Gurevich Magnetomorphic Oscillations in Single Crystals of Aluminum at Helium Temperatures*

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Oscillations periodic in the magnetic field have been observed in the Hall resistivity of two electropolished single crystals of aluminum for the case of a magnetic field along the [111] direction. Both period and amplitude are dependent upon sample thickness in the field direction. The phenomena are interpreted in terms of Gurevich theory as providing a measure of the radius of Gaussian curvature of the second-zone hole surface in the [111] direction. This radius is about three times the radius of the free-electron sphere and is much larger than the values of that radius inferred by other laboratories from experiments upon polycrystalline samples.

EXPRESSIONS have been derived by Sondheimer[1] for the elements of the isothermal electrical conductivity tensor $\hat{\sigma}$ for the case of a free-electron metal in the form of a thin plate with magnetic field directed parallel to the thin dimension. His work shows that when the mean free path is of the order of the sample thickness, diffuse boundary scattering gives $\hat{\sigma}$ a magneto-oscillatory component whose period is determined by the ratio of the radius of the Fermi sphere to the crystal thickness. Grenier *et al.*[2] have generalized this result for the case of Fermi surfaces which are surfaces of revolution about the magnetic field. In this case the radius of interest is the radius of curvature at the apex of the Fermi surface in the direction of the magnetic field. Gurevich[3] has treated arbitrary energy surfaces and, specifically for the case of convex surfaces, the radius which enters is given by $1/\sqrt{K}$, where K is the Gaussian curvature at the elliptic point where the Fermi surface normal is parallel to the magnetic field. The period P_0 is given by

$$P_0=2\pi c/ea\sqrt{K}\,, \qquad (1)$$

where a is the sample thickness. Forsvoll and Holwech have reported oscillations in the magnetoresistance[4] ρ_{11} and Hall resistivity[5] ρ_{21} of thin polycrystalline foils of aluminum. The period was shown to correspond essentially to the mean radius of the second zone hole surface, and the data were therefore interpreted in terms of Sondheimer's free-electron theory. Zytveld *et al.*[6] have confirmed this result. However, the use of polycrystalline samples prohibits the determination of any specific details of the Fermi surface, because of the averaging effect of randomly oriented crystallites. This paper reports the result of experiments on oriented single crystals of electropolished aluminum at 1.1°K.

A monocrystal of 69 grade zone-refined aluminum[7] was prepared by spark planning followed by electropolishing to give a thin dimension of 0.22 mm. The surface normal was parallel to within two degrees to the [111] direction. Alutin 51-S solder and Alutin-51 flux[8] were used to attach electrical probes to the sample. The crystal was oriented in the magnetic field to within 5° by the use of small mirrors and a telescopic cathetometer. After data were taken on this sample, the thin dimension was further reduced by electropolishing to 0.07 mm and the experiment was repeated.[9]

The magnetoresistance of both samples was observed to increase monotonically in fields up to 24 kG, contrary to the results of Forsvoll and Holwech,[4] who found ρ_{11} saturated at 2–5 kG. The gross Hall resistivity ρ_{21} was positive and increased nearly linearly as a function of magnetic field, with slope corresponding to a carrier density of approximately 1.2×10^{23} cm^{-3}. The result of subtracting a least-squares quadratic from each gross ρ_{21} is presented in Fig. 1. It is seen that a very low

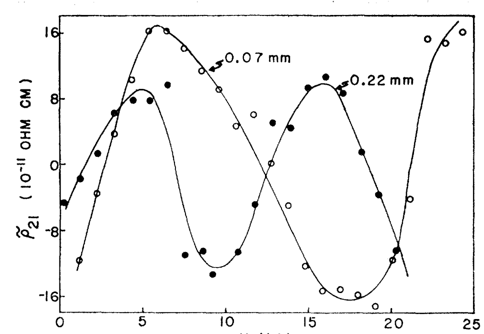

FIG. 1. The magneto-oscillatory component $\tilde{\rho}_{21}$ of the Hall resistivity is shown for the two samples studied. Smooth curves have been drawn through the data points for clarity.

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[1] E. H. Sondheimer, Phys. Rev. **80**, 401 (1950).

[2] C. G. Grenier, K. R. Efferson, and J. M. Reynolds, Phys. Rev. **143**, 406 (1966).

[3] V. L. Gurevich, Zh. Eksperim. i Teor. Fiz. **35**, 668 (1958) [English transl.: Soviet Phys.—JETP **8**, 464 (1959)].

[4] K. Forsvoll and I. Holwech, Phil. Mag. **9**, 435 (1964).

[5] K. Forsvoll and I. Holwech, Phil. Mag. **10**, 921 (1964).

[6] J. Van Zytveld, J. Bass, and F. J. Blatt, Bull. Am. Phys. Soc. **12**, 397 (1967).

[7] The aluminum was obtained from Cominco Products, Inc., Spokane, Washington.

[8] Available from Eutectic Welding, Dallas, Texas.

[9] The details of the electropolishing procedure and crystal-holding device are similar to those detailed in H. J. Mackey, J. R. Sybert, and J. T. Fielder, Phys. Rev. **157**, 578 (1967). The relationship between ρ_{21} and σ_{12} is also detailed.

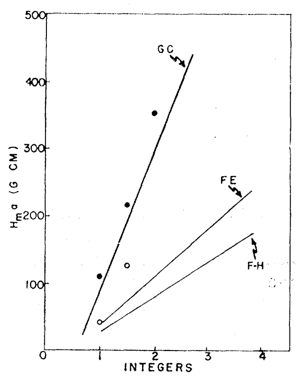

FIG. 2. The product of the field values H_m at which maxima and minima occur in $\tilde{\rho}_{21}$, with the corresponding sample thicknesses a is plotted against the integers for each sample. The closed and open circles indicate the data for the 0.22-mm and the 0.07-mm samples, respectively. The line GC chosen to represent these data has a slope corresponding to a Gaussian radius of curvature 5.3×10^{-19} g cm/sec. The line FE corresponds to the Fermi radius of 1.84 $\times10^{-19}$ g cm/sec, and the line F-H corresponds to the mean radius of the second-zone hole surface used by Forsvoll and Holwech (See Ref. 4). The vertical intercepts of the lines FE and F-H have been chosen arbitrarily and are not intended to show relative phase.

amplitude oscillatory component, apparently periodic in the magnetic field, is present in both samples. The oscillation in the thinner sample is larger in amplitude and longer in period than in the thicker sample. Although theory gives expressions for the oscillatory conductivity $\tilde{\sigma}$, one may show[9] that the oscillatory Hall resistivity may be expressed as

$$\tilde{\rho}_{21}=|\tilde{\rho}_{21}|\cos(2\pi H/P_0+\delta). \quad (2)$$

Then the field values for which maxima occur may be written as

$$H_m=(n-\delta/2\pi)P_0. \quad (3)$$

In order to normalize the data belonging to different thicknesses one may substitute P_0 from Eq. (1) into Eq. (3) to obtain

$$H_m a=(2\pi c/e\sqrt{K})n-c\delta/e\sqrt{K}. \quad (4)$$

Equation (4) shows that a plot of $H_m a$ against the integers should yield a straight line if δ is independent of field. The slope of the line determines the Gaussian curvature K, and the vertical intercept determines the phase δ. Figure 2 is such a plot for the two samples studied. Maxima have been plotted at the integers and minima at the half-integers. The line chosen to represent the data corresponds to a radius $1/\sqrt{K}=5.3\times10^{-19}$ g cm/sec. Because of the long period only a few maxima and minima were observed up to 24 kG. The horizontal separation of the data belonging to the two samples indicates δ may depend on a. The line indicating the free-electron slope and the line representing the slope corresponding to the data of Forsvoll and Holwech[5] are exhibited for comparison. The radius of Gaussian curvature of the Fermi surface in the [111] direction is seen to be about three times the radius of the Fermi sphere. This may be expected because of the lifting of the central symmetry points on the free-electron second-zone surface toward the zone faces, and the lowering of points near the zone corners and edges, due to the lattice potential.[10–12]

In order to obtain more complete and more accurate data it is necessary to use very thin samples, since the amplitude of the effect grows exponentially as thickness is reduced. However, reduction in thickness means a corresponding increase in period, thus necessitating the use of very high fields. This laboratory will soon have access to a 100-kG magnet, and the experiments will be repeated and extended.

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[11] W. A. Harrison, Phys. Rev. **118**, 1182 (1960).
[12] B. Segal, Phys. Rev. **131**, 121 (1963).