Low-Frequency Permeabilities of a Superconductor due to Surface Currents*

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Superconductors which can carry critical persistent currents around the circumference of a specimen may dissipate energy when an alternating magnetic field $h_0 \cos\omega t$ is superimposed on a magnetic field $\mathbf{H}_0$ which is swept at a constant rate dH_0/dt. We have calculated, for a long macroscopic superconducting cylinder, the real and imaginary parts of the permeabilities due to the surface currents alone for the first three Fourier components of the internal field. The results are complicated and depend on two parameters, namely, $(dH_0/dt)/\omega h_0$ and $h_c(H_0)/h_0$. The function $h_c(H_0)$ is a measure of the current-carrying capacity of the surface of the superconductor.

I. INTRODUCTION

IT has been known for several years[1–12] that type-II superconductors are, in general, lossy materials at low frequencies when an alternating magnetic field $h(t)=h_0 \cos\omega t$ is superimposed on a static magnetic field H_0. At present it is believed that these losses are mainly associated with the surface of the superconductor, in particular for magnetic fields H_0 between the upper critical field H_{c2} and the surface nucleation field H_{c3}. To explain these losses, essentially two different models have been proposed, namely, an effective conductivity model[1,3,6,13] of the surface, and a model[9,14] which is based on a well-defined critical current[15] associated with the surface sheath. This current can be calculated[15] from energy considerations and from the inherent properties of the superconductor. The conductivity and the critical-state models have been compared in detail by Rollins and Silcox.[12] Paskin *et al.*'s model[3] assumed also a critical current (of the kind Abrikosov[16,17] has calculated) but does not take into account hysteresis loops in the B–H_0 plane.[18] Based upon this model[13] Schwartz and Maxwell[19] have calculated the losses for a type-II superconductor for swept magnetic fields H_0 with the assumption that the amplitude of the alternating driving field h_0 is small and no hysteresis loops in the B–H_0 plane are swept out. They find fair agreement with the experimental results of Maxwell and Robbins.[8] They conclude that the losses depend only on a single parameter $q=(dH_0/dt)/\omega h_0$. The critical-state model,[9,12,14,15] however, leads to the conclusion when H_0 is not swept that the losses depend on $p=h_c/h_0$, where h_c is a magnetic-field parameter which is related to the critical state of the surface sheath and which is a function[15] of H_0. There appears to be a discrepancy as to the parameters on which the low-frequency losses depend. It is the purpose of this work to calculate the real and imaginary parts of the permeabilities under the most general conditions of the swept-field case. It is found that in general the permeabilities depend on both p and q. When, however, $p\geq p_0\leq 1$ (p_0 is a parameter which will be defined below) the response depends on q only, which probably was the case in Maxwell and Robbins's[8] experiment when H_0 was between approximately H_{c1} and H_{c2}.

II. FORMULATION OF THE PROBLEM

It has been recognized for some time that the low-frequency magnetic behavior of type-II superconductors is quite different[20] from static magnetization measurements. This behavior has been correlated by LeBlanc[21] to hysteresis loops in the static magnetization. Because of the hysteretic behavior of the real part of the microwave impedance, which was observed by Cardona *et al.*[22] in the mixed state of a type-II superconductor, it was suggested[23] that the surface sheath, which might also exist in the mixed state,[24] could carry

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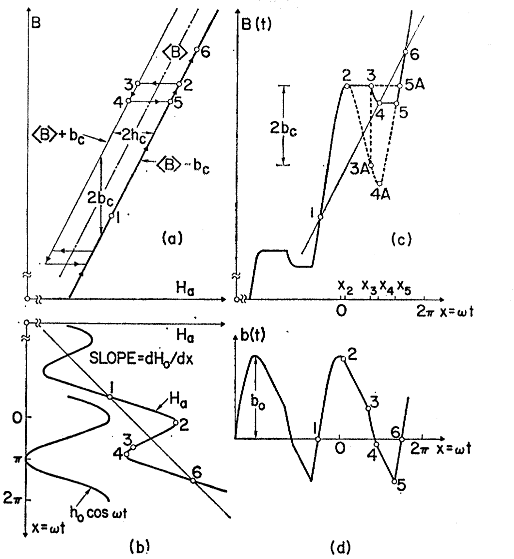

FIG. 1. The response of the internal field of a long superconducting cylinder to an applied magnetic field H_a for the critical-state model. (a) B as a function of H_a. The term b_c is due to critical surface currents and $\langle B \rangle$ is the magnetic induction which is assumed to be magnetically reversible. (b) The applied field H_a as a function of time. H_a is the sum of the magnetic field $h_0 \cos \omega t$ and the field H_0, which is swept at constant rate dH_0/dt. (c) The time variation of the internal field due to the applied field H_a. (d) The alternating component of the internal field as a function of time. $b(t)$ is here defined as $B(t)-(\langle B \rangle - b_c)$. For details see text.

persistent currents.[15] This was confirmed by Fischer *et al.*[25] for low κ materials with a surface sheath. For magnetic fields larger than H_{c2} the observed hysteresis loops in the static magnetization[26,27] could be readily correlated[27] to currents in the surface sheath.[15] Below but near H_{c2} it has been proven definitely[28] that the hysteresis in the mixed state is due to surface currents, by plating the specimen surface with chromium (antiferromagnetic), though the total current appeared to be somewhat larger than one would anticipate from an extension of the theory of the surface sheath[15,24,29] to fields below H_{c2}, when the mixed state is neglected. There are other experiments[30–35] which support the idea of surface currents, but these currents are not entirely associated with the surface sheath alone but can be partially explained by the Bean[36]–London[37] model. Currents which are associated with the surface sheath and the mixed state in the form of longitudinal transport currents have also been measured.[38,39]

In order to simplify the calculations we assume that the low-frequency losses can be adequately described by the critical-state model of the surface sheath alone.[14,15] We assume that this surface sheath exists for magnetic fields between H_{c1} and H_{c3} (or H_c and H_{c3} for type-I superconductors). We neglect contribution to the losses from the mixed state and the normal state. We disregard also surface currents other than those associated with the surface sheath, though this is not a necessary assumption as long as these currents behave qualitatively in the same fashion as the currents in the surface sheath. Then the parameter $p=h_c/h_0$ has to be interpreted more broadly than due to the surface-sheath currents alone.

We also assume that the sample is at constant temperature at all times and that temporary heating effects[40] may be ignored. The zero-frequency hysteresis loop is assumed to exist also in the same way at low frequencies, which means that we ignore possible relaxation effects (inherent as well as due to defects). At the present it is not known whether relaxation effects exist and/or can be ignored. In essence, possible relaxation effects will determine what is meant by low frequencies. The energy dissipated per unit volume per cycle is equal to the area enclosed by the B–H_0 loop divided by 4π when H_0 is not swept ($q=0$) and it is equal to $\mu'' h_0^2/4$ for a linearly polarized driving signal where μ'' is the imaginary part of the fundamental component of the permeability $\mu=\mu'-i\mu''$.

We now make further simplifying assumptions by considering Fig. 1. The magnetic induction $\langle B(H_0) \rangle$, which we call the average internal field, is shown in Fig. 1(a) with the average internal field $(\langle B \rangle - b_c)$ which exists in the superconductor when the external field H_0 is increased. When H_0 is decreased the average field is $(\langle B \rangle + b_c)$. $\langle B \rangle$ is thought to be magnetically reversible and corresponds to the magnetic induction of the lowest energy state of the specimen at a given value of H_0. For $H_{c1}<H_0<H_{c2}$ the value of $\langle B \rangle$ is smaller than H_0 and for $H_{c2}<H_0<H_{c3}$ the value of $\langle B \rangle$ is H_0. The magnetic induction (in cgs Gaussian units) b_c is due to critical surface currents and the fields $(\langle B \rangle - b_c)$ and $(\langle B \rangle + b_c)$ are magnetically irreversible owing to the surface contributions. When the external field H_0 is swept at a constant rate and an alternating field $h(t)=h_0 \cos\omega t$ is superimposed on the swept field

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as shown in Fig. 1(b), the internal field $B(H_a)$ changes over one cycle of the external field [points 1 to 6 in Fig. 1(b)], as shown in Fig. 1(a) by the lines connecting the points 1 to 6. We assume that we may linearize $\langle B\rangle$ over one cycle of the small alternating field $h(t)$. This is exact for $H_{c2}<H_0<H_{c3}$ provided the thickness of the surface sheath is small compared to the diameter of the cylinder. We disregard induced very low-frequency components owing to the nonlinear behavior of $\langle B\rangle$ in the mixed state when H_0 is swept. Further it is assumed that b_c is constant over the period of one cycle, though it may change slowly as a function of H_0 (which it actually does) when H_0 is swept at constant rate. As in most instances $h_c(H_0)\ll(H_{c3}-H_{c2})$ or $h_c(H_0)\ll(H_{c2}-H_{c1})$, a sufficient condition for the latter assumption is that h_0 is also small compared to $H_{c3}-H_{c2}$ or $H_{c2}-H_{c1}$. In other words h_0 should not be very large compared to h_c when $q<1$. When $q>1$ no low-frequency losses occur even when $h_0\gg h_c$ as the instantaneous magnetic field proceeds only along $(\langle B\rangle-b_c)$ when $dH_0/dt>0$ or along $(\langle B\rangle+b_c)$ when $dH_0/dt<0$. Also, possible rounding effects at the corners of the hysteresis loop due to magnetic-field penetration changes are assumed not to exist.

III. SOLUTION

The instantaneous applied magnetic field is

$$H_a(t)=(dH_0/dt)t+h_0\cos\omega t+C_1. \qquad (1)$$

The origin of ωt may be chosen arbitrary, which is compensated for by adjusting the constant C_1 in Eq. (1). We assume that the field is swept at a constant rate $dH_0/dt=a$. When surface effects are included and only H_0 is changed at a constant rate with $h_0=0$, then the average internal field is

$$B(H_0)=\langle B(H_0)\rangle\pm b_c. \qquad (2)$$

The sign in front of b_c depends on whether the external field H_0 is decreased or increased at a constant rate a.

If an alternating field $h(t)=h_0\cos\omega t$ is superimposed on the increasing field H_0, and *if* Eq. (2) would be reversible (which it is not), then the internal field B as a function of time would be

$$B(t)=\frac{d[\langle B(H_0)\rangle-b_c]}{dH_0}\frac{dH_0}{dt}t+b_0\cos\omega t+C_2, \qquad (3)$$

where the constant C_2 is determined by the origin of ωt and

$$b_0=h_0(d\langle B(H_0)\rangle/dH_0).$$

This would correspond to the curve 1, 2, 4A, 6 in Fig. 1(c). As b_c is assumed not to vary over the period of one cycle it follows that $db_c/dH_0=0$, and Eq. (3) reduces to

$$B(t)=\frac{d\langle B(H_0)\rangle}{dH_0}\left[\frac{dH_0}{dt}t+h_0\cos\omega t\right]+C_2$$

$$=H_a\frac{d\langle B\rangle}{dH_0}+C_3, \qquad (4)$$

where we have assumed that $\langle B(H_0)\rangle$ can be linearized over the period of one cycle. For $H_{c2}<H_0<H_{c3}$ the value of $d\langle B\rangle/dH_0=1$. In the mixed state for magnetic fields near H_{c2} where Abrikosov's theory[41] applies,

$$\langle B\rangle=H_0-(H_{c2}-H_0)/(2\kappa^2-1)\beta, \qquad (5)$$

with $\beta=1.16$ for a triangular[42] lattice and hence

$$d\langle B\rangle/dH_0=(1+1/(2\kappa^2-1)\beta). \qquad (6)$$

Near H_{c1} in the mixed state some other approximation for $d\langle B\rangle/dH_0$ has to be found. In the Meissner state Eq. (4) is zero.

Because of surface effects the internal field $B(H_0)=\langle B(H_0)\rangle\pm b_c$ is not reversible and a hysteresis loop, as shown in Fig. 1(a), is swept out over one cycle of the applied alternating field, which leads in general to a variation of the internal field with time as shown in Fig. 1(c) by the wave form 1, 2, 3, 4, 5, 6.

When the distance between the point 3 and 4 of the hysteresis loop in Fig. 1(a) shrinks to zero when, for example, ω or h_0 is decreased or (dH_0/dt) is increased, the response of the internal field as a function of time corresponds to a wave shape 1, 2, 5*A*, 6 in Fig. 1(c). When $dH_0/dt>\omega h_0$ no part of the "hysteresis loop" is traced out and $B(t)$ is described by Eq. (4). For the latter case no losses occur as the alternating component of $B(t)$ is sinusoidal and is in phase with the alternating driving field $h(t)$.

Figure 1(d) shows the alternating component $b(t)=[B(t)-(\langle B\rangle-b_c)]$ of $B(t)$ of Fig. 1(c) when from Fig. 1(c) the drift component (which is the straight line which goes through the points 1 and 6) is subtracted. This drift component was assumed to generate a very low-frequency Fourier component in the nonlinearized approach and is zero in the linearized approach. This Fourier component will be disregarded in further considerations.

The parameters x_2, x_3, x_4, and x_5 and their relation to $dH_0/dt=a$, ω, h_c, and h_0 are found from Fig. 1(a) to 1(c). They determine the wave shape of Fig. 1(d), which we shall Fourier analyze.

x_2 and x_4 are obtained from the extreme values of H_a in Fig. 1(b). They are

$$\sin x_2=a/h_0\omega, \qquad (7)$$

$$x_4=\pi-x_2. \qquad (8)$$

We define

$$q=a/h_0\omega. \qquad (9)$$

x_3 is determined from $B(t_2)-B(t_{3A})=2b_c$ which leads to

$$\cos x_3-\cos x_2+q(x_3-x_2)+2p=0, \qquad (10)$$

where

$$p=h_c/h_0. \qquad (11)$$

The increment of B between points 3 and 4 in Fig. 1(c)

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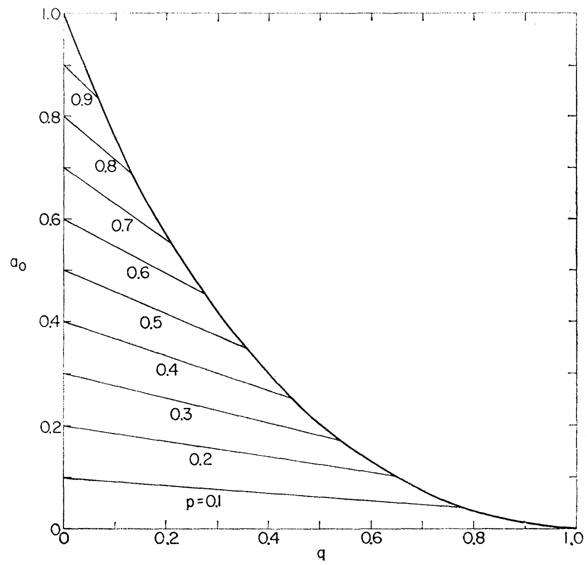

FIG. 2. The constant Fourier coefficient a_0 of the alternating internal field $b(t)$ of Fig. 1(d) as a function of $q=(dH_0/dt)/h_0\omega$ and the parameter $p=h_c(H_0)/h_0$. For details see text.

is equal to the same increment between points $3A$ and $4A$ and it is

$$\Delta B(t)\,\Big|_4^3=2(d\langle B\rangle/dH_0)h_0(p_0-p), \qquad (12)$$

where we have defined p_0 as that value of p which, for a given value of q, makes $\Delta B(t)\,|_4^3=0$. It is

$$p_0=\cos x_2+q(x_2-\pi/2). \qquad (13)$$

In other words, once we have chosen $q=a/h_0\omega$ the value of x_2 is determined from Eq. (7), which in turn determines $p_0=(h_c/h_0)_0$. For this and larger values of p the wave shape 1, 2, 3, 4, 5, 6 in Fig. 1(c) which exist for $p<p_0$ changes to the wave shape 1, 2, $5A$, 6. As long as q is smaller than unity and is held constant, the wave shape for all values of $p\geq p_0$ is the same. For $q\geq 1$ the wave shape in Fig. 1(c) is independent of p and obeys Eq. (4).

The point x_5 is found from $B(t_2)-\Delta B(t)\,|_4^3=B(t_5)$, which leads to

$$\cos x_5+\cos x_2+q(x_5+x_2-\pi)=2p. \qquad (14)$$

Hence the wave shape of Fig. 1(d) can be completely described by Eqs. (7), (8), (10) and (14) as a function of the two parameters p and q. Inherent in these parameters are the experimental parameters dH_0/dt, h_0, ω and the parameter h_c, which is related to the critical current capacity of the surface of the superconductor.

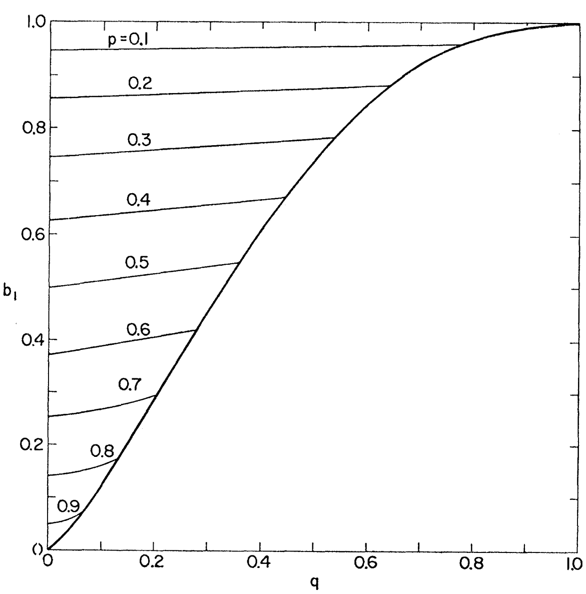

FIG. 3. The in-phase first-harmonic Fourier coefficient b_1 of $b(t)$ of Fig. 1(d) as a function of q and the parameter p. b_1 is a measure of the dispersion [Eq. (18)].

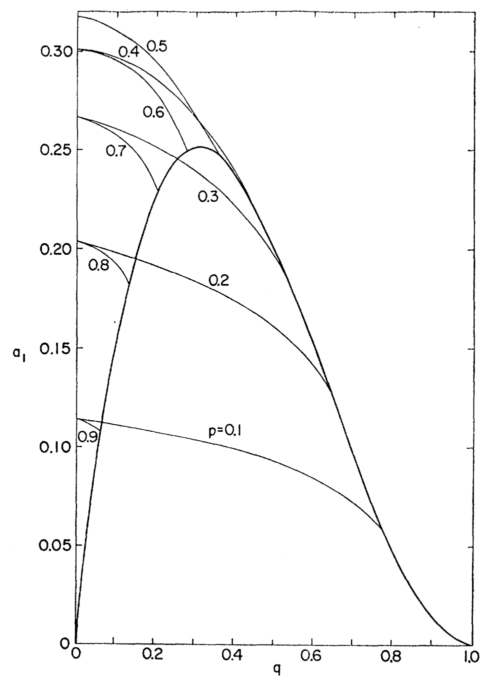

FIG. 4. The out-of-phase first-harmonic Fourier coefficient a_1 of $b(t)$ of Fig. 1(d) as a function of q and the parameter p. a_1 is a measure of the absorption [Eq. (19)]. For details see text.

The general expression for the wave form of Fig. 1(d) is

$$b(\omega t)=b(x)=b_0\{a_0+\sum_{n=1}^{\infty}(b_n\cos nx+a_n\sin nx)\} \qquad (15)$$

$$=h_0\{c_0+\sum_{n=1}^{\infty}(\mu_n'\cos nx+\mu_n''\sin nx)\}, \qquad (16)$$

when $h(t)=h_0\cos\omega t$ and where a_0, a_n and b_n are the Fourier coefficients. The constant term and the perme-

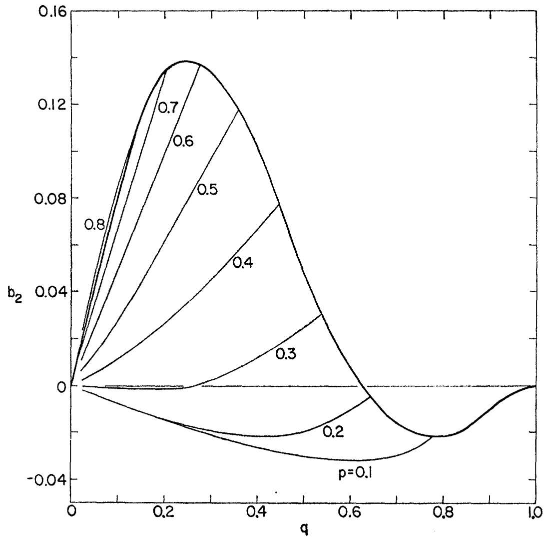

FIG. 5. The in-phase second-harmonic Fourier coefficient b_2 of $b(t)$ of Fig. 1(d) as a function of q and the parameter p.

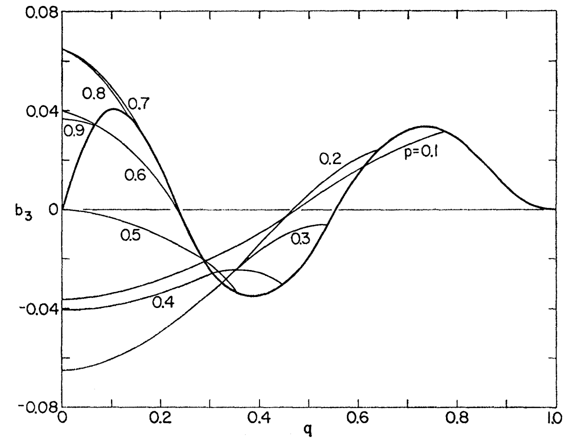

FIG. 7. The in-phase third-harmonic Fourier coefficient b_3 of $b(t)$ of Fig. 1(d) as a function of q and the parameter p.

abilities in Eq. (16) are defined by

$$c_0 = a_0(d\langle B\rangle/dH_0), \tag{17}$$

$$\mu_n' = b_n(d\langle B\rangle/dH_0), \tag{18}$$

$$\mu_n'' = a_n(d\langle B\rangle/dH_0). \tag{19}$$

The susceptibilities have been calculated previously[12,14] when $dH_0/dt=0$ and $H_{c2}<H_0<H_{c3}$. In Ref. 14 the shape of the wave form of $b(t)$ was linearized. The calculations of Rollins and Silcox[12] are more accurate, though both[12,14] calculations make the same simplifying assumption, as was done here, that b_c is constant over the extent of a loop in the B–H_0 plane. This is a good approximation when p is near p_0 but a poor approximation when $p \ll p_0$. It should be remembered that $b_c = b_c(H_0)$ and the above analysis breaks down if $h_0 \sim (H_{c2}-H_{c1})$ or $(H_{c3}-H_{c2})$. This is probably one reason why the agreement between experiment[12] and theory[12,14] is only fair when $h_0/h_c \gtrsim 10$ when $dH_0/dt=0$.

We have calculated on an analog computer the Fourier coefficients a_0, a_n and b_n for $n=1$, 2, and 3 for all possible values of p and q. Figures 2 to 8 show the results as a function of q with p as a parameter. Figure 2 shows only the constant component a_0 of Fig. 1(d). In an experiment one would measure an additional constant component due to the swept field dH_0/dt. From Fig. 3 and Eq. (18) one obtains the dispersion. Figure 4 in conjunction with Eq. (19) gives the quantity which is directly related to the low-frequency power loss of the surface sheath. When H_0 is decreased through H_{c2} the losses increase at H_{c2} as $d\langle B\rangle/dH_0=1$ for $H_0>H_{c2}$, but $d\langle B\rangle/dH_0>1$ for $H_0<H_{c2}$ as can be seen readily from Eq. (6). This sudden increase in μ_n' and μ_n'' is larger the smaller the κ value of the type-II super-

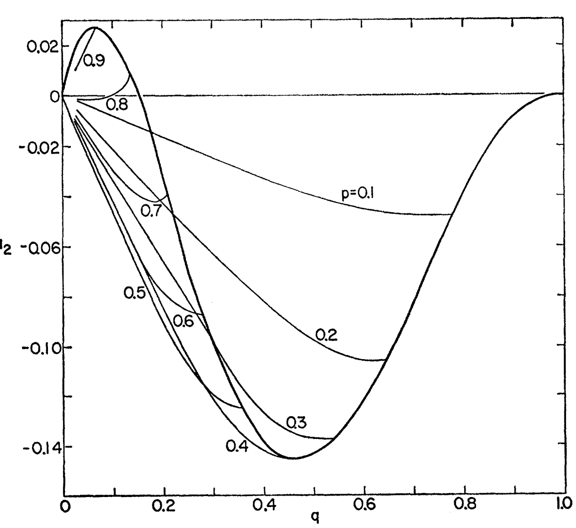

FIG. 6. The out-of-phase second-harmonic Fourier coefficient a_2 of $b(t)$ of Fig. 1(d) as a function of q and the parameter p.

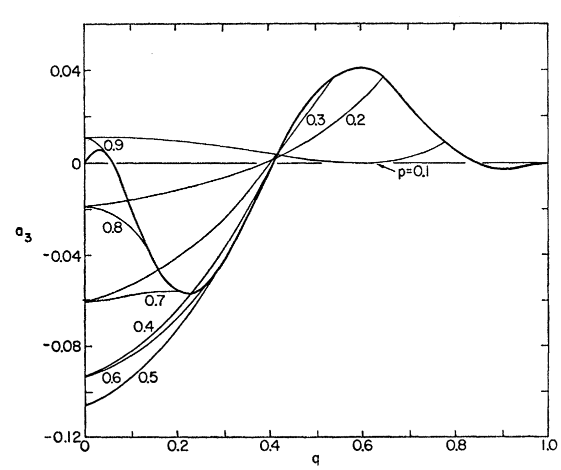

FIG. 8. The out-of-phase third-harmonic Fourier coefficient a_3 of $b(t)$ of Fig. 1(d) as a function of p and q.

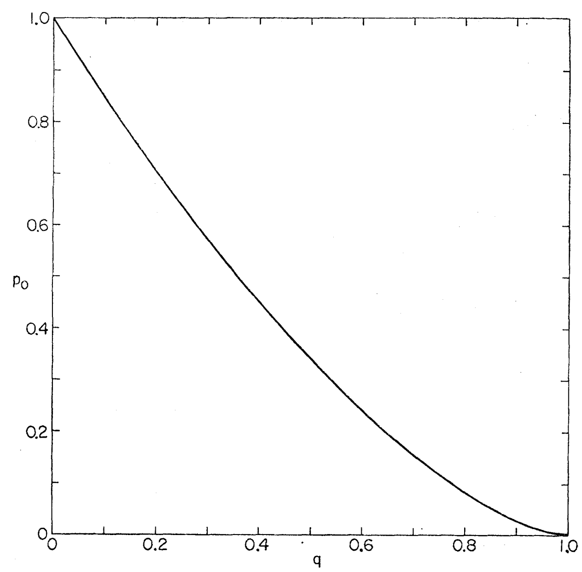

FIG. 9. p_0 [Eq. (13)] as a function of q. When $p \geq p_0$, μ_n' and μ_n'' are not any longer functions of p but only functions of q. For $p < p_0$ both μ_n' and μ_n'' are functions of p and q. For details see text.

conductor is. Though for $H_0 < H_{c2}$ the average internal field is smaller than H_0, the internal alternating-field variations are amplified below H_{c2} as compared to above H_{c2}, hence, the observed[12] increase of μ_1' and μ_1''. The Fourier coefficients of the higher harmonic components are shown in Figs. 5 to 8 and might be a useful guide for certain types of experiments which could compare the finer details of various proposed models. The results are quite complicated.

When $p \geq p_0$ (p_0 is shown in Fig. 9 as a function of q) the Fourier coefficients are the same for all values of p for a given value of q and are shown in Figs. 2 to 8 by the boundary curves at which the various curves for $p < 1$ terminate. These boundary curves correspond to the value of $p = p_0$. The boundary curves in Figs. 3 and 4 are of the same shape as those calculated by Schwartz and Maxwell[19] though some of the details are not the same. In our case, a_1 for $p = p_0$ did not become negative for $0.83 \lesssim q \leq 1$. Their results[19] are plotted in arbitrary units so that no absolute comparison can be made.

IV. CONCLUSIONS

A type-II or type-I superconductor with a surface sheath, which can carry critical persistent currents around the circumference of the specimen, is in general a lossy superconductor when an alternating magnetic field is superimposed on a magnetic field which is swept at a constant rate. The losses are very complicated and depend on two parameters, $p = h_c(H_0)/h_0$ and $q = (dH_0/dt)\,\omega h_0$. The function $h_c(H_0)$ is a measure of the total critical surface current and is the quantity of significance in any experiment. In the above calculations $h_c(H_0)$ is not necessarily assumed to be only associated with the surface sheath but currents other than the latter could contribute to h_c. When $dH_0/dt = 0$ the imaginary part of the permeability μ_1'' can be simply related to the area of the hysteresis loop in the $B - H_0$ plane. This simple relation does not hold any longer when $|dH_0/dt| > 0$. For values of q between zero and unity the hysteresis loop in the $B - H_0$ plane disappears when $p \geq p_0 (\leq 1)$, though the permeabilities are finite.

In order to visualize the low-frequency losses as hysteresis loops for $q > 0$ and $p \geq p_0$, one has to introduce a moving frame coordinate system with one coordinate as $h(t) = H_a(t) - [(dH_0/dt)t + C_1]$ and the other as $b(t) = B(t) - [(d\langle B\rangle/dt)t + C_2]$, where the terms in the brackets correspond to the swept external and swept internal fields, respectively. In the $b(t) - h(t)$ plane the points 1 and 6 of Fig. 1 are the same and the hysteresis loop is closed. When $q > 1$ the permeabilities are all zero (except μ_1') and no losses occur, provided the mixed state and the normal state can be ignored.

The main simplification in the above calculations was made by the assumption that the actual average internal field as a function of H_0 is parallel to the internal equilibrium field $\langle B(H_0)\rangle$ [see Fig. 1(a)] over one cycle of the driving signal. This means that p should not be small compared to p_0. μ_1'' reaches a maximum of about $0.252(d\langle B\rangle/dH_0)$ at $q \approx 0.310$ when $p \geq p_0 \approx 0.562$ (see Fig. 4).

In the above model the mixed state and the normal state were neglected, so that we have disregarded possible absorption due to vibrations of the quantize flux tubes in the mixed state. Without these and other possible relaxation mechanisms, the energy which is absorbed per unit time is directly proportional to the first power of the frequency.

The above calculations are based on the phenomenological aspects of irreversible thermodynamics. As such they predict low-frequency losses in a superconductor. The microscopic interpretation of these losses is beyond the scope of this approach.

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