Anomalous Level Order in ^{22}Ne: Solution by Means of Two Concurrent Effects

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(Received 25 April 1967)

The experimental order of the ^{22}Ne levels is reproduced by increasing the amount of Majorana exchange as well as the ratio b/a between the harmonic-oscillator length parameter and the range of the residual interaction. Better argeement with experiment is also obtained for the lowest ^{20}Ne band.

I. INTRODUCTION

THREE years ago we suggested[1] that the shortening of the range of the effective interaction could bring about the experimental order in ^{22}Ne levels. This fact was mentioned at the Paris Congress in 1966,[2] but as far as we know nobody has as yet tried to check it. In the meantime new data about ^{22}Ne have been accumulating.[3–5] These data corroborate and amplify the results[6] which motivated our previous work.[1] In fact a calculation along the lines we suggested has been made by Stover.[7] He uses our effective interaction and varies the range, but keeps both the range and the strength of the interaction as free parameters and ignores, in his attempts to fit the experimental results, the data published during the last four years. He concludes that the residual interaction used is inadequate to fit the spectrum of ^{22}Ne. We stress again that we are interested in the order of the levels and not in their spacing. As previously explained,[1] we believe that by mixing in other configurations, the spacing of the levels, and not their order, will change.

II. INCREASE OF b/a IN THE RESIDUAL INTERACTION

In this paper we shall adopt the notations and definitions of our previous work[1] (hereafter referred to as I) in order to avoid repetition. We have here carried out the program outlined in I, that is, we have calculated the two lower $SU(3)$ bands of ^{22}Ne with a Yukawa-Serber potential, varying the range but keeping the product $(a/b)^2V_0$ constant.

Our results for the excitation energy of the three lowest ^{22}Ne excited states are shown in Fig. 1. The effect we anticipated, namely, the relative shift of the two lower bands, appears but is small. The expected inversion of the order of the second 2^+ and the first 4^+ levels occurs only at $b/a=2.6$. This value seems too high, even if one remembers that the determination of b is to a large extent arbitrary.[8] We started from $b/a=1.2$, which is the value used by Elliott and Flowers[9] in their basic work on ^{19}F. (The "normal" increase would bring the b/a value for ^{22}Ne only up to 1.26.)

III. INCREASE OF THE AMOUNT OF MAJORANA EXCHANGE

At this point the natural question is "what further change in the effective interaction could lower the b/a value at which the inversion occurs?" A change which is both easy to handle and quite reasonable is to increase the amount of Majorana exchange. (In fact we have previously used the Serber exchange only because of its convenient simplicity.)

Our next form of the two-body effective interaction is therefore

$$V(r)=V_0(b/a)[e^{-r/a}/(r/a)][W+MP^x], \quad (1)$$

$$V_0(1.2)=-45\ \text{MeV}, \quad (a/b)^2\times V_0(b/a)=\text{const}, \quad W+M=1. \quad (2)$$

We recall[1,10] that for a two-body interaction of the general form $V(r)[W+MP^x]$, the Hamiltonian of an

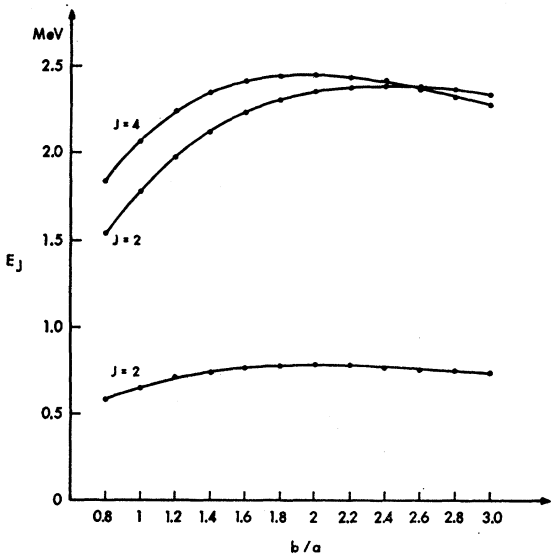

FIG. 1. Excitation energy of the three lowest ^{22}Ne excited states versus the ratio b/a for a Yukawa-Serber residual interaction.

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[3] C. Broude and M. E. Eswaran, Can. J. Phys. **42**, 1300 (1964).
[4] C. Broude and M. A. Eswaran, Can. J. Phys. **42**, 1311 (1964).
[5] S. Buhl, D. Pelte, and B. Povh, Nucl. Phys. **A91**, 319 (1967).
[6] D. Pelte, B. Povh, and W. Sholz, Nucl. Phys. **52**, 333 (1964).
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n-nucleon system in the space of the s, d shell can be written as

$$\mathbf{H}=\sum_{i<j} V(r_{ij})=\sum_{\rho=1}^{10} M_\rho H_\rho. \tag{3}$$

In (3) a separation of effects is achieved, namely, the M_ρ depend *only* on the form of the two-body interaction, while the intrinsic matrix elements of the H_ρ and of the connected operators

$$F_{15}H_\rho,\quad F_{411}H_\rho,\quad F_{-455}H_\rho,\quad F_{1155}H_\rho,\quad F_{45111}H_\rho,$$
$$F_{-41555}H_\rho,\quad F_{441111}H_\rho,\quad F_{-4-4555}H_\rho \tag{4}$$

depend *only* on the nuclear configuration. [The operators $F_{\alpha\beta\gamma}=F_\alpha F_\beta F_\gamma$ are defined in Ref. 10 as products of $SU(3)$ generators.]

For a Serber potential, $W=M=0.5$ and $M_8=M_9=M_{10}=0$. The increase of M in expression (1) does not affect the first seven M_ρ, which stay constant and negative, while $M_8M_9M_{10}$ are positive and increase. The change is, of course, similar to a decrease of the strength of the interaction, and therefore the anticipated general effect would be to reduce the level spacing—which is already too small. However, in this situation every change which goes against the general trend will be strongly felt; therefore, even a little shift up of the $K=2$ band could bring about the wanted inversion of levels. We have stated that the constant (independent of L) coefficient in each band is essentially given by the intrinsic matrix element, and that this matrix element is independent of K for $H=H_8$, H_9, or H_{10}. We are therefore looking for an "unessential" effect caused by the contribution of the matrix elements of the operators listed in (4) to the constant coefficient of the bands. Instead of the usual notation[10] $\langle[f](\lambda\mu)\epsilon_{\max}K|F_{\alpha\beta\gamma}H|[f](\lambda\mu)\epsilon_{\max}K\rangle$, we simply write the intrinsic matrix elements as $\langle K|F_{\alpha\beta\gamma}H|K\rangle$, because K is the only quantum number which differentiates between the two bands. The constant coefficients $c(K)$ of the two bands are written below.

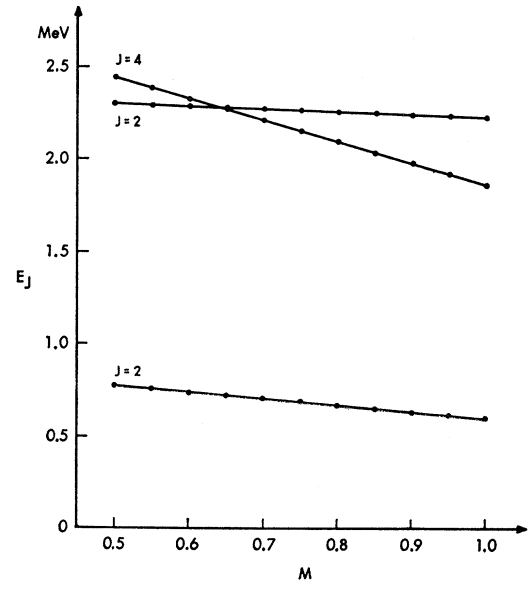

FIG. 2. Excitation energy of the three lowest ^{22}Ne excited states versus the exchange factor M ($b/a=1.8$).

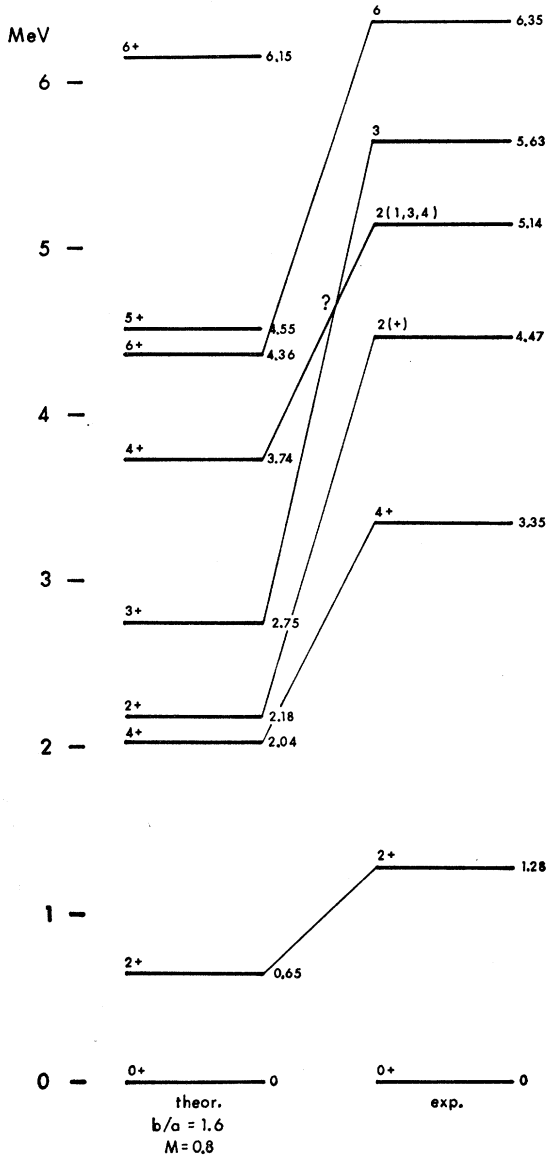

FIG. 3. ^{22}Ne energy levels.

$$c(0)=\langle 0|H|0\rangle-\frac{9}{70}\langle 0|F_{15}H|0\rangle+\frac{2}{70}\langle 0|F_{411}H|0\rangle$$
$$+\frac{4}{350}\langle 0|F_{1155}H|0\rangle-\frac{1}{350}\langle 0|F_{45111}H|0\rangle \tag{5}$$

[10] M. K. Banerjee and C. A. Levinson, Phys. Rev. **130**, 1036 (1963).

$$c(2)=\langle 2|H|2\rangle-\frac{97}{616}\langle 2|F_{15}H|2\rangle+\frac{13}{1232}\langle 2|F_{411}H|2\rangle$$
$$+\frac{43}{2464}\langle 2|F_{1155}H|2\rangle-\frac{5}{1848}\langle 2|F_{45111}H|2\rangle$$
$$+\frac{1}{9856}\langle 2|F_{441111}H|2\rangle. \quad (6)$$

To find out how $c(2)-c(0)$ changes, with the inclusion of H_8, H_9, and H_{10} to H, we used our table of the intrinsic matrix elements of the $F_{\alpha\beta\gamma}H_\rho$. The parameters appearing in the table have been recently recomputed as quotients of integers, thus removing the inaccuracy due to round-off errors. The table will be published shortly.[11]

It turns out that for $H=H_8$, H_9, and H_{10} only the first three matrix elements appearing in Eqs. (5) and (6) differ from zero, and only the second is K-dependent. Due to the equality $\langle 0|H_\rho|0\rangle=\langle 2|H_\rho|2\rangle$ and to the difference between the numerical coefficients appearing in Eqs. (5) and (6), the relevant factor is no more the K dependence of the matrix elements, but their total value. We have calculated once for all the $c(2)-c(0)$ value for every H_ρ and have found, respectively,

$$c(2)-c(0)=0.0642,\ 0.0909,\ -0.1551 \quad \text{for } H=H_8,\ H_9,\ H_{10}. \quad (7)$$

Therefore the $K=2$ band is shifted up by the inclusion of H_8 and H_9 in the Hamiltonian, and down by the inclusion of H_{10}. The net results should, however, be in the desired direction, because both M_8 and M_9 are greater than M_{10} for all the b/a values considered.

To check our predictions, we have recomputed the two lower $SU(3)$ bands of ^{22}Ne for interaction (1) with b/a varying from 0.8 to 3.0, and M from 0.50 to 1.00.

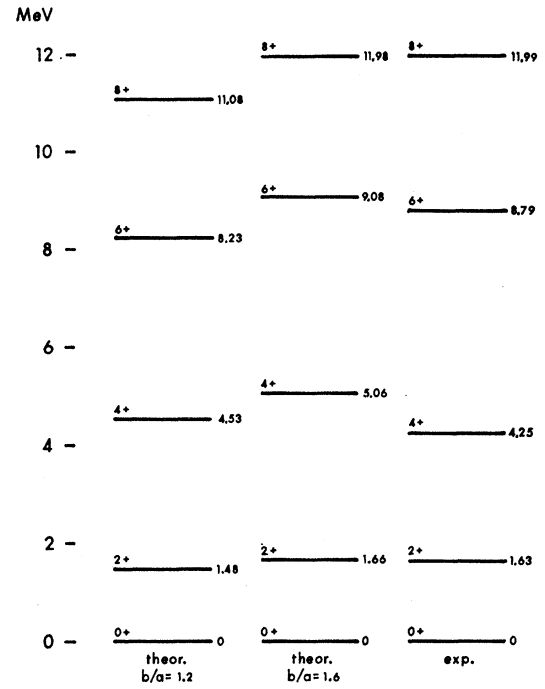

FIG. 4. ^{20}Ne energy levels—the lowest band.

In Fig. 2 we report the excitation energy of the three lowest ^{22}Ne excited states as function of the exchange factor M for an intermediate b/a value ($b/a=1.8$).

As expected the increase of M causes the moment of inertia of the two bands to increase by equal amounts, but for the lowest level of the $K=2$ band this effect is almost balanced by the increase of the difference $c(2)-c(0)$. The inversion is thus achieved with $b/a=1.8$ if $M>0.65$.

The b/a value at which the inversion occurs is given below as function of the exchange factor M:

M	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
$(b/a)_{\text{inv}}$	2.6	2.4	2.2	1.9	1.6	1.3	1.0	<0.8	<0.8	<0.8	<0.8

(8)

IV. REEXAMINATION OF THE LOWEST ^{20}Ne BAND: FINAL CHOICE OF THE PARAMETERS

To sum up: in Fig. 3 we report, besides the latest experimental results,[5] one of the 132 calculated spectra of ^{22}Ne. The choice of the reported spectrum was not based on a best-fit calculation, which would be meaningless; rather we chose the highest amount of Majorana exchange which seemed reasonable, and the b/a value which gives best agreement for the first ^{20}Ne band. In fact the much-quoted[1,12,13] good agreement between the first $SU(3)$ band and the first experimental band in ^{20}Ne is appreciably improved by changing b/a from 1.2 to 1.6—as shown in Fig. 4. (In the case of the first ^{20}Ne band $[f]=[4]$, and therefore the exchange mixture has no influence.)

A similar improvement in the ^{20}Ne case could be achieved by increasing the strength of the interaction (by 8%), but this change would not bring about the inversion of levels in ^{22}Ne.

The experimental data in Fig. 4 are from Kuehner and Ollerhead's paper.[14] Their identification of the 8^+ level at 11.99 MeV together with the previous identifi-

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[14] J. A. Kuehner and R. W. Ollerhead, Phys. Letters **20**, 301 (1966).

cation[15] of the 6^+ state at 8.79 MeV shift the center of interest away from the 4^+ level (the only one for which the agreement is better with $b/a=1.2$ than with $b/a=1.6$). As a matter of fact, it is reasonable to expect a better agreement for the $J=2$ and $J=8$ levels than for the $J=4$ and $J=6$ levels. In general, the mixing of configurations affects more strongly the higher J levels; in the ^{20}Ne case, however, there is no other $J=8$ wave function which could be mixed in within the $2s$, $1d$ shell. As for the $J=4$ level we found in a previous work[16] that configuration mixing brings it down by just about 1 MeV.

V. CONCLUSION

In the case of ^{22}Ne a calculation should now be made with a larger basis to find out if the configuration mixing adequately increases the spacing. As of now it certainly is too early to speculate about possible inversion of higher levels. (See question mark in Fig. 3.)

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^{19}F$(d,n)^{20}$Ne Reaction from 2.5 to 6.5 MeV*

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The excitation functions of the ground- and first-excited-state neutron groups from the reaction ^{19}F$(d,n)^{20}$Ne have been measured at laboratory angles of 0° and 30° in the deuteron energy range of 2.5 to 6.5 MeV with energy steps of about 80 keV. In general, the fluctuations observed in the excitation functions are relatively small. The angular distributions of both neutron groups have also been measured at 3.062, 3.554, 3.851, 4.400, 4.938, 5.601, and 6.065 MeV. The ground-state group peaks strongly at 0° for energies above 4 MeV, while the first-excited-state group peaks at 30° for all energies, as expected from stripping theory for the known spins and parities of the levels of ^{20}Ne. The angular distributions measured at incident energies above 4.4 MeV have been fitted by distorted-wave Born approximation calculations. The spectroscopic factors determined in this way vary by approximately a factor of 2, becoming larger at the higher energies. The energy-averaged value of the spectroscopic factor for the ^{20}Ne ground state is found to be 0.40, and that for the first-excited state is 0.38. When compared with the predictions of various nuclear models, these values are found to be in agreement with the hypothesis that ^{20}Ne is a deformed nucleus.

1. INTRODUCTION

THE (d,n) reaction has been successfully used in nuclear spectroscopy for a number of years. The shapes of the angular distributions of the neutron groups can be used to fix the parities and give information about the spins of the levels of the residual nuclei.[1–3] The intensities of the neutron groups provide a measure of the spectroscopic factors of the corresponding levels,[4,5] and these spectroscopic factors can be compared to the predictions of various nuclear models.

The primary purpose of this study of the ^{19}F$(d,n)^{20}$Ne reaction was to get reliable values of the spectroscopic factors for the ground- and first-excited states of ^{20}Ne from distorted-wave Born approximation (DWBA) fits to the data. The neutron groups to the ground- and first-excited states of ^{20}Ne were easily resolved in this work. The second-excited-state group could also be identified in some of the neutron spectra, but no useful information concerning it could be extracted from the data.

The spins and parities for these three levels in ^{20}Ne are well known, and it has been suggested that they can be explained by a rotational model.[6] On the assumption that ^{20}Ne is a deformed nucleus, spectroscopic factors have been calculated[5] using Nilsson wave functions,[7] and also using Bishop's modification[8] of the Nilsson functions. Two shell-model calculations have been published in recent years which also give a rotational-type level structure for ^{20}Ne. The first of these, by Elliott,[9] used the SU_3 classification of shell-

* This work was supported in part by the National Science Foundation.

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