$M2$ Transitions in Nuclei*

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A survey of experimental data about $M2$ transitions between low-lying nuclear states shows that the great majority of transitions in nuclei with $A>30$ is severely inhibited. For heavy nuclei with a permanent deformation, the inhibition can be ascribed either to K forbiddenness or to an asymptotic selection rule. Even when a permanent deformation is not present, the effect of this selection rule may persist, and it can lead to $M2$ transitions whose lifetimes are several hundred times the single-particle estimate. In light nuclei, some transitions are inhibited by an isobaric-spin effect. Theoretical values are obtained for comparison with some recently measured $M2$ lifetimes in the region around $A=40$. Both quadrupole deformation and the isobaric-spin effect can contribute to inhibitions in this region. The isobaric-spin effect is sufficient to account for the lifetimes in K^{39} and Ca^{41}. For nuclei further removed from Ca^{40}, the effect of deformation is also necessary. In some simple examples we show that such a trend results from the nature of the particle-hole interaction.

I. INTRODUCTION

A SURVEY of the measured $M2$ lifetimes in nuclei with $A\geqslant 30$ shows that transitions between low-lying nuclear states are severely inhibited, often by several orders of magnitude. In this paper we show that for deformed nuclei there is an asymptotic selection rule—similar to the asymptotic selection rule for $E1$ decay[1]—which inhibits the emission of this type of radiation. Even when a permanent deformation does not exist, the effect of this selection rule still persists for nuclei only slightly removed from a closed shell. This is apparent from the lifetime measurements of the hole states in the scandium isotopes.[2,3] In these latter nuclei, the residual nucleon-nucleon force leads to an eigenfunction for the hole state in which the hole couples not only to the ground state of the core nucleus but also to one of the low excited states of the core. Since these components interfere destructively in the matrix element governing the transition,[4,5] the lifetime of the state is longer than predicted by the single-particle model. Thus in addition to the well-known cancellation in the matrix element which occurs in self-conjugate nuclei when the transition is between states of the same isobaric spin,[6] there is an additional cancellation which arises because of the strong quadrupole force in nuclei.

Recently, six new $M2$ lifetime measurements have been made for nuclei that do not have a permanent deformation, i.e., for nuclei in the $1f_{7/2}$ shell.[7,8] With the exception of the partial $M2$ lifetime for the $\frac{3}{2}^+\rightarrow\frac{5}{2}^-$ transition[9] in Ca^{43}, and perhaps the $\frac{3}{2}^+\rightarrow\frac{7}{2}^-$ transition[8] in K^{39}, these transitions are all severely inhibited. It is shown that the degree of inhibition encountered in these transitions can be quantitatively understood if effects resulting from the strong quadrupole force are taken into account in the wave functions used.

II. ASYMPTOTIC SELECTION RULE

In this section we shall show that for nuclei with large permanent deformations there is an asymptotic selection rule which in most cases prohibits the emission of $M2$ radiation between low-lying nuclear states. Following the notation of Nilsson,[10] we may write the deformed oscillator potential as

$$V(r)=\tfrac{1}{2}m\omega^2r^2-\tfrac{4}{3}\delta(\pi/5)^{1/2}m\omega^2r^2Y_0{}^2. \qquad (1)$$

As discussed by Nilsson in Appendix B of his paper, if the deformation δ is very large, then it may be convenient to characterize the spatial eigenfunctions of the Hamiltonian in terms of the asymptotic quantum numbers N, n_z, and Λ, where N is the principal oscillator quantum number, n_z is the oscillator quantum number along the symmetry axis of the nucleus, and Λ is the orbital angular-momentum component along the symmetry axis. In the limit in which the spin-orbit interaction is small in comparison with the deformation energy, the lowest state for $\delta>0$ with principal quantum number N will be the one in which $n_z=N$, $\Lambda=0$, and the highest state in the shell will have $n_z=0$, $\Lambda\leqslant N$. Thus for positive deformation the Nilsson level whose n_z is at its maximum value (and whose Λ is at its minimum value) will fill first. For negative deformation, the situation will be just reversed.

Low-lying states of parity opposite to that of the states that arise during filling of the oscillator shell N can occur in one of two ways: (1) by exciting a particle out of a closed shell and up to the valence shell N,

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[1] D. Strominger and J. O. Rasmussen, Nucl. Phys. 3, 197 (1957).
[2] R. E. Holland, F. J. Lynch, and K.-E. Nystén, Phys. Rev. Letters 13, 241 (1964).
[3] T. T. Bardin, K. Runge, and C. S. Wu, Nucl. Phys. 88, 169 (1966).
[4] R. D. Lawson and M. H. Macfarlane, Phys. Rev. Letters 14, 152 (1965).
[5] R. K. Bansal and J. B. French, Phys. Letters 14, 230 (1965).
[6] E. K. Warburton, Phys. Rev. Letters 1, 68 (1958).
[7] *Nuclear Data Sheets*, edited by K. Way, *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 15, D. C.).
[8] R. E. Holland and F. J. Lynch (to be published).
[9] T. Lindqvist, Arkiv Fysik 12, 495 (1957).
[10] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 29, No. 16 (1955).

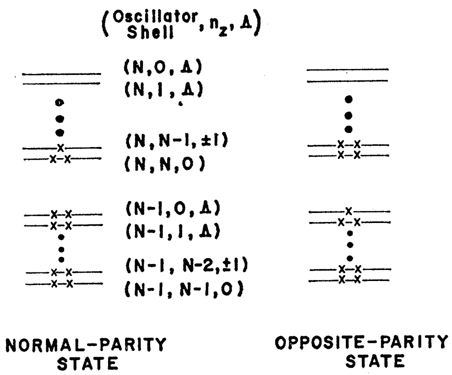

FIG. 1. The configuration of a normal-parity state and an opposite-parity state at the beginning of a shell, illustrated for prolate deformation. The quantum numbers (N,n_z,Λ) characterize the spatial part of the eigenfunction when the deformation is large. Λ can take on the values $0, \pm 2, \cdots, \pm(N-n_z)$ for $(N-n_z)$ even or $\pm 1, \pm 3, \cdots \pm(N-n_z)$ for $(N-n_z)$ odd. An x signifies that the level is occupied.

or (2) by exciting a valence nucleon from the shell N up to the shell $N+1$.

At the beginning of the shell, the normal-parity states are those that would arise from filling the lowest Nilsson orbits with given N; and if positive deformation is assumed these orbits have n_z near its maximum value and Λ near zero. The opposite-parity states will arise from exciting a nucleon out of one of the uppermost levels in the closed shell to one of the valence orbits. Thus the hole in the closed shell will have n_z near zero. This situation is illustrated in Fig. 1. Therefore, a γ transition between these two states involves changing a particle with n_z near its maximum value into a particle with n_z near zero. When the shell is more than half-filled, the opposite-parity state will arise from the excitation of a valence nucleon to the next higher shell; and again a γ transition to the normal-parity states will require a large change in n_z.

If one neglects the small contribution from the orbital angular momentum, then the $M2$ operator for the transition between states has the form

$$(M2)_\lambda = 10^{1/2}\mu(e\hbar/2mc)[\sigma_m \times rY_{m'}^1]_\lambda^2, \qquad (2)$$

where σ_m is the spin operator, $rY_{m'}^1$ is the spatial part of the operator, μ is the magnetic moment of the neutron or proton, and the notation $[\times]_\lambda^2$ means that the space and spin operators are coupled to angular momentum 2 with z component λ. Now the quantity $rY_{m'}^1$ is precisely the $E1$ operator. Hence any selection rule deduced for this operator which depends only on the space coordinates of the particle must carry over and apply to the $M2$ matrix element. Naturally, the required spin dependence can ordinarily be accommodated to the Σ values of the Nilsson states to give $\Delta\Sigma = 0, \pm 1$. Strominger and Rasmussen[1] have shown that the $E1$ operator can only induce transitions in which $\Delta n_z = \pm 1$, $\Delta\Lambda = 0$ or $\Delta n_z = 0$, $\Delta\Lambda = \pm 1$. Consequently, if one neglects the small contribution from the $[l_m \times rY_{m'}^1]_\lambda^2$ part of the $M2$ operator, then the same selection rules that apply to $E1$ transitions should apply to $M2$ decay. Thus $M2$ transitions between low-lying nuclear states should be inhibited in much the same way that $E1$ transitions are slowed down.

TABLE I. $M2$ lifetimes for $A>30$. Except where noted, the experimental lifetimes (column 5) were taken from the tables of Lindskog, Sundström, and Sparrman (Ref. 13). Columns 1, 2, 3, and 4 give the nucleus under consideration, the initial spin state I_i, the final spin state I_f, and the energy of the emitted γ ray in MeV, respectively. Column 6 gives a single-nucleon estimate for the lifetime [Eq. (3)]. In the last column, the ratio of the observed lifetime to the estimated lifetime is tabulated.

Nucleus	I_i	I_f	E (MeV)	τ_γ (sec)	τ_{est} (sec)	τ_γ/τ_{est}
P^{31}	$\frac{3}{2}^-$	$\frac{1}{2}^+$	7.768	1.3×10^{-11}	4.3×10^{-14}	300
	$\frac{3}{2}^-$	$\frac{1}{2}^+$	4.635	4.5×10^{-11}	5.6×10^{-13}	80
	$\frac{3}{2}^-$	$\frac{1}{2}^+$	8.546	4.6×10^{-13}	2.6×10^{-14}	18
	$\frac{3}{2}^-$	$\frac{1}{2}^+$	9.036	$>9.2\times10^{-12}$	2.0×10^{-14}	>460
	$\frac{3}{2}^-$	$\frac{1}{2}^+$	9.165	4.0×10^{-13}	1.9×10^{-14}	21
	$\frac{1}{2}^-$	$\frac{3}{2}^+$	7.95	2.0×10^{-13}	1.9×10^{-14}	11
Ar^{39}	$\frac{3}{2}^+$	$\frac{7}{2}^-$	1.520	2.7×10^{-9}[a]	5.0×10^{-11}	54
K^{39}	$\frac{7}{2}^-$	$\frac{3}{2}^+$	2.820	$<8\times10^{-11}$[b]	4.5×10^{-12}	<18
K^{41}	$\frac{7}{2}^-$	$\frac{3}{2}^+$	1.291	9.5×10^{-9}[a]	2.2×10^{-10}	43
Ca^{41}	$\frac{3}{2}^+$	$\frac{7}{2}^-$	2.014	8.0×10^{-10}[b]	1.2×10^{-11}	57
Ca^{43}	$\frac{3}{2}^+$	$\frac{7}{2}^-$	0.993	5.0×10^{-8}[b]	3.9×10^{-10}	130
	$\frac{3}{2}^+$	$\frac{5}{2}^-$	0.619	1.8×10^{-9}[c]	2.5×10^{-8}	0.07
Sc^{43}	$\frac{3}{2}^+$	$\frac{7}{2}^-$	0.150	6.6×10^{-4}[d]	5.0×10^{-6}	130
Sc^{45}	$\frac{3}{2}^+$	$\frac{7}{2}^-$	0.0125	140[d]	1.2	120
Sc^{47}	$\frac{3}{2}^+$	$\frac{7}{2}^-$	0.766	4.0×10^{-7}[d]	1.3×10^{-9}	300
Ga^{72}	1^+	3^-	0.100	9.6×10^{-2}	6.9×10^{-5}	1390
Ge^{71}	$\frac{9}{2}^+$	$\frac{5}{2}^-$	0.023	4.3	7.6×10^{-2}	56
Ge^{73}	$\frac{1}{2}^-$	$\frac{5}{2}^+$	0.0539	8.4	5.0×10^{-4}	17 000
As^{73}	$\frac{9}{2}^+$	$\frac{5}{2}^-$	0.359	8.8×10^{-6}	8.0×10^{-8}	110
As^{75}	$\frac{9}{2}^+$	$\frac{5}{2}^-$	0.025	4.2	4.8×10^{-2}	87
Br^{78}	3^-	1^+	0.149	2.2×10^{-4}	8.9×10^{-6}	25
Br^{81}	$\frac{9}{2}^+$	$\frac{5}{2}^-$	0.272	5.6×10^{-5}	3.0×10^{-7}	190
Rb^{85}	$\frac{9}{2}^+$	$\frac{5}{2}^-$	0.514	8.7×10^{-7}	1.2×10^{-8}	72
Tc^{101}	$\frac{5}{2}^-$	$\frac{9}{2}^+$	0.191	1.8×10^{-3}	9.1×10^{-7}	2000
Pd^{105}	$11/2^-$	$\frac{7}{2}^+$	0.183	8.9×10^{-5}	1.7×10^{-6}	52
Sb^{124}	5^+	3^-	0.010	$\sim$90 000	4.8	$\sim1.9\times10^4$
Eu^{147}	$11/2^-$	$\frac{7}{2}^+$	0.395	$>1.2\times10^{-6}$	2.9×10^{-8}	>43
Eu^{149}	$11/2^-$	$\frac{7}{2}^+$	0.347	$>4.4\times10^{-6}$	5.6×10^{-8}	>78
Eu^{151}	$11/2^-$	$\frac{7}{2}^+$	0.175	$>3.4\times10^{-4}$	1.7×10^{-6}	>200
Ho^{165}	$\frac{3}{2}^+$	$\frac{7}{2}^-$	0.3167	2.8×10^{-6}	2.5×10^{-8}	110
W^{181}	$\frac{5}{2}^-$	$\frac{9}{2}^+$	0.3655	3.0×10^{-5}	2.4×10^{-8}	1300
W^{183}	$11/2^+$	$\frac{7}{2}^-$	0.103	>300	2.1×10^{-5}	$>1.4\times10^7$
Os^{190}	10^-	8^+	0.038	9.5×10^5	4.5×10^{-3}	2×10^8
Pb^{205}	$13/2^+$	$\frac{9}{2}^-$	0.0262	74	1.8×10^{-2}	4100
Po^{205}	$13/2^+$	$\frac{9}{2}^-$	0.160	1.8×10^{-2}	2.1×10^{-6}	8700
Po^{207}	$13/2^+$	$\frac{9}{2}^-$	0.310	1.6×10^{-4}	7.6×10^{-8}	2100

[a] Reference 7.
[b] Reference 8.
[c] Reference 9.
[d] Reference 2.

Table I lists the known $M2$ transitions for $A \geqslant 30$ and compares the experimental mean γ-ray lifetimes with single-particle estimates. For the single-particle estimate we use

$$\tau_{est} = [8.4\times10^7 A^{2/3} E^5 S]^{-1}, \qquad (3)$$

where $S=1$ for even A, while for odd A the statistical factor S depends on the spins involved. Moszkowski's definition[11] is

$$S=(2I_f+1)(I_iI_f\tfrac{1}{2}-\tfrac{1}{2}|20)^2$$

if I_i and I_f are half-integral. Here $(I_iI_f\tfrac{1}{2}-\tfrac{1}{2}|20)$ is a Clebsch-Gordan coefficient. The experimental data for P^{31} were taken from the recent compilation of Skorka.[12] Except where noted, the other experimental data were taken from the compilation of Lindskog, Sundström, and Sparrman.[13]

In most cases, the $M2$ lifetime is 10 to 1000 times the single-particle estimate. There are only four cases in Table I involving nuclei with a large stable quadrupole deformation. Of these, the large inhibitions in W^{183} and Os^{190} can be attributed to K forbiddenness. The cases of Ho^{165} and W^{181} involve transitions between Nilsson levels such that $\Delta n_z=1$ and $\Delta\Lambda=2$, so that only the weak orbital part of the $M2$ operator contributes. This is consistent with the fact that the odd-neutron transition in W^{181} is 10 times weaker than the odd-proton transition in Ho^{165}. The majority of the nuclei listed do not have a permanent deformation, yet they exhibit a substantial reduction in the transition matrix element. It has been shown previously in the case of the scandium isotopes[4] that a cancellation in the $M2$ matrix element results if one projects, from the appropriate Nilsson eigenfunction, states with good angular momentum and then calculates the $M2$ transition rates by use of these projected wave functions. Thus, even when the nucleus does not possess a permanent deformation, the effect of the quadrupole force is to give eigenfunctions corresponding to a mixture of shell-model configurations; and this mixture is such that there is destructive interference in the $M2$ transition amplitude.

In certain other cases, where the nuclei do not possess a permanent deformation, there may be some simple configuration selection rules which will slow down the $M2$ decay of a state For example, the nature of the configuration may require that the transition involves a change in the single-particle orbital angular momentum which is greater than one unit or that $\Delta j>2$. Such changes are not possible for an $M2$ decay. Alternatively, one of the states involved in the decay may correspond to a particle weakly coupled to a 2^+ vibrational state, whereas the other state may correspond to a particle coupled to the 0^+ state of the vibrator. An $M2$ transition would be forbidden between these configurations. The inhibition in the $13/2^+$ to $\frac{9}{2}^-$ decay observed in the Pb, Po region may be attributable to one or both of these effects.

For $A\geqslant 30$ there is only one example of a transition that does not show an inhibition in the $M2$ transition probability. This occurs for the $\frac{3}{2}^+\rightarrow\frac{5}{2}^-$ transition in Ca^{43}. The $M2$ component of this radiation was estimated from γ-γ correlation studies—which indicate, moreover, that the transition proceeds at roughly 14 times the single-particle speed. We have found no way to explain this enhanced $M2$ transition rate.

There is one other case in which experiment does not show a clear-cut inhibition. This is the case of K^{39}, for which the $M2$ transition may proceed at close to the single-particle speed. Since this nucleus has a single hole in a doubly magic core, one would expect that the low-lying states should be almost spherical. Therefore, contributions to the $\frac{7}{2}^-$ wave function in which the $f_{7/2}$ nucleon couples to excited states of the core should be small, and indeed the experimental $M2$ lifetime tends to bear this out. On the basis of this, one might also argue that the transition rate in Ca^{41} should be close to the single-particle estimate. However, the resulting transition matrix element is proportional to $2\langle\nu_{3/2}\|M2\|\nu_{7/2}\rangle+\langle\pi_{3/2}\|M2\|\pi_{7/2}\rangle$, where the double-barred quantity is the reduced matrix element of the $M2$ operator and π_j and ν_j are proton and neutron wave functions of angular momentum j, respectively. Since the neutron-reduced matrix element is proportional to μ_n and the proton-reduced matrix element to $(\mu_p-\frac{1}{3})$, one sees that a cancellation results. This is an example of an isobaric-spin effect, which may produce inhibition in addition to that due to deformation. In Ca^{41}, therefore, even if the $\frac{3}{2}^+$ hole state is attributed to the $d_{3/2}$ hole coupling exclusively to the $J=0$, $T=1$ state of the core, the transition rate is only 1/47 of the single-particle estimate. The observed ratio of 1/67 only requires the wave function to contain a very small admixture that corresponds to the $d_{3/2}$ hole coupled to an excited-core state. Thus, despite the observed cancellation in the $M2$ matrix element, the nuclear states in Ca^{41} can be interpreted as being quite close to spherical.

For $A<30$ most of the data pertain to the p-shell nuclei, for which the inhibition is marked only when the $M2$ transition takes place between states of the same isobaric spin in a self-conjugate nucleus.[6] The known experimental data, taken from Skorka's compilation,[12] are shown in Table II. The fact that transitions for which the isobaric-spin selection rule does not apply proceed at roughly the single-particle speed indicates that the opposite-parity particle in these nuclei is only weakly coupled to the core. It has already been shown that this must be the case in order to explain the $E1$ lifetimes[14,15] in C^{13}. That is, in C^{13} the $\frac{5}{2}^+$ state arises almost entirely from the coupling of the $d_{5/2}$ particle to the C^{12} ground state; only a small component in the wave function corresponds to the coupling of a $d_{5/2}$ or $d_{3/2}$ neutron to the excited 2^+ state at 4.43 MeV in C^{12}. Thus the situation in light nuclei is quite different from that in heavier nuclei, for which the strong-coupling

[11] S. A. Moszkowski, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), Vol. 2, p. 863.
[12] S. J. Skorka (unpublished).
[13] J. Lindskog, T. Sundström, and P. Sparrman, in Ref. 11, Vol. 2, p. 1599.
[14] D. Kurath and R. D. Lawson, Nucl. Phys. **23**, 5 (1961).
[15] F. C. Barker, Nucl. Phys. **28**, 96 (1961).

TABLE II. $M2$ lifetimes for $A<30$. The experimental values in column 6 are all taken from the recent compilation by Skorka (Ref. 12). Columns 1–5 give the nucleus under consideration, the isobaric spin change ΔT in the transition, the initial spin state I_i, the final spin state I_f, and the energy of the emitted γ ray in MeV, respectively. Column 7 gives the single-nucleon estimate for the lifetime [Eq. (3)]. In the last column the ratio of the observed lifetime to the estimated lifetime is tabulated.

Nucleus	ΔT	I_i	I_f	E (MeV)	τ_γ (sec)	$\tau_{\rm est}$ (sec)	$\tau_\gamma/\tau_{\rm est}$
Be^9	...	$\frac{3}{2}^+$	$\frac{1}{2}^-$	16.650	2.2×10^{-15}	2.2×10^{-15}	1.0
	...	$\frac{3}{2}^+$	$\frac{1}{2}^-$	17.500	0.94×10^{-15}	1.7×10^{-15}	0.55
B^{10}	0	2^-	3^+	5.105	1.8×10^{-12}	7.4×10^{-13}	2.5
	1	2^-	0^+	3.366	2.2×10^{-13}	5.9×10^{-12}	0.04
	0	1^-	3^+	6.880	$>6.2\times10^{-15}$	1.7×10^{-13}	>0.04
	0	2^-	0^+	6.011	$>3.2\times10^{-15}$	3.3×10^{-13}	>0.02
C^{13}	0	$\frac{5}{2}^+$	$\frac{1}{2}^-$	3.850	1.1×10^{-11}	2.5×10^{-12}	4.5
N^{13}	0	$\frac{5}{2}^+$	$\frac{1}{2}^-$	3.560	$>3.3\times10^{-13}$	3.7×10^{-12}	>0.09
N^{14}	0	2^-	1^+	5.104	6.1×10^{-10}	5.9×10^{-13}	1000
	1	2^-	0^+	2.792	4.0×10^{-11}	1.2×10^{-11}	3.3
	1	3^-	1^+	8.903	4.1×10^{-14}	3.7×10^{-14}	1.1
	0	3^-	1^+	5.832	$<1.9\times10^{-9}$	3.0×10^{-13}	<6300
					$>8.6\times10^{-11}$		>290
N^{15}	0	$\frac{5}{2}^+$	$\frac{1}{2}^-$	5.276	$>1.0\times10^{-12}$	4.8×10^{-13}	>2.1
O^{15}	0	$\frac{5}{2}^+$	$\frac{1}{2}^-$	5.247	$>1.0\times10^{-12}$	4.9×10^{-13}	>2.0
O^{16}	0	2^-	0^+	8.880	2.3×10^{-10}	3.4×10^{-14}	6800
	0	2^-	0^+	12.52	3.9×10^{-13}	6.1×10^{-15}	64
Ne^{20}	0	2^-	2^+	3.338	7.5×10^{-10}	3.9×10^{-12}	190
	0	1^-	2^+	4.153	$>4.1\times10^{-11}$	9.6×10^{-12}	>4.3

model is usually appropriate. The reason for this difference is that in many cases the normal-parity states in light nuclei are fairly widely spaced, so that it is difficult for the extra-core particle to excite the nucleus and couple to an excited state. On the other hand, for heavier nuclei the excitation energy of the first 2^+ level is 1 MeV or less and hence it is fairly easy for the opposite-parity particle (or hole) to couple to an excited-core state.

III. LIFETIMES IN THE $d_{3/2}$-$f_{7/2}$ REGION

A. General Remarks

In this section we turn to a discussion of the $M2$ lifetimes of the $d_{3/2}$ hole states in the region of the calcium isotopes. For nuclei slightly away from the closure of the $d_{3/2}$ shell, it has already been shown that a reasonable estimate for lifetimes can be obtained on the assumption that the opposite-parity particle is strongly coupled to the core.[4] Therefore, to interpret the new lifetime measurements in this region we shall again take the model in which the $d_{3/2}$ and $f_{7/2}$ single-particle levels are assumed isolated from all other states. Moreover, we assume that an adequate approximation to the nuclear wave functions can be obtained by projecting states of good angular momentum from a Nilsson determinant or sum of Nilsson determinants that correspond to the desired parity. Very near the shell closure at $N=Z=20$ it would not be surprising if this strong-coupling model were to break down, since then the core is difficult to excite. In this case, eigenfunctions in which the opposite-parity particle is weakly coupled to the core may lead to predictions that are closer to experiment.

Before discussing particular cases, we show that $M2$ transitions will be inhibited in the strong-coupling model even though only the $d_{3/2}$ and $f_{7/2}$ orbits are involved. Consider the usual Nilsson model for the case in which there is an odd number of neutrons in the single-particle levels (Fig. 2). The intrinsic ground state is formed by filling the $d_{3/2}$ levels and putting the remaining nucleons in the $f_{7/2}$ levels with an odd neutron in a particular level $k_f=k$. The intrinsic excited state is formed by raising a neutron from the uppermost positive-parity level, $k_a=k^+$, into the level k. The reduced matrix element for an $M2$ transition from an excited state I^+ of the k^+ band to a state I^- of the k band is given by[10]

$$\langle I^-(k)\|M2\|I^+(k^+)\rangle=[(2I^++1)/(2I^-+1)]^{1/2}\times[(I^+2k^+k-k^+|I^-k)\langle\chi_k|M_{k-k^+}{}^2|\chi_{k^+}\rangle+(-1)^{3/2-I^+}(I^+2-k^+k+k^+|I^-k)\langle\chi_k|M_{k+k^+}{}^2|\chi_{-k^+}\rangle]. \quad (4)$$

Since the levels (Fig. 2) refer to eigenstates of angular momentum, the intrinsic matrix elements are easily evaluated in terms of the reduced matrix element for a single-neutron transition from the $d_{3/2}$ state to the $f_{7/2}$ state. The hindrance factor[4] $\mathfrak{h}$ is the ratio of the single-neutron reduced matrix element to the reduced matrix element of Eq. (4). The case wherein $I^-=\frac{7}{2}$ and $I^+=\frac{3}{2}$ is the simplest for illustrating the k dependence of the hindrance factor. For this case,

$$\mathfrak{h}^{-2}=\left[\frac{\langle\frac{7}{2}(k)\|M2\|\frac{3}{2}(k^+)\rangle}{\langle\nu_{7/2}\|M2\|\nu_{3/2}\rangle}\right]^2 =2[(\tfrac{7}{2}\,2kk^+-k|\tfrac{3}{2}k^+)^2+(\tfrac{7}{2}\,2-kk^++k|\tfrac{3}{2}k^+)^2]^2. \quad (5)$$

If we refer to the levels of Fig. 2, it is clear that $k^+=\frac{3}{2}$ for prolate deformation, and that $k_f=k$ increases with increasing number of neutrons in the $f_{7/2}$ shell. The corresponding hindrance factors to the nearest integer are $\mathfrak{h}^2=98$, 25, 6, and 2 for $k=\frac{1}{2},\frac{3}{2},\frac{5}{2}$, and $\frac{7}{2}$, respectively.

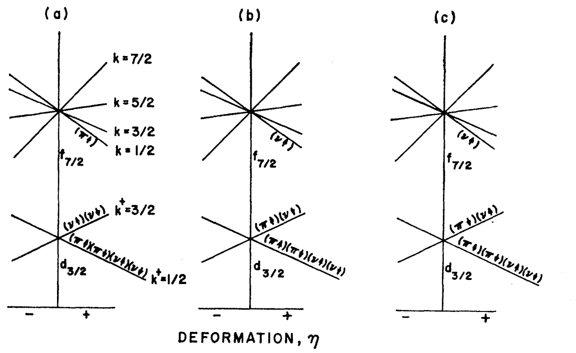

FIG. 2. Splitting of the $d_{3/2}$ and $f_{7/2}$ single-particle states when the nucleus is deformed. The diagrams, drawn for positive deformation, show the Nilsson configurations from which the $T=\frac{1}{2}$ $J=\frac{7}{2}^-$ state of K^{39} is generated. In order to get an eigenfunction of the appropriate isospin, a linear combination $(6)^{-1/2}[2(a)-(b)-(c)]$ of the diagrams must be taken. In the diagram, π stands for a proton and ν signifies a neutron. The arrow attached to the nucleon specifies the z component of angular momentum: $\uparrow=+k$, $\downarrow=-k$.

For oblate deformation, $k^+=\frac{1}{2}$ and $k_f=k$ decreases as the number of neutrons increases. Here $\mathfrak{h}^2=\infty$, 11, 4, and 3 for $k=\frac{7}{2}, \frac{5}{2}, \frac{3}{2}$, and $\frac{1}{2}$, respectively. It is clear that a strong remnant of the asymptotic selection rule remains, even though the Nilsson wave functions have been taken to be $1f_{7/2}$ and $1d_{3/2}$ eigenfunctions. Therefore, strong $M2$ inhibitions are possible in this region.

In the following particular cases, many-particle wave functions have been obtained by projection from Nilsson configurations. It is not always possible, however, to select a particular Nilsson configuration as lying lowest in energy. Therefore, the properties of the $d_{3/2}$, $f_{7/2}$ particle-hole interaction are used as an added criterion in selecting the most likely configurations. The possibility that weak coupling is a better description is also investigated, and the various calculations are compared with the experimental evidence.

To facilitate the discussion, we make a more general definition of the hindrance factor. The single-proton and single-neutron reduced matrix elements are of nearly the same magnitude since

$$\langle\pi_{7/2}\|M2\|\pi_{3/2}\rangle=[(\mu_p-\tfrac{1}{3})/\mu_n)]\langle\nu_{7/2}\|M2\|\nu_{3/2}\rangle. \quad (6)$$

If one uses the free-nucleon values $\mu_p=2.79$, $\mu_n=-1.91$, one obtains a value of -1.286 for the ratio. Therefore, we shall compare the many-particle matrix elements with the matrix element for the neutron transition, namely,

$$\mathfrak{h}^{-2}(I_1,I_2)\equiv\frac{(2I_2+1)\langle I_2\|M2\|I_1\rangle^2}{8\langle\nu_{7/2}\|M2\|\nu_{3/2}\rangle^2}. \quad (7)$$

Since

$$(2I_2+1)\langle I_2\|M2\|I_1\rangle^2=(2I_1+1)\langle I_1\|M2\|I_2\rangle^2,$$

the ratio is independent of which state is the initial state. For transitions between $\frac{7}{2}^-$ and $\frac{3}{2}^+$ states, $\mathfrak{h}^{-2}$ reduces to the ratio of the squares of the reduced matrix elements and the lifetime for the many-particle transition is

$$\tau(\tfrac{7}{2}^-\leftrightarrow\tfrac{3}{2}^+)=\mathfrak{h}^2\tau(\nu_{7/2}\leftrightarrow\nu_{3/2}). \quad (8)$$

If one assumes that $\tau(\nu_{7/2}\leftrightarrow\nu_{3/2})$ is equal to the Moszkowski estimate, $\mathfrak{h}^2$ represents the calculated value to be compared with the numbers in the last column of Table I. The $M2$ lifetimes in the $d_{3/2}$, $f_{7/2}$ region involving spins other than $\frac{3}{2}^+$ and $\frac{7}{2}^-$ will also be compared with the $\langle\nu_{7/2}\|M2\|\nu_{3/2}\rangle$ matrix element since this is more meaningful than the Moszkowski estimate in this region.

B. ${}_{19}K_{20}{}^{39}$

In K^{39} there is a $\frac{7}{2}^-$ state lying at an excitation of 2.820 MeV above the $\frac{3}{2}^+$ ground state. The lifetime for transition between these two states is observed[8] to be less than 8×10^{-11} sec. Thus the transition is inhibited by a factor less than 18 with respect to the single-particle estimate, and may in fact have nearly the single-particle lifetime.

The ground state is pictured as a single $1d_{3/2}$ hole with respect to Ca^{40}, and the $\frac{7}{2}^-$ excited state consists of two holes in the $1d_{3/2}$ subshell and a particle in the $1f_{7/2}$ level. Bansal and French[16] and Zamick[17] have shown that the nature of the $d_{3/2}$, $f_{7/2}$ particle-hole interaction is such that the energetically lowest state of the system will be that in which both particles and holes are separately coupled to maximum isobaric spin. In K^{39}, this means that the two $d_{3/2}$ holes are coupled to $T_0=1$, and the weak-coupling approximation is obtained by assuming that their angular momentum is $I_0=0$. The weak-coupling approximation, which would seem appropriate for K^{39}, leads to a calculated lifetime that is 4.85 times the single-neutron estimate. This weak-coupling result is consistent with the experimental evidence.

There may be other components in the $\frac{7}{2}^-$ wave function corresponding to coupling the $1f_{7/2}$ nucleon to other components (I_0,T_0) of the core. For this simple nucleus, the general expression for the reduced matrix element can be expressed in terms of the coefficients $\alpha_{I_0T_0}$ of the components as

$$\begin{aligned}\langle\psi^{3/2+}(K^{39})\|M2\|\psi^{7/2-}(K^{39})\rangle=&-\tfrac{1}{6}\sqrt{3}[\alpha_{01}+(15/7)^{1/2}\alpha_{21}]\\&\times(\langle\nu_{3/2}\|M2\|\nu_{7/2}\rangle+2\langle\pi_{3/2}\|M2\|\pi_{7/2}\rangle)\\&-(9/70)(35)^{1/2}[\alpha_{10}+(11/27)^{1/2}\alpha_{30}]\langle\nu_{3/2}\|M2\|\nu_{7/2}\rangle. \quad (9)\end{aligned}$$

One can apply the strong-coupling model to calculate the coefficients, remembering that we still wish the $1d_{3/2}$ core to have isobaric spin $T_0=1$. The Nilsson function from which we generate the $\frac{7}{2}^-$ wave function for positive deformation is then a linear combination of the possibilities shown in Fig. 2. The appropriate linear combination is determined by the requirement that the intrinsic state have $T=T_z=\frac{1}{2}$. This leads to $\chi=(6)^{-1/2}$

[16] R. K. Bansal and J. B. French, Phys. Letters 11, 145 (1964).
[17] L. Zamick, Phys. Letters 19, 580 (1965).

$\times(2\chi_a-\chi_b-\chi_c)$ in terms of the configurations of Fig. 2. The resultant coefficients are $\alpha_{01}=(21/26)^{1/2}$ and $\alpha_{21}=-(5/26)^{1/2}$, which produces a cancellation in Eq. (9). From Eqs. (6)–(8) the hindrance factor is found to be $\mathfrak{h}^2=73.6$. This is much greater than one observes experimentally and is 15 times the value for the weak-coupling case ($\alpha_{01}=1$).

If the $\frac{7}{2}^-$ eigenfunction is generated on the assumption of negative deformation, the transition is K forbidden since the odd nucleon must go from a $k=\frac{7}{2}$ orbit to a $k=\frac{1}{2}$ orbit in making the transition. This is not possible for $M2$ radiation, so the result again disagrees strongly with the observed rate of transition.

Therefore, since the K^{39} transition does not exhibit a substantial inhibition, the weak-coupling model seems most suitable. One might wonder about the effect of admixtures in which the $1d_{3/2}$ core has isobaric spin $T_0=0$, since this has an appreciable amplitude in Eq. (9). The hindrance factor for K^{39} is

$$\mathfrak{h}^{-1}=0.454[\alpha_{01}+(15/7)^{1/2}\alpha_{21}]-0.761[\alpha_{10}+(11/27)^{1/2}\alpha_{30}]. \quad (10)$$

Erné[18] has made a fit to the energies of the negative-parity states in the upper end of the $1d$, $2s$ shell and thereby obtained a set of matrix elements characterizing the $d_{3/2}$, $f_{7/2}$ and $d_{3/2}$, $d_{3/2}$ interactions. These matrix elements were used to determine the coefficients of Eq. (10). The results are $\alpha_{01}=0.957$, $\alpha_{21}=0.052$, $\alpha_{10}=-0.080$, and $\alpha_{30}=0.275$. The effect of the fairly large value of α_{30} is partially cancelled by α_{10}, the resultant inhibition being $\mathfrak{h}^2=6.4$, close to the value $\mathfrak{h}^2=4.8$ for the weak-coupling case $\alpha_{01}=1$.

For the corresponding transition in the mirror nucleus Ca^{39}, the role of the proton and neutron single-particle matrix elements are exchanged in Eq. (9). If their relative values are given correctly by Eq. (6), the ratio of hindrance factors that results from using the Erné values for $\alpha_{I_0T_0}$ is $\mathfrak{h}^2(Ca^{39})=11\mathfrak{h}^2(K^{39})$. If suppressed intrinsic moments are used in Eq. (6), one expects the Ca^{39} transition to be hindered more, so that its lifetime is probably in the range suitable for experimental measurement.

C. $_{20}Ca_{21}{}^{41}$

In Ca^{41} the lifetime of the $\frac{3}{2}^+$ excited state at 2.014 MeV is measured[8] to be 8.0×10^{-10} sec. This decay to the $\frac{7}{2}^-$ ground state is therefore inhibited by a factor of 67 from the single-neutron estimate.

The $\frac{3}{2}^+$ state is pictured as arising from coupling a $1d_{3/2}$- hole to the various two-particle states (I_0,T_0) of $(1f_{7/2})^2$. The general expression for the reduced matrix element of the transition can be given simply in terms of the admixture coefficients $\alpha_{I_0T_0}$, namely,

$$\begin{aligned}\langle\psi^{7/2-}(Ca^{41})\|M2\|\psi^{3/2+}(Ca^{41})\rangle &=\tfrac{1}{12}(6)^{1/2}[\alpha_{01}+(15/7)^{1/2}\alpha_{21}](2\langle\nu_{7/2}\|M2\|\nu_{3/2}\rangle\\&\quad+\langle\pi_{7/2}\|M2\|\pi_{3/2}\rangle)\\&\quad+(9/140)(70)^{1/2}[\alpha_{10}+(11/27)^{1/2}\alpha_{30}]\\&\quad\times\langle\pi_{7/2}\|M2\|\pi_{3/2}\rangle. \quad (11)\end{aligned}$$

As in K^{39}, the nature of the $d_{3/2}$, $f_{7/2}$ particle-hole interaction suggests that the $\frac{3}{2}^+$ state is formed predominantly by coupling to states with $T_0=1$. The weak-coupling approximation is obtained by setting $\alpha_{01}=1$ and the other coefficients equal to zero. The calculated hindrance factor for this case is $\mathfrak{h}^2=47$, fairly close to the observed inhibition.

The strong-coupling result is obtained by generating from a Nilsson configuration wherein the two $f_{7/2}$ nucleons are in a state with $T_0=1$. For prolate deformation, one obtains $\alpha_{01}=(21/26)^{1/2}$ and $\alpha_{21}=-(5/26)^{1/2}$. As in K^{39}, this produces more inhibition than in the weak-coupling case, the result being $\mathfrak{h}^2=713$. For negative deformation, the transition is again K forbidden. Therefore, for both signs of deformation the strong-coupling calculation produces a hindrance factor much greater than is experimentally observed.

One can obtain an independent estimate of the coefficients $\alpha_{I_0T_0}$ of Eq. (11) by using Erné's[18] matrix elements for the $d_{3/2}$, $f_{7/2}$ interaction and the matrix elements of McCullen, Bayman, and Zamick[19] for the $(1f_{7/2})^2$ interaction. These interactions lead to $\alpha_{01}=0.995$, $\alpha_{21}=-0.018$, $\alpha_{10}=0.034$, and $\alpha_{30}=0.095$ and give a calculated inhibition factor $\mathfrak{h}^2=39$. This calculation supports the weak-coupling approximation, but the net effect of the small admixtures is to reduce the hindrance factor from the weak-coupling value.

The inhibitions for both the weak-coupling and strong-coupling assumptions are considerably greater in Ca^{41} than the inhibitions obtained from the corresponding calculations in K^{39}. The source of this difference is evident if we compare Eq. (11) with Eq. (9). Aside from a factor of $\sqrt{2}$ which arises from differences in angular-momentum coupling, the main difference is that the roles that the neutron and proton had in K^{39} have been interchanged in Ca^{41}. The interchange increases the inhibition by a factor of 5 in Ca^{41} if only $T_0=1$ parents are involved, with the over-all result that the inhibition in Ca^{41} is 10 times that in K^{39}. The basic reason is that Ca^{41} is an odd-neutron nucleus, while K^{39} is an odd-proton nucleus. On the other hand, if one compares Sc^{41} with K^{39}, the calculated inhibitions differ only by a factor of 2.

D. $_{18}Ar_{21}{}^{39}$

In Ar^{39} the $\frac{3}{2}^+$ state at 1.52 MeV decays to the $\frac{7}{2}^-$ ground state with a lifetime[7] of 2.7×10^{-9} sec. The transition is therefore inhibited by a factor of about 54 with respect to the single-neutron estimate.

[18] F. C. Erné, Nucl. Phys. 84, 91 (1966).

[19] J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. 134, B515 (1964).

The configuration of the $\frac{3}{2}^+$ excited state is $(d_{3/2})^5(f_{7/2})^2$. For nuclei several particles removed from a closed shell, the nature of the wave function is determined by competition between the $d_{3/2}$, $f_{7/2}$ particle-hole interaction and the particle-particle interaction. According to Zamick,[17] the $d_{3/2}$, $f_{7/2}$ interaction favors the state with $T_f=1$ for $(f_{7/2})^2$ and $T_d=\frac{3}{2}$ for $(d_{3/2})^5$ by about 4.3 MeV over the state with $T_f=1$ and $T_d=\frac{1}{2}$. On the other hand, the interaction among the $d_{3/2}$ nucleons favors the state with $T_d=\frac{1}{2}$. The energy difference can be estimated from tables of binding energies[20] if we assume that the $\frac{3}{2}^+$ ground states of Ar^{37} and Cl^{37} both arise from the $(d_{3/2})^5$ configuration. The Coulomb energy difference is taken to be the difference in binding of the mirror nuclei Cl^{35} and Ar^{35}. The result is that the nuclear interactions within the $(d_{3/2})^5$ subshell favor the $T_d=\frac{1}{2}$ state over the $T_d=\frac{3}{2}$ state by 5.1 MeV. From these estimates we expect two low-lying $\frac{3}{2}^+$ states in Ar^{37}, separated in zeroth order by about 800 keV. The lower state ψ_1 is described by the expression

$$\psi_1=\sum_{J_dJ_f}\alpha_{J_dJ_f}[(d_{3/2})_{J_d,T_d=1/2}{}^5\times(f_{7/2})_{J_f,T_f=1}{}^2]^{I=3/2=T}, \tag{12}$$

while the upper state is

$$\psi_2=\sum_{J_dJ_f}\beta_{J_dJ_f}[(d_{3/2})_{J_d,T_d=3/2}{}^5\times(f_{7/2})_{J_f,T_f=1}{}^2]^{I=3/2=T}, \tag{13}$$

where the brackets indicate coupling of the angular momenta and isobaric spins of the components to give the resultant values $I=\frac{3}{2}$ and $T=\frac{3}{2}$.

In our discussion of K^{39} and Ca^{41} we found that weak coupling is appropriate to describe the transitions for these nuclei. However, it has been shown[21] that a strong-coupling description of the $\frac{7}{2}^-$ ground state of Ar^{39} is necessary in order to explain its strongly inhibited probability for β decay. As a check on these empirical conclusions, we use Erné's $d_{3/2}$, $f_{7/2}$ interaction to investigate the degree of coupling in the two similar wave functions, namely, the excited $\frac{7}{2}^-$ state of K^{39} and the $\frac{7}{2}^-$ ground state of Ar^{39}. Both wave functions are formed by coupling an $f_{7/2}$ nucleon to the $T_d=1$ states of the $(d_{3/2})^6$ configuration. The difference is that in K^{39} the resultant has $T=\frac{1}{2}$, while in Ar^{39} it has $T=\frac{3}{2}$. The degree of coupling is determined by the matrix element that connects the $J_d=2$ component and the $J_d=0$ component. This matrix element is

$$\begin{aligned}\langle[(d_{3/2})_{J_d=2,T_d=1}{}^6\times f_{7/2}]^{7/2-,T}&\\ \times|\mathcal{V}|[(d_{3/2})_{J_d=0,T_d=1}{}^6\times f_{7/2}]^{7/2-,T}\rangle&\\ =\tfrac{3}{4}(10)^{1/2}\sum_{J_3T_3}(-1)^{J_3+1}(2J_3+1)(2T_3+1)&\\ \times W^2(\tfrac{1}{2}T\tfrac{1}{2}\,\tfrac{1}{2};1T_3)W(\tfrac{7}{2}\,\tfrac{7}{2}\,\tfrac{3}{2}\,\tfrac{3}{2};2J_3)\mathcal{E}(J_3,T_3).&\end{aligned} \tag{14}$$

Here $\mathcal{E}(J_3,T_3)$ is the $d_{3/2}$, $f_{7/2}$ interaction energy in the state (J_3,T_3) and the W's are Racah coefficients. If we use Erné's values of $\mathcal{E}(J_3,T_3)$ in Eq. (14), the matrix element for K^{39} for which $T=\frac{1}{2}$ has the value 0.014 MeV, whereas the matrix element for Ar^{39}, for which $T=\frac{3}{2}$, has the value -0.819 MeV. It is reasonable to assume that the $J_d=2$ state of the $(d_{3/2})^6$ configuration lies about 2 MeV above the $J_d=0$ state. Then because of the small matrix element, the lower $\frac{7}{2}^-$, $T=\frac{1}{2}$ state of K^{39} will have a wave function near that predicted by weak coupling. However for Ar^{39}, the lower $\frac{7}{2}^-$, $T=\frac{3}{2}$ wave function is

$$\psi^{7/2\,3/2}=0.942[(d_{3/2})_{J_d=0,T_d=1}{}^6\times f_{7/2}]^{7/2\,3/2}+0.336[(d_{3/2})_{J_d=2,T_d=1}{}^6\times f_{7/2}]^{7/2\,3/2}.$$

This wave function is similar to the strong-coupling generated $\frac{7}{2}^-$ wave function of Ar^{39}. The latter has coefficients $(21/26)^{1/2}$ for α_{01} and $(5/26)^{1/2}$ for α_{21}, giving an overlap of 0.994 with the Erné state. Therefore, the $d_{3/2}$, $f_{7/2}$ interaction supports the change from weak coupling in K^{39} to strong coupling as one goes to two more particles away from the closed shell.

Since the strong-coupling approximation is appropriate for the $\frac{7}{2}^-$ ground state, we assume strong coupling for the $\frac{3}{2}^+$ states as well since they are further removed from the closed shell. For positive deformation, the generator for the $\frac{7}{2}^-$ state has two protons and two neutrons in the $k^+=\frac{1}{2}$ level of Fig. 2, two neutrons in the $k^+=\frac{3}{2}$ level, and one neutron in the $k=\frac{1}{2}$ level of the $f_{7/2}$ group. The generator for the $\frac{3}{2}^+$ state with the $(d_{3/2})^5$ core in a $T_d=\frac{1}{2}$ state is formed by promoting a neutron from the $k^+=\frac{3}{2}$ level to the $k=\frac{1}{2}$ level. The resulting $M2$ matrix element between these states is appreciably reduced, as one would expect, and leads to the hindrance factor $\mathfrak{h}^2=71.4$. This is slightly bigger than the observed value $\mathfrak{h}^2=54$ for the decay of the 1.52-MeV state.

Our energy considerations showed that the $\frac{3}{2}^+$ state which has the $(d_{3/2})^5$ core in a $T_d=\frac{3}{2}$ state is expected to lie only about 800 keV above the state with the $T_d=\frac{1}{2}$ core. The generator for this $\frac{3}{2}^+$ state is formed from the generator for the $\frac{7}{2}^-$ ground state by promoting a nucleon from the $k^+=\frac{1}{2}$ level to the $k=\frac{1}{2}$ level in a manner to assure $T_d=\frac{3}{2}$ for the $(d_{3/2})^5$ core. In this case the $M2$ transition to the ground state would not be expected to be as strongly inhibited since it connects the lower level of the $d_{3/2}$ system to the lowest level of the $f_{7/2}$ system. The calculated inhibition factor is $\mathfrak{h}^2=18.6$. Therefore, if we form a $\frac{3}{2}^+$ state that has mainly a $T_d=\frac{1}{2}$ core for $(d_{3/2})^5$ together with a small admixture of $T_d=\frac{3}{2}$, we can easily produce the experimental hindrance factor.

In this case the strong-coupling effect on the $M2$ matrix elements shows up dramatically, since if one assumed weak coupling for all the states involved the inhibition factors would be $\mathfrak{h}^2=4.8$ for the state with the $T_d=\frac{1}{2}$ core and $\mathfrak{h}^2=70$ for the state with the $T_d=\frac{3}{2}$ core. One can, therefore, also obtain the observed

[20] L. A. Konie, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. 31, 18 (1962).
[21] S. Cohen and R. D. Lawson, Phys. Letters 17, 299 (1965).

hindrance in weak coupling. However, since other evidence requires strong coupling for the $\frac{7}{2}^-$ ground state, it seems much more reasonable to use strong coupling in all the states. The results from negative deformation do not resemble observation, a feature which is true in general for all the transitions we discuss. Therefore it will not be mentioned for the remaining cases.

E. $_{19}K_{22}{}^{41}$

There is an observed $M2$ transition from a $\frac{7}{2}^-$ level at 1.291 MeV in K^{41} to the $\frac{3}{2}^+$ ground state. The lifetime[7] of 9.5×10^{-9} sec corresponds to an inhibition by a factor of about 43 from the single-particle estimate.

The configuration of the $\frac{7}{2}^-$ excited state is $(d_{3/2})^6(f_{7/2})^3$. While we expect that it is energetically favorable to have the $(d_{3/2})^6$ component in a $T_d=1$ configuration, the question is whether the $(f_{7/2})^3$ component has $T_f=\frac{1}{2}$ or $\frac{3}{2}$. The $d_{3/2}$, $f_{7/2}$ interaction favors $T_f=\frac{3}{2}$ by about 4.3 MeV, but the interaction among $f_{7/2}$ nucleons favors $T_f=\frac{1}{2}$ by about 4.3 MeV, so both possibilities must be considered.

For positive deformation, the generator for the $\frac{3}{2}^+$ ground state has a proton and two neutrons in the $k^+=\frac{3}{2}$ level of Fig. 2, and two neutrons in the $k=\frac{1}{2}$ level of the $f_{7/2}$ group. The generator for the $\frac{7}{2}^-$ state that has $T_f=\frac{1}{2}$ and $T_d=1$ is formed by promoting the odd proton from the $k^+=\frac{3}{2}$ level to the $k=\frac{1}{2}$ level of the $f_{7/2}$ shell. The resultant $M2$ transition is then hindered by the factor $\mathfrak{h}^2=27$ with respect to the single-neutron transition. Alternatively, since this is an odd-proton nucleus, it may be more appropriate to compare with the single-proton transition; this gives $\mathfrak{h}^2=44.5$.

On the other hand, the generator for the $\frac{7}{2}^-$ case with $T_d=1$ and $T_f=\frac{3}{2}$ is formed from the generator for the $\frac{3}{2}^+$ ground state by promoting a nucleon from the $k^+=\frac{3}{2}$ level to the $k=\frac{3}{2}$ level of the $f_{7/2}$ group. The hindrance factor for this transition is large, the value being $\mathfrak{h}^2=1780$ with respect to the single-proton transition. The chief reason for this abnormally large hindrance factor is a cancellation of the neutron and proton contributions to the matrix element. This cancellation, which is an isobaric-spin effect, will be discussed in more detail later. It is important to note, however, that this effect is the same for either strong or weak coupling.

Since the $\frac{7}{2}^-$ state that has $T_f=\frac{1}{2}$ for the $(f_{7/2})^3$ component leads to about the observed hindrance factor, we conclude that it is dominant part of the $\frac{7}{2}^-$ state at 1.29 MeV. This does not preclude an appreciable admixture of the $T_f=\frac{3}{2}$ state since the $M2$ transition would be a very insensitive indicator for such a component, provided, of course, that it does not dominate the wave function.

F. $_{20}Ca_{23}{}^{43}$

In Ca^{43}, a $\frac{3}{2}^+$ state at 993 keV decays to three negative-parity states, namely, the $\frac{7}{2}^-$ ground state, a $\frac{5}{2}^-$ state at 374 keV, and a $\frac{3}{2}^-$ state at 594 keV. The lifetime for the $M2$ transition to the ground state[8] is 5×10^{-8} sec, which shows an inhibition of 130 relative to the single-neutron estimate. An angular-correlation measurement[9] for the $\frac{3}{2}^+\rightarrow\frac{5}{2}^-\rightarrow\frac{7}{2}^-$ chain is consistent with a pure $M1$ transition for the lower link and a mixed $E1$-$M2$ transition for the upper link with $\delta=0.3$. The experimental lifetime[22] and branching ratios[23] have been determined for the 993-keV state, so one can extract a lifetime of 1.8×10^{-9} sec for the $M2$ transition from $\frac{3}{2}^+$ to $\frac{5}{2}^-$. This $M2$ transition is not inhibited; in fact, its transition probability is 14 times the single-particle estimate. There is no experimental information about the strength of the $M2$ component in the $\frac{3}{2}^+\rightarrow\frac{3}{2}^-$ transition.

In strong coupling, the negative-parity wave functions are all obtained from the same generator, one that has a full $d_{3/2}$ shell plus two neutrons in the $k=\frac{1}{2}$ level and one neutron in the $k=\frac{3}{2}$ level of Fig. 2. The nature of the $\frac{3}{2}^+$ level has been investigated by Zamick[24] in connection with the β decay of K^{43}. Zamick finds that the main component of the $\frac{3}{2}^+$ state is one in which the $d_{3/2}$ hole couples to the $T_f=2$ states of the $(f_{7/2})^4$ configuration. However, in order to fit the observed $\log ft$ value, he also admixes a state[25] wherein the $d_{3/2}$ hole couples to $T_f=1$ states. With the $T_f=1$ component as described in Ref. 25, the linear combination of $\frac{3}{2}^+$ wave functions that leads to the observed value of $\log ft=5.6$ is

$$\psi^{3/2+}=0.813\psi(T_f=2)-0.582\psi(T_f=1). \qquad (15)$$

The resulting inhibition factor for the $M2$ transition from $\frac{3}{2}^+$ to $\frac{7}{2}^-$ is $\mathfrak{h}^2=551$, a factor of 4 greater than is observed. Alternatively, we find that the observed inhibition $\mathfrak{h}^2=130$ is obtained with the wave function

$$\psi^{3/2+}=0.929\psi(T_f=2)-0.371\psi(T_f=1), \qquad (16)$$

so that the result is quite sensitive to the wave function. The wave function that fits the $M2$ inhibition leads to $\log ft=4.8$ for the β-decay. Destructive interference between the $T_f=2$ and $T_f=1$ components is required to fit both the $M2$ and β-decay observations. However, the desired amount of interference differs slightly in the two cases.

Recently, the spectroscopic factor for the $Ca^{44}(d,t)Ca^{43}$ reaction leading to the 993-keV state has been measured[26] and found to be $\mathcal{S}=2$. This transition measures the probability that the $d_{3/2}$ hole is coupled to the ground state of Ca^{44}. The strong-coupling wave function

[22] R. S. Weaver and R. Barton, Can. J. Phys. **40**, 660 (1962).
[23] N. Benczer-Koller, A. Schwarzschild, and C. S. Wu, Phys. Rev. **115**, 108 (1959).
[24] L. Zamick, Phys. Letters **20**, 168 (1966).
[25] In checking Zamick's result, we disagree with his $T_f=1$ wave function. Because the off-diagonal Coriolis term is negative, the lower state should be $\frac{1}{2}\sqrt{2}(\chi_1+\chi_2)$. This means that $M_{GT}(T_0=1)=-0.47$, $\alpha_1=-0.80$, and $\alpha_2=-0.58$ in the first line of his Table I. Only a $T_f=1$ admixture with $\alpha_2=-0.58$ satisfies the β observation and gives a reasonable $M2$ transition to the ground state.
[26] J. L. Yntema, Phys. Rev. (to be published).

for positive deformation gives 51.6% for the probability that the $d_{3/2}$ hole couples to the $J=0$ $T=2$ state of the $(f_{7/2})^4$ configuration. Combining this with the probability, deduced from $M2$ decay, that the hole state actually corresponds to the $(f_{7/2})^4$ configuration with $T=2$, we would predict that $\mathcal{S}=4\times0.516\times(0.929)^2=1.78$.

The strong-coupling result for the $M2$ transition from $\frac{3}{2}^+$ to $\frac{5}{2}^-$ gives a probability about 1/100 of the observed transition strength. The observed result, which shows an enhanced $M2$ transition, cannot be explained by any combination of the components in Eq. (16). The calculated $M2$ strength for the $\frac{3}{2}^+\rightarrow\frac{3}{2}^-$ transition is about 0.16% of the total strength deduced from the measured branching ratios and lifetime. The weak transitions from the $\frac{3}{2}^+$ level to both the $\frac{3}{2}^-$ and the $\frac{5}{2}^-$ appear to be predominantly $E1$ transitions. These are forbidden for $d_{3/2}$, $f_{7/2}$ configurations, so that a more complicated picture including $2p_{3/2}$ and $1f_{5/2}$ particles seems necessary. Nevertheless, the strong $M2$ transition from $\frac{3}{2}^+$ to $\frac{5}{2}^-$ will probably remain difficult to understand.

IV. DISCUSSION

Our survey of the experimental evidence on $M2$ transitions shows that most of these transitions are inhibited relative to the single-particle estimate, often by large factors. We have attempted to show that there are two factors that can lead to $M2$ inhibition. One factor is an asymptotic selection rule associated with the presence of strong quadrupole deformation. This factor is more likely to be effective in heavy nuclei. The second factor is the effect of the isobaric-spin dependence of the $M2$ operator. This is most effective in light nuclei and occurs in its simplest form in the inhibition of $M2$ transitions with $\Delta T=0$ in self-conjugate nuclei. In intermediate cases, such as the transitions we discussed in the $d_{3/2}$, $f_{7/2}$ region, both factors can be effective, Finally, there are transitions (e.g., $\Delta T=1$ transitions in light nuclei) for which neither factor is effective; these proceed with nearly the single-particle strength.

In heavy nuclei, which have a large neutron excess, neutrons and protons are filling levels in different regions of the shell model. Therefore, the isobaric-spin factor is not important since the $M2$ transition between low-lying nuclear states can usually be associated with a transition between two nearby shell-model levels for one type of nucleon. If these are levels of a deformed shell model, there usually is inhibition due either to the asymptotic selection rule or to K forbiddenness.

In light nuclei, however, both neutrons and protons are filling the same shell-model levels. The isobaric-spin effect is due to the fact that the isobaric part of the $M2$ operator is usually much smaller than the isovector part. Warburton[6] has pointed out that this general property of magnetic multipole operators inhibits magnetic transitions with $\Delta T=0$ in self-conjugate nuclei. Our examples of $M2$ transitions in odd-A nuclei in the $d_{3/2}$, $f_{7/2}$ region show that the isobaric-spin effect can also be important in such cases. We define the isoscalar and isovector matrix elements for this region as

$$2\langle S\rangle\equiv\langle\nu_{7/2}\|M2\|\nu_{3/2}\rangle+\langle\pi_{7/2}\|M2\|\pi_{3/2}\rangle\,,$$
$$2\langle V\rangle\equiv\langle\nu_{7/2}\|M2\|\nu_{3/2}\rangle-\langle\pi_{7/2}\|M2\|\pi_{3/2}\rangle\,, \quad (17)$$

in terms of the reduced matrix elements of Eq. (6). From Eq. (6) we see that

$$\langle S\rangle/\langle V\rangle=(\mu_n+\mu_p-\tfrac{1}{3})/(\mu_n-\mu_p+\tfrac{1}{3})\,. \quad (18)$$

For $\mu_p=2.793$ and $\mu_n=-1.913$, one obtains $\langle V\rangle\approx-8\langle S\rangle$, which is the value used in our calculations. It may be that suppressed intrinsic moments would be more appropriate, as seems to be the case for $M1$ transitions in the $d_{3/2}$, $f_{7/2}$ region. However, even with $\mu_p=2.0$ and $\mu_n=-1.1$ we still have $\langle V\rangle\approx-5\langle S\rangle$, so there would be no major changes except in cases in which use of the free-nucleon values predicts near cancellation between the $\langle V\rangle$ and $\langle S\rangle$ contributions.

The isobaric-spin factor can be isolated as a multiplicative factor for the general case of a transition

$$[(d_{3/2})_{T_{1d}}{}^{n_d}\times(f_{7/2})_{T_{1f}}{}^{n_f}]_{T_3}{}^{T}\leftrightarrow [(d_{3/2})_{T_{2d}}{}^{n_d-1}\times(f_{7/2})_{T_{2f}}{}^{n_f+1}]_{T_3}{}^{T} \quad (19)$$

from one configuration to another. The matrix element governing this transition will be proportional to a linear combination of the reduced proton and neutron matrix elements. That is, there will be a proportionality factor F given by

$$F=A\langle\nu_{7/2}\|M2\|\nu_{3/2}\rangle+(1-A)\langle\pi_{7/2}\|M2\|\pi_{3/2}\rangle\,,$$
$$F=\langle S\rangle+(2A-1)\langle V\rangle\,, \quad (20)$$

which can give rise to the isospin inhibition discussed in Sec. III. For either $T_{1f}+T_{1d}=T$ or $T_{2f}+T_{2d}=T$ (a condition satisfied in all our calculations), A will lie between zero and one. Since the quantity A can be expressed in terms of the isospin quantum numbers of Eq. (19), the expression for F can be rewritten in the form

$$F=\langle S\rangle+[T_3/T(T+1)][(T_{2f}-T_{1f})(2T_{2f}+1)-(T_{2d}-T_{1d})(2T_d+1)]\langle V\rangle\,. \quad (21)$$

Equation (21) is quite general[27] and holds for a single-particle transition between any two configurations such as those described in Eq. (19). There are, of course, other terms having to do with isospin and angular-momentum coupling and the effects of deformation by which Eq. (21) must be multiplied to obtain the complete matrix element. However, Eq. (21) can be used to see whether or not there will be inhibition due to the isobaric-spin factor F. For example, it is clear that for

[27] A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press Inc., New York, 1963). Equation (21) can be derived from their Eq. (37.35), p. 488.

self-conjugate nuclei for which $T_3=0$, only the weak isoscalar $\langle S\rangle$ can contribute. The importance of the factor F was strikingly illustrated in the case of the K^{41} transitions for which $T=\frac{3}{2}=T_3$, $T_{1d}=\frac{1}{2}$, $T_{2d}=1$, and $T_{1f}=1$. There were two possibilities for T_{2f}: $T_{2f}=\frac{1}{2}$ and $T_{2f}=\frac{3}{2}$. In this case Eq. (21) gives

$$\begin{aligned} F&=\langle S\rangle-\langle V\rangle=+9.0\langle S\rangle \quad \text{for} \quad T_{2f}=\tfrac{1}{2}, \\ F&=\langle S\rangle+\tfrac{1}{5}\langle V\rangle=-0.6\langle S\rangle \quad \text{for} \quad T_{2f}=\tfrac{3}{2}, \end{aligned} \tag{22}$$

and accounts for the large difference in magnitude between our computed hindrance factors for the two possible configurations in this nucleus.

Therefore we see that in the $d_{3/2}$, $f_{7/2}$ region both the isobaric-spin effect and the strong-coupling effect can produce $M2$ inhibitions. We have demonstrated that the isobaric-spin effect can account for transitions in those nuclei that are only one particle away from a closed shell. For nuclei three particles removed from a closed shell, strong coupling and prolate deformation are needed to account for the observed inhibitions. This conclusion has been supported with theoretical calculations in K^{39}, Ca^{41}, and Ar^{39}, empirical energy matrix elements being used to evaluate the degree of coupling in the ground states. In the case of excited states, we have assigned likely configurations that can account for the observed $M2$ lifetimes. Some of these assignments can be checked by nucleon-transfer reactions For instance in Ar^{39} we would expect two $\frac{3}{2}^+$ excited states, separated by about 1 MeV. The lower of these states should have a large component of the $[(d_{3/2})_{3/2\,1/2}{}^5 \times (f_{7/2})_{01}{}^2]^{I=3/2=T}$ configuration, that is, two $f_{7/2}$ particles coupled to the low-lying states of Ar^{37}. The upper $\frac{3}{2}^+$ state should be predominantly $[(d_{3/2})_{3/2\,3/2}{}^5 \times (f_{7/2})_{01}{}^2]^{I=3/2=T}$. Thus the higher state should be populated strongly in the reaction $Cl^{37}(He^3,p)Ar^{39}$ whereas the lower state should be weakly populated.

Although $M2$ transitions between low-lying states are usually inhibited, there are a few exceptions in which relatively strong $M2$ transitions have been observed. In the heavy nuclei the spin-orbit coupling can lower a single-particle level with large l and principal quantum number N, so that it lies among the levels $N-1$ of Fig. 1. The quantum numbers (N,n_z,Λ,Σ) may then be such as to allow a transition to one of the nearby $N-1$ levels. An example of this has recently been observed[28] in $_{71}Lu^{175}$, where an $M2$ component with close to single-particle strength is found between the levels $(514\uparrow)$ and $(404\downarrow)$. The $E1$ transition is severely inhibited, not by the orbital selection rules but because the spin must be flipped in order for the transition to proceed. This feature does not inhibit the $M2$ transition.

In the $d_{3/2}$, $f_{7/2}$ region, the strong transition from $\frac{3}{2}^+$ to $\frac{5}{2}^-$ in Ca^{43} is a striking exception to the general rule of $M2$ inhibition. There is no apparent mechanism to explain the observed enhancement factor of more than 10 over the single-particle estimate.

For light nuclei, the $M2$ transitions among low-lying levels do not appear to be inhibited, except those with $\Delta T=0$ in self-conjugate nuclei. This implies that in nuclei with $A\lesssim 30$, an adequate description of non-normal-parity states is obtained by having a particle of opposite parity weakly coupled to the core.

There is one possible anomaly concerning the decay of the 5.105-MeV $(I=2^-, T=0)$ level in B^{10}. The angular distribution observed in the $Li^6(\alpha,\gamma)B^{10}$ reaction[29] is consistent with a 2^- assignment for this state only if the γ transition to the $(I=3^+, T=0)$ ground state contains an $M2$ component. Meyerhof and Chase[30] propose a 1% $M2$ component to explain the angular distribution, but this implies a $\Delta T=0$ transition with approximately single-particle strength. This is in disagreement with other observed $\Delta T=0$ transitions in self-conjugate nuclei. A recent revision[31] of the γ branching ratio for the decay of the 5.105-MeV level may resolve this problem.

While $M2$ transitions among low-lying levels are inhibited, there are enhanced $M2$ transitions with $\Delta T=1$ to higher states. These have been observed in inelastic electron scattering, and their existence has been predicted in particle-hole calculations.[32] The subject is discussed in a recent review by de Forest and Walecka,[33] who have investigated the nature of such giant $M2$ resonances.

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[28] G. T. Emery and M. L. Perlman, Phys. Rev. **151**, 984 (1966).

[29] L. Meyer-Schützmeister and S. S. Hanna, Phys. Rev. **108** 1506 (1957).

[30] W. E. Meyerhof and L. F. Chase, Jr., Phys. Rev. **111**, 1348 (1958).

[31] P. D. Forsyth, H. T. Tu, and W. F. Hornyak, Nucl. Phys. **82**, 33 (1966).

[32] N. Vinh-Mau and G. E. Brown, Nucl. Phys. **29**, 89 (1962); V. Gillet and N. Vinh-Mau, Phys. Letters **1**, 25 (1962).

[33] T. deForest, Jr., and J. D. Walecka, Advan. Phys. **15**, 1 (1966).