Energy Loss and Range of Fission Fragments in Solid Media*

J. B. CUMMING AND V. P. CRESPO†
Chemistry Department, Brookhaven National Laboratory, Upton, New York
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Energy losses and ranges in Mylar have been measured for some representative fission fragments; these data and existing stopping-power measurements for bromine and iodine ions in media from carbon to gold are shown to be consistent with a semiempirical theory which treats electronic stopping processes in terms of an effective charge Z_1^* which depends on the velocity of the ion. The fractional effective charge Z_1^*/Z_1 is shown to be a universal function of the variable $v/v_0Z_1^{2/3}$, where v_0 is the Bohr velocity and Z_1 is the nuclear charge of the fragment. This semiempirical treatment, when combined with existing theoretical estimates for nuclear stopping powers, correctly predicts ranges of fission-fragment-like ions.

INTRODUCTION

A KNOWLEDGE of range-energy relations for heavy ions in various substances is of considerable practical importance for the interpretation of a variety of experiments[1] in which mean ranges or range distributions are measured. In this laboratory, range distributions have been measured[2–4] for a number of products of the interaction of high-energy protons with heavy-element targets. Mylar, empirical formula $C_{10}H_8O_4$, was used as a stopping material in these experiments because it contains only elements of low atomic number and is available commercially in the form of strong uniform foils having thicknesses down to $\approx$0.5 mg/cm^2.[5] The present paper describes absolute range and energy-loss measurements in Mylar for some fragments from the thermal-neutron-induced fission of U^{235} and from spontaneous fission of Cf^{252}. The results are discussed in terms of a semiempirical theory which then may be used to obtain stopping powers and hence ranges of fragments of different masses and charges in Mylar and other stopping media.

There have been numerous theoretical discussions of heavy-ion stopping powers.[6–8] In such discussions it is convenient to consider three velocity regions. Ions having velocities v much greater than $v_0Z_1^{2/3}$, where Z_1 is the atomic number of the ion and v_0 is the Bohr velocity [0.2227 (MeV/amu)$^{1/2}$], are expected to be fully charged and to lose energy by electronic excitation and ionization of the stopping medium. The electronic stopping power $(dE/dR)_e$ is expected to follow the Bethe-Bloch equation

$$(dE/dR)_e=0.286(Z_1^2Z_2/v^2A_2)\ \ln(2mv^2/I). \quad (1)$$

In this equation and subsequently we will use Z and A to denote atomic numbers and mass numbers with subscripts 1 and 2 indicating the moving ion or stopping medium, respectively. The logarithmic term of Eq. (1) depends on the ion velocity and the properties of the stopping medium (as represented by a mean excitation energy I), but not on the charge carried by the ion. The Bethe-Bloch equation predicts that the electronic stopping power should be proportional to the square of the charge on the ion. Ions having masses typical of fission fragments ($80\leq A$, ≤ 150) are not available in this velocity region; however, Eq. (1) has been used extensively for low-mass ions, i.e., protons, α particles, etc.

As the fragment velocity becomes comparable to $v_0Z_1^{2/3}$, the ion begins to pick up electrons and its charge decreases with decreasing velocity. It has been suggested[6] that the effective charge Z_1^* on a nondegraded fission fragment ($v\approx 0.5v_0Z_1^{2/3}$) will be equal to $Z_1^{1/3}v/v_0$. Replacement of Z_1 in Eq. (1) by this value for Z_1^* leads to the prediction that $(dE/dR)_e$ will depend on $Z_1^{2/3}$ with a velocity dependence entering only through the logarithmic term. In this region, energy loss to atomic nuclei, although increasing with decreasing velocity, is still a minor contribution, e.g., a fragment with $A_1=140$ and $Z_1=54$ must have its energy reduced to 17 MeV before nuclear stopping is 10% of the total.

At still lower velocities, $v\approx v_0$, nuclear stopping becomes dominant. Lindhard, Scharff, and Schiott[9] (hereafter referred to as LSS) have given a theoretical treatment for the nuclear stopping power based on a Thomas-Fermi model for the interacting atoms. We will use their estimates for nuclear stopping in our subsequent discussions. LSS also obtained an estimate for electronic stopping power from the same model

$$\left(\frac{dE}{dR}\right)_e=\xi_e\frac{Z_1Z_2}{ZA_2}\left(\frac{v}{v_0}\right) \quad \text{for } v\leq v_0Z_1^{2/3}. \quad (2)$$

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† Present address: Estudos Gerais Universtarios de Mozambique, C. P. 257 Lourenzo Marques, Portuguese East Africa.
[1] J. M. Alexander, in *Nuclear Chemistry*, edited by L. Yaffe Academic Press Inc., New York, (to be published).
[2] J. B. Cumming, R. J. Cross, Jr., J. Hudis, and A. M. Poskanzer, Phys. Rev. **134**, B167 (1964).
[3] V. P. Crespo, J. B. Cumming, and A. M. Poskanzer (unpublished).
[4] V. P. Crespo, J. B. Cumming, and J. M. Alexander (unpublished).
[5] Small batches of $\approx$0.2 mg/cm^2 Mylar have been prepared by E. I. du Pont de Nemours and Company, Inc.
[6] N. Bohr, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **18**, No. 8 (1948).
[7] U. Fano, Ann. Rev. Nucl. Sci. **13**, 1 (1963).
[8] L. C. Northcliffe, Ann. Rev. Nucl. Sci. **13**, 67 (1963).
[9] J. Lindhard, M. Scharff, and H. E. Schiott, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **33**, No. 14 (1963).

TABLE I. Energies of three fission fragments after passage through various thicknesses of Mylar.

Thickness of Mylar (mg/cm²)	Fragment energy (MeV) $A=99$, $Z=38.8$[a]	$A=111$, $Z=43.8$[a]	$A=140$, $Z=53.9$[a]
0.000	102.9	104.1	80.0
0.197	89.2	89.5	66.5
0.387	77.4	76.0	53.7
0.489	71.3	69.3	48.3
0.698	58.7	56.2	37.8
0.853	49.8	47.8	32.1
0.997	42.5	39.6	26.9
1.060	39.5	36.8	25.1

[a] These charges were obtained from the results of H. R. Bowman, S. G. Thompson, R. L. Watson, S. S. Kapoor, and J. O. Rasmussen [*Physics and Chemistry of Fission* (International Atomic Energy Agency, Vienna, 1965), Vol. II, p. 125].

Here $Z=(Z_1^{2/3}+Z_2^{2/3})^{3/2}$ and ξ_e is a factor which is expected[10] to lie between 1 and 2 and may vary as $Z_1^{1/6}$. A consequence of Eq. (2) is that, in a stopping medium of low atomic number, $(dE/dR)_e$ should depend approximately on $Z_1^{1/6}$ since the term Z_1/Z is relatively insensitive to Z_1. For fission fragments this is at variance with the conclusions drawn from Eq. (1) and an effective charge, $Z_1^*=Z_1^{1/3}v/v_0$.

In the following sections, we will present experimental data on the stopping of fission fragments in Mylar and compare the results with treatments based on Eqs. (1) and (2).

EXPERIMENTAL

Mean ranges were obtained for Mo^{99} and Ba^{140} produced by the thermal-neutron-induced fission of U^{235}. The arrangement for this experiment consisted of a stack of $\approx$0.5 mg/cm² Mylar foils in a 2π geometry. The thickness of the foils was determined by direct weighing. On one of the foils, a $\frac{1}{2}$-in. square target was prepared by vacuum deposition of U^{235} F_4 to a thickness of $\approx$0.005 mg/cm². The foils and target were clamped between graphite plates and irradiated in the Brookhaven National Laboratory graphite reactor. After irradiation, the Mo^{99} and Ba^{140} content of each foil was assayed by conventional radiochemical techniques. The specific activity of Ba^{140} was constant within errors in the first three foils out from the target. It decreased in the fourth and fifth and was essentially zero in the sixth. The distribution of Mo^{99} was similar but was shifted to greater ranges by about one foil. From the fractional activities per unit thickness in the region of constant specific activity, mean ranges in Mylar of Ba^{140} and Mo^{99} were determined to be (2.11±0.04) and (2.57±0.05) mg/cm², respectively. Statistical uncertainties (reproducibility) and estimated systematic effects contribute equally to the indicated 2% errors.

[10] J. Lindhard and M. Scharff, Phys. Rev. **124**, 128 (1961).

Information on the width of a range distribution can be obtained from the fractional activity observed in the last foil of a 2π range curve. Convenient data relating this fraction to the full width at half-maximum were calculated assuming a Gaussian distribution of ranges and are given in the Appendix. Subject to this assumption, the experimental data were consistent with relative full widths at half-maximum of (8±3)% for Ba^{140} and (13±3)% for Mo^{99}. While this resolution is poorer than that observed[11] for fission fragments stopping in gases of low atomic number, it is significantly better than the 17% full width at half-maximum (FWHM) reported[12] for aluminum. Since range is nearly proportional to momentum for fission fragments in Mylar, the FWHM of the momentum distributions will be $\approx$10% and those of the energy distributions $\approx$20%.

Energy losses in Mylar for fragments from the spontaneous fission of Cf^{252} were measured with silicon surface-barrier detectors. A two-counter coincidence procedure was used in which the heights of pulses from both detectors were digitized and stored in a 128- by 32-channel array. Energy scales were calibrated by the procedure of Schmitt *et al.*[13] The energy-energy array was transformed into a mass-channel array using momentum conservation and corrections for prompt neutron emission based on the data of Bowman *et al.*[14] Examination of the latter array showed that selection of a particular channel on the 32-channel side resulted in selection of a rather broad distribution of masses incident on the other detector. The distributions were nearly Gaussian in the vicinity of their peaks and had full widths at half-maximum of $\approx$8 and $\approx$12 amu for the light- and heavy-mass peaks, respectively.

Energy spectra were then obtained with various thickness of Mylar between the source and the detector corresponding to the 128-channel side. These absorbers were prepared from nominal 0.9-, 0.5-, and 0.2-mg/cm² foil. Surface densities of the 0.9- and 0.5-mg/cm² foils were obtained by direct weighing of known areas. The thinner foils were compared to the thicker ones with an α-particle thickness gauge which was also used to check the foil uniformity. Various combinations of the absorbers were used to obtain different total thicknesses up to $\approx$1 mg/cm². Each two-parameter spectrum was then analyzed to give the mean energy of the fragments emerging from the absorber as a function of fragment mass.

Since the mass resolution was poor and since the observed changes with mass were quite gradual, we have presented data for three representative masses in Table I. The fragment velocities are plotted as a

[11] K. A. Petrzbak, Ya. G. Petrov, and F. A. Shlyamin, Zh. Eksperim. i Teor. Fiz. **38**, 1723 (1960) [English transl.: Soviet Phys.—JETP **11**, 1244 (1960)].

[12] F. Brown and B. H. Oliver, Can. J. Chem. **39**, 616 (1961).

[13] H. W. Schmitt, W. E. Kiker, and C. W. Williams, Phys. Rev. **137**, B837 (1965).

[14] H. R. Bowman, J. C. D. Milton, S. G. Thompson, and W. J. Swiatecki, Phys. Rev. **129**, 2133 (1963).

function of Mylar thickness t in Fig. 1. The curves in this figure are least-squares fits to a function of the form

$$t=a+bv+c/v. \quad (3)$$

A linear dependence of t (or range) on v is expected if the electronic stopping power is proportional to velocity and if nuclear stopping contributes only at lower velocities. The third term in Eq. (3) was introduced to correct approximately for nuclear stopping at velocities in the range covered by our data. The fits of the data to Eq. (3) are quite good, the standard deviation of a point being $\approx$0.005 mg/cm^2 if all errors are forced into the t dimension. This is consistent with estimated uncertainties of $\pm$0.5 MeV in the energies at each point. The points for masses 99 and 111 in Fig. 1 fall close to straight lines. However, the least-squares analyses indicate a slight downward concavity for both masses (more significant for $A=99$ than $A=111$) which is in the direction expected for a residual contribution from nuclear stopping. The curve for $A=140$ is quite definitely concave upwards and presumably indicates a deviation from proportionality of electronic stopping power to velocity.

Values of the stopping power of Mylar can be obtained from these data in two ways. They can be derived analytically from the parameters a, b, and c, taking advantage of the data smoothing in the least-squares fits. However, as a consequence of the form of Eq. (3), the stopping powers must then vary monotonically with velocity. The other method, that which we have used, is by direct calculation from the data in Table I. This emphasizes statistical fluctuations which may be estimated, based on errors of $\pm$0.5 MeV in energy or $\approx$0.005 mg/cm^2 in thickness at each point, to be $\approx$5–10% for typical values of dE/dR.

Total stopping powers for $A=99$, 111, and 140 are plotted as histograms in Fig. 2. For comparison, the predictions of the LSS theory are shown in each case as dashed lines. These fall up to a factor of 2 below our experimental values. Also included in Fig. 2 as solid curves are data from Moak and Brown[15] on the stopping of I^{127} and $Br^{79,81}$ in carbon. Agreement between their results and those from the present experiment is reasonably good, considering the differences in the stopping media and the ions and the experimental errors of both measurements.

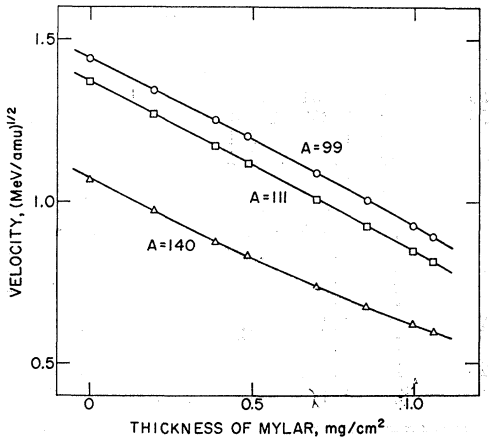

FIG. 1. Dependence of velocity on added thickness of Mylar for three fragments from the spontaneous fission of Cf^{252}.

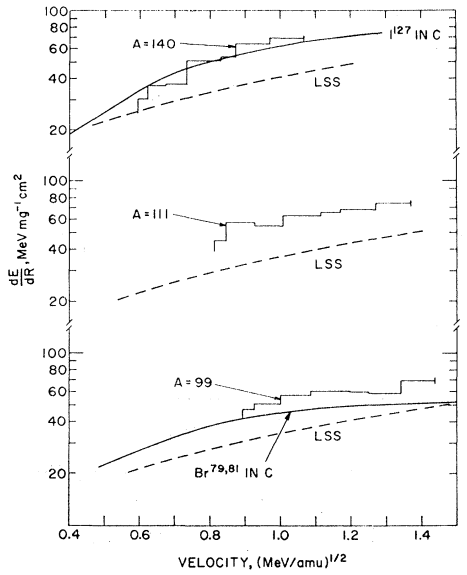

FIG. 2. Dependence of stopping power of Mylar on velocity. The histograms represent the results of this work. The solid curves are based on experimental results of Moak and Brown (Ref. 16). The dashed curves are estimates from the theory of LSS (Ref. 9).

DISCUSSION

In their theory, Lindhard, Scharff, and Schiott (LSS)[9] used a Thomas-Fermi statistical model for interacting atoms to predict both nuclear and electronic stopping powers for heavy ions. It was their intent to provide a general framework for the discussion of a wide variety of stopping phenomena. Considering that the theory is absolute in the sense there are no adjustable parameters, the agreement between experiment and theory is surprisingly good. However, real disagreements exist, e.g., as shown in Fig. 2, and they are generally in one direction. The LSS theory tends to underestimate stopping powers and hence overestimates ranges. It has been pointed out by Lindhard and Scharff[10] that the treatment of electronic stopping power by Eq. (2) is an approximation. In particular, there is considerable uncertainty in the quantity ξ_e, and in attempts to improve the theory, ξ_e has been treated as an adjustable parameter. It has been observed[16] that increasing the value of the exponent in

$$\xi_e=Z_1^n \quad (4)$$

from the LSS value of 0.167 to 0.22 results in correct

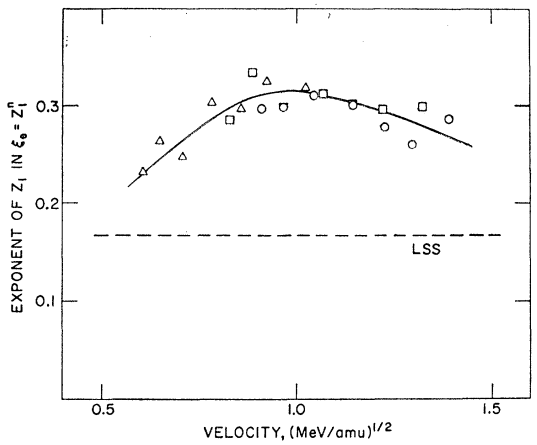

FIG. 3. Comparison of experiment and theory. The points are derived from data for $A=99$ (○), $A=111$ (□), and $A=140$ (△). The solid line indicates their general trend. The theoretical prediction (Ref. 9) is shown by the horizontal dashed line.

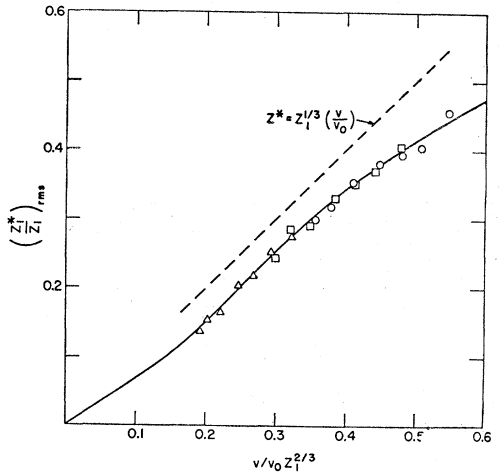

FIG. 4. Fractional effective charge in Mylar as a function of reduced velocity. Points are for $A=99$ (○), $A=111$ (□), and $A=140$ (△). A theoretical estimate from Ref. 6 is shown by the dashed line.

prediction of the ranges for full-energy fission fragments in aluminum.

We have used our stopping-power data in Mylar, corrected for a small (<5%) contribution of nuclear stopping based on LSS theory, in conjunction with Eq. (2) to obtain values of the exponent n of Z_1 in Eq. (4). These are plotted in Fig. 3 and indicate n is not independent of velocity. Data for masses 99, 111, and 140 define the solid curve which approaches the LSS estimate for both low and high velocities. A similar behavior is observed in the extensive stopping-power data of Moak and Brown.[15] The rise from low velocities to ≈ 1 (MeV/amu)$^{1/2}$ reflects an increase in the charge dependence of electronic stopping from the relative weak dependence predicted by LSS toward the stronger dependence expected from Eq. (1) and the relation $Z_1^* = Z_1^{1/3} v/v_0$. This trend is expected to continue, hence the decrease of n for velocities above ≈ 1 (MeV/amu)$^{1/2}$ probably is due to an additional breakdown of Eq. (2). It may reflect an additional velocity dependence; e.g., Moak and Brown[15] have noted that the stopping power for high-energy bromine ions appears to increase somewhat more slowly than proportional to velocity. It is interesting to note that a rather small velocity-independent increase in n such as used by Aras *et al.*[16] might be expected to fit range data and yet the slope of the range-energy curve could be erroneous for full-energy fragments. We have incorporated a dependence of n on v such as indicated in Fig. 3 into a LSS-type calculation. This will clearly fit our energy-loss data and, with some adjustment at low velocities, our absolute ranges as well. However, it does not predict correctly stopping powers of different materials. We have chosen to proceed in a different way towards what we will show is a more universal treatment of the stopping of fission-fragment-like ions.

Electronic stopping powers for fission-fragment-like ions should be obtainable from the Bethe-Bloch equa-

TABLE II. Comparison of calculated and observed ranges.

Stopping medium	Z^A Fragment	Energy (MeV)	Range (mg/cm²) Experimental	Calculated	% Difference
Mylar	39.8^{99}	101.0	2.57	2.62	+2
	54.6^{140}	67.5	2.11	2.09	−1
	65.0^{149}	10.0	0.60[a]	0.52	−13
Aluminum	39.8^{99}	101.0	3.98[b]	3.77	−5
	54.6^{140}	67.5	2.98[b]	3.00	+1
	66.0^{149}	60.0	2.60[c]	2.60	0

[a] Range in Formvar from Ref. 4.
[b] From Ref. 16.
[c] From J. Natowitz and J. M. Alexander (unpublished).

TABLE III. Fractional effective charges for $Br^{79,81}$ and I^{127} ions in various stopping media.

$v/v_0 Z_1^{2/3}$	Ion	Stopping medium Be	C	Al	Ni	Ag	Au
0.65	Br	0.467	0.462	0.442	0.441	0.467	0.429
0.60	Br	0.431	0.429	0.410	0.407	0.430	0.399
0.55	Br	0.398	0.401	0.397	0.379	0.403	0.372
0.50	Br	0.365	0.371	0.348	0.345	0.373	0.347
0.45	Br	0.330	0.332	0.317	0.312	0.337	0.314
0.40	Br	0.297	0.301	0.285	0.277	0.296	0.274
	I	0.300	0.310	0.283	0.277	0.299	0.283
0.35	Br	0.262	0.262	0.254	0.240	0.256	0.233
	I	0.260	0.268	0.251	0.239	0.259	0.250
0.30	Br	0.227	0.219	0.226	0.206	0.218	0.206
	I	0.221	0.228	0.217	0.201	0.217	0.210
0.25	Br	0.190	0.182	0.195	0.182	0.195	0.176
	I	0.182	0.185	0.185	0.162	0.176	0.173
0.20	I	0.141	0.141	0.140	0.133	0.142	0.138
0.15	I	0.106	...	0.112	0.100	0.112	0.109

[15] C. D. Moak and M. D. Brown, Phys. Rev. **149**, 244 (1966).

[16] N. K. Aras, M. P. Menon, and G. E. Gordon, Nucl. Phys. **69**, 337 (1965).

TABLE IV. Percent of activity penetrating a given foil thickness as a function of the width of the range distribution.

Foil thickness $t/\langle r\rangle$	Full width at half-maximum (%) 2	4	6	8	10	12	16	20
0.95	2.497	2.487	2.482	2.502	2.550	2.619	2.794	2.996
0.96	1.997	1.989	2.002	2.051	2.127	2.219	2.431	2.658
0.97	1.496	1.500	1.547	1.631	1.735	1.849	2.093	2.342
0.98	0.998	1.037	1.132	1.252	1.381	1.514	1.783	2.049
0.99	0.522	0.635	0.775	0.922	1.069	1.215	1.501	1.779
1.00	0.168	0.331	0.492	0.650	0.804	0.955	1.249	1.532
1.01	0.024	0.142	0.285	0.435	0.585	0.734	1.026	1.309
1.02	0.001	0.048	0.150	0.276	0.412	0.551	0.832	1.109
1.03		0.013	0.070	0.165	0.279	0.403	0.665	0.932
1.04		0.002	0.030	0.093	0.182	0.288	0.525	0.776
1.05			0.011	0.049	0.114	0.200	0.408	0.640
1.06			0.004	0.024	0.069	0.135	0.312	0.523
1.07			0.001	0.011	0.040	0.088	0.235	0.424
1.08				0.005	0.022	0.056	0.174	0.339
1.09				0.002	0.012	0.035	0.127	0.269
1.10				0.001	0.006	0.021	0.091	0.212
1.11					0.003	0.012	0.064	0.164
1.12					0.001	0.007	0.045	0.126
1.13						0.004	0.030	0.096
1.14						0.002	0.020	0.072
1.15						0.001	0.013	0.054

tion [Eq. (1)] provided the nuclear charge Z_1 is replaced by an effective charge Z_1^* carried by the moving ion. Semiempirical treatments[8,17] which emphasize the role of this effective charge have had marked success in describing the stopping behavior of lower-mass ($A \leq 40$) heavy ions.

The logarithmic term of Eq. (1) is not expected to depend on the effective charge, but will depend on the properties of the stopping medium. Lindhard and Scharff[18] have proposed on statistical grounds that this term should be a universal function of a parameter $x = v^2/v_0^2 Z_2$. For low velocities, $x \leq 10$, they suggest

$$L(x) = 1.36x^{1/2} - 0.016x^{3/2}. \quad (5)$$

Fano,[7] on examining more closely the recent experimental data, has concluded that Eq. (5) overestimates $L(x)$ for stopping media of low atomic numbers. This is not unexpected since the statistical treatment was not expected to be valid for atoms containing only a few electrons. Despite the lack of universality, we have used for calculational simplicity an approximation for Mylar

$$L(x) = 1.30x^{1/2} + 0.014x^{3/2}, \quad (6)$$

based on the average of values for beryllium and aluminum in Fano's graph. Our electronic stopping data have been used in conjunction with Eq. (1) and this approximation to calculate ratios (Z_1^*/Z_1). We have used mean values for Mylar, $Z_2 = 6.46$ and $A_2 = 12.87$. It is expected that an ion moving with a velocity v will lose those electrons moving with velocities less than or comparable to v. Hence, values of Z_1^*/Z_1 are expected to fall on a universal curve when plotted as a function of a reduced velocity, $v_r = v/v_0 Z_1^m$, which corrects for the dependence of electron velocities on nuclear charge. At velocities where only a few electrons are carried by an ion, the value of m is expected to approach unity as predicted from the K-electron velocities in a hydrogenic ion. For systems of many electrons, Thomas-Fermi considerations indicate that $m = \frac{2}{3}$ should be a good approximation. The emulsion data of Heckman *et al.*[17] show clearly a transition from a dependence on $Z_1^{2/3}$ to one on Z_1 as the velocity, and hence the charge, of lower-mass ($A \leq 40$) heavy ions increases. This transition is at higher velocities and values of Z_1^*/Z_1 than those encountered in the present experiment; hence, we have plotted our values of the effective charge as a function of $v_r = v/v_0 Z_1^{2/3}$ in Fig. 4. Examination of this figure indicates that the fractional effective charges for the three mass-charge combinations fall on a single curve within their errors. The values, however, are the order of 25% lower than the prediction of Bohr which is shown as the dashed line.

We have reversed the above procedure and used the solid curve in Fig. 4 to obtain values of $(dE/dR)_e$ for fragments having $A_1 = 99$, $Z_1 = 39.8$ and $A_1 = 140$, $Z_1 =$

[17] H. H. Heckman, B. L. Perkins, W. G. Simon, F. M. Smith, and W. H. Barkas, Phys. Rev. **117**, 544 (1960).
[18] J. Lindhard and M. Scharff, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 15 (1953).

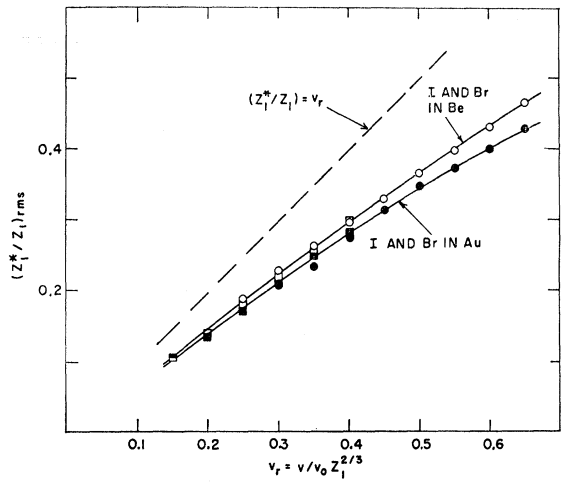

FIG. 5. Fractional effective charge aa a function of reduced velocity for bromine (○, ●) and iodine (□, ■) ions derived from the data of Ref. 16. Open symbols are for stopping in beryllium, filled ones for gold.

54.6.[19] After addition of the nuclear stopping power from the LSS theory, integrations were performed to give range-energy curves. Kinetic energies for fragments from the thermal fission of U^{235} were obtained from the measurements of Schmitt, Neiler, and Walter[20] in combination with the prompt neutron data of Apalin *et al.*[21] Comparison in Table II shows excellent agreement between the experimental and calculated ranges; hence, both energy-loss and range measurements in Mylar are consistent with this effective-charge treatment. Agreement for lower-energy ions, e.g., the 10-MeV Tb^{149} shown in Table II, is less satisfactory and reflects uncertainty in the curve in Fig. 4 at low velocities. Undoubtedly, some adjustment could improve the agreement. Ranges for three fragments in aluminum calculated from the same effective-charge curve are also seen to be in satisfactory agreement with experimental values in Table II. This is in part fortuitous, as the use of a mean charge and mass for Mylar and the approximation for the logarithmic term are expected to lead to errors when the curve in Fig. 4 is used to predict stopping powers in other materials. However, it would appear that a semiempirical treatment which emphasizes the role of effective charge offers considerable promise for the description of stopping phenomena for fission-fragment-like ions.

A critical test of this approach is afforded by the stopping-power measurements of Moak and Brown[15] for bromine and iodine ions in stopping media from Be to Au. Their analysis of the results indicated substantial deviations from LSS theory. We have reanalyzed their data in terms of the effective-charge approach. Values of (dE/dR) were read from the Moak and Brown curves, corrected for nuclear stopping by the LSS theory, and then used to calculate effective charges. Rather than use analytical forms for the log term as was done above, a direct comparison with stopping powers for protons[22] of the same velocities in the same medium were used. The results are given in Table III. Data for stopping by Be and Au are also plotted in Fig. 5 in a manner analogous to Fig. 4. Results for the different ions fall on the same curve for a given medium. The curves for Be and Au are less than 10% apart and the remainder of the data in Table IV generally fall between these extreme cases. Considering the 2–10% errors on the proton-stopping powers, the 5–10% errors of the iodine and bromine data, and the fact that results for Be, C, and Ag appear to be high while those for Al, Ni, and Au are low, there is no evidence for a dependence of Z^*/Z on the medium. At least, the dependence must be the same as that experienced by protons. The curves in Fig. 5 fall well below the Bohr estimate and somewhat below the results for Mylar shown in Fig. 4. However, a major part of the apparent difference between the Mylar data and that of Fig. 5 can be attributed to the approximations for the logarithmic term. When proton stopping powers are used, the Mylar results are close to those for Be in Fig. 5. It is interesting to note that recent direct measurements[23] of the charges on iodine and bromine ions emerging from foils give substantially higher values than would have been expected from the Z^*/Z values in Table III. The observed values are closer to the Bohr estimates, which poses an interesting problem: The charge necessary to account for energy loss of a fission fragment in a condensed medium appears to be significantly lower than that observed on the fragment after it emerges from the foil. It has been pointed out[17] that incomplete screening of the nucleus would lead one to expect the reverse effect.

CONCLUSIONS

Data on electronic stopping powers of Mylar for fission fragments are shown to be consistent with a universal dependence of the fractional effective charge carried by the ion on a reduced variable, $v/(v_0 Z_1^{2/3})$. When combined with theoretical estimates for nuclear stopping powers, this semiempirical treatment predicts ranges in Mylar and aluminum to an accuracy of better than 10% for fission-fragment-like ions having energies from 25 to 100 MeV. Extension of this treatment to stopping powers in solid media from Be to Au is successful, and suggests this approach will be useful in discussing a variety of fission-fragment and heavier-ion stopping phenomena.

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APPENDIX: 2π RANGE CURVES

In a 2π-range-curve measurement, the percent of the total activity of a given isotope which will penetrate a given thickness of foil t (measured in units of the mean range) will be given by

$$F(t)=100\int_{x=t}^{\infty}\int_{r=x}^{\infty}[P(r)/2r]drdx,$$

where $P(r)$ gives the distribution of ranges. We have assumed a Gaussian distribution and have evaluated this integral numerically. Values of $F(t)$ are given in Table IV for values of t from 0.95 to 1.15 and for relative full widths at half-maximum from 2 to 20%. Data in this table are convenient for the evaluation of the widths of range distribution from the activity experimentally observed in the last foil of a 2π range measurement.

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Nuclear Magnetic Resonance in Beryllium*

W. T. ANDERSON, JR., M. RUHLIG,† AND R. R. HEWITT
University of California, Riverside, California

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NMR of ^{9}Be in beryllium metal powder has been measured from 5 to 20 kG at 295°K. The isotropic Knight shift is measured as $(-0.0027\pm0.0006)\%$; the anisotropic Knight shift $|a|<0.0003\%$; and the quadrupole coupling e^2qQ/h is measured as 56.4 ± 0.3 kc/sec.

I. INTRODUCTION

THE nearly zero Knight shift in beryllium metal as first reported by Townes *et al.*[1] has been the subject of a number of theoretical attempts[2] at explanation with no success to date. Our interest in the measurement reported here arose from our earlier measurements in metals in the presence of large quadrupole interactions where it is apparent that the position of the $(\frac{1}{2}-\frac{1}{2})$ transition is affected by that quadrupole interaction and that these apparent shifts are magnetic-field-dependent. As will be shown in the following, the quadrupolar energy in beryllium metal is so small that the apparent shift of the $(\frac{1}{2}-\frac{1}{2})$ line from the second-order broadening is less than the precision of measurement.

Our initial measurements did show a measurable Knight shift, and as a result we continued the series of measurements as a function of magnetic field. These measurements provide the opportunity to fit the second-order quadrupole broadening of the $(\frac{1}{2}-\frac{1}{2})$ line with the magnetic field dependence of that linewidth.

The treatment of NMR in polycrystalline metal samples with quadrupolar interactions has been discussed extensively in recent literature[3,4] and is not repeated here. The NMR spectrum is composed of the central $(\frac{1}{2}-\frac{1}{2})$ transition and two symetrically placed satellites that are powder pattern discontinuities of the $(\frac{1}{2}-\frac{3}{2})$ transitions. The maxima of absorption from these discontinuities occur approximately where the angle between the electric field gradient axis and the applied magnetic field is 90°. As shown in Ref. 4, some care must be taken in the measurement of the quadrupole coupling from the position of these discontinuities because of their apparent shifting from the Gaussian dipolar broadening σ.

II. EXPERIMENTAL DESCRIPTION

The NMR apparatus was a Varian broadline induction spectrometer. The electromagnets used in the measurements were Magnion instruments with FFC-4 rotating-coil field controls. The measurements were made at fixed field, and the swept rf frequencies were measured with a Hewlett-Packard 5245L frequency counter. In each run, the field was calibrated with the deuteron resonance in D_2O; the salt reference of Be in BeO was measured; the Be-metal NMR was measured; and finally the deuteron resonance was remeasured to check for any drift in the magnetic field. This procedure

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† National Science Foundation Undergraduate Research Participant.

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