# Production and Detection of Solitary Macroscopic Quantized Vortices in Helium II†

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Ultrasonic cavitation was induced in liquid helium using plane and cylindrical standing-wave systems resonant at 89 and 53 kc/sec, respectively. The threshold voltage for cavitation noise was measured at different rotation rates of a shaft, with and without an attached paddle, rotating in the vicinity of the sound field. Below the λ point the threshold was found to be lowered sharply at a critical rotation rate $\omega_c$. The experiments were repeated with five different shaft sizes, and $\omega_c$ was found to be inversely proportional to $r^2$, where $r$ is the radius of the shaft. The constant of proportionality was found to be $\hbar/m$, where $m$ is the mass of one helium atom. Further, smaller threshold reductions were observed at rotation rates of $2\omega_c$, with the largest shaft sizes. These results are consistent with the theory of quantized vortex formation and appear to indicate that such vortices play a role in the cavitation nucleation process.

## I. INTRODUCTION

IT has been found[1] that cavitation noise can be induced ultrasonically in liquid helium with threshold pressure amplitudes of only a few millibars. However, below the λ point the threshold for visible cavitation was at least an order of magnitude higher than the threshold for noise.[2] A curious finding was that the noise threshold was virtually independent of static pressure.[2,3] It was concluded,[3] because of the very low noise thresholds, that the cavitation must have been nucleated, although evidence pointed against most of the agencies commonly believed to operate in normal liquids. Two possible mechanisms, however, have not been investigated: first, cosmic radiation, and second, free vortices possibly quantized. It was also noted that the independence of static pressure might be explained if nucleation involved an accumulation of submicroscopic structures, by steady forces arising from fluid flow or sonic radiation pressure. The experiments to be described were originally conceived to determine if liquid flow had any effect on the noise threshold, whether through vortex formation or through some accumulation process.

The vortex-nucleation hypothesis has received support from the work of Edwards, Cleary, and Fairbank,[4] who used a bubble chamber to produce visible cavities in He II and found nucleation to occur readily after vortices had evidently been formed within the chamber. Their techniques, however, did not permit them to measure the circulation of the vortices generated.

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[1] R. D. Finch, R. Kagiwada, M. Barmatz, and I. Rudnick, Phys. Rev. **134**, A1425 (1964).

[2] R. D. Finch and T. G. J. Wang, J. Acoust. Soc. Am. **39**, 511 (1966).

[3] R. D. Finch, T. G. J. Wang, R. Kagiwada, M. Barmatz, and I. Rudnick, J. Acoust. Soc. Am. **40**, 211 (1966).

[4] M. H. Edwards, R. M. Cleary, and W. M. Fairbank, in *Quantum Fluids*, edited by D. F. Brewer (North-Holland Publishing Company, Amsterdam, 1966), pp. 140–145.

## II. EFFECT OF A ROTATING PADDLE

The original object was then to study the effect of fluid velocity on the threshold of cavitation noise. It was felt that a convenient method of inducing a flow in the liquid was to rotate a paddle in the vicinity of the cavitation region. The cavitation system employed was the plane-wave arrangement previously described,[1] and which consisted of two identical disk transducers of PZT4 ceramic, each being $\frac{1}{2}$ in. in radius and in thickness with electrodes on the flat surfaces. The resonant frequencies of both transducers were within 0.2% of 89.11 kc/sec in liquid helium. They were supported by threads from brass suspension rings with their opposed faces parallel, there being a separation of some 4.5 cm between them.

The rotatable stainless-steel shaft was $\frac{1}{8}$ in. in diameter and some 5 ft in length, terminating in a rectangular blade (5 cm in length and 1 cm in breadth, either welded or epoxied to the shaft). The blade was positioned just above and to one side of the upper transducer, as shown in Fig. 1. This whole assembly was suspended vertically in a standard double-Dewar system, and could be viewed through parallel vertical slits in the silvering of the Dewar. The cryostat employed Wallace and Tiernan gauges for temperature measurement. The upper end of the shaft, finishing outside the Dewar, could be coupled through a system of pulleys to a variable speed motor with rotation speeds controllable to ±0.001 rpm. Rotation speed was monitored continuously by a tacho-generator coupled to the shaft of the motor and registered by an electronic counter. A schematic of the whole setup, including circuitry, is shown in Fig. 2.

The threshold was determined by raising the voltage on the driver transducer until cavitation noise started, as heard on the loudspeaker and seen on an oscilloscope trace. As mentioned before, a difficulty sometimes arose in that different observers tended to choose somewhat different criteria for the noise threshold and these criteria were not readily available for comparison when using earphones. With the present system employing a loudspeaker, however, a more careful study of this problem was possible. It was found that the criterion which gave the most reproducible results was the onset of a relatively loud "popping" noise. Also as mentioned previously, the threshold was not obtained at exactly the same value of driving voltage on repetition, and the error bars shown in the present results represent the spread of at least five different determinations.

Above the λ point, rotation of the paddle had no apparent effect on the cavitation threshold. However, below the λ point, the first tests performed showed a marked reduction of the threshold with even the lowest

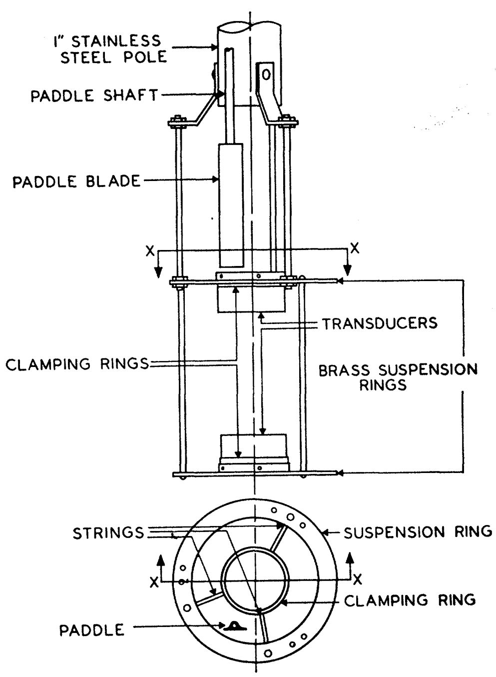


FIG. 1. Plane-standing-wave apparatus. The transducers were resonant at about 89 kc/sec, one being used as a driver, the other as a microphone.

rotation speeds attainable from the motor (Fig. 3). It was after this that the pulley system was installed to obtain a substantially lower speed range to explore the nature of this reduction in more detail. It was then found that at very low speeds the threshold remained constant, and that the reduction came sharply at a certain critical rotation rate ($\omega_c$), with a further sharp reduction at twice this value, followed by an apparently more gradual decline at higher speeds (Fig. 4). More than ten such runs were made with the $\frac{1}{8}$-in. shaft and in every case the same pattern occurred, the value of $\omega_c$ proving to be consistently reproducible at 0.06 rpm.

## III. EVIDENCE FOR QUANTIZED VORTEX FORMATION

The fact that the threshold reductions occurred only below the λ point and at multiples of a critical rotational

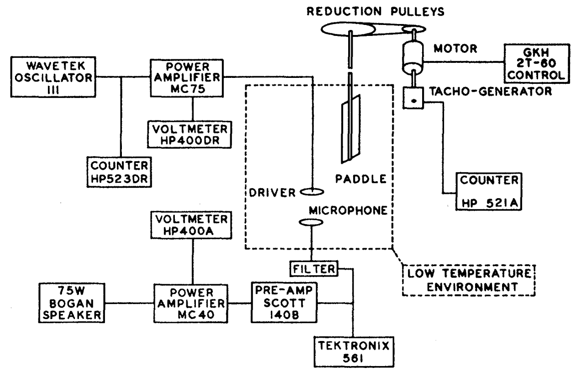


FIG. 2. Schematic diagram of circuitry.

rate suggested the formation of a quantized superfluid vortex around the shaft or paddle. In this event the circulation of the vortex according to Feynman would have to be given by[5]

$$\oint \mathbf{v}\cdot d\mathbf{l} = \frac{nh}{m},$$

where $v$ is the superfluid velocity, $h$ is Planck's constant, $m$ is the mass of one helium atom, $n$ is an integer, and $dl$ is an element of the path of integration. Taking a circular path of integration of radius $r$, the angular rotation rate for the formation of a vortex is

$$\omega_c = (n\hbar/m)(1/r^2). \tag{1}$$

It was supposed that $r$ would be some dimension typical of the rotating system, such as the paddle radius or thickness, or the shaft radius, and that Eq. (1) might only determine $\omega_c$ to an order of magnitude. However, for $r=\frac{1}{16}$ in. (shaft radius) and $n=1$, Eq. (1) yields $\omega_c=0.057$ rpm, which is very close to the experimental value, the dashed line in Fig. 4 corresponding to this theoretical speed.

This indication of the dependence of $\omega_c$ on the shaft radius according to Eq. (1) was confirmed by repeating the experiment with shafts of different sizes but with a constant size of paddle. For a given shaft diameter, a

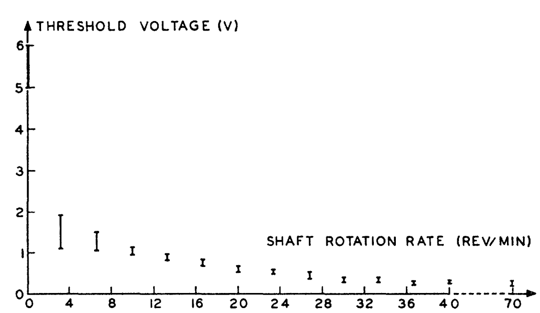


FIG. 3. Threshold voltage (V) versus shaft rotation rate (rpm) for a $\frac{1}{8}$-in. diam shaft. Temperature=2.10°K.

[5] R. P. Feynman, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1955), Vol. I, pp. 17–53.

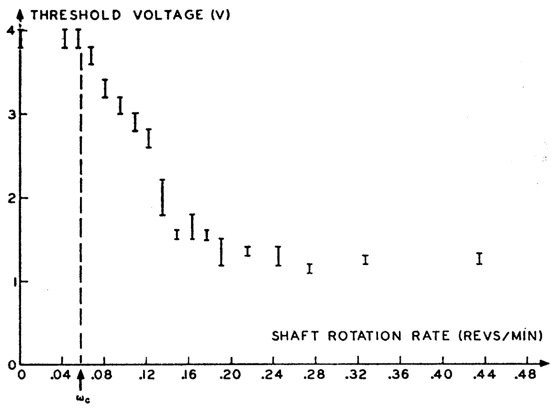


FIG. 4. Threshold voltage (V) versus shaft rotation rate (rpm). Plane-wave apparatus, paddle on $\frac{1}{8}$-in.-diam shaft. Temperature = 2.16°K. Dashed line at 0.057 rpm is the theoretical critical angular speed $\omega_c$ calculated from Eq. (1) with $n=1$. Note also the possible threshold reduction at about $2\omega_c$.

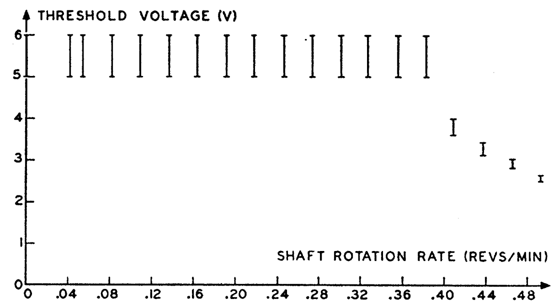


FIG. 6. Threshold voltage (V) versus shaft rotation rate (rpm). Plane-wave apparatus, paddle on 0.043-in.-diam shaft. Temperature = 2.16°K.

run was made first at 2.16°K, a given speed being maintained constant for 2 or 3 min, in order to allow the system to attain a steady state. The threshold was measured, as previously described, some five times at each speed before increasing the speed to a new setting. A second run was performed with each shaft size at 2.10°K. Figures 5 and 6 show typical results obtained with shafts 0.0625 in. and 0.043 in. in diameter. Figure 7 shows the values of $\omega_c$ for the two temperatures for various shaft radii plotted against the reciprocal of the square of the shaft radius. These values may be seen to lie very close to a straight line of slope $\hbar/m$, as given by Eq. (1) with $n=1$.

Early in the investigations it was suspected that low-frequency vibrations of the 1-in. pole supporting the transducers could cause a reduction of the threshold by shedding vortices; to eliminate this, the top of the pole was vibration isolated with taut wires tied to the laboratory walls. In addition, the motor and pulley assembly was mounted on a frame mechanically isolated from the cryostat. Vibration from the vacuum pump was reduced by using a long rubber connection to the main metal pumping line, which was also vibration isolated with wires.

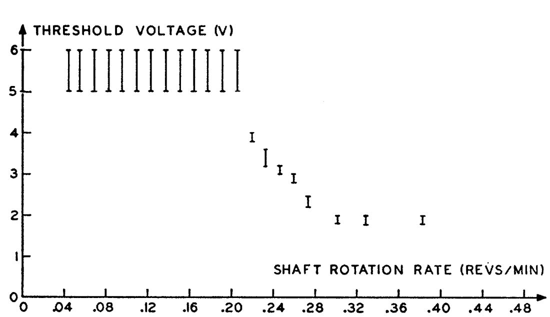


FIG. 5. Threshold voltage (V) versus shaft rotation rate (rpm). Plane-wave apparatus, paddle on $\frac{1}{16}$-in.-diam shaft. Temperature = 2.16°K.

There still remained to be resolved the question of the role played by the paddle blade. A shaft without a paddle was tried, but no conclusive effect on the threshold was observed, even at very high rotation speeds. In order to resolve this issue a new apparatus was constructed.

## IV. CYLINDRICAL STANDING-WAVE SYSTEM

In this new assembly the driver transducer employed was a hollow PZT4 cylinder $\frac{3}{2}$-in. long and $\frac{3}{2}$-in. o.d. with $\frac{1}{8}$-in.-thick walls. The cylinder was operated in its breathing mode, which corresponded to a resonant frequency of about 53 kc/sec in liquid helium. The cylinder was supported with its axis vertical by tightly stretched nylon strings tied to two $\frac{1}{2}$-in. vertical stainless-steel tubes, to permit alignment of the rotating shaft with the transducer axis, as shown in Fig. 8. A PZT4 disk, $\frac{1}{2}$ in. in radius and in thickness, mounted below the cylinder, served as a microphone. The cir-

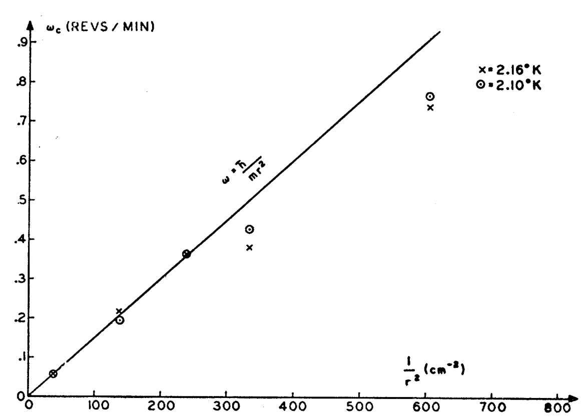


FIG. 7. Measured values of critical speed $\omega_c$ (rpm) versus $1/r^2$ (cm$^{-2}$), where $r$ = shaft radius. Plane-standing-wave apparatus, paddles on shafts. Straight line is theoretical expression according to Eq. (1) with $n=1$.

cuitry and motor-drive system were the same as those used with the plane-wave system.

Following exactly the same procedures as in the previous experiments, a clear repeatable effect was now obtained using a $\frac{1}{8}$-in. shaft without a paddle blade. The critical rotation rates found in the previous experiments were exactly reproduced. When a paddle of the same size as that used in the previous investigations was attached to the end of the shaft the same results were obtained, but the noise of cavitation seemed to be enhanced at its onset.

A series of runs was made with shafts of different sizes without paddles, and the results (Fig. 9) again showed a dependence on $1/r^2$, the agreement with Eq. (1) with $n=1$ being rather better in this case than for the plane-wave system (Fig. 7). It was found that a

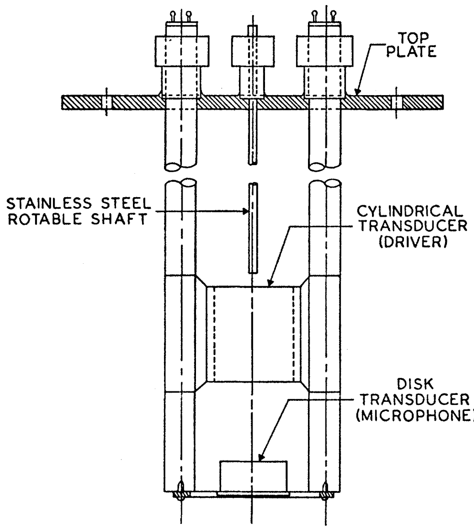


FIG. 8. Cylindrical standing-wave apparatus. Driving cylinder resonant at 53 kc/sec.

waiting time of some 10 to 15 min was required for the threshold to return to its original value after stopping the shaft.

It was concluded then that the paddle blade acted as a perturbation to the vortex around the shaft. The vortex flow cannot separate from the shaft above the paddle blade, since this would lead to a discontinuity in velocity, and consequently it must encompass the blade. But this would probably lead to an instability of flow, particularly around the free "tail" coming off the bottom of the blade. In the absence of a blade, however, the flow might be relatively stable with the free "tail" possibly terminating on the transducer suspension ring in the plane-wave system (Fig. 1) or on the microphone in the cylindrical system (Fig. 8). Thus, for the plane-wave system the blade is necessary to break up or stretch the free tail so that it penetrates the sound field, while for the cylindrical system no blade is neces-

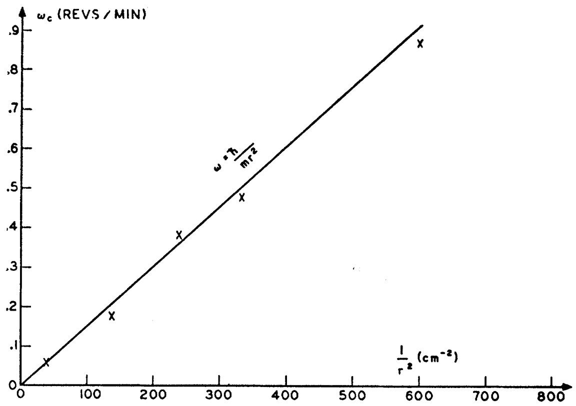


FIG. 9. Measured values of critical speed $\omega_c$ (rpm) versus $1/r^2$ (cm$^{-2}$), where $r$=shaft radius. Cylindrical standing-wave apparatus. No paddles on shafts. Straight line is theoretical expression according to Eq. (1) with $n=1$.

sary since the unperturbed tail passes directly through the region of maximum sonic intensity. This presumes, of course, that it is the free tail which is responsible for nucleation, an idea which was supported by the following experiments.

## V. EVIDENCE FOR NUCLEATION BY A FREE VORTEX, AND FOR THE SOLITARY NATURE OF THE VORTEX

A solid steel cone was constructed to be $\frac{1}{4}$ in. in diameter at its base and about $\frac{1}{4}$ in. in height, the apex being sharpened to a point. The base of the cone was stuck to the upper face of the microphone (Fig. 10) with silicone grease, which afforded an excellent bond at low temperatures. At the end of a $\frac{1}{8}$-in.-diam shaft a small socket was made which was fitted over the apex of the cone with a slight pressure to keep the cone's point in contact with the shaft. The shaft was then rotated on this bearing, so that no length of free vortex tail could exist above the microphone; the background

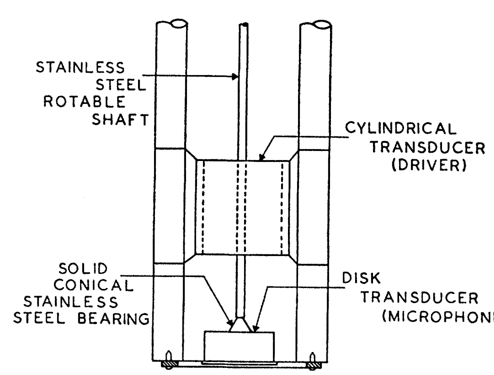


FIG. 10. Arrangement to eliminate any length of free vortex tail above microphone. With this arrangement no threshold reductions were observed.

noise was enhanced, but it was still possible to distinguish the cavitation-noise threshold without difficulty.

Under these conditions, with or without a blade, no reduction of the threshold was observed even with rotation rates several times in excess of $\omega_c$. However, when the experiment was carried out with a gap of only $\frac{1}{8}$ in. between the end of the shaft and the microphone, a clear threshold reduction was obtained at the usual value of $\omega_c$. The actual values of the thresholds, as measured by the driving voltage, were less in this case, because the presence of the solid shaft along the cylinder's axis represented a change in the boundary conditions of the sound field. Moreover, the shaft acted as a waveguide, conveying sound to the short length of free tail just above the microphone face.

Several inferences are to be drawn from these experiments with the point bearing: Firstly, that the free tail of the vortex is responsible for nucleation, as already proposed; secondly, that the vortex flow does not separate from the shaft when a blade is attached (also already suggested); thirdly, that a blade does not generate vortices off its edges as it might in a normal liquid; and finally, although not least, that no free vortices are generated by the rotating shaft, except for the tail of the vortex around the shaft. It is for this reason that the vortex around the shaft, even at speeds greater than twice $\omega_c$, is said to be solitary.

## VI. DISCUSSION

It has been shown that the nucleation occurs on the "free" portion of the vortex and this suggests that an interaction occurs between the applied sound field and the core. It has been pointed out by Dean[6] that an empty or vapor-filled vortex core of radius $r$ will experience a contractile pressure $P$ given by

$$P=\sigma/r-(\Gamma^2\rho/8\pi^2r^2),$$

where $\sigma$ is the macroscopic surface tension, $\Gamma$ is the circulation ($=nh/m$, for a quantized vortex), and $\rho$ is the liquid density, which should be replaced by $\rho_s$, the superfluid density, in the present case. It follows that there is an equilibrium radius $r_e$ given by

$$r_e=\Gamma^2\rho/8\pi^2r^2\simeq 0.5\ \text{Å} \qquad \text{for } n=1.$$

Considering the crudeness of the model, this is not an unreasonable value for the core radius. When the core radius is $r_m$, where

$$r_m=2r_e,$$

the contractile pressure has a maximum value $P_m$ given by

$$P_m=2\pi^2\sigma^2/\Gamma^2\rho_s\simeq 20\ \text{atm} \qquad \text{for } n=1.$$

This value is close to the estimates of the ultimate tensile strength of liquid helium[1] and is considerably greater than the experimental values of the cavitation-noise threshold. However, an applied sound field would induce a forced pulsation of the vortex core so that it is conceivable that the application of a sinusoidally varying pressure of amplitude much less than $P_m$ could produce relatively large-amplitude nonlinear core pulsations. Such a nonlinearity could afford one explanation for the generation of the audio-frequency noise which has been observed, and which the authors have called cavitation noise. Moreover, if the amplitude of the pulsation were increased sufficiently, it is possible that the vortex core might explode and break up into visible cavities. This admittedly crude picture obviously requires further clarification, both from theoretical and experimental studies, and efforts in this direction are currently proceeding.

In summary, it is felt that these experiments have resulted in the discovery of a method of creating solitary macroscopic quantized vortices, and a new technique for their detection through a nonlinear interaction with sound, although the exact details of this process remain unclear. The results further show that cavitation below the $\lambda$ point can be nucleated by quantized vortices.

## ACKNOWLEDGMENTS

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[6] R. B. Dean, J. Appl. Phys. 15, 446 (1944).