that is, as the difference between a positive operator of unit norm, and a positive-definite operator of norm less than 1, we see that the norm of K itself is less than 1, and so the Neumann series for $(1-K)^{-1}$ converges.

We briefly recapitulate the results of this Appendix: we have shown that no reformulation, of the Faddeev,[5] Weinberg,[8] or Blankenbecler and Sugar[9] type, of the scattering equation for $g(W)$, yields a tractable integral equation at positive, real energies. They all evidently suffer from the same disease as the two-body Lippmann-Schwinger equation for the Coulomb Green's function, namely the scattering amplitude diverges in the forward direction. This strongly implies that an entirely new approach is required to construct the Coulomb Green's function for three charged particles at positive energies. Secondly, we have shown how a method suggested by Sugar and Blankenbecler[9] may be applied to the case of repulsive potentials, at negative energies. This particular form of the solution [Eq. (A12)] has the advantage of being nearly in the product form, which Amado has found more closely resembles the true solution than the partial-sum form.[26] Evidently there is some merit to Amado's contention, since the kernel K has norm less than 1, indicating that the product $G_0k_1k_2k_3$ is the first term in a convergent series expansion for $g(-|E|)$.

[26] R. D. Amado, Phys. Rev. **158**, 1414 (1967).

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Migdal's Quasiparticle Model and Partial Muon Capture in O^{16}

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By means of the process involving the capture of muons in O^{16} (g.s.) leading to definite final states in N^{16}, we examine simultaneously (a) the *quasiparticle* model of nuclear structure developed by Migdal and (b) the pseudoscalar coupling generated by the axial-vector coupling in the effective weak-interaction Hamiltonian. In (a) we clarify the basic assumptions essential for the model and the connection between this model and other better-known (nuclear) models. In (b), it is shown that the Migdal model successfully eliminates the well-known discrepancy between theory and experiment in $\mu^-+O^{16}(0^+) \to \nu_\mu+N^{16}(2^-)$ and also in $e^-+O^{16}(0^+) \to e^-+O^{16*}(2^-)$. This in turn enables us to make use of the nuclear model to obtain a reasonable estimate of $C_P=m_\mu F_P/F_A$. The conclusion is that the one-pion-pole dominance hypothesis is compatible with all available data in O^{16} and that there seems to be no urgent need to introduce the tensor coupling as some people have suggested.

I. INTRODUCTION

THE major difficulty with the use of complex nuclei as a means of studying the muon capture process is the inherent uncertainty associated with the nuclear structure. One process which does not involve nuclear physics is capture in hydrogen. But since the capture takes place mainly in muon-hydrogen molecules, there is some uncertainty associated with the molecular structure. Moreover, not all the necessary information on the weak-coupling constants can be deduced from this muon-hydrogen experiment.

The μ capture process involves a large momentum transfer $q\sim m_\mu\sim 100$ MeV/c, and for this reason it can provide valuable information about effects which are not found in processes like β decay; e.g., the induced pseudoscalar (P) coupling generated by the axial-vector coupling.[1,2] Goldberger and Trieman (GT) have obtained a theoretical estimate of the P coupling constant by relating the constant to the pion lifetime and pion-nucleon coupling constant in the one-pion-pole dominance hypothesis.[2] At this moment, there is no clear experimental verification of the GT result. There is evidence, however, which suggests that the actual P constant might be much larger than the GT estimate. One set of experiments which seems to indicate this is the measurement of the asymmetry of the neutrons emitted after the capture of partially polarized muons.[3] The other is the radiative μ capture in complex nuclei.[4] These two seem to require a larger P coupling constant than the theoretical estimate.

Here we are concerned with another type of experiment, which seems to have been proposed originally by Shapiro and Blokhintsev.[5] This is to look at the partial-

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[1] H. Primakoff, Rev. Mod. Phys. **31**, 802 (1959).
[2] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).
[3] See the review by H. P. C. Rood [Cern Report, 1965 (unpublished)], where other references are given.
[4] M. Conversi, R. Diebold, and L. di Lella, Phys. Rev. **136**, B1077 (1964); H. W. Fearing, *ibid.* **146**, 723 (1966).
[5] I. S. Shapiro and L. D. Blokhintsev, Zh. Eksperim. i Teor. Fiz. **39**, 1112 (1960) [English transl: Soviet Phys.—JETP **12**, 775 (1961)].

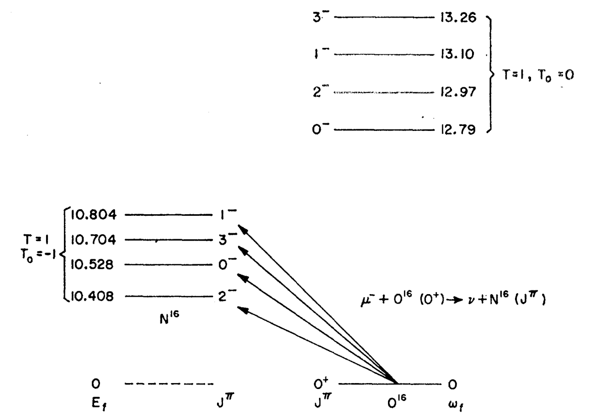

FIG. 1. The energy-level diagrams and isobaric analog-states in O^{16} and N^{16}. The arrows indicate the muon capture transitions. E_f and ω_f are energies in MeV measured relative to the O^{16} ground state.

transition rates in O^{16}. By a partial transition, we mean a transition which involves definite initial and final states. The process we are interested in is

$$\mu^- + O^{16}(0^+, T=0) \to \nu_\mu + N^{16}(0^-, 1^-, 2^-, 3^-, T=1), \quad (1)$$

where we have indicated the J^π of the states of interest in addition to the isotopic spin T. The level schemes involved are shown in Fig. 1, where the isobaric analog states in O^{16} are also exhibited.

Reaction (1) presents an interesting combination of processes. First of all, the transitions $0^+ \to 1^-$ and 3^- do not involve the P coupling constant. If one assumes that the vector and axial-vector coupling constants are known, these processes can be used to determine the validity of the nuclear wave functions. Once the nuclear wave functions are reliably determined, then the transitions $0^+ \to 0^-$ and 2^- can be employed as an experimental probe for the P coupling constant. With these in mind, several experiments[6] have been performed on reaction (1). At the same time, however, it has been found in various theoretical calculations[7] that the $0^+ \to 1^-$ and 2^- cannot be fitted no matter what nuclear wave functions are used, so that doubt is cast on the only result of calculation—namely, the $0^+ \to 0^-$ process—which seems to agree with the GT value. In view of impressive success obtained with the universal Fermi interaction (UFI) picture [namely, the conserved vector current hypothesis (CVC) for the vector coupling[8] and the equality of the muon coupling to the electron coupling for the axial-vector part[9]], such a discrepancy is most likely to come from the nucear-structure uncertainty. It is the major concern of this paper to propose a "nuclear model" which can eliminate this discrepancy.[10]

If we assume that the defect in the nuclear wave functions is the cause of this discrepancy, then we can think of two possibilities: (1) the states involved in the transition may be deformed, in which case the spherical basis which has been used so far is no longer good,[11] and/or (2) the effective nuclear interaction used may not be correct. These two conditions are not completely independent of each other, however, and the relationship between the two is a currently interesting subject. Since there is no conclusive evidence for the deformation of the O^{16} ground state or the low-lying states in N^{16}, we shall attempt to treat the second effect and see whether the defect can be remedied. We shall thus confine ourselves to the spherical j-j coupling scheme throughout this work. The model[12] (or method) we are going to use is that which has been developed by Migdal with the aim of taking into account correctly the interactions between quasiparticles.[13] The main point of this approach is that the renormalization effect due to configurations more complicated than two-quasiparticle ones can in principle be properly taken into account through *both* the "effective" single-particle (transition) operator *and* the "effective" quasiparticle interaction (hereafter denoted by Γ^R), which is described by a set of constants to be taken from various experiments. Whether or not one can obtain such constants theoretically is hard to answer. In this paper, we shall not try to justify Migdal's assumptions, but shall apply the method with the same constants as determined from magnetic moments,[14] β decay,[15] the total μ capture,[16] etc. to the partial transitions (1) and show that the difficulty with the $0^+ \to 2^-$ transition can indeed be eliminated.[17]

In Sec. II the essence of Migdal's approach is reviewed. The assumptions involved are clearly indicated. We discuss in Sec. III the problem of muon capture and

[6] R. C. Cohen, S. Devons, and A. D. Kanaris, Phys. Rev. Letters **11**, 134 (1963); Nucl. Phys. **57**, 255 (1964), referred to as the Columbia measurement; A. I. Astbury, L. D. Auerbach, D. Cutts, R. J. Esterling, D. A. Jenkins, N. H. Lipman, and R. E. Schafer, Nuovo Cimento **33**, 1020 (1964), referred to as the Berkeley measurement.

[7] I. Duck, Nucl. Phys. **35**, 27 (1962); T. Ericson, J. C. Sens, and H. P. C. Rood, Nuovo Cimento **34**, 52 (1964); V. Gillet and D. A. Jenkins, Phys. Rev. **140**, B32 (1965); H. Ohtsubo, Phys. Letters **22**, 480 (1966).

[8] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); for a review on this, see T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **15**, 381 (1965), especially p. 402 ff.

[9] C. P. Bhalla, Phys. Letters **19**, 691 (1966).

[10] From the point of view of weak-interaction physicists, the nuclear-structure complication is an undesirable feature. In this connection, C. W. Kim and H. Primakoff [Phys. Rev. **140**, B566 (1965)] have developed a scheme whereby nuclear physics can be avoided by treating the nuclei as "elementary particles."

[11] For this, see G. E. Brown and A. M. Green, Nucl. Phys. **85**, 87 (1966).

[12] The reason for calling the Migdal's approach a model is stated at the end of Sec. II.

[13] A. B. Migdal, Enrico Fermi Summer School, Varenna, 1965 (unpublished); also Nucl. Phys. **57**, 29 (1966) and references given therein.

[14] A. B. Migdal, Zh. Eksperim i Teor. Fiz. **46**, 1680 (1964) [English transl.: Soviet Phys.—JETP **19**, 1136 (1964)]; Nucl. Phys. **75**, 441 (1966).

[15] A. B. Migdal and V. A. Khodel, Soviet J. Nucl. Phys. **2**, 20 (1966); Y. V. Gaponov, *ibid.* **2**, 714 (1966).

[16] V. M. Novikov and M. G. Urin, Soviet J. Nucl. Phys. **3**, 302 (1966); G. G. Bunatyan, *ibid.* **2**, 619 (1966); **3**, 613 (1966).

[17] A preliminary result was reported before; M. Rho, Phys. Rev. Letters **18**, 671 (1967).

derive the capture rate in terms of Migdal's amplitudes. The details of calculation are given in Sec. IV. Sections V and VI contain discussions of the results as well as some interesting conclusions on both the weak-coupling constants and the nuclear structure. The Appendices should be consulted for explicit formulas and the proofs of two essential theorems.

II. THE NUCLEAR MODEL

In this section, we present in a representation suitable for our purpose what we shall call "Migdal's quasiparticle model of the nucleus." We consider specifically the case where the total isospin T is a good quantum number and where the j-j coupling scheme is applicable.

Let us suppose that an external disturbance, for instance the weak-interaction current, is applied to a doubly closed-shell nucleus in its ground state ($J^\pi=0^+$, $T=0$). If a transition is induced via some single-particle operator t, and the final state $|f\rangle$ has the isospin $T=1$, then to the lowest order in the nuclear interaction the excitation will correspond to a simple particle-hole type, and the transition element $\langle f|t|0^+\rangle$ should involve an isospin flip. To higher orders, not only other one-particle–one-hole (1p-1h) configurations, but also np-nh where $n>1$ can intermix. The aim of Migdal is then to write an equation for an *exact* operator T in terms of another operator t^R which already contains a good part of information about the np-nh configurations. This is illustrated diagrammatically in Fig. 2. This T, which is to be evaluated in the pure 1p-1h configuration, can be simply related to the exact transition matrix element $\langle f|t|0^+\rangle$.

We define $G_{\lambda\lambda'}(\omega)$ to be the single-particle Green's function,[18] where λ represents the appropriate quantum number (for example, $\lambda=n_\lambda l_\lambda j_\lambda m_\lambda$). We write $\langle\alpha\beta|U|\gamma\delta\rangle$ for an irreducible vertex part[19] which in general depends upon energy ω. Let us further define an *exact* transition operator T as

$$\langle\alpha\beta|T(\omega)|0\rangle=\langle\alpha\beta|t|0\rangle+\sum_{\gamma\delta\mu\nu}\langle\alpha\beta|U|\gamma\delta\rangle \times Q_{\gamma\delta\mu\nu}(\omega)\langle\mu\nu|T(\omega)|0\rangle, \quad (2a)$$

where

$$Q_{\gamma\delta\mu\nu}(\omega)=(2\pi i)^{-1}\int d\epsilon\, G_{\gamma\mu}(\epsilon+\omega)G_{\delta\nu}(\epsilon).$$

Suppressing indices and the summation, we shall write this as a matrix equation[20]

$$\mathbf{T}=\mathbf{t}+\mathbf{UQT}. \quad (2b)$$

It will be seen later that $\langle\alpha\beta|T(\omega)|0\rangle$ has a pole corresponding to the transition energy $\omega=E_f$, where $|\alpha\beta\rangle$ is the dominant (1p-1h) configuration of the final state $|f\rangle$; and the residue at that pole is related to the transition matrix element.

We state the first assumption which goes into the model and which we call the Landau quasiparticle hypothesis.[21,22]

(a) Near the Fermi surface, corresponding to low-lying excitations, a single-particle Green's function can be separated into two parts, i.e.,

$$\begin{aligned}G_{\lambda\lambda'}(\omega)&=G_{\lambda\lambda'}{}^{P}(\omega)+G_{\lambda\lambda'}{}^{NP}(\omega),\\ G_{\lambda\lambda'}{}^{P}(\omega)&=Z_\lambda[\omega-\epsilon_\lambda+i\delta\ \mathrm{sign}\epsilon_\lambda]^{-1}\delta_{\lambda\lambda'},\end{aligned} \quad (3)$$

where $Z_\lambda<1$ is known as the Green's-function renormalization, and $G_{\lambda\lambda'}{}^{NP}(\epsilon)$ is a regular function of ϵ near the Fermi surface, but can have poles corresponding to 3-quasiparticle, 5-quasiparticle, etc. excitations. Use of Eq. (3) is a common practice in the usual shell-model calculation. One example is the choice of unperturbed particle or hole energies from neighboring odd-A nuclei. One more simplification is made at this stage by assuming the state independence of Z_λ; i.e., $Z_\lambda=Z$. Then it can be shown that this renormalization constant Z appears only in the constants describing interactions between quasiparticles [see Eq. (8) below]. Therefore, everywhere else, we can set $Z=1$ without loss of generality.

Now if we define another (reducible) vertex part $\mathbf{\Gamma}^R$ which contains Z^2 as[23]

$$\mathbf{\Gamma}^R=\mathbf{U}+\mathbf{UD\Gamma}^R,\quad \mathbf{D}\equiv\mathbf{Q}-\mathbf{A} \quad (4)$$

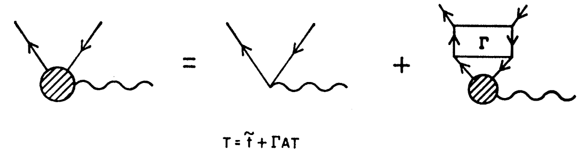

FIG. 2. A diagrammatic representation of Eq. (6). The shaded circle represents the exact operator T, the square represents the amplitude Γ^R (denoted here as Γ) given in Eq. (8), and the internal lines on the second diagram on the right-hand side represent the pole parts of the Green's function. $\tilde{t}$ is the same as t^R of the text.

[18] In terms of the creation and destruction operators $(a^\dagger,a)$ in Heisenberg representation, the Green's function $G_{\lambda\lambda'}(\epsilon)$ is the Fourier transform in time of $G_{\lambda\lambda'}(t)$ where

$$G_{\lambda\lambda'}(t)=-i\langle T(a_\lambda(t)a_{\lambda'}{}^\dagger(0))\rangle_{0^+},$$

$T\equiv$time ordering operator. For a reference, see D. J. Thouless, Nucl. Phys. **22**, 78 (1961).

[19] An "irreducible vertex part" is defined as the part of graph which is connected to the rest of a diagram by one incoming and one outgoing line and cannot be separated into two parts by cutting two Fermion lines horizontally.

[20] The convention for matrix index is as follows: a 1p-1h state is labeled by $(\alpha\beta)$, which therefore makes up one index; **T**, **A**, **t** etc. are to be considered as diagonal matrices or column matrices, and $\mathbf{\Gamma}^R$, **Q**, and **D** are square matrices, since they involve two sets of labels $(\alpha\beta)(\gamma\delta)$.

[21] L. D. Landau, Zh. Eksperim. i Teor. Fiz. **35**, 97 (1958) [English transl: Soviet Phys.—JETP **38**, 70 (1959)].

[22] The validity of Eq. (3) has been discussed extensively by V. Gillet, B. Giraud, and M. Rho, Nucl. Phys. (to be published), and a model in which the assumption apparently breaks down has been studied by S. T. Belyaev and V. G. Zelvinskii, Soviet J. Nucl. Phys. **2**, 442 (1966). See also, E. Werner, D. Müller, and K. Emrich, Z. Physik **188**, 385 (1965); E. Werner, *ibid.*, **191**, 381 (1966).

[23] We shall call A the "pole part" and D the "regular part" of the Green's functions.

or explicitly

$$\langle\alpha\beta|\Gamma^R|\gamma\delta\rangle=\langle\alpha\beta|U|\gamma\delta\rangle+\sum_{\lambda\eta\mu\nu}\langle\alpha\beta|U|\lambda\eta\rangle\times D_{\lambda\eta\mu\nu}\langle\mu\nu|\Gamma^R|\gamma\delta\rangle, \quad (5)$$

then we get

$$\mathbf{T}=\mathbf{t}^R+\mathbf{\Gamma}^R\mathbf{A}\mathbf{T}, \quad (6)$$

where A is a diagonal matrix whose elements are given in terms of occupation probabilities n_α by

$$\delta_{\alpha\gamma}\delta_{\beta\delta}A_{\alpha\beta}(\omega)=\frac{1}{2\pi i}\int d\epsilon\, G_{\alpha\gamma}{}^P(\epsilon+\omega)G_{\beta\delta}{}^P(\epsilon)=\frac{\mathrm{n}_\alpha-\mathrm{n}_\beta}{\epsilon_\alpha-\epsilon_\beta-\omega}\delta_{\alpha\gamma}\delta_{\beta\delta}$$

and

$$\mathbf{t}^R=\mathbf{t}+\mathbf{U}\mathbf{D}\mathbf{t}^R=(1+\mathbf{\Gamma}^R\mathbf{D})\mathbf{t}=e(t)\mathbf{t}. \quad (7)$$

Equation (7) with the neglect of possible energy dependence defines the effective charge $e(t)$ for an operator t. In deriving Eq. (6), Γ^R is assumed to be a slowly varying function of energy. Migdal has given a momentum-space argument which seems to justify such an assumption, but it is not clear how the same argument can be applied to a finite system. As is implied in Eq. (5), Γ^R includes contributions from all configurations which are far away from Fermi surface, and it therefore may be considered to be nearly the same for all nuclei except for the very light ones. This and the above remarks are implied in the following anastz of Migdal, Eq. (8).

(b) The matrix element of Γ^R is given by the energy-independent quantity

$$\langle\alpha\beta|\Gamma^R|\gamma\delta\rangle=V_0\int d\mathbf{r}\Big[f_0(\phi_\alpha{}^*\phi_\beta\phi_\gamma{}^*\phi_\delta)+f_0'(\phi_\alpha{}^*\boldsymbol{\tau}\phi_\beta)\cdot(\phi_\gamma{}^*\boldsymbol{\tau}\phi_\delta)+\frac{f_1}{p_F{}^2}\mathbf{J}_{\alpha\beta}{}^1(\mathbf{r})\cdot\mathbf{J}_{\gamma\delta}{}^1(\mathbf{r})+\frac{f_1'}{p_F{}^2}\mathbf{J}_{\alpha\beta}{}^\tau(\mathbf{r})\cdot\mathbf{J}_{\gamma\delta}{}^\tau(\mathbf{r})$$
$$+g_0(\phi_\alpha{}^*\boldsymbol{\sigma}\phi_\beta)\cdot(\phi_\gamma{}^*\boldsymbol{\sigma}\phi_\delta)+g_0'(\phi_\alpha{}^*\boldsymbol{\sigma\tau}\phi_\beta)\cdot(\phi_\gamma{}^*\boldsymbol{\sigma\tau}\phi_\delta)+\frac{g_1}{p_F{}^2}\mathbf{J}_{\alpha\beta}{}^\sigma(\mathbf{r})\cdot\mathbf{J}_{\gamma\delta}{}^\sigma(\mathbf{r})+\frac{g_1'}{p_F{}^2}\mathbf{J}_{\alpha\beta}{}^{\sigma\tau}(\mathbf{r})\cdot\mathbf{J}_{\gamma\delta}{}^{\sigma\tau}(\mathbf{r})+\cdots\Big], \quad (8)$$

where p_F is the Fermi momentum which we shall take to be 1.36 F^{-1}, and ϕ_α is a single-particle wave function which in the j-j coupling scheme is of the form

$$\phi(\mathbf{r})=R_{nl}(r)\psi_{lsj}(\Omega,\sigma_z)\chi(\tau_z) \quad (9)$$

($R\equiv$radial wave function, $\psi\equiv$angular part including spin, and $\chi\equiv$isospin wave function),

$$\mathbf{J}_{\alpha\beta}{}^\Sigma(\mathbf{r})\equiv\frac{1}{2i}(\phi_\alpha{}^*\nabla\mathbf{\Sigma}\phi_\beta-\phi_\beta\nabla\mathbf{\Sigma}\phi_\alpha{}^*), \quad \mathbf{\Sigma}=1,\ \boldsymbol{\sigma},\ \boldsymbol{\tau},\ \boldsymbol{\sigma\tau}. \quad (10)$$

The quantity V_0 is a normalization factor which acts as an over-all strength normalization of the interaction (analogous to the force strength in an effective force) and is usually taken to be

$$V_0=d\epsilon_F/d\rho, \quad (11)$$

where ϵ_F is the Fermi energy, ρ the nuclear-matter density. With this strength normalization, f, f', g, and g' are dimensionless coupling constants of order unity which also contain the information on the Green's-function renormalization Z^2.

The origin of the terms in Eq. (8) can be seen as follows: Consider the amplitude Γ^R in momentum space; Γ^R can be related to the scattering amplitude. For quasi-particle interactions with small momentum transfer near the Fermi surface, the following expansion is assumed to hold:

$$\Gamma^R=V_0\sum_\kappa[f_\kappa+g_\kappa\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+(f_\kappa'+g_\kappa'\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2]\times P_\kappa\left(\frac{\mathbf{p}_1\cdot\mathbf{p}_2}{p_F{}^2}\right), \quad (12)$$

where P_κ is the Legendre polynomial. If the expansion converges rapidly, one may restrict the sum to $\kappa=0$ and 1. The validity of this remark can be tested only phenomenologically. The Fourier transform of Eq. (12) gives δ-function-type interaction in coordinate space ($\kappa=0$ part), and derivatives of δ functions ($\kappa=1$ part) which give rise to the current-current type of interaction.[24] Although it follows from the Fourier transform of Eq. (12), Eq. (8) is not derived. Our attitude in this paper is to take it as an ansatz. Once one takes Eq. (8), then one sees immediate advantages. Suppose we consider a system with good isospin quantum number T. For $T=1$ states, only the amplitudes with τ operator (f_0',g_0',f_1',g_1') survive, while $T=0$ states involve the f_0, g_0, f_1, and g_1 amplitudes. That reduces the number of constants by a factor of 2. Now if we further assume that LS coupling is sufficiently valid, only the terms with $\boldsymbol{\sigma}$ remain for $S=1$, and the terms without $\boldsymbol{\sigma}$ for $S=0$. Hence, by analyzing different experiments, most of the constants can be determined reliably. In a case where spin-orbit coupling is important, of course, the freedom with the S quantum number is lost.

We shall now express Eq. (8) in a j-j-coupled p-h representation. For this, we need the notation for the particle-hole state.

$$|S_{\alpha\beta}\rangle\equiv|\alpha\beta^{-1}S\rangle\equiv|\alpha\beta^{-1}JT\rangle, \quad (13)$$

where, as before, $|\alpha\rangle=|j_\alpha m_\alpha\rangle|\tfrac{1}{2}m_{\tau_\alpha}\rangle$ and $|\beta^{-1}\rangle\equiv\langle\bar{\beta}|$.

[24] The spin and isospin dependence follows from the rotational invariance in spin and isospin space. Noncentral types of interaction are neglected here. The Fourier transform of Eq. (12) can be given, for example, as

$$f_0\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_1')\delta(\mathbf{r}_2-\mathbf{r}_2')-(f_1/p_f{}^2)(\nabla_1-\nabla_1')\delta(\mathbf{r}_1-\mathbf{r}_1')\cdot(\nabla_2-\nabla_2')\delta(\mathbf{r}_2-\mathbf{r}_2')\delta((\mathbf{r}_1+\mathbf{r}_1')/2-(\mathbf{r}_2+\mathbf{r}_2')/2).$$

$$|\bar{\beta}\rangle \equiv \mathcal{K}|\beta\rangle = (-)^{j_\beta+m+1/2+m_{\tau\beta}}|j_\beta - m_\beta\rangle|\tfrac{1}{2}-m_{\tau\beta}\rangle \quad (14)$$

and $\mathcal{K}$ is the time-reversal operator. For convenience, we shall lump JT together and call it S. For $T=1$, the second and sixth terms in Eq. (8) (denoted by $\Gamma_0{}^R$) can be given as

$$\langle\alpha\beta^{-1}S|\Gamma_0{}^R|\gamma\delta^{-1}S\rangle = 2(2J+1)^{-1}V_0\mathfrak{F}_{\alpha\beta\gamma\delta}\delta_{\lambda J}$$
$$\times\{(-)^{\lambda+1}f_0'\langle j_\alpha j_\beta{}^{-1}J\|Y_\lambda\|0\rangle\langle 0\|Y_\lambda\|j_\gamma j_\delta{}^{-1}J\rangle$$
$$+g_0'\sum_k(-)^k\langle j_\alpha j_\beta{}^{-1}J\|[\sigma\times Y_k]_\lambda\|0\rangle$$
$$\times\langle 0\|[\sigma\times Y_k]_\lambda\|j_\gamma j_\delta{}^{-1}J\rangle\}, \quad (15)$$
$$\mathfrak{F}_{\alpha\beta\gamma\delta} = \int r^2dr\, R_\alpha(r)R_\beta(r)R_\gamma(r)R_\delta(r),$$
$$[\sigma\times Y_k]_{JM} \equiv \sum_{\mu\nu}(1\mu k\nu|JM)\sigma_\mu Y_{k\nu}(\Omega),$$

where $Y_{\lambda\mu}(\Omega)$ are the normalized spherical harmonics. Here the reduced matrix element with respect to the isospin has already been evaluated.

The $\kappa=1$ terms (momentum-dependent amplitudes) cannot be given in a compact form as in Eq. (15), becasue of the more complicated radial integrals. We write them as

$$\langle\alpha\beta^{-1}S|\Gamma_1{}^R|\gamma\delta^{-1}S\rangle = \frac{V_0}{p_F{}^2}\sum_{m_\alpha m_\beta m_\gamma m_\delta}(j_\alpha m_\alpha j_\beta m_\beta|JM)$$
$$\times(j_\gamma m_\gamma j_\delta m_\delta|JM)\int d\mathbf{r}$$
$$\times[f_1'\mathbf{J}_{\alpha\beta}{}^\tau(\mathbf{r})\cdot\mathbf{J}_{\gamma\delta}{}^\tau(\mathbf{r})+g_1'\mathbf{J}_{\alpha\beta}{}^{\sigma\tau}(\mathbf{r})\cdot\mathbf{J}_{\gamma\delta}{}^{\sigma\tau}(\mathbf{r})] \quad (16)$$

in the notation of Eq. (10). The angular momentum algebra is straightforward, and the explicit forms of Eqs. (15) and (16) are given in Appendix A.

A. Polarization Operator

The next thing to do is to express the transition matrix element $|\langle f|t|0^+\rangle|^2 \equiv M_F{}^2$ in terms of the interaction amplitude Γ^R, the pole parts of the Green's function A, and the renormalized quasiparticle operator t^R. In order to do so, we return to the uncoupled representation and introduce a function called the "polarization operator" $\mathcal{P}(\omega)$

$$\mathcal{P}(\omega) = \sum_{\alpha\beta\gamma\delta}\langle 0|t|\alpha\beta\rangle\mathcal{G}(\alpha,\beta;\gamma,\delta;\omega)\langle\gamma\delta|t|0\rangle, \quad (17)$$

where $\mathcal{G}$ is a particle-hole Green's function.[25] Now writing the integral equation satisfied by $\mathcal{G}$ and its expansion in a perturbation series, one obtains [by means of Eq. (2)]

$$\mathcal{P}(\omega) = \sum_{\alpha\beta\gamma\delta}\langle 0|t|\alpha\beta\rangle Q_{\alpha\beta\gamma\delta}(\omega)\langle\gamma\delta|T(\omega)|0\rangle. \quad (18)$$

It is shown in Appendix B in a schematic notation how one can go from Eq. (18) to

$$\mathcal{P}(\omega) = \langle\langle(\mathbf{t}^R\mathbf{D}\mathbf{t}+\mathbf{t}^R\mathbf{A}\mathbf{T})\rangle\rangle, \quad (19)$$

where the double bracket means sum over all indices as in Eqs. (17) and (18). The second term of Eq. (19) now contains only the quantities t^R, A, and T. It is well known that $\mathcal{P}(\omega)$ has poles at ω corresponding to the excitation spectrum (collective and noncollective).[26] Suppose we specialize to a state described by $|f\rangle$ and the eigenenergy ω_f measured relative to $|0^+\rangle$. We have

$$M_f{}^2 = |\langle f|t|0^+\rangle|^2 = [\mathrm{Res}\mathcal{P}(\omega)]_{\omega=\omega_f}. \quad (20)$$

If $|f\rangle$ is a low-lying excitation, it is easy to see that there is no contribution from the first term of Eq. (19). Notice that the pole at $\omega=\omega_f$ is found in the two-particle Green's function in Eq. (17), while it is contained in T in Eq. (19).

B. A Solution

We shall now determine the residue of $\mathcal{P}(\omega)$ for $\omega=\omega_f$ corresponding to a definite state $|f\rangle$. Let us assume that a particular configuration can be assigned to $|f\rangle$. In our case, it would be a (1p-1h) configuration which has the largest amplitude in $|f\rangle$ (in other words, a dominant configuration). We denote it by $|D\rangle \equiv |\alpha_D\beta_D\rangle$.

A useful trick which Migdal has used is to introduce the following quantities[27]:

$$B_{\alpha\beta} \equiv A_{\alpha\beta}(1-\delta_{\alpha\alpha_D}\delta_{\beta\beta_D}),$$

or in matrix form

$$\mathbf{B} = \begin{bmatrix} a_{\alpha_1\beta_1} & & 0 \\ & a_{\alpha_D\beta_D}=0 & \\ 0 & & \ddots \; a_{\alpha_n\beta_n} \end{bmatrix},$$
$$a_{\alpha\beta} = \frac{n_\alpha - n_\beta}{\epsilon_\alpha-\epsilon_\beta-\omega}, \quad (21)$$

and

$$\mathbf{\Pi} = \mathbf{\Gamma}^R + \mathbf{\Gamma}^R\mathbf{B}\mathbf{\Pi}, \quad (22)$$
$$\mathbf{T}' = \mathbf{t}^R + \mathbf{\Pi}\mathbf{B}\mathbf{t}^R. \quad (23)$$

Recalling that T satisfies [see Eq. (6)]

$$\mathbf{T} = \mathbf{t}^R + \mathbf{\Gamma}^R\mathbf{A}\mathbf{T},$$

one gets from Eqs. (22) and (23)

$$\mathbf{T} = \mathbf{T}' + \mathbf{\Pi}(\mathbf{A}-\mathbf{B})\mathbf{T}. \quad (24)$$

We consider the matrix element of Eq. (24) connecting $|0\rangle$ and $|D\rangle$;

$$\langle D|T|0\rangle = \langle D|T'|0\rangle + \langle D|\Pi|D\rangle A_D\langle D|T|0\rangle \quad (25)$$

[25] A particle-hole Green's function $\mathcal{G}(\alpha\beta\gamma\delta;\epsilon)$ is defined by time Fourier transform of
$\mathcal{G}(\alpha\beta\gamma\delta;t) = (-i)^2\{\langle T(a_\beta{}^\dagger(t)a_\alpha(t)a_\gamma{}^\dagger(0)a_\delta(0))\rangle_{0^+} - \langle a_\beta{}^\dagger(0)a_\alpha(0)\rangle_{0^+}\langle a_\gamma{}^\dagger(0)a_\delta(0)\rangle_{0^+}\}$.

[26] See D. J. Thouless, Ref. 18.

[27] The method given here is described in Ref. 13 and also in A. B. Migdal, A. A. Lushnikov, and D. F. Zaretsky, Nucl. Phys. **66**, 193 (1965).

from which we obtain

$$\langle D|T|0\rangle=\langle D|T'|0\rangle[1-\langle D|\Pi|D\rangle A_D]^{-1}. \quad (26)$$

In Appendix C it is shown how one can express $\mathcal{P}(\omega)$ in terms of Eq. (26) and that the singularity (pole) of $\mathcal{P}(\omega)$ coincides with that of Eq. (26). The resulting equation is

$$M_f{}^2=N(\omega_f)^{-2}|\langle D|T'|0\rangle|_{\omega=\omega_f}{}^2, \quad (27)$$

where

$$N(\omega_f)^2=\left[1+(n_{\alpha D}-n_{\beta D})\left\langle D\left|\frac{\partial\Pi}{\partial\omega}\right|D\right\rangle\right]_{\omega=\omega_f} \quad (28)$$

and ω_f is the solution of

$$\omega_f=\epsilon_{\alpha_D}-\epsilon_{\beta_D}-(n_{\alpha_D}-n_{\beta_D})\langle D|\Pi(\omega_f)|D\rangle. \quad (29)$$

Continuing in matrix notation, we have from Eqs. (22) and (23)

$$\mathbf{\Pi}=[1-\mathbf{\Gamma}^R\mathbf{B}]^{-1}\mathbf{\Gamma}^R=\mathbf{\Gamma}^R[1-\mathbf{B}\mathbf{\Gamma}^R]^{-1}, \quad (30)$$

$$\mathbf{T}'=\mathbf{\Pi}\mathbf{A}\mathbf{t}^R. \quad (31)$$

Noting that

$$\frac{\partial\mathbf{B}}{\partial\omega}=\mathbf{S}^{-1}\mathbf{B}^2, \quad (32)$$

where $S_{\alpha\beta}=n_\alpha-n_\beta$, we obtain

$$\begin{aligned}\frac{\partial\mathbf{\Pi}}{\partial\omega}&=\mathbf{\Gamma}^R\mathbf{S}^{-1}\mathbf{B}^2\mathbf{\Gamma}^R(1-\mathbf{B}\mathbf{\Gamma}^R)^{-2}\\&=\mathbf{\Gamma}^R\mathbf{S}^{-1}\mathbf{B}^2(1-\mathbf{\Gamma}^R\mathbf{B})^{-2}\mathbf{\Gamma}^R\\&=\mathbf{\Gamma}^R(1-\mathbf{B}\mathbf{\Gamma}^R)^{-1}\mathbf{S}^{-1}\mathbf{B}^2(1-\mathbf{\Gamma}^R\mathbf{B})^{-1}\mathbf{\Gamma}^R. \quad (33)\end{aligned}$$

This is a matrix operator, the matrix element of which is to be substituted into Eq. (28). Equation (27) together with Eqs. (28), (29), and (33) constitutes the desired solution. To put these equations in the p-h representation, the following substitutions are made

$$|D\rangle=|\alpha_D\beta_D{}^{-1}S\rangle,\quad n_{\alpha_D}-n_{\beta_D}=-1\ (\text{TDA}). \quad (34)$$

C. Connection with Other Methods

It is of interest to compare the solution Eq. (27) with other, better-known methods of nuclear-structure calculation such as the diagonalization of a Hamiltonian matrix in the Tamm-Dancoff approximation (TDA) or random-phase approximation (RPA). For this purpose, let us show first that $N(\omega_f)^{-1}$ defined in Eq. (28) can be related to a normalization constant of the wave function with the eigenvalue $\omega=\omega_f$. This we do in the representation of Eq. (34). Though the proof is quite simple, it does not seem to have been pointed out explicitly in the literature. Writing out explicitly the intermediate states (denoting the complete set of 1p-1h states by $|n\rangle$),

$$\langle D|T'|0\rangle=\sum_n\langle D|\Pi|n\rangle A_n\langle n|t^R|0\rangle. \quad (35)$$

The quantity

$$C_n\equiv\langle D|\Pi|n\rangle A_n \quad (36)$$

can be considered as the *amplitude of the wave function* (i.e., $|f\rangle=\sum_n C_n|n\rangle$). We need to evaluate the sum of $|C_n|^2$ at $\omega=\omega_f$,

$$\begin{aligned}\sum_n|C_n|^2&=\sum_n\langle D|\Pi|n\rangle A_n{}^2\langle n|\Pi|D\rangle\\&=\langle D|\Pi A^2\Pi|D\rangle\\&=\langle D|(1+\Pi B^2\Pi)|D\rangle\\&=\langle D|[1+\Gamma^R(1-B\Gamma^R)^{-1}A^2\\&\qquad\times(1-\Gamma^RB)^{-1}\Gamma^R]|D\rangle.\end{aligned}$$

Comparing Eqs. (28) and (33), we see that

$$\sum_n|C_n|^2=N(\omega_f)^2 \quad (37)$$

which is what we desired to show.[28]

Now to make the connection to the diagonalization procedure, let us return to the polarization operator given by the second term of Eq. (19). Expanding the series for T

$$\begin{aligned}\mathcal{P}(\omega)&=\langle\langle\mathbf{t}^R\mathbf{A}\mathbf{T}\rangle\rangle\\&=\langle\langle\mathbf{t}^R\mathbf{A}(\mathbf{t}^R+\mathbf{\Gamma}^R\mathbf{A}\mathbf{t}^R+\cdots)\rangle\rangle\\&=\langle\langle\mathbf{t}^R(\mathbf{A}+\mathbf{A}\mathbf{\Gamma}^R\mathbf{A}+\cdots)\mathbf{t}^R\rangle\rangle\\&=\langle\langle\mathbf{t}^R\mathbf{K}'\mathbf{t}^R\rangle\rangle, \quad (38)\end{aligned}$$

where $\mathbf{K}'$ satisfies

$$\mathbf{K}'=\mathbf{A}(1+\mathbf{\Gamma}^R\mathbf{K}'). \quad (39)$$

Or more explicitly

$$K'_{\alpha\beta\gamma\delta}=\frac{n_\alpha-n_\beta}{\epsilon_\alpha-\epsilon_\beta-\omega}(\delta_{\alpha\gamma}\delta_{\beta\delta}+\sum_{\mu\nu}\langle\alpha\beta|\Gamma^R|\mu\nu\rangle K'_{\mu\nu\gamma\delta}), \quad (40)$$

which is exactly the same as Eq. (5) of Thouless,[18] except that his Γ differs from ours. Therefore the analysis of Thouless on the RPA will be applicable to Eq. (40) without modification. We shall devote a little more time to this problem in a future paper. Here we merely remark that as long as we deal with particle-hole configurations, the approach of Migdal seems in spirit to be the same as the RPA or TDA with the following notable differences: (1) the appearance of the renormalized operator t^R in Eq. (38) as opposed to t in the Thouless's equation (31); (2) the different nature of determining the effective force Γ^R; the matrix elements Eq. (8) are essentially determined from experiments, while those of usual shell-model forces cannot be uniquely determined.

[28] There seems to be an inconsistency in Migdal's analysis. In his lecture notes, he points out that for a very low excitation, one can take $N^2(\omega_f)\approx 1$. That this is incorrect in general can be seen by considering the low-lying collective 2^+ states in spherical nuclei. Since according to Eq. (23) the wave function is normalized so that the dominant configuration has unit amplitude, large configuration mixing in the 2^+ states implies that $N^2(\omega_f)>1$.

It should be noted that as far as the momentum-independent terms are concerned, there is an apparent connection between the matrix elements Γ^R as defined by Eqs. (8) and (15) and those of the usual shell-model forces; i.e., the $\kappa=0$ terms in Eq. (8) may be compared to the direct or annihilation term of particle-hole matrix elements taken with a zero-range (shell-model) force. Equation (15) shows this manifestly. It is also clear why the τ-independent terms vanish for $T=1$ states; for instance, the term proportional to f_0 in Eq. (8) is $\langle J',T'\|Y_\lambda\|J=T=0\rangle\langle J=T=0\|Y_\lambda\|J',T'\rangle\delta_{J'\lambda}=0$ if $T\neq 0$.

At first sight it may appear that the Migdal matrix elements of Eq. (8) do not contain terms corresponding to the exchange (or scattering) terms of the shell-model matrix elements. That this is not so can be seen as follows: For the zero-range interaction to which our $\kappa=0$ terms correspond, the exchange terms can be written in terms of the direct terms, since the space exchange operator $P_r=+1$. Therefore the Migdal constants f, g, f', and g' which exhibit explicitly the spin and isospin dependence can be given in terms of the force parameters a_0, a_σ, a_τ, and $a_{\sigma\tau}$ for the operators 1, $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$, $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$, and $(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$ in the shell-model force. This shows that Eq. (8) contains terms corresponding both to the direct and to the exchange terms. Although the σ and τ dependence can be shown in the same way for the $\kappa=1$ case as for the $\kappa=0$ case, the connection of these momentum-dependent terms with shell-model matrix elements is not quite clear. A counterpart of such terms may be absent in the latter.

III. PARTIAL TRANSITIONS IN MUON CAPTURE

In this section, we apply the method presented above to the particular situation where a muon is captured into the ground state of the doubly closed-shell nucleus O^{16} and induces transitions to low lying (final) states $J^\pi(T=1)$ in N^{16}. Before proceeding further, we describe the weak-interaction (WI) part of the process.

A. Weak Interaction

The following assumptions are made for the WI process:

1. The validity of the conserved-vector-current hypothesis[8]; this provides the vector part of the coupling constant, and the presence of the weak magnetism term.
2. The absence of G-parity irregular terms[29]; that is, possible tensor and scalar couplings are ignored.
3. Time-reversal invariance; this implies that the weak-coupling constants can be taken to be real.

We make one more assumption: That the effective weak Hamiltonian obtained by Fujii and Primakoff[30] (up to linear in p/M) is sufficiently reliable for the process.

[29] S. Weinberg, Phys. Rev. **112**, 1375 (1958).
[30] A. Fujii and H. Primakoff, Nuovo Cimento **12**, 327 (1959); J. R. Luyten, H. P. C. Rood, and H. A. Tolhoek, Nucl. Phys. **41**, 236 (1963).

TABLE I. Effective weak-coupling constants.

Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
G_V^2	G_A^2	$G_P^2-2G_PG_A$	$-2G_Vg_V$	$-2g_A(G_A-G_P)$	$-2G_Ag_V$
1.038[a]	1.957	−1.428	−1.980	−1.716	2.720

[a] Values obtained with $\nu=90$ MeV, $C_P=8$. The form factors (CVC, β decay) are $F_1=0.972$, $F_2=3.602/2M$, and $F_A=-1.18$ as given in the text. The numbers are in unit of G^2.

Summing over the magnetic quantum number of the final states, averaging over the initial state, and integrating over all directions of neutrino momentum, one obtains for the capture transition[30] $|0^+\rangle\to|f\rangle$ $(\hbar=c=1)$

$$\Lambda(f)=\mathcal{R}\sum_{n=1}^{6}\Omega_nM_n\,, \tag{41}$$

where Ω_n(n=1, $\cdots$, 6) are combinations of effective weak-coupling constants G_V, G_A, G_P, g_V, g_A and are given in Table I; and

$$\mathcal{R}\equiv\frac{\nu^2}{2\pi}\left(\frac{1}{1+\nu/AM}\right)|\phi_\mu|_{\text{av}}{}^2$$
$$=\frac{\nu^2}{2\pi}\left(\frac{1}{1+\nu/AM}\right)\frac{(Z\alpha m_\mu)^3}{\pi}\left(\frac{Z_{\text{eff}}}{Z}\right)^4, \tag{42}$$

where the K-shell muon wave function has been taken out as an average and Z_{eff} has been introduced following Sens.[31] The nuclear matrix elements M_i evaluated with the exact final state $|f\rangle$ are

$$M_n=\sum_{M_f}\int\frac{d\hat{\nu}}{4\pi}\mathfrak{M}_n,\quad n=1,\,2,\,\cdots,\,6 \tag{43}$$

with

$$\mathfrak{M}_1=|\textstyle\int 1|^2,\quad \mathfrak{M}_2=|\int\boldsymbol{\sigma}|^2,\quad \mathfrak{M}_3=|\hat{\nu}\cdot\int\boldsymbol{\sigma}|^2,$$
$$\mathfrak{M}_4=(\textstyle\int 1)^*(\hat{\nu}\cdot\int\mathbf{p}/M),\quad \mathfrak{M}_5=(\hat{\nu}\cdot\int\boldsymbol{\sigma})^*(\int\boldsymbol{\sigma}\cdot\mathbf{p}/M),$$
$$\mathfrak{M}_6=i\hat{\nu}\cdot(\textstyle\int\boldsymbol{\sigma})\times(\int\mathbf{p}/M)^*,$$

where

$$\int\mathbf{t}\equiv\langle f|\sum_{i=1}^{A}\tau_i^{(-)}\exp(-i\boldsymbol{\nu}\cdot\mathbf{r}_i)\mathbf{t}_i|0^+\rangle,$$

$\mathbf{t}=(1,\boldsymbol{\sigma},\mathbf{p},\mathbf{p}\cdot\boldsymbol{\sigma})$, $|\boldsymbol{\nu}|=\nu=$momentum carried away by neutrino$=m_\mu-E_f-E_B$, E_B is the binding energy of the muon in the atom, and E_f is the transition energy.

[31] J. C. Sens, Phys. Rev. **113**, 679 (1958).

For the weak-coupling constants, we follow the notation of Foldy and Walecka,[32] and define

$$G_V/G=F_1\left(1+\frac{\nu}{2M}\right),$$

$$G_A/G=F_A-(\nu/2M)(F_1+2MF_2),$$

$$G_P/G=[F_A(C_P-1)-(F_1+2MF_2)](\nu/2M),$$

$$g_V/G=F_1,\quad g_A/G=F_A,\quad C_P=m_\mu F_P/F_A,$$

and G is the fundamental weak constant equal to $1.02\times10^{-5}/M^2$.

In accordance with the CVC, the vector coupling constant F_1 and the weak magnetism term F_2 are taken to be $F_1=g_V/G=0.972$ and $F_2=(\mu_p-\mu_n)F_1/2M=3.602/2M$. From neutron β decay[9] comes the value for the axial-vector coupling constant $F_A=-1.18$.

The only coupling constant we consider unknown in the WI part is the induced pseudoscalar coupling constant F_P, or equivalently, C_P. The theoretical estimate for this based on the Goldberger-Treiman relation ranges roughly[33] between the following limits:

$$6.5\leqslant C_P\leqslant 7.5. \qquad (44)$$

The analyses based on the radiative muon capture[34] in Ca^{40} and the neutron asymmetry parameter[35] seem to indicate a larger value of C_P (if one neglects the tensor coupling).

In this paper, C_P will be determined by means of the partial capture rates. Just to give an idea of the magnitude, we set $C_P=8$ and $\nu=90$ MeV to obtain the numerical values of the effective coupling constants given in Table I.

B. Quasiparticle Residual Interaction

To write the transition matrix elements in terms of T' [Eq. (23)] and Π [Eq. (22)], we need the following replacements.

(a) $\langle f|\rightarrow N(\omega_f)^{-1}\sum_n\langle D|\Pi|n\rangle A_n\langle n|$;

the normalization $N(\omega_f)$ was defined in Eq. (28).

(b) $\mathbf{t}=(1,\boldsymbol{\sigma},\mathbf{p},\mathbf{p}\cdot\boldsymbol{\sigma})\rightarrow \mathbf{t}^R=e(t)\mathbf{t}$

in terms of the effective charge $e(t)$ for $T=1$. With the definition $C_n\equiv\langle D|\Pi|n\rangle A_n$, we can now write Eq. (44) as

$$\Lambda(f)=\Re\left[1+(n_{\alpha D}-n_{\beta D})\left\langle D\left|\left(\frac{\partial\Pi}{\partial\omega}\right)_{\omega=\omega_f}\right|D\right\rangle\right]^{-1}\times\sum_{i=1}^{6}\Omega_iM_i', \qquad (45)$$

where

$$M_1'=4\pi|\sum_n C_nT_n{}^J(1)|^2,\quad M_2'=4\pi\sum_l|\sum_n C_nT_n{}^{Jl}(\boldsymbol{\sigma})|^2,$$

$$M_3'=4\pi|\sum_l i^{3l}(10J0|l0)\sum_n C_nT_n{}^{Jl}(\boldsymbol{\sigma})|^2,$$

$$M_4'=-4\pi\sum_l i^{3l+J-1}(10J0|l0)\times[\sum_n C_nT_n{}^{Jl}(\mathbf{p})][\sum_n C_nT_n{}^J(1)]^*,$$

$$M_5'=4\pi\sum_l i^{3l+J-1}(10J0|l0)\times[\sum_n C_nT_n{}^{Jl}(\boldsymbol{\sigma})]^*[\sum_n C_nT_n{}^J(\boldsymbol{\sigma}\cdot\mathbf{p})],$$

$$M_6'=-4\pi(\sqrt{6})\sum_{ll'} i^{l+l'+2J+1}(2l+1)^{1/2}(10l0|l'0)\times W(11l'l;1J)[\sum_n C_nT_n{}^{Jl}(\boldsymbol{\sigma})][\sum_n C_nT_n{}^{Jl'}(\mathbf{p})]^*.$$

We have used the notations

$$T_n{}^J(1)=\langle n\|\sum_i \tau_i{}^{(-)}j_J(\nu r_i)Y_J(\Omega_i)\|0\rangle,$$

$$T_n{}^{Jl}(\boldsymbol{\sigma})=\langle n\|\sum_i \tau_i{}^{(-)}j_l(\nu r_i)[Y_l\times\sigma]_J{}^R\|0\rangle,$$

$$T_n{}^{Jl}(\mathbf{p})=\langle n\|i\sum_i \tau_i{}^{(-)}j_l(\nu r_i)[Y_l\times\mathbf{p}/M]_J{}^R\|0\rangle$$

and

$$T_n{}^J(\boldsymbol{\sigma}\cdot\mathbf{p})=\langle n\|i\sum_i \tau_i{}^{(-)}j_J(\nu r_i)Y_J(\Omega_i)(\boldsymbol{\sigma}\cdot\mathbf{p}/M)J^R\|0\rangle.$$

The explicit forms of these reduced matrix elements can be found in the paper by Rose and Osborn.[36]

IV. CALCULATION

A. Available Experimental and Theoretical Results

Two measurements on the partial muon capture in O^{16} are available[6]:

1. The Columbia measurements by Cohen *et al.*, who looked at the processes $N^{16}\rightarrow O^{16*}+e^-+\bar{\nu}_e$, $O^{16*}\rightarrow O^{16}$

[32] L. L. Foldy and J. D. Walecka, Nuovo Cimento **34**, 1206 (1964); Phys. Rev. **140**, B1339 (1965).

[33] J. C. Taylor, Phys. Letters **11**, 77 (1964).

[34] It is difficult to understand why improving the wave function as Fearing did requires larger values of C_P (i.e., $C_P=16.5\pm3.1$ compared with 13.3 ± 2.7 of Conversi *et al.*). In view of the compatibility of other experiments (Ref. 35) with C_P between 6 and 10, it is not unlikely that the basic mechanism of the radiative capture is still not understood well.

[35] See, however, M. K. Akimova, L. D. Blokhintsev, and E. I. Dolinsky, Nucl. Phys. **23**, 369 (1961); these authors use the direct-interaction picture for the neutron emission and an optical potential for the interaction between the neutron and the nucleus.

[36] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1326 (1954). Note, however, that our definition for the reduced matrix element differs from theirs by a statistical factor $(2j_\alpha+1)^{1/2}$.

TABLE II. Particle-hole energies in MeV.[a]

Conf.	$1d_{5/2}(1p_{1/2})^{-1}$	$2s_{1/2}(1p_{1/2})^{-1}$	$1d_{3/2}(1p_{1/2})^{-1}$	$1d_{5/2}(1p_{3/2})^{-1}$	$2s_{1/2}(1p_{3/2})^{-1}$	$1d_{3/2}(1p_{3/2})^{-1}$
$\epsilon_p-\epsilon_h$	11.52	12.39	16.60	17.68	18.55	22.76

[a] These are neutron-particle–neutron-hole energies taken from Ref. 32.

$+\gamma$. The results are[37]

$$\Lambda(0^-)=(1.10\pm0.20)\times10^3\ \text{sec}^{-1}, \tag{46}$$

$$\Lambda(1^-)=(1.88\pm0.10)\times10^3\ \text{sec}^{-1}, \tag{47}$$

$$\Lambda(2^-)=(6.3\pm0.7)\times10^3\ \text{sec}^{-1}. \tag{48}$$

2. The Berkeley measurements by Astbury *et al.*, who studied directly the γ transitions in N^{16}. Their results differ disturbingly from the Columbia results:

$$\Lambda(0^-)=(1.6\pm0.2)\times10^3\ \text{sec}^{-1}, \tag{49}$$

$$\Lambda(1^-)=(1.4\pm0.2)\times10^3\ \text{sec}^{-1}. \tag{50}$$

There have been several calculations[5,7] with the aim of deducing C_P from the experimental results. We make a brief comment on each.

1. Shapiro and Blokhintsev originally proposed that $\Lambda(0^-)/\Lambda(1^-)$ is independent of the nuclear model, and could be used to determine C_P. The model-independence argument is not correct, as was argued by Ericson *et al.*

2. Duck and Ericson *et al.* have used various nuclear models and the Fujii-Primakoff Hamiltonian. Because of their failure to fit the $\Lambda(1^-)$ and $\Lambda(2^-)$ (the latter by more than a factor of 2), no conclusion can be reached from their calculations. Their conclusion that the relativistic terms are dominant for $\Lambda(0^-)$ remains correct, although they have treated them inconsistently by including terms $(p/M)^2$ in Λ with a Hamiltonian valid up to p/M.

3. Gillet and Jenkins have improved the nuclear wave functions by means of the RPA and have also used the Morita-Fujii Hamiltonian.[38] Their results do not show any improvement over the previous calculations as far as $\Lambda(2^-)$ is concerned. $\Lambda(1^-)$ is still too high by more than 25%.

4. Ohtsubo[7] has taken into account $(p/M)^2$ terms correctly and has used various nuclear wave functions, including Gillet's. He introduces further the tensor coupling. Though he gets better results for $\Lambda(0^-)$, the tensor coupling does not help for $\Lambda(1^-)$ and $\Lambda(2^-)$.

5. Kim[39] has used the Stanford data[40] on the transverse form factor $F^2(q)$ for the inelastic electron scattering $e^-+O^{16}(0^+)\to e^-+O^{16*}(2^-)$ (at 180°), and the elementary-particle method of Kim and Primakoff.[10] He has obtained (with the one-pion-pole dominance hypothesis) reasonable agreement: $\Lambda(2^-)=(5.8\pm2.3)\times10^3$ sec^{-1}. Despite the large error assigned to the calculation, which is due to the limited experimental information on the electron scattering itself, Kim's model-independent calculation shows that the failure in the previous calculations stems indeed from the defect in the nuclear wave function.

In the remainder of this section, we describe essential points of our calculations. The questions we ask are: (1) Can the nuclear-structure defect be eliminated? (2) Can the theoretical Λ's be used to determine C_P? (3) Can the process be used to determine some of Migdal's coupling constants which have not been determined before?

B. The Method of Solution

The equations to be solved are Eqs. (29), (30), (31), and (33). These are to be considered as matrix equations to be solved by successive matrix inversion in the particle-hole representation of Eq. (34).[41] The specific forms of the "dominant" configuration $|D\rangle=|\alpha_D\beta_D{}^{-1}S\rangle$ will be given below. We now need to determine the complete set of the p-h configurations for $J^\pi=0^-, 1^-, 2^-, 3^-$ and their p-h energies.

C. Particle-Hole Energies

The neutrino momentum defined above requires the transition energy E_f which is the energy difference between the final state of N^{16} and the O^{16} ground state. The unperturbed quasiparticle-quasihole energies $(\epsilon_\alpha-\epsilon_\beta)$ can be chosen to be the experimental neutron-particle–proton-hole energies. In such a case, $\omega_f=E_f$. For convenience, we use a somewhat different convention. We choose $(\epsilon_\alpha-\epsilon_\beta)$ to be the experimental neutron p-h energies obtained from O^{17} and O^{15}, in which case ω_f corresponds to the excitation energy of $|f\rangle$ in O^{16}. Such a procedure is equivalent to a rotation in isospin space, and is valid for the system in consideration.[32] The neutrino energy can then be given by

$$\nu=m_\mu-\omega_f-E_B+(\omega_f-E_f)\cong108.0\ \text{MeV}-\omega_f, \tag{51}$$

where $\omega_f-E_f=2.56$ MeV is obtained from $[E(2^-,O^{16})-E(2^-,N^{16})]_{T=1}$. It should be noted that for more general cases where T is not a good quantum number, one

[37] The $0^+\to1^-$ Columbia measurement is scaled as by Gillet and Jenkins (Ref. 7). The originally reported value is 1.73 ± 0.10. The scaling factor used in Gillet and Jenkins (i.e., 0.75/0.69) is based on the $1^-\to0^-$ γ branching ratio.

[38] M. Morita and A. Fujii, Phys. Rev. **118**, 606 (1960).

[39] C. W. Kim, Phys. Rev. **146**, 692 (1966).

[40] G. J. Vanprat and W. C. Barber, Nucl. Phys. **79**, 550 (1966).

[41] This method of solution differs in appearance from that of Migdal. He has obtained with some simplifying approximations the exact operators for the magnetic moment calculation. Such a technique does not seem to be feasible here unless one makes a drastic approximation. Besides, the connection to other models is more easily clarified in the method we use.

TABLE III. Transition matrix elements M_i, $i=1, \cdots, 6$ defined by Eq. (45). The numbers correspond to $10\times M_i'$.

J^π		M_1'	M_2'	M_3'	M_4'	M_5'	M_6'
0^-	(a)	0.0	0.608	0.608	0.0	−0.303	0.0
	(b)	0.0	0.412	0.412	0.0	−0.220	0.0
	(c)	0.0	0.415	0.415	0.0	−0.221	0.0
1^-	(a)	0.751	1.216	0.0	−0.0680	0.0	−0.275
	(b)	0.499	0.650	0.0	−0.0474	0.0	−0.183
	(c)	0.661	0.582	0.0	−0.0614	0.0	−0.199
2^-	(a)	0.0	9.425	3.622	0.0	0.0325	−0.118
	(b)	0.0	3.257	1.230	0.0	−0.0438	−0.109
	(c)	0.0	3.266	1.232	0.0	−0.0439	−0.108
3^-	(a)	0.0455	0.0491	0.0	−0.0040	0.0	−0.0077
	(b)	0.0295	0.0312	0.0	−0.0028	0.0	−0.0050
	(c)	0.0324	0.0315	0.0	−0.0030	0.0	−0.0053

[a] $V_0=0$ (Independent quasiparticle approximation).
[b] $V_0/4\pi=35$ MeV F^3, $f_0'=0.35$, $g_0'=0.50$, $f_1'=g_1'=0$.
[c] $V_0/4\pi=35$ MeV F^3, $f_0'=0.35$, $g_0'=0.50$, $f_1'=-0.40$, $g_1'=-0.10$.

should work directly with the neutron-particle–proton-hole representation.

The particle-hole configurations and their energies are given in Table II. The use of the experimental values of $\epsilon_\alpha-\epsilon_\beta$ in the pole part of the Green's function may partially justify the Landau hypothesis which has been used [i.e., Eq. (3)]. It is also believed most likely to be valid near the doubly closed-shell nuclei.

From Table II, it is easy to see that the configurations corresponding to $|D\rangle$ are $|2s_{1/2}1p_{1/2}^{-1}\rangle$ for $J^\pi=0^-$ and 1^-, and $|1d_{5/2}1p_{1/2}^{-1}\rangle$ for $J^\pi=2^-$ and 3^-. The dimensionality N of the matrices Π and $\partial\Pi/\partial\omega$ is 2 for $J=0^-$, 5 for $J=1^-$ and 2^-, and 3 for $J=3^-$.

D. Radial Wave Function

For convenience, we take the harmonic-oscillator wave function for the radial part $R_\alpha(r)$. The error in using such a function is minimized by taking the oscillator length parameter $b=(\hbar/M\omega)^{1/2}=1.75$ F, consistent with electron scattering data. The same value has also been used by the authors in Ref. 7.

E. Coupling Constants for Γ^R

For $T=1$, one has four nuclear constants f_0', g_0', f_1', and g_1'. The corresponding constants for the case of free space and of nuclear matter have been estimated. In finite nuclei, because of the surface, one expects the constants to lie in between the free-scattering and nuclear-matter cases. The essential feature of the present model is that those constants are to be taken from experiments. Without going into the details of how to obtain them (see Refs. 13 through 16), we give the results.

First we need to settle the strength constant V_0. Migdal *et al.* have taken

$$V_0=\frac{d\epsilon_f}{d\rho}=\frac{2\epsilon_f}{3\rho}. \tag{52}$$

Choosing $\epsilon_f=40$ MeV, $r_0=1.25$ F, they have obtained $V_0=4\pi\times35$ MeV F^3. Since this has been used by Bunatyan[16] for the total μ capture in O^{16} and Ca^{40}, we shall take it for the strength normalization.

The comparison with the magnetic moments[14] yields $g_0'=0.50$. This value is consistent with the Gamow-Teller matrix element in β decay[15] and the axial-vector matrix element in total μ capture.[16,42] A rough estimate of f_0' may be obtained from the symmetry term in the Weizsäcker mass formula, but a somewhat more accurate value may be obtained from the electric quadrupole moments.[13] So far, the estimate is $0.35\leqslant f_0'\leqslant 0.40$. Although there is some uncertainty here, the capture rate is rather insensitive to f_0', as we shall see later.

So far there is no information available on the constants f_1' and g_1'. The reason for this is likely to be that the static or even (nonrelativistic) transition moments do not sensitively depend on these terms. Since, however, the partial muon capture, especially the $0^+\rightarrow0^-$ transition, is sensitive to the p/M term in the weak Hamiltonian, it seems necessary to include them. In the absence of other information, we shall infer the signs and magnitudes of those constants from the free-space values; i.e., $f_1'<0$ and $|g_1'|\ll1$. We choose

$$f_1'\cong-0.40, \tag{53}$$
$$g_1'\cong-0.01,-0.1,0.01,0.1$$

and examine the consistency of these values with the data (both the transition rate and transition energy).

F. Effective Charge

The effective charge $e(t)$ for an operator t has been defined to take into account the renormalization due to complicated quasiparticle configurations. This follows from the Landau hypothesis [Eq. (3)], the constancy of Z_λ, and conservation laws (i.e., the Ward identity). The following discussion is based on the assumption that $e(t)$ *does not depend on energy* (nor, therefore, on the state λ). The validity of this assumption is certainly an open question, and probably has to be considered together with the question whether the energy dependence of Γ^R can indeed be neglected.

With the operators $t=(1, \boldsymbol{\sigma},\mathbf{p},\mathbf{p}\cdot\boldsymbol{\sigma})$ which appear in Λ, all except 1 undergo renormalization. The operator $\boldsymbol{\sigma}$ has no conservation law associated with it. Therefore we introduce a parameter ζ such that

$$e(\boldsymbol{\sigma})=1-2\zeta \quad \text{for} \quad T=1. \tag{54}$$

From magnetic moments,[43] Migdal deduces $\zeta=0.05$. *We shall thus take $e(\boldsymbol{\sigma})=0.90$ for all single-particle transi-*

[42] Bunatyan obtains two uncoupled equations corresponding to Eq. (6), one for the $\tau^{(-)}$ operator (vector matrix element) and the other for $\tau^{(-)}\sigma$ (axial-vector matrix element). Since he uses the j-j coupling scheme in his second paper, the separation can at best be approximate. It is not clear how he managed to uncouple them without going to LS coupling, in which case his equations are correct.

[43] This comes from the paramagnetic susceptibility (or the contribution to the magnetic moment from the σ operator); see Ref. 14.

TABLE IV. Capture rates in 10^3 sec^{-1} with $V_0=4\pi\times 35$ MeV F^3.

C_P	0^- a	b	c	d	2^- a	b	c	1^-	3^-
−14	6.79	4.40	4.48	⋯	45.3	13.0	13.0	4.27[a]	0.223[a]
−10	5.57	3.62	3.69	3.88	39.5	11.3	11.4	2.18[b]	0.126[b]
− 8	⋯	3.26	3.32	3.52	⋯	10.6	10.6	2.16[e]	0.121[e]
− 4	⋯	2.59	2.63	2.85	⋯	9.22	9.27	1.96[f]	0.119[f]
0	3.00	1.99	2.02	2.25	28.2	8.07	8.12	2.22[c]	0.133[c]
4	2.18	1.46	1.48	1.72	24.9	7.12	7.16		
6	1.81	1.23	1.24	1.48	23.5	6.72	6.75		
8	1.47	1.01	1.02	1.26	22.3	6.36	6.40		
12	0.881	0.618	0.623	0.874	20.5	5.80	5.83		
16	0.406	0.302	0.301	0.558	19.4	5.43	5.46		
Expt. g	1.1±0.2				6.3±0.7			1.88±0.10	⋯
h	1.6±0.2				⋯			1.4 ±0.2	⋯

[a] Noninteracting quasiparticle approximation ($V_0=0$).
[b] $f_0'=0.35$, $g_0'=0.50$, $f_1'=g_1'=0$.
[c] $f_0'=0.35$, $g_0'=0.50$, $f_1'=-0.40$, $g_1'=-0.10$.
[d] Same as (b) with $(p/M)^2$ correction.
[e] $f_0'=0.40$, $g_0'=0.50$, $f_1'=g_1'=0$.
[f] $f_0'=0.35$, $g_0'=0.50$, $f_1'=0.10$, $g_1'=0.30$.
[g] Columbia measurement.
[h] Berkeley measurement.

tions. This means that for $T=1$

$$\langle n|\boldsymbol{\sigma}^R|0\rangle=(1-2\zeta_n)\langle n|\boldsymbol{\sigma}|0\rangle=(1-2\zeta)\langle n|\boldsymbol{\sigma}|0\rangle. \quad (55)$$

Since we have chosen the j-j coupling scheme, we may drop, in $\langle n|\mathbf{p}^R|0\rangle$, the term corresponding to the spin-orbit correction and write

$$e(\mathbf{p})=1-\tfrac{2}{3}(f_1-f_1'), \quad T=1 \quad (56)$$

with f_1 and f_1' defined in Eq. (12). Though an account of $e(\mathbf{p})$ can be important for the relativistic WI terms, we have no estimate of (f_1-f_1'), and we shall assume that $e(\mathbf{p})\approx 1$. On this basis, we may also take

$$e(\boldsymbol{\sigma}\cdot\mathbf{p})\cong e(\boldsymbol{\sigma})=0.90. \quad (57)$$

A more correct treatment of this term would be important for the $0^+\to 0^-$ transition.

V. RESULTS

The transition matrix elements M_n' $(n=1, 2, \cdots, 6)$ are given in Table III, and the capture rates are summarized in Table IV and in Figs. 3, 4, and 6. Table V contains the transition energy E_f, and Table VI lists Π, A, $\partial\Pi/\partial\omega$, which can be translated into the wave function by means of Eqs. (36) and (37). Finally, Fig. 7 shows $F^2(q)$ for $e^-+O^{16}(0^+)\to e^-+O^{16*}(2^-)$ calculated with the wave function in Table VI. We now discuss each transition.

A. $0^+\to 0^-$ Transition

Only the configurations $2s_{1/2}1p_{1/2}^{-1}$ and $1d_{3/2}1p_{3/2}^{-1}$ contribute to Π matrix. Since the particle-hole partners involve a spin flip, it is easy to see that the spin-independent amplitudes f_0' and f_1' vanish. If one takes Eq. (57), one finds that $\Lambda(0^-)$ is not sensitive to g_1'. The reliability of g_0' determination, as discussed before, is then expected to be important with this transition.

The rate (as shown in Table IV) is very sensitive to C_P through the matrix elements M_3' and M_5'. Since $M_2'=M_3'$ (see Table III), the nonrelativistic contribution is

$$(G_A-G_P)^2M_2'. \quad (58)$$

For $C_P=8$, Table II shows that $(G_A-G_P)^2\approx 0.53$ while $2g_A(G_A-G_P)\approx 1.72$. In such a case the relativistic term Ω_5M_5' becomes dominant. For C_P sufficiently large, say $C_P\approx 20$, both Ω_3 and Ω_5 approach zero, in which case, the higher-order terms (in p/M) are obviously needed.

Friar[44] has shown recently that in order to make the result accurate to order $(p/M)^2$, it is sufficient to add one more term,

$$g_A^2\left|\int \boldsymbol{\sigma}\cdot\mathbf{p}/M\right|^2. \quad (59)$$

The remaining terms turn out to contribute about 3% for $C_P\approx 8$. The results obtained with the addition of Eq. (59) are given in Table IV and Fig. 3. We estimate from the figure the following ranges of C_P compatible with experiments (see M' in Fig. 3):

$3\leqslant C_P\leqslant 7$ for the Berkeley data,

$7\leqslant C_P\leqslant 12$ for the Columbia data.

There is a definite need for more accurate measurement.

Although g_1' may not play an important role for Λ, there is evidence that $g_1'<0$ helps to bring E_f in the right direction (Table V). This possibility will be discussed further below in connection with the 1^- state.

B. $0^+\to 1^-$ Transition

This process which is independent of C_P, has the important role in this calculation of verifying the consistency of the constants f_0' and g_0'. It can also shed some light on the sign and magnitude of f_1' and g_1'. We believe that the relativistic terms of order $(p/M)^2$ can be reliably neglected for this transition and hence our equation for Λ has a negligible error as far as the weak Hamiltonian is concerned.

We find from the calculations (Table IV) that (a) Λ changes insignificantly from $f_0'=0.35$ to $f_0'=0.40$; (b)

[44] J. L. Friar, Nucl. Phys. 87, 407 (1966).

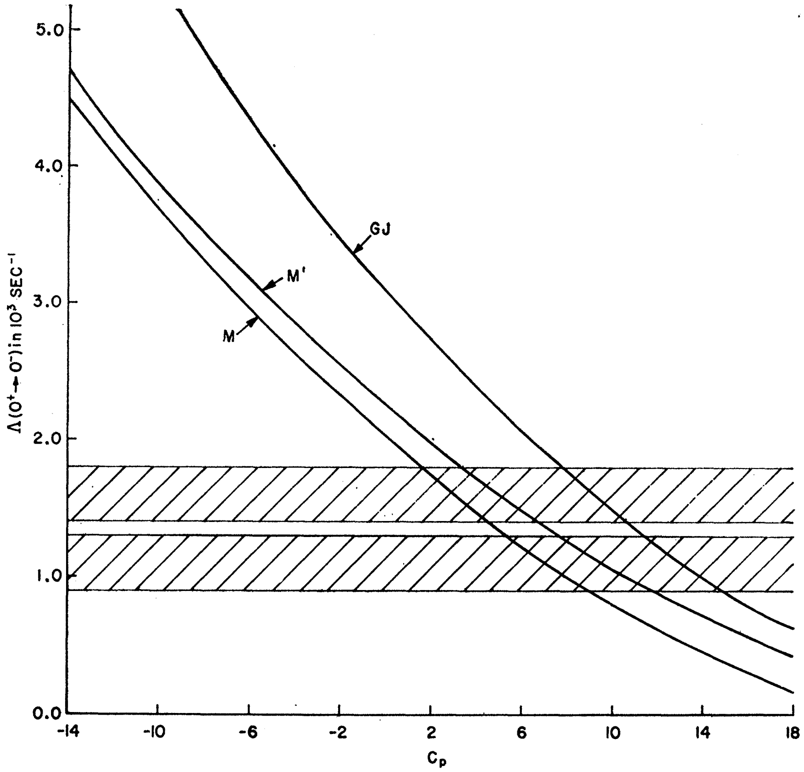

FIG. 3. The capture rates in 10^3 sec^{-1} versus C_P for the transition $\mu^- + O^{16}(0^+) \rightarrow \nu_\mu + N^{16}(0^-)$. The shaded areas are experimental ranges: the upper for the Berkeley measurement, the lower for the Columbia measurement. The curve M represents the Migdal model with $g_0'=0.50$, $g_1'=0$ correct to order p/M, M' the same with $g_1'=0$ correct to order $(p/M)^2$, and GJ stands for the RPA calculation of Gillet and Jenkins ($b=1.75$ F) of Ref. 7.

the result with $f_0'=0.35$ or 0.40 and $g_0'=0.50$ agrees reasonably with the Columbia datum (though higher than the Berkeley one). The capture rate is insensitive to f_1' and also to g_1' within the range given by Eq. (53). This feature is shown in Fig. 4.

Let us now examine the effect on the transition energy (Fig. 5 and Table V). Whereas $\Lambda(1^-)$ is not sensitive to the $l=1$ amplitudes, E_f is; without those terms, E_f is in general too high, as is the case with the 0^- state.

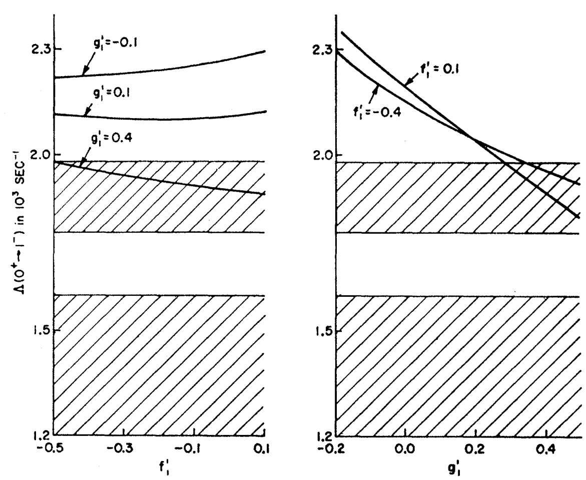

FIG. 4. The capture rates in 10^3 sec^{-1} versus f_1' and g_1' (with $f_0'=0.35$ and $g_0'=0.50$) for $\mu^- + O^{16}(0^+) \rightarrow \nu_\mu + N^{16}(1^-)$. The upper shaded area is the Columbia experimental range, the lower one the Berkeley range.

Figure 5 shows that E_f can be brought into agreement with experiment by taking $f_1' \approx -0.4$ and $g_1' \approx -0.1$. These values are of course not unique; a larger negative g_1' and an $f_1' > -0.4$ will also give the right energy of the 1^- state, and better transition energies for the 0^-, 2^-, and 3^- states. We leave the problem of the energy levels to a future study which will include all other J^π states in addition to the lowest ones. We merely mention here that *negative f_1' and g_1' seem to be necessary for the transition energy to come out correctly.*

C. $0^+ \rightarrow 2^-$ Transition

The f_0' and f_1' amplitudes can be shown to vanish in the Γ^R matrix. Since the g_1' amplitude is very small for this transition, the major role is played by the g_0' term. This is a consequence of the fact that the axial-vector matrix element M_2' dominates the transition (Table

TABLE V. Eigenenergy E_f in MeV for $O^{16} + \mu^- \rightarrow N^{16}$ (lowest states) $+\nu_\mu$ obtained[a] with $f_0'=0.35$, $g_0'=0.50$, and $(1/4\pi)\ V_0=35$ MeV F^3.

	0^-	1^-	2^-	3^-
b	12.15	11.88	10.86	11.21
c	12.19	11.32	10.86	11.11
d	11.74	10.93	10.84	11.07
Expt.	10.53	10.80	10.41	10.70

[a] Values given here are true excitation energies of N^{16} ground and low excited states relative to the O^{16} ground state.
[b] $f_1' = g_1' = 0$.
[c] $f_1' = -0.40$, $g_1' = +0.10$.
[d] $f_1' = -0.40$, $g_1' = -0.1$.

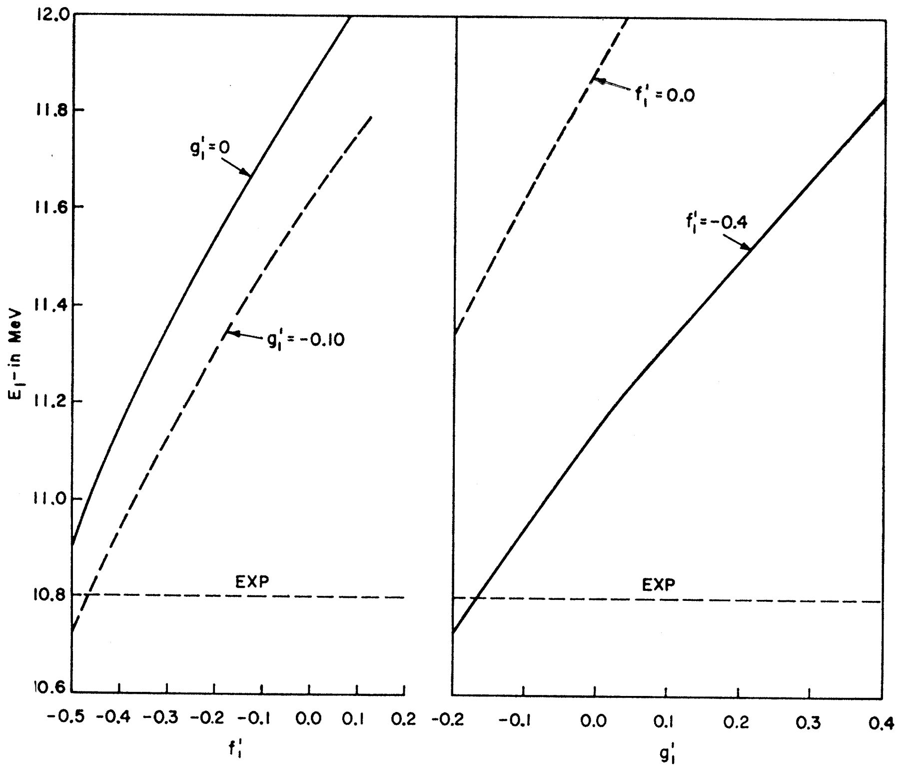

FIG. 5. The transition energy E_f versus f_1' and g_1' for the process $O^{16}(0^+) \to N^{16}(1^-)$. E_f corresponds to the excitation energy of the 1^- state in N^{16} relative to the ground state of O^{16}. The dashed line indicates the experimental value.

III). The relativistic terms (M_5' and M_6') are small. Since major cancellation does not occur between M_2' and M_3', the capture rate $\Lambda(2^-)$ is not as sensitive to C_P near $C_P=8$ as $\Lambda(0^-)$ is.

The crucial test of the nuclear model used here is to see whether it succeds in predicting $\Lambda(2^-)$ correctly. This is the more so in that no other nuclear-model-dependent calculation has made such a prediction consistently with that of $\Lambda(0^-)$. The result is given in Table IV, and agrees with experiment for $C_P \approx 8$. In order to see how this large reduction of Λ from the IQP prediction occurs,[45] it is instructive to examine the matrix elements in Table III. Notice that the quasiparticle interaction reduces the dominant matrix elements by a factor of 3. This can be best illustrated in terms of a wave function. Using Eqs. (36) and (37), and Table VI,

TABLE VI. Effective amplitude Π, propagator A, and residue function $\partial\Pi/\partial\omega$ for the lowest $J^\pi = 0^-, 1^-, 2^-$, and 3^- states of N^{16}: $V_0/4\Pi = 35$ MeV F^3, $f_0' = 0.35$, $g_0' = 0.50$.

J^π	$\partial\pi/\partial\omega$		$1d_{5/2}(1p_{1/2})^{-1}$	$2s_{1/2}(1p_{1/2})^{-1}$	$1d_{3/2}(1p_{1/2})^{-1}$	$1d_{5/2}(1p_{3/2})^{-1}$	$2s_{1/2}(1p_{3/2})^{-1}$	$1d_{3/2}(1p_{3/2})^{-1}$
0^-	-0.0047^a	π		2.32^a				-0.553
	-0.0046^b			1.91^b				-0.573
		A		0.431^a				-0.124
				0.524^b				-0.118
1^-	-0.0125^a	π		2.05^a	-0.030	0.305	-0.225	0.186
	-0.0342^b			1.10^b	$+0.0063$	0.375	-0.812	0.180
		A		0.488^a	-0.463	-0.309	-0.243	-0.120
				0.909^b	-0.322	-0.239	-0.198	-0.108
2^-	-0.150^a	π	1.90^a	...	-0.161	-1.56	-0.287	-0.993
	-0.147^b		1.88^b		-0.154	-1.55	-0.309	-0.983
		A	0.529^a		-0.313	-0.234	-0.195	-0.107
			0.532^b		-0.313	-0.234	-0.194	-0.107
3^-	-0.0765^a	π	2.26^a			1.03		0.759
	-0.0604^b		2.11^b			0.926		0.827
		A	0.442^a			-0.256		-0.111
			0.474^b			-0.247		-0.110

[a] Calculated with $f_1' = g_1' = 0$. [b] Calculated with $f_1' = -0.40$, $g_1' = -0.1$.

[45] We have checked the accuracy of taking the average value of ϕ_μ by comparing the IPM value calculated by Gillet and Jenkins, and our IQPM value with $e(\sigma)$ set equal to unity. They agree within a few percent.

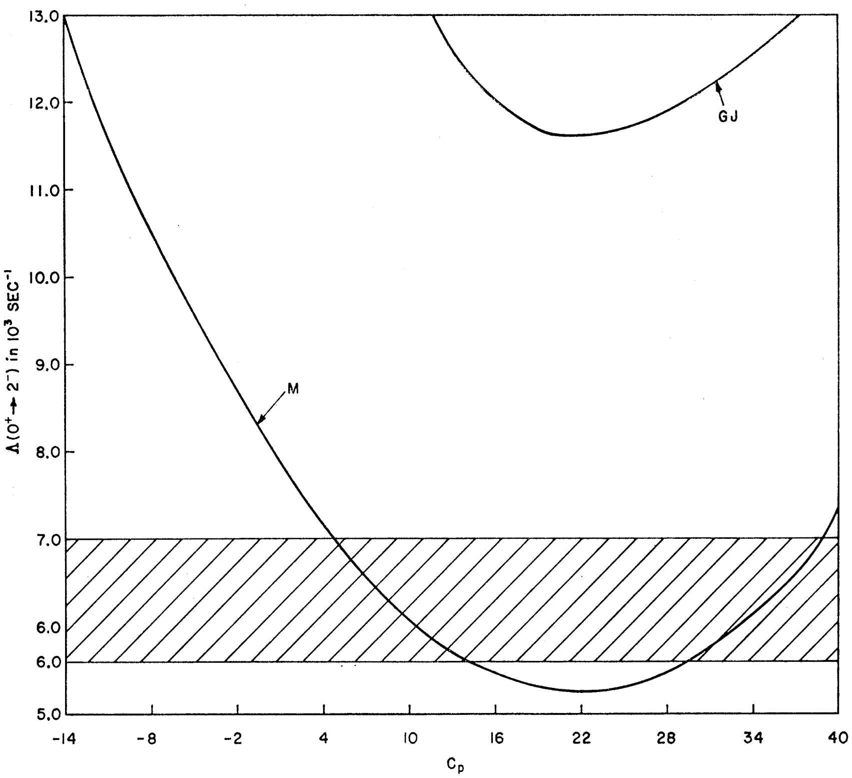

FIG. 6. The capture rates in 10^3 $\sec^{-1}$ versus C_P for the transition $\mu^- + O^{16}(0^+) \to \nu_\mu + N^{16}(2^-)$. The shaded area is the experimental range of the Columbia measurement. M represents the Migdal model with $g_0' = 0.50$ and $g_1' = 0$, GJ the RPA calculation of Gillet and Jenkins.

we can easily construct the normalized wave function, which for $g_0' = 0.50$. and $g_1' = = 0$ is

$$|2^-\rangle = 0.932|d_{5/2}p_{1/2}^{-1}\rangle + 0.340|d_{5/2}p_{3/2}^{-1}\rangle + 0.047|d_{3/2}p_{1/2}^{-1}\rangle + 0.057|s_{1/2}p_{3/2}^{-1}\rangle + 0.099|d_{3/2}p_{3/2}^{-1}\rangle. \quad (60)$$

This is to be compared with the Gillet wave function, the amplitudes of which are 0.983, 0.174, 0.007, 0.054, 0.035 in the order of Eq. (60). Note that Eq. (60) has the same sign as, but much more mixing than, the Gillet wave function.

Consider M_2' which involves the matrix element

$$L_k(n) \equiv \langle n \| \sum_i \tau_i^{(-)} j_k(\nu r_i)[Y_k \times \sigma]_2 \| 0 \rangle, \quad K = 1, 3 \quad (61)$$

which also occurs in M_3' and M_5'. Evaluating for $\nu = 94.6$ MeV, one finds the numerical values

$$L_1(d_{5/2}p_{1/2}^{-1}) = -0.271, \quad L_3(d_{5/2}p_{1/2}^{-1}) = 0.0042,$$
$$L_1(d_{5/2}p_{3/2}^{-1}) = 0.254, \quad L_3(d_{5/2}p_{3/2}^{-1}) = 0.0045, \quad (62)$$
$$L_1(d_{3/2}p_{3/2}^{-1}) = 0.111, \quad L_3(d_{3/2}p_{3/2}^{-1}) = -0.0103,$$
$$L_1(d_{3/2}p_{1/2}^{-1}) = -0.0554, \quad L_3(d_{3/2}p_{1/2}^{-1}) = -0.0205.$$

It is now clear from Eqs. (60) and (62) that a large cancellation of the matrix elements occurs. The dominant matrix elements in Eq. (62) being about equal in magnitude but with opposite sign, the matrix elements are expected to decrease with increasing mixing of the first two configurations in Eq. (60).

The squared transverse magnetic form factor $F^2(q)$, which can be related to inelastic electron scattering cross section at 180°, has been computed by de Forest[46] for

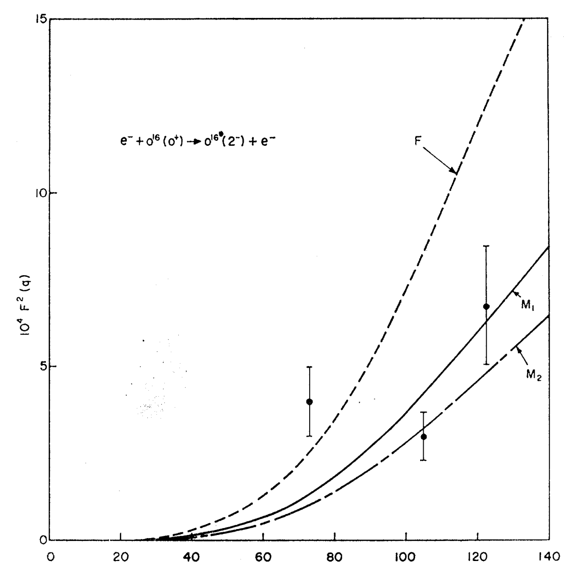

FIG. 7. The squared transverse form factor $F^2(q)$ for the inelastic electron scattering $e^- + O^{16}(0^+) \to e^- + O^{16*}(2^-)$. The experimental points are from the Stanford data (Ref. 40); F was calculated by de Forest; M_1 and M_2 are results of the Migdal model, respectively, without and with the renormalization of the operator $\boldsymbol{\sigma}$, for $g_1' = 0$.

[46] T. deForest, Jr., Phys. Rev. **139**, B1217 (1965).

the process $O^{16}(0^+)+e^- \to O^{16*}(2^-)+e^-$, and it was found that the wave function computed in the ordinary model gave too large values for $F^2(q)$ (roughly by a factor of 2, although there are not enough experimental points to be precise). We have computed $F^2(q)$ using Eq. (60) with and without the renormalization of the $\boldsymbol{\sigma}$ operator. The result is consistent[47] with the experiment and also with Kim's model-independent calculation.[39]

With a reasonable confidence in the nuclear model, we can now estimate the range of C_P. Because of the large error in the experimental datum, we can at best obtain two widely separated ranges, one for $5 \leqslant C_P \leqslant 14$ and the other for $30 \leqslant C_P \leqslant 38$ [see Fig. 6]. The former is quite consistent with that obtained for $\Lambda(0^-)$. We tend to believe that the second range can be ruled out, as it does not seem to be supported by any other information.

D. $0^+ \to 3^-$ Transition

Since the transition is highly forbidden, the matrix element may be too small to measure. However, if it could be measured reliably, it would be useful in checking the nuclear constants, since the C_P-dependent terms do not contribute. Notice that the quasiparticle interaction reduces $\Lambda(3^-)$ much more in our model than in the others: While Gillet and Jenkins find less than 3% reduction from IPM, we get over 40% reduction due to the interaction.

VI. SUMMARY

We have used the coupling constants for the quasiparticle amplitudes and an effective charge $e(\boldsymbol{\sigma})$ which give correctly the magnetic moments, the β-decay rates and other nuclear properties; without unknown parameters, we obtain all three measured partial rates in O^{16} within experimental accuracy. That the present model predicts $\Lambda(2^-)$ correctly is further supported by the satisfactory fit of the transverse form factor for inelastic electron scattering $O^{16}(0^+)(e,e')O^{16}(2^-)$, as can be seen in Fig. 7.

The momentum-dependent Migdal amplitudes do not affect the capture rates, but they seem to be required if the transition energies are to come out correctly and also if more correct treatment of the relativistic WI terms is desired.

We find that the ratio C_P based on the one-pion-pole-dominance hypothesis is quite compatible with the O^{16} data, and there seems to be no necessity to introduce G-parity irregular terms in the WI Hamiltonian.

Finally, the difference between the present model and other models has been pointed out in the case of doubly closed-shell nuclei; it is found to be in the matrix element of the effective force and the effective charge of the transition operators. The success of Migdal's calculation lies most likely in how these quantities are obtained. These last points are being studied further and a report will be given at a later time.

ACKNOWLEDGMENTS

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APPENDIX A: MATRIX ELEMENTS OF Γ^R

Let us explicitly take the particle-hole configuration as

$$|n\rangle = |j_1 j_2^{-1} JT\rangle, \quad |m\rangle = |j_3 j_4^{-1} JT\rangle.$$

Then Eq. (15) is given by

$$\frac{4\pi}{V_0}\langle n|\Gamma_0^R|m\rangle = (-)^{j_1+j_2+j_3+j_4}\hat{l}_1\hat{l}_2\hat{l}_3\hat{l}_4\hat{j}_1\hat{j}_2\hat{j}_3\hat{j}_4 \times 2$$

$$\times\Bigg\{ f_0'(2J+1)^{-1}(-)^{j_2+j_4+1}(l_20l_10|J0)(l_40l_30|J0)W(l_1j_1l_2j_2;\tfrac{1}{2}J)W(l_3j_4l_4j_4;\tfrac{1}{2}J)+6g_0'(-)^{l_2+l_4}$$

$$\times\sum_k (l_20l_10|k0)(l_40l_30|k0)\begin{Bmatrix} l_2 & \frac{1}{2} & j_2 \\ l_1 & \frac{1}{2} & j_1 \\ k & 1 & J\end{Bmatrix}\begin{Bmatrix} l_4 & \frac{1}{2} & j_4 \\ l_3 & \frac{1}{2} & j_3 \\ k & 1 & J\end{Bmatrix}\Bigg\}\mathfrak{F}_{1234}. \quad \text{(A1)}$$

For the momentum-dependent terms, we define

$$H_\lambda{}^k(12,34) \equiv \int r^2dr[(-)^{l_3+l_4+\lambda}h_\lambda{}^k(12)h_\lambda{}^k(34) - h_\lambda{}^k(12)h_\lambda{}^k(43)$$

$$+(-)^{l_1+l_2+\lambda}h_\lambda{}^k(21)h_\lambda{}^k(43) - (-)^{l_1+l_2+l_3+l_4}h_\lambda{}^k(21)h_\lambda{}^k(34)], \quad \text{(A2)}$$

[47] The large reduction is also obtained in the photopion production process $\gamma+O^{16}(0^+) \to \pi^+ + N^{16}(2^-)$; this is consistent with experiment: V. Devanathan, M. Rho, K. S. Rao, and S. C. K. Nair, Nucl. Phys. (to be published).

where

$$h_\lambda{}^k(12)=i(-)^{\lambda+l_1}\frac{\hat{\lambda}\hat{l}_1}{(4\pi)^{1/2}}\Big\{[(l_2+1)(2l_2+3)]^{1/2}$$
$$\times W(l_1l_2k1;\lambda l_2+1)(l_10l_2+10|k0)R_1(r)$$
$$\times\left(\frac{d}{dr}-\frac{l_2}{r}\right)R_2(r)-[l_2(2l_2-1)]^{1/2}W(l_1l_2k1;\lambda l_2-1)$$
$$\times(l_10l_2-10|k0)R_1(r)\left(\frac{d}{dr}+\frac{l_2+1}{r}\right)R_2(r)\Big\}.$$

Then we obtain for the f_1' amplitude in Eq. (16)

$$\frac{p_F{}^2}{V_0}\langle n|\Gamma_1{}^R(f_1')|m\rangle=2f_1'(2J+1)^{-1}$$
$$\times(-)^{j_1+j_3+l_2+l_3+J}\hat{j}_1\hat{j}_2\hat{j}_3\hat{j}_4W(l_1j_1l_2j_2;\tfrac{1}{2}J)$$
$$\times W(l_3j_3l_4j_4;\tfrac{1}{2}J)\sum_k(-)^kH_J{}^k(12{,}34),\quad\text{(A3)}$$

and for the g_1' amplitude

$$\frac{p_F{}^2}{V_0}\langle n|\Gamma_1{}^R(g_1')|m\rangle=12g_1'(-)^{j_1+j_2+j_3+j_4}\hat{j}_1\hat{j}_2\hat{j}_3\hat{j}_4$$
$$\times\sum_\lambda(-)^\lambda\begin{Bmatrix}l_2&l_1&\lambda\\ \frac{1}{2}&\frac{1}{2}&1\\ j_2&j_1&J\end{Bmatrix}\begin{Bmatrix}l_4&l_3&\lambda\\ \frac{1}{2}&\frac{1}{2}&1\\ j_4&j_3&J\end{Bmatrix}$$
$$\times\sum_k H_\lambda{}^k(12{,}34).\quad\text{(A4)}$$

Here we have used the notation $\hat{j}=(2j+1)^{1/2}$, and $\{\ \}$ is a 9-j symbol. Equations (A2) through (A4) can be straightforwardly obtained by means of gradient formulas.

APPENDIX B: DERIVATION OF EQ. (19)

The derivation of this equation is given in Migdal's paper, but since it is not familiar to many nuclear physicists, we repeat it here.

For simplicity, we shall use the matrix notation throughout. Define an exact (reducible) vertex $\mathbf{\Gamma}_{\rm ex}$ as

$$\mathbf{\Gamma}_{\rm ex}=\mathbf{U}+\mathbf{UQ}\mathbf{\Gamma}_{\rm ex}\quad\text{(B1)}$$

and write

$$\mathbf{Q}=\mathbf{A}+\mathbf{D}.\quad\text{(B2)}$$

Recall that

$$\mathbf{\Gamma}^R=\mathbf{U}+\mathbf{UD}\mathbf{\Gamma}^R,\quad\text{(B3)}$$
$$\mathbf{T}=\mathbf{t}+\mathbf{UQT}.\quad\text{(B4)}$$

Using Eqs. (B1) and (B4), we have

$$\mathbf{T}=\mathbf{t}+\mathbf{\Gamma}_{\rm ex}\mathbf{Qt}.\quad\text{(B5)}$$

From Eqs. (1) and (3),

$$\mathbf{\Gamma}_{\rm ex}=\mathbf{\Gamma}^R+\mathbf{\Gamma}^R\mathbf{A}\mathbf{\Gamma}_{\rm ex}.\quad\text{(B6)}$$

With Eqs. (B3) and (B4), we get

$$\mathbf{T}=\mathbf{t}^R+\mathbf{\Gamma}^R\mathbf{AT}=\mathbf{t}^R+\mathbf{t}^R\mathbf{A}\mathbf{\Gamma}_{\rm ex},\quad\text{(B7)}$$

where we have used

$$\mathbf{t}^R=\mathbf{t}+\mathbf{\Gamma}^R\mathbf{Dt}.\quad\text{(B8)}$$

Using Eq. (B7), we can write Eq. (18) of the text as

$$\mathcal{P}=\langle\langle\mathbf{TQt}\rangle\rangle$$
$$=\langle\langle(\mathbf{t}^R\mathbf{Qt}+\mathbf{t}^R\mathbf{A}\mathbf{\Gamma}_{\rm ex}\mathbf{Qt})\rangle\rangle.\quad\text{(B9)}$$

From Eq. (B5), we have $\Gamma_{\rm ex}Qt=T-t$, which we substitute into (B9) to obtain

$$\mathcal{P}=\langle\langle\mathbf{t}^R\mathbf{Dt}+\mathbf{t}^R\mathbf{AT}\rangle\rangle,$$

the desired equation.

APPENDIX C: RESIDUE OF $\mathcal{P}(\omega)$

We wish to show here that the pole of Eq. (26) coincides with the pole of $\mathcal{P}(\omega)$. Recalling the configuration $|\alpha_D\beta_D\rangle\equiv|D\rangle$, we write explicitly

$$\mathcal{P}(\omega)=\langle 0|t^R|D\rangle A_D\langle D|T|0\rangle$$
$$+\sum_n\langle 0|t^R|n\rangle B_n\langle n|T|0\rangle.\quad\text{(C1)}$$

Substituting Eq. (25) into (C1),

$$\mathcal{P}(\omega)=\langle 0|t^R|D\rangle A_D\langle D|T|0\rangle$$
$$+\sum_n\langle 0|t^R|n\rangle B_n\langle n|T'|0\rangle$$
$$+\sum_n\langle 0|t^R|n\rangle B_n\langle n|\Pi|D\rangle A_D\langle D|T|0\rangle$$
$$=\langle 0|T'|D\rangle A_D\langle D|T|0\rangle$$
$$+\sum_n\langle 0|t^R|n\rangle B_n\langle n|T'|0\rangle.\quad\text{(C2)}$$

The proof is completed by noting that the second term does not have a pole at the transition energy $\omega=\omega_f$. The pole appears in $\langle D|T|0\rangle$ in the first term.