from this average. In the present situation it is convenient to include the broad structure in the energy averaged amplitude using the single level form of the S matrix, i.e.,

$$S_{ba}(E)=S_{ba}{}'(E)+e^{i[\lambda_b(E)+\lambda_a(E)]}\left\{\frac{i[\Gamma_b(E)\Gamma_a(E)]^{1/2}}{E_\lambda-E+\Phi_\lambda(E)}\right\}. \tag{5.2}$$

This expression is easily deduced from Eqs. (2.7a) and (2.7b) if we define $\lambda_c(E)$ as the channel phase $(\delta+\omega_c-\phi_c)$, and $S_{ba}{}'(E)$ as the S matrix in the absence of the special state $|\lambda\rangle$. The single-level parameters δ_c, ϕ_c, ω_c, $\Gamma_c(E)$, $\Phi_\lambda(E)$, and E_λ are identical to those defined in Sec. II except that we have indicated the energy dependence of the parameters explicitly in the present situation. The energy averaged amplitude $\bar{S}_{ba}$ can be obtained by the use of complex energies $E+i|F|$, where $|F|$ is made large enough so that the fine structure is effectively damped out. The relation

$$\bar{S}_{ba}(E)=S_{ba}(E+i|F|) \tag{5.3}$$

holds rigorously if the energy average is performed[21] using a Lorentzian weighting factor with a half-width of $|F|$. However, the use of $S_{ba}(E+i|F|)$ is expected to be an accurate representation of $\bar{S}_{ba}(E)$ for any reasonable energy averaging procedure.

As a convenient notation we write

$$\Gamma_b=|\exp[i\lambda_b(E+i|F|)]\Gamma_b(E+i|F|)|,$$

since by definition it should now be essentially constant within the energy interval containing the gross structure. Energy dependence arising from threshold effects will not be damped out by using $E+i|F|$ in place of E, but in any case the energy dependence from this source is monotonic and can easily be treated in terms of conventional penetrability arguments. If we use a similar notation for the other parameters we obtain

$$\bar{S}_{ba}=S_{ba}{}'+ie^{i(\lambda_b+\lambda_a)}\frac{(\Gamma_b\Gamma_a)^{1/2}}{E_R-E-\frac{1}{2}i(\Gamma+2|F|)}, \tag{5.4}$$

where $E_R=E_\lambda+\mathrm{Re}\Phi_\lambda(E+i|F|)$ and $\Gamma=-2\,\mathrm{Im}\Phi_\lambda(E+i|F|)$ define the resonance energy and width, respectively, for the broad structure. Usually $|F|\ll\Gamma$, so that $\Gamma+2|F|\simeq\Gamma$ can be used.

Expressions for the differential cross section averaged over an energy interval characterized by F are given by a sum of two terms, i.e.,

$$\langle d\sigma_{\alpha'\alpha}/d\Omega\rangle_{\mathrm{av}}=d\bar{\sigma}_{\alpha'\alpha}/d\Omega+\langle d\sigma_{\alpha'\alpha}{}^{\mathrm{fl}}/d\Omega\rangle_{\mathrm{av}}, \tag{5.5}$$

where $d\bar{\sigma}_{\alpha'\alpha}/d\Omega$ is the "shape" cross section obtained by using $\bar{S}_{ba}$ in the scattering amplitudes and $\langle d\sigma_{\alpha'\alpha}{}^{\mathrm{fl}}/d\Omega\rangle_{\mathrm{av}}$ is the energy averaged contribution from the fluctuating amplitude $S_{ba}{}^{\mathrm{fl}}$. If the fluctuations from the average are regarded as being random then $\langle d\sigma^{\mathrm{fl}}/d\Omega\rangle_{\mathrm{av}}$ can be estimated in a similar fashion to the Hauser-Feshbach results[25] except that the transmission coefficients T_c are modified[9] so that in the present case

$$T_c=1-\sum_{c'}|\bar{S}_{c'c}|^2, \tag{5.6}$$

which involves all channels c' for which shape reactions are not negligible. $\bar{S}_{c'c}$ is given by Eq. (5.4).

In the Hauser-Feshbach theory the differential cross section involves the terms $T_cT_{c'}(\sum_{c''}T_{c''})^{-1}$ which in the general case considered here will not reduce to a simple Breit-Wigner form as exemplified by the shape amplitude itself. In the most complicated cases *each* transmission coefficient involves an interfering Breit-Wigner type of energy dependence, so that the cross section may well have a complicated energy dependence. Some insight into $\langle d\sigma^{\mathrm{fl}}/d\Omega\rangle_{\mathrm{av}}$ is obtained in the

[25] W. Hauser and H. Feshbach, Phys. Rev. 87, 366 (1952).

situation where $S_{ba}' \approx e^{2i\lambda_a}\delta_{ba}$, with λ_a=real. One finds in this approximation that

$$\frac{T_bT_a}{\sum_c T_c}=\left(\frac{\Gamma_b\Gamma_a}{\sum_c \Gamma_c}\right)\times\frac{(\Gamma+2|F|-\sum_c \Gamma_c)}{(E_R-E)^2+\frac{1}{4}(\Gamma+2|F|)^2}, \quad (5.7)$$

in which the first factor gives the fractional probability of the resonant transition $a \rightarrow b$ and the second factor corresponds to the contribution out of the total strength of the broad resonance arising from fluctuations. This follows from the usual definition of a strength function and from the fact that $\Gamma-\sum_c \Gamma_c=W_0$ is the spreading width discussed in Sec. III. Usually $F \ll \Gamma$ can be used so that $\langle d\sigma^{fl}/d\Omega\rangle_{av}$ becomes directly proportional to W_0 in the present example.

The above cross-section formulas appear to be best suited for the analysis of experiments on isobaric analog resonances as discussed in Sec. VI.

B. Cross Sections with Observable Fine Structure

Within a particular broad resonance the mean fine structure width may be large enough so that a high-resolution experiment can observe the oscillatory behavior of the cross section. When the level density is small enough, the fine structure may be separated into a series of essentially isolated levels. In such cases it is appealing to try to discuss each particular fine-structure state explicitly. On the other hand, as the level density increases the fine-structure resonances will overlap, and it becomes impractical to discuss particular resonances. It is convenient to discuss these two experimental possibilities separately.

1. Separated Fine Structure

This situation is in principle the simpler of the two possibilities and can arise experimentally when the complicated states $|\mu\rangle$ surrounding the special state $|\lambda\rangle$ have a large enough level spacing so that after the mixing via h is included, each new state remains well separated from its neighbor. In the case of isobaric analog resonances such a situation appears to have been approximately realized.[26] Since we are involved with single resonances, we again have recourse to the general single-level formula. Now, however, it is appropriate to single out a particular fine-structure state $|\mu\rangle$ and introduce a slight change in notation, i.e.,

$$\mathcal{G}=\mathcal{G}_1+|g_\mu\rangle\langle\tilde{g}_\mu|/(E_\mu-E+\Phi_\mu), \quad (5.8)$$

where

$$|g_\mu\rangle=(1-\mathcal{G}_1h)|\mu\rangle, \quad (5.9)$$

$$\Phi_\mu=\langle\mu|h|g_\mu\rangle, \quad (5.10)$$

$$\mathcal{G}_1=1(+\mathcal{G}_1{}^0h)^{-1}\mathcal{G}_1{}^0, \quad (5.11)$$

and

$$\mathcal{G}_1{}^0=\frac{|\lambda\rangle\langle\lambda|}{E_\lambda-E}+\sum_{\mu'\neq\mu}\frac{|\mu'\rangle\langle\mu'|}{E_{\mu'}-E}$$

$$=\mathcal{G}_\lambda{}^0+\mathcal{G}^0{}_{\infty,-\mu}. \quad (5.12)$$

In writing the above results, we have assumed $H^0+\mathfrak{L}^0$ to be Hermitian and energy-independent, so that the ideal states $|\lambda\rangle$ and $|\mu\rangle$ are real and independent of energy. The S matrix or T matrix can be obtained as in Sec. II, which yields a simple Breit-Wigner amplitude for the resonance part as given in form by Eq. (2.11). The reduced width $|\alpha_{\mu c}|^2$ arising from $|g_\mu\rangle$ is given by an equation like (2.12), but it is more convenient to manipulate this into the interesting form

$$\alpha_{\mu c}=\alpha_{\mu c}{}^0-\frac{\langle\lambda|h|g_\mu{}^0\rangle}{E_\lambda-E+\langle\lambda|h|g_\lambda{}^0\rangle}\alpha_{\lambda c}{}^0, \quad (5.13)$$

where

$$|g_\mu{}^0\rangle=(1-\mathcal{G}_{\infty,-\mu}h)|\mu\rangle, \quad (5.14)$$

$$|g_\lambda{}^0\rangle=(1-\mathcal{G}_{\infty,-\mu}h)|\lambda\rangle, \quad (5.15)$$

$$\mathcal{G}_{\infty,-\mu}=(1+\mathcal{G}_{\infty,-\mu}{}^0h)^{-1}\mathcal{G}_{\infty,-\mu}{}^0, \quad (5.16)$$

and

$$\mathcal{G}_{\infty,-\mu}{}^0=\mathcal{G}_\infty{}^0-|\mu\rangle\langle\mu|/(E_\mu-E) \quad (5.17)$$

relates $\mathcal{G}_{\infty,-\mu}{}^0$ to the complete fine-structure operator $\mathcal{G}_\infty{}^0$ used in earlier sections.

Unfortunately, the behavior of the partial widths $\Gamma_{\mu c}$ given by

$$\Gamma_{\mu c}=2P_c|\alpha_{\mu c}|^2$$

will depend on the peculiarities of the individual states. It is possible to ask what happens, however, if we regard each $|\mu\rangle$ state as essentially the same and consider what happens as one moves across the energy region affected by the special state. For those channels satisfying the condition $\gamma_{\lambda c}{}^2 \gg \gamma_{\mu c}{}^2$ for all states $|\mu\rangle$ in the vicinity of the broad structure, we see from Eq. (5.13) that the fine-structure widths $\Gamma_{\mu c}$ will tend to be broadened near the center of the broad resonance. The actual energy behavior of these fine-structure widths can be described by a simple Breit-Wigner shape only if $\langle\lambda|h|g_\mu{}^0\rangle$ and $\alpha_{\mu c}{}^0$ are uncorrelated in their relative phase (or, of course, if $\alpha_{\mu c}{}^0 \approx 0$ for all μ). In general, the contributions from $\alpha_{\mu c}{}^0$ and $\alpha_{\lambda c}{}^0$ in Eq. (5.13) which produce $\alpha_{\mu c}$ will interfere, and if the phase of $\langle\lambda|h|g_\mu{}^0\rangle$ and $\alpha_{\mu c}{}^0$ is in some way correlated, the interference will be observable.

One interesting possibility involving a correlation between $\langle\lambda|h|g_\mu{}^0\rangle$ and $\alpha_{\mu c}{}^0$ has been suggested[11] in the case of isobaric analog resonances. Such correlations can be phrased more generally. For example, if we assume h has a surface form

$$h=\sum_{c'}|c'\rangle\delta(r_{c'}-a_{c'})\frac{\hbar^2a_{c'}}{2m_{c'}}M_{c'}\langle c'|, \quad (5.18)$$

[26] G. A. Keyworth, G. C. Kyker, Jr., E. G. Bilpuch, and H. W. Newson, Phys. Letters **20**, 281 (1966).

we obtain

$$\alpha_{\mu c}=\alpha_{\mu c}{}^0[(E_1-E-\tfrac{1}{2}i\Gamma_1)/(E_2-E-\tfrac{1}{2}i\Gamma_2)], \quad (5.19)$$

where

$$E_1-\tfrac{1}{2}i\Gamma_1=E_\lambda+\sum_{c'\neq c} M_{c'}\gamma_{\lambda c'}\alpha_{\lambda c'}{}^0\left(1-\frac{\alpha_{\lambda c}{}^0\alpha_{\mu c'}{}^0}{\alpha_{\lambda c'}{}^0\alpha_{\mu c}{}^0}\right), \quad (5.20)$$

and

$$E_2-\tfrac{1}{2}i\Gamma_2=E_\lambda+\sum_{c'} M_{c'}\gamma_{\lambda c'}\alpha_{\lambda c'}{}^0. \quad (5.21)$$

In general, the usefulness of Eq. (5.19) depends upon how the parameters E_1, Γ_1. E_2 and Γ_2 depend on the individual fine-structure levels. The parameters E_2 and Γ_2 are unlikely to depend sensitively on $|\mu\rangle$ since a $|\mu\rangle$ dependence enters only via $\alpha_{\lambda c'}{}^0$. On the other hand, the parameters E_1 and Γ_1 could be sensitive in this respect. There are two obvious cases where E_1 and Γ_1 become independent of μ. Firstly, when $M_{c'}\gamma_{\lambda c'}\approx 0$ except for $c'=c$, one obtains the single-channel result

$$\alpha_{\mu c}=\alpha_{\mu c}{}^0[(E_\lambda-E)/(E_2-E-\tfrac{1}{2}i\Gamma_2)], \quad (5.22)$$

which shows a striking interference pattern, as has been discussed elsewhere.[27] The zero at $E=E_\lambda$ appears to be a characteristic only of the single-channel approximation within the present framework.

Secondly, if many channels contribute and the fine-structure amplitudes $\alpha_{\mu c'}$ and $\alpha_{\mu c''}$ for the various channels are uncorrelated in phase, then the series involving $\alpha_{\mu c'}{}^0/\alpha_{\mu c}{}^0$ will tend to zero and E_1, Γ_1 will become as insensitive to μ as E_2, Γ_2 are expected to be. In this situation, however, Γ_1 need not be zero, because for the special state $|\lambda\rangle$ it is unlikely that the phases of the various amplitudes $\alpha_{\lambda c}{}^0$ will also be uncorrelated.

Note that the phase of the reduced width is essentially determined by the slowly varying term in square brackets in the examples discussed above, only if $\alpha_{\mu c}{}^0\approx\gamma_{\mu c}$. This approximation is likely to be valid for the fine-structure states near isobaric analog resonances, but in general $\alpha_{\mu c}{}^0$ will have a non-negligible imaginary part.

2. *Overlapping Fine Structure*

In the situation where $\Gamma_{\rm fs}\gg D_{\rm fs}$, (with $D_{\rm fs}$ being the average level spacing of the fine structure observed), the previous analysis becomes difficult to apply because all quantities $g_\mu{}^0$, $g_\lambda{}^0$ become energy dependent in a nontrivial manner. In this situation we only wish to know how the fluctuations behave on the *average* (even when the fine structure is separated the average behavior is of some interest). This result has already been obtained in part A of this section, i.e.,

$$\langle\sigma_{c'c}{}^{\rm fl}\rangle_{\rm av}\propto T_cT_{c'}/\sum_{c''}T_{c''},$$

with $T_c=1-\sum_{c''}|\bar{S}_{cc''}|^2$ and $\bar{S}_{cc''}$ given by Eq. (5.4).

[27] P. Richard, C. F. Moore, D. Robson, and J. D. Fox, Phys. Letters **13**, 343 (1964).

Only in the single-channel case and elastic scattering is a simple result obtained, i.e.,

$$\langle\sigma_{cc}{}^{\rm fl}\rangle_{\rm av}\propto T_c=1-|\bar{S}_{cc}|^2.$$

For this case we write from Eq. (5.4) (assuming $|F|\ll\Gamma$),

$$\bar{S}_{cc}=e^{2i\xi_c}[e^{-2\mu_c}+e^{2i\phi_c{}^R}i\Gamma_c/(E_R-E-\tfrac{1}{2}i\Gamma)], \quad (5.23)$$

where the background function $S_{cc}{}'=e^{2i\xi_c}e^{-2\mu_c}$ and $\phi_c{}^R=\lambda_c-\xi_c$ have been substituted so that all parameters are real. The meaning of $\phi_c{}^R$ becomes clear if we evaluate $T_c=1-|\bar{S}_{cc}|^2$ (which is valid in the single-channel case), i.e.,

$$T_c=A_c+\frac{B_c(E_R-E)+C_c\Gamma_c}{(E_R-E)^2+\frac{1}{4}\Gamma^2}, \quad (5.24)$$

with

$$\begin{aligned} A_c&=1-e^{-4\mu_c}=T_c', \\ B_c&=2\Gamma_c\sin 2\phi_c{}^Re^{-2\mu_c}, \\ C_c&=\Gamma\cos 2\phi_c{}^Re^{-2\mu_c}-\Gamma_c. \end{aligned} \quad (5.25)$$

Clearly the term involving (E_R-E) gives an interference pattern (or asymmetric Breit-Wigner pattern) and arises only if $\phi_c{}^R\neq 0$. One can understand the general relationship between T_c and the fine structure better by the result

$$T_c\approx 2\pi\langle\bar{\Gamma}_{\mu c}\rangle/D, \quad (5.26)$$

where $\langle\bar{\Gamma}_{\mu c}\rangle$ is the *average* partial width of the underlying fine structure and D is the average level spacing. To prove (5.26) we introduce the notation

$$|c\rangle=(\hbar^2a_c/2m_c)^{1/2}|r_c{}^{-2}\delta(r_c-a_c)\varphi_c\rangle, \quad (5.27)$$

so that the definition (5.6) of T_c may be written as

$$\begin{aligned} T_c&=1-\langle c|\bar{S}\bar{S}^\dagger|c\rangle \\ &=4P_c\langle c|(\mathrm{Im}\langle\mathcal{G}\rangle)-\langle\mathcal{G}\rangle P\langle\mathcal{G}\rangle^\dagger|c\rangle, \end{aligned} \quad (5.28)$$

where $\langle\mathcal{G}\rangle=\mathcal{G}(E+i|F|)$ is the smoothed Green's operator. In writing (5.28) we have assumed that h is independent of energy for the energy interval ΔE encompassing the broad structure. The operator P in Eq. (5.28) is given by

$$P=\sum_{c'}|c'\rangle P_{c'}\langle c'|, \quad (5.29)$$

with P_c being the appropriate penetrability in channel c. Noting the relation

$$\langle\mathcal{G}\rangle=(1+\langle\mathcal{G}^0\rangle h)^{-1}\langle\mathcal{G}^0\rangle, \quad (5.30)$$

we obtain

$$T_c=4P_c\langle c|(1+\langle\mathcal{G}^0\rangle h)^{-1}\,\mathrm{Im}\langle\mathcal{G}^0\rangle(1+h^\dagger\langle\mathcal{G}^0\rangle^\dagger)^{-1}|c\rangle, \quad (5.31)$$

where

$$\langle\mathcal{G}^0\rangle\approx\frac{|\lambda\rangle\langle\lambda|}{E_\lambda-E}+\sum_{\mu'\,(\rm distant)}\frac{|\mu'\rangle\langle\mu'|}{E_{\mu'}-E}+\sum_{\mu\,(\rm nearby)}\frac{|\mu\rangle\langle\mu|}{E_\mu-E-i|F|}. \quad (5.32)$$

The smoothed $\langle \mathcal{G}^0 \rangle$, it should be noted, only requires a smoothing for the narrow nearby fine-structure states and consequently

$$\mathrm{Im}\langle \mathcal{G}^0 \rangle \approx \sum_{\mu\,(\mathrm{nearby})} \mathrm{Im} \frac{|\mu\rangle\langle\mu|}{E_\mu - E - i|F|}, \tag{5.33}$$

since the first two terms are real in Eq. (5.32) by construction. Finally, then we have

$$T_c = 4P_c \sum_\mu \frac{|\bar{\alpha}_{\mu c}|^2}{E_\mu - E - i|F|}, \tag{5.34}$$

with

$$\bar{\alpha}_{\mu c} = \langle c | (1 + \langle \mathcal{G}^0 \rangle h)^{-1} | \mu \rangle, \tag{5.35}$$

and if $|F|$ is large enough and the level density high enough we can evaluate the sum over μ by turning it into an integral. This results in Eq. (5.26) if we make the identifications commonly used:

$$\langle \bar{\Gamma}_{\mu c} \rangle = 2P_c \langle |\bar{\alpha}_{\mu c}|^2 \rangle, \tag{5.36}$$

$$\mathrm{Im} \sum_{\mu\,(\mathrm{nearby})} (E_\mu - E - i|F|)^{-1} = \pi/D. \tag{5.37}$$

Experiments involving overlapping fine structure in the case of isobaric analog resonances have been discussed earlier.[27] The above approach yields a more exact result for the energy dependence of the fluctuations across the analogue resonance. In the experiments cited,[27] provided that h can be represented by a surface delta function[11] in channel c, one obtains[28] the transmission coefficient in channel c as

$$T_c = T_c'(E_\lambda - E)^2 / [(E_R - E)^2 + \tfrac{1}{4}\Gamma^2]. \tag{5.38}$$

Note that this equation actually does include the terms $c' \neq c$, as required from the definition (5.6) of T_c. This result involves E_R, Γ in place of the natural energy E_A and width Γ_A introduced in the earlier work.[11] The present expression is more accurate and consistent with the conventional theory of average cross sections; moreover, it removes the earlier difficulties associated with matrix inversion. The present approach is now being applied to isobaric analog resonances in the reactions $Mo^{92}+p$, $Zr^{90}+p$ and will be reported elsewhere.

The important suggestion which the above discussion introduces is that fine structure and gross structure may be correlated in an observable fashion. It should be emphasized, however, that such a correlation exists only for levels of the same spin and parity, if we assume H^0 and H' are rotationally invariant. In most nuclear reactions, levels of many values of spin and parity will contribute, so that the effects discussed above will be masked. Certain selective reactions such as (p,γ_0) decaying to even-even nuclei, or reactions involving isobaric analogue resonances look most favorable for examining such phenomena. A certain amount of selectivity can be obtained in some elastic-scattering experiments by measuring at particular scattering angles; e.g., at 90° in the elastic scattering of α particles from even-even nuclei only states with even spin and positive parity can be excited.

[28] D. Robson, J. D. Fox, P. Richard, and C. F. Moore, Phys. Letters **18**, 86 (1965).

VI. ISOBARIC ANALOG RESONANCES

The theory of isobaric analog resonances has been treated[11] in some detail using essentially the conventional R-matrix formalism. It is a phenomenon which is a remarkable illustration of several spects of the present formalism; e.g., the observed resonances[10,29] show all the expected features of the giant resonance discussed in Sec. III. We consider a pair of nuclei having the same nucleon number but differing by one unit of the isobaric spin projection T_Z; i.e., the two nuclei differ by the interchange of one neutron for one proton. For obvious reasons we refer to these two nuclei as the neutron nucleus and the proton nucleus. For simplicity we consider the situation wherein the low-lying bound states of the neutron nucleus (N,Z) are assumed to have pure isobaric spin $T = T_Z = \frac{1}{2}(N-Z)$. The eigenstates $\psi_{\alpha Z}$ are solutions of a Hamiltonian H and satisfy bound-state boundary conditions expressed via an $\mathcal{L}$ operator $\mathcal{L}_Z$ which involves a sum over all closed channels of the (N,Z) system. Analog states in the "ideal" proton nucleus $(N-1, Z+1)$ are then defined[11,12] by

$$\psi_{\alpha,Z,1} = \bar{T}_- \psi_{\alpha Z} = (2T)^{-1/2} T_- \psi_{\alpha Z},$$

with T_- being the usual lowering operator. The extension of the formalism to the case where T is not a good quantum number is straightforward[30] but will not be considered here because the basic ideas remain unaltered.

The states $\psi_{\alpha,Z+1}$ are not physical eigenstates of the proton system but are solutions of the operator $\bar{T}_- H \bar{T}_+$ with boundary conditions $\bar{T}_- \mathcal{L}_Z \bar{T}_+$. It is convenient for the present discussion to introduce projection operators in isobaric spin space of the form

$$D = |T_< T_Z - 1\rangle\langle T_< T_Z - 1|,$$
$$U = |T_> T_Z - 1\rangle\langle T_> T_Z - 1|,$$

where $T_< = T_Z - 1$, $T_> = T_< + 1$ are the usual values of isobaric spin used to characterize "normal" states and analogue states, respectively. In the energy region close to $T_>$ states of interest, we assume $D+U=1$, which corresponds to neglecting all states with isobaric spin T larger than $T_>$.

We are now in a position to relate the four operators in I to the operators describing isobaric analog reso-

[29] G. A. Jones, A. M. Lane, and G. C. Morrison, Phys. Letters **15**, 329 (1964).

[30] D. Robson, in *Proceedings of the International Conference on Isobaric Spin in Nuclear Physics* (Academic Press Inc., New York, 1966), p. 411.

nances. The necessary relations are

$$H=H^0+H',$$
$$\mathfrak{L}=\mathfrak{L}^0-L=\mathfrak{L}_{Z+1},$$
$$H^0=DHD+\bar{T}_-H\bar{T}_+,$$
$$\mathfrak{L}^0=D\mathfrak{L}_{Z+1}D+\bar{T}_-\mathfrak{L}_Z\bar{T}_+,$$

where $\mathfrak{L}_{Z+1}$ is the physical boundary matching operator for the $(N-1, Z+1)$ nucleus. The difference between H and H^0, $\mathfrak{L}$ and $\mathfrak{L}^0$ are given by H', $-L$, respectively, i.e., $h=H'-L$ and,

$$H'=DHU+UHD+U[H-\bar{T}_-H\bar{T}_+]U,$$
$$-L=D\mathfrak{L}_{Z+1}U+U\mathfrak{L}_{Z+1}D+U[\mathfrak{L}_{Z+1}-\bar{T}_-\mathfrak{L}_Z\bar{T}_+]U,$$

although the terms involving operators UXU will be small when the analogue states are widely spaced. The terms of type UXD or DXU involve the mixing via Coulomb interactions or the change of boundary conditions due to Coulomb interactions, as discussed below. The interaction H', as can be seen from its definition, is the charge-dependent (usually Coulomb) interaction of the converted neutron. The eigenstates of H^0 are of two types, one set being the normal $T_<$ states and the other et being the analogue states.

Mixing occurs between an "isolated" $T_>$ state and the myriad of $T_<$ states usually surrounding it in the case of heavy nuclei. This mixing is contained in the single-level formula of Sec. II if we associate the state $|\lambda\rangle$ with the $T_>$ state and the remainder with essentially the surrounding $T_<$ states. The giant-resonance state $|g_\lambda\rangle$ is given from Eq. (2.5a), i.e.,

$$|g_\lambda\rangle=|\lambda\rangle-\mathcal{G}_\infty h|\lambda\rangle, \tag{6.1}$$

which yields

$$\alpha_{\lambda c}=\gamma_{\lambda c}-\sum_{\mu\neq\lambda}\beta_{\lambda\mu}\gamma_{\mu c}, \tag{6.2}$$

where

$$\beta_{\lambda\mu}=\sum_{\mu'\neq\lambda}\langle\mu|\mathcal{G}_\infty|\mu'\rangle\langle\mu'|h|\lambda\rangle \tag{6.3}$$

gives the mixing coefficients essentially in terms of $\langle\mu'|h|\lambda\rangle$.

In the R-matrix-type formalism the matrix elements are limited to a volume $r_c\leqslant a_c$ and consequently the matrix elements $\langle\mu|h|\lambda\rangle$ have two contributions

(i) Internal or interaction mixing. This is expressed by the matrix elements of $H-H^0=H'$, i.e.,

$$I_{\mu\lambda}=\langle\mu|H'|\lambda\rangle \tag{6.4}$$

and is the type of mixing which several groups[12] have been particularly interested in calculating. In these treatments the states $|\mu\rangle$ and $|\lambda\rangle$ are replaced by harmonic oscillator functions and boundary condition differences $(\mathfrak{L}-\mathfrak{L}^0)$ ignored. The difficulty is that orthogonality over the range $r_c=0\rightarrow a_c$ is not ensured by such methods, so that the results need not correspond to $I_{\mu\lambda}$.

(ii) External or boundary condition mixing. This, of course, is simply the matrix elements of $(\mathfrak{L}-\mathfrak{L}^0)$, viz.,

$$S_{\mu\lambda}=-\langle\mu|L|\lambda\rangle, \tag{6.5}$$

which is important because of the long-range character of Coulomb forces which mixes isobaric states at large values of r_c. This type of mixing was emphasized in Ref. 11 on the grounds that a constant interaction H' for $r_c\leqslant a_c$ does not cause any mixing provided that $|\mu\rangle$ and $|\lambda\rangle$ are orthogonal over this range. Actual calculations of the average value of $S_{\mu\lambda}$ yield results which account for the observed[10,27,28] mixing effects in heavy nuclei and suggests that radii a_c exist which minimize the effects of internal mixing.

One important difference between H' and L is that H' is not in general diagonal in channel space, whereas L by construction is diagonal, i.e.,

$$\langle\mu|L|\lambda\rangle=\sum_c L_c\gamma_{\lambda c}\gamma_{\mu c}, \tag{6.6}$$

where $L_c=b_c-b_c^0$ and for proton channels $b_p=S_p+iP_p$, $b_p^0=S_n(E-\Delta_c)$ as indicated in Ref. 11. Consequently, if only one term in the sum over c is important then we have the identity

$$\langle\mu|L|\lambda\rangle\gamma_{\lambda c}=\langle\lambda|L|\lambda\rangle\gamma_{\mu c}. \tag{6.7}$$

If as in Ref. 11, we choose $\langle\lambda|H'|\lambda\rangle=0$ (by simple modification of H^0 to $H^0+\langle\lambda|H'|\lambda\rangle$) and assume $\langle\lambda|H'|\mu\rangle=0$, then we obtain a relation like Eq. (5.22) of Sec. V. Such a result was applied in Ref. 11 to explain the observed[24] damping of fluctuations near the analog resonance being investigated.

Finally, in this section we note that the reduced-width relation Eq. (6.2) explains the existence of a resonance in isobaric-spin-forbidden reactions, e.g., (p,n) or (p,α) reactions leading to the low-lying levels of the respective residual nuclei. In particular, boundary-condition mixing does give rise to a neutron width in (p,n) reactions via the component $\mathcal{G}_\infty L|\lambda\rangle$ of Eq. (6.1) and has been shown[28] to be a reasonable explanation of the large S-wave resonance observed in $Zr^{92}(p,n)Nb^{92}$. If there is a significant fine-structure contribution then isobaric-spin-forbidden reactions averaged over energy will also resonate via the term $\langle\sigma^{fl}\rangle_{av}$ due to the resonance terms contained in the nonforbidden transmission coefficients. Such a possibility was first pointed out by Jones, Lane, and Morrison[29] in the case of $Y^{89}(p,p')Y^{89*}$. In general one should consider the inclusion of both the shape contribution $\bar{\sigma}$ and the fluctuation term $\langle\sigma^{fl}\rangle_{av}$, although, as pointed out earlier, this may be a complicated problem and difficult to apply in practice.

VII. CONCLUSIONS

The most striking feature of the present work is the way in which several apparently unrelated phenomena are described by the same formalism simply by algebraic rearrangements of the same basic entities. This

flexibility arises by virtue of the arbitrariness of the operators H^0, $\mathfrak{L}^0$ which are to be chosen in each instance so that the significant parts of the mathematical spectrum of H^0, $\mathfrak{L}^0$ are most simply associated with their physical counterparts. In some instances this "mapping" procedure becomes a matter of personal taste. The choice which is obviously "most physical" to one person is not always so evident to someone else.

In the applications given here we have usually employed R-matrix-type boundary conditions for the operator $\mathfrak{L}^0$, the reasons for this choice being discussed already in I. Nevertheless, the formalism allows for multiple choices of the basic operators and only after a careful evaluation of each choice will it be possible to decide which choice is most appropriate for the particular problem being investigated. General criteria for choosing a set of operators are not obvious due to the necessity of compromising between mathematical convenience and "physical" significance. Fortunately, many of the interesting results may be obtained without specifying H^0 and $\mathfrak{L}^0$ too precisely so that the basic formalism is reasonably universal in its applicability to nuclear reactions. Applications of the formalism to direct reactions and nuclear-model calculations will be reported at a later time.

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^{14}N(^{14}N,^{13}N)^{15}N Reaction at Low Energies and the Elastic Scattering of ^{14}N by ^{14}N†

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The molecular-viewpoint form of nucleon tunneling theory is used in the two-level approximation and with neglect of the dynamic reaction terms for a partial-wave analysis allowing the inclusion of the effects of wave function absorption through the use of an imaginary part of the potential. The equations are used in an analysis of improved measurements of the differential and total cross sections of the reaction ^{14}N(^{14}N,^{13}N)^{15}N, with special attention to laboratory energies $E_l \leq 16$ MeV which are below the Coulomb barrier. At the lowest energies, the analysis involves only the Coulomb interaction between the heavy particles. Fits to data are improved at the higher energies through the introduction of optical potentials. The principal function of these in the present work is to modify the wave function at distances larger than those corresponding to definite contact between ^{14}N and ^{14}N. The transfer function $\beta(R)$ is cut off at small values of the internuclear distance R to avoid the inclusion of unrealistic contributions to neutron transfer when the two nuclei are no longer distinct. The potential has been adjusted for best fits to neutron-transfer data. The long distance tail of the potentials tried was made to agree, regarding relative values at different distances, with that calculated by McIntosh, Rawitscher, and Park in their work on the elastic scattering of ^{14}N by ^{14}N, and depends therefore on nucleon-nucleus scattering information. The potentials were adjusted to represent the elastic-scattering ^{14}N+^{14}N data simultaneously with neutron-transfer data. These combined requirements are met best by potentials referred to as 2 and 3 in the text. The reduced width of the transferred neutron obtained from transfer data depends on the potential only weakly. The same reduced width from elastic-scattering information is sensitive to the choice of potential. The best agreement of the elastic-scattering and neutron-transfer reduced widths is obtained for potential 3, the disagreement being less than by a factor 2. The combined uncertainty of the two ways of arriving at the reduced width is believed to be large and to make the discrepancy insignificant. The combined treatment of neutron transfer and of elastic scattering is self-consistent in the sense described. The neutron transfer reduced width is slightly smaller than the single-particle reduced width calculated with the nucleon-nucleus potential employed in obtaining the proportionality constant of the long-distance nucleus-nucleus potential tail.

I. INTRODUCTION

THE treatment of nucleon tunneling proposed by one of the writers[1–3] is applied to the analysis of the ^{14}N(^{14}N,^{13}N)^{15}N reaction data of Becker and McIntyre,[4] of Hiebert, McIntyre, and Couch[5] as well as Gaedke, Toth, and Williams.[5] The treatment, in its main part, avoids schematic and questionable replacements of many particle systems by single particles or

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