Meson Spectrum and Superconvergence Sum Rules for the $J^{PC}=2^{++}$ Mesons*

R. J. RIVERS
Physics Department, Imperial College, London, England

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We classify the mesons with masses $m \leqslant 2000$ MeV by means of a simple quark model and calculate the contributions that individual resonances make to meson superconvergence sum rules. To a good approximation we find that only the two lowest supermultiplets are needed. For the case of elastic scattering of pseudoscalar mesons with $J^{PC}=2^{++}$ mesons, we find that in some sum rules one of these supermultiplets is suppressed for kinematic reasons. Predictions are made and compared with both higher-symmetry predictions and experiment.

1. INTRODUCTION

On the basis of plausible assumptions about the asymptotic behavior of scattering amplitudes, many authors[1,2] have recently derived superconvergence sum rules for strong interactions. Approximating the relevant dispersion integrals by a sum of single-particle intermediate states (in a narrow-width approximation) enables predictions to be made that are in reasonable agreement with experiment. (However, the predictive power of many of the sum rules in Refs. 1 and 2 has been considerably weakened by the omission of Regge branch-point behavior which invalidates some of them.[3])

There are two major difficulties associated with the single-particle intermediate-state approximation. Firstly, the sum rules are fixed-t dispersion relations, valid for all values of the momentum transfer, and it is difficult to construct an approximation scheme that is valid for more than a very limited range of t. Secondly, the more intermediate states we include, the less predictive the sum rules become. These difficulties are clearly related, since the dependence of the sum rules on the momentum transfer is determined by the spins and parities of the single-particle states included.

It may be that the first difficulty has a parallel in the difficulties of consistently saturating moment sum rules in current algebra with a finite number of single-particle intermediate states. In this paper, we shall be more concerned with the question of which states we ought to include in the sum rules.

The general philosophy has been to evaluate the sum rules at $t=0$ and to take as intermediate states the relevant members of the lowest $U(6)\times U(6)$ representation appropriate.[4] This procedure has in part been justified for the case of πN elastic scattering[2] (although Regge cuts are now seen to invalidate the superconvergence relation) by the realization that for values of the momentum transfer $|t|\ll m_N^2$, the sum rules essentially uncouple for each value of the orbital angular momentum.[5] This behavior does not hold in general, and Taylor and Frampton[6] have shown that for $\rho\pi$ scattering the contributions from one-particle states not belonging to the lowest multiplet can be large.

Now one of the interesting features of the sum rules is that the predictions are often in agreement or near-agreement with the predictions of $U(6,6)$ or $U(6)_W$ provided the intermediate states belong to the lowest $U(6)\times U(6)$ multiplet. It may be that under some higher symmetry group classification individual multiplets approximately satisfy the sum rules, but until this can be shown to be the case, there is little justification for neglecting those states lying outside the lowest multiplet whose individual contributions to the sum rules can be shown to be large.

We shall be examining the contributions of known and speculative meson resonances to meson superconvergence sum rules. This means that we need a scheme for classifying meson resonances at least up to masses of 2000 MeV. In the following section we shall consider a classification scheme based on the group $U(6)\times U(6)\times O(3)$.[7] Although we find it a very convenient scheme for classification, the conclusions that we draw about the magnitude of the resonance contributions to the $\rho\pi$ sum rules (in Sec. 3) do not depend crucially upon the classification, but rather upon the resonance masses and widths.

In Sec. 4 we consider sum rules for the scattering of pseudoscalar mesons with the $J^{PC}=2^{++}$ meson nonet. Using the conclusions of the previous section and further kinematic arguments, we are able to make predictions which are compared both with group-theoretical predictions and with experiment.

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[1] V. de Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters 21, 576 (1966); H. F. Jones and M. D. Scadron, Nuovo Cimento 48, 545 (1965); *ibid.* (to be published).

[2] B. Sakita and K. Wali, Phys. Rev. Letters 18, 29 (1967); G. Altarelli, F. Bucella, and R. Gatto, Phys. Letters **24B**, 57 (1967); P. Babu, F. J. Gilman, and M. Suzuki, *ibid.* **24B**, 65 (1967). (Babu *et al.* include higher baryon states in the sum rule.)

[3] R. J. N. Phillips, Phys. Letters **24B**, 342 (1967).

[4] A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) **284A**, 146 (1965); A. Salam, R. Delbourgo, M. A. Rashid, and J. Strathdee, *ibid.* **285A**, 312 (1965); K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 14, 48 (1965); B. Sakita and K. Wali, *ibid.* 14, 404 (1965).

[5] That the range is not larger can be seen, for example, by writing $R_{1+}(t)$ [in the notation of B. Sakita and K. Wali, Ref. 2] as

$$R_{1+}(t)=\frac{4\pi}{q^3}\frac{\{12M^2(p^2-\frac{1}{2}t)-[(M+m)^2-\mu^2]^2\}}{[(M+m)^2-\mu^2]}.$$

[6] P. H. Frampton and J. C. Taylor, Clarendon Laboratory report (unpublished).

[7] M. Gell-Mann, Phys. Rev. Letters 14, 77 (1965). Also see Ref. 7.

2. CLASSIFICATION OF MESONS

The classification of most of the known mesons by $U(6)\times O_L(3)$ or $U(6)\times U(6)\times O_L(3)$ has been discussed by several authors in considerable detail.[8] We shall classify the mesons with mass $m<2000$ MeV according to the $(6,\bar{6},2L+1)$ representations of $U(6)\times U(6)\times O_L(3)$, corresponding to the quark-antiquark pair with orbital angular momentum L.

We shall adopt the nonrelativistic model of Sinanoğlu[8] with some minor alterations, taking the $(q\bar{q})$ potential [in the absence of $SU(3)$ breaking] to be

$$V(r)\cong V_C(r)+V_S(r)\mathbf{S}_1\cdot\mathbf{S}_2+V_{LS}(r)\mathbf{L}\cdot\mathbf{S}+V_T(r)S_{12}, \quad (2.1)$$

where V_C, V_S, V_{LS}, V_T are the central, spin, spin-orbit and tensor potentials, respectively, satisfying $V_C\gg V_S$, V_{LS}, V_T.

This leads to a linear mass-formula[9]

$$m=m_0+\Delta m_D(n',L)+a(n',L)\mathbf{L}\cdot\mathbf{S}+b(n',L)G_t^{(1)}+c(n',L)\mathbf{S}_1\cdot\mathbf{S}_2, \quad (2.2)$$

where $n'=0, 1, 2, \cdots$ and $L=0, 1, 2, \cdots$ are the radial and angular dynamical quantum numbers. The term Δm_D determines the spacing between the (n',L) supermultiplets and a, b, c determine the splitting between the $SU(3)$ nonets within the supermultiplets. In common with the authors of Ref. 7 we take the lowest supermultiplets as the (0,0) [$n'=L=0$, consisting of the $J^{PC}=0^{-+}, 1^{--}$ nonets] and the (0,1) [$n'=0, L=1$, consisting of the $J^{PC}=2^{++}, 1^{++}, 1^{+-}, 0^{++}$ nonets]. Their contents are listed in Table I.[9,10]

For the (0,0) supermultiplet, the tensor and spin-orbit forces give no splitting, the splitting coming entirely from $c(0,0)$. As in Ref. 8, we suppose that $V_S(r)$ is strongly dependent on r and large only for small values of r so that

$$\langle n'L|V_S(r)|n'L\rangle\approx 0 \quad \text{for} \quad n', L\neq 0. \quad (2.3)$$

Thus for the (0,1) supermultiplet we take $c(0,1)\approx 0$. If in addition we take $b(0,1)\approx 0$, we obtain two relations between the nonet central masses. These are

$$\bar{m}(2^{++})+\bar{m}(1^{++})=2\bar{m}(1^{+-})$$

and

$$\bar{m}(2^{++})+2\bar{m}(0^{++})=3\bar{m}(1^{++}). \quad (2.4)$$

Assuming that Eqs. (2.4) are approximately valid for $I=\frac{1}{2}$, 1 separately, we have

$$m_{A_1}+m_{A_2}\cong 2m_B \quad (2.5a)$$

and

$$m_{A_2}+2m_{\pi_V}\cong 3m_A. \quad (2.5b)$$

Experimentally, Eqs. (2.5a) and (2.5b) give

$$2385\pm 16 \text{ MeV}\cong 2416\pm 24 \text{ MeV} \quad (2.5a')$$

and

$$3312\pm 8(?) \text{ MeV}\cong 3237\pm 24 \text{ MeV}, \quad (2.5b')$$

respectively (no quoted error on m_π), in reasonable agreement. For the isospin doublets, Eq. (2.4) predicts $K'(1230\pm 25)$, $J^P=1^+$ and $K''(1108\pm 50)$, $J^P=0^+$. Because of the possibility of different single-octet mixing in the nonets, it is difficult to say anything about the isospin singlets.

Using the known and predicted masses, we take the (0,0) supermultiplet center at

$$\bar{m}(0,0)=605\pm 50 \text{ MeV}, \quad (2.6)$$

and the (0,1) supermultiplet center at

$$\bar{m}(0,1)=1220\pm 100 \text{ MeV}. \quad (2.7)$$

Let us consider Δm_D. The assumption that $V_c(r)$ is a deep-well "hard-core" potential gives[8]

$$\Delta m_D(n',L)\cong m_1'n'+m_2L(L+1). \quad (2.8)$$

With the above central masses, Eq. (1.8) gives the (0,2) supermultiplet center as

$$\bar{m}(0,2)\cong 2450\pm 150 \text{ MeV}. \quad (2.9)$$

The n' excitations might be expected to lie as high. Although an exchange potential could lower these masses to some extent, such a spectrum would still not include the great variety of conjectured resonances with masses $m\approx 1650$ MeV that we have not accommodated so far.

Therefore, unlike Sinanoğlu, we reject a "cored" potential in favor of a "coreless" potential, giving a typical spectrum

$$\Delta m_D(n',L)\cong 2m_1n'+m_1L. \quad (2.10)$$

This would give the (0,2) supermultiplet center at

$$\bar{m}(0,2)\cong 1830\pm 150 \text{ MeV}. \quad (2.11)$$

Under spin-orbit and tensor forces this would split into $J^{PC}=3^{--}$, 2^{--}, 1^{--}, and 2^{-+} nonets. Superimposed upon this spectrum would be the (1,0) supermultiplet (consisting of $J^{PC}=0^{-+}$ and 1^{--} nonets) with center

$$\bar{m}(1,0)\cong 1830\pm 150 \text{ MeV} \quad (2.12)$$

and small spin-splitting because of assumption (2.3).

We would therefore expect a very rich mass spectrum in this range (possibly lowered by exchange potentials). There is some evidence that such a spectrum does

[8] E. Borchi and R. Gatto, Phys. Letters **14**, 352 (1965); R. Gatto, L. Maiani, and G. Preparata, Phys. Rev. **140**, B1579 (1965); R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966); O. Sinanoğlu, Phys. Rev. **145**, 1205 (1966); E. G. Goldhaber, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967); R. H. Dalitz, *ibid.*

[9] $G_t^{(1)}=[(2\mathbf{s}_1\cdot\mathbf{s}_2+\frac{3}{2})\mathbf{L}^2-\frac{3}{2}(\mathbf{L}\cdot\mathbf{S})-3(\mathbf{L}\cdot\mathbf{S})^2]/(2L+3)(2L-1)$.

[10] Unless stated otherwise, all data will be taken from A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

TABLE I. The (0,0) and (0,1) supermultiplets.[a]

J^{PC}	n'	L	S	$I=1$	$I=\frac{1}{2}$	$I=0$	$I=0$
0^{-+}	0	0	0	$\pi(140)$	$K(495)$	$\eta'(958)$	$\eta(550)$
1^{-}	0	0	1	$\rho(760)$	$K^*(890)$	$\omega(783)$	$\phi(1020)$
1^{+}	0	1	0	$B(1208\pm12)$	$K_A(1320\pm12)$	$H(1000)$?	$[H']$
0^{++}	0	1	1	$\pi_V(1003)$?[b]	$[K''(1138\pm50)]$[c]	$S(720)$?	$S(1050)$?
1^{++}	0	1	1	$A_1(1079\pm8)$	$[K'(1230\pm25)]$[c]	$D(1285\pm4)$	$E(1424\pm7)$?[d]
2^{++}	0	1	1	$A_2(1306\pm8)$	$K_v(1411\pm5)$	$f(1254\pm12)$	$f'(1514\pm16)$

[a] Bracketed states denote no experimental evidence. The masses of such states are predicted from the mass formula of Sec. 2.
[b] An alternative candidate is the $\delta(965)$.
[c] The K' and K'' are possibly the $K_c(1215)$ and the $K^*(1080)$, respectively.
[d] The E may be pseudoscalar. See Refs. 10 and 11.

exist.[11] Although definite experimental evidence is scanty, there is evidence for $\pi_A(1640)$, $K_A(1800)$, $g(1650)$, and $R_1(1632\pm10)$, $R_2(1699\pm10)$, $R_3(1748\pm10)$. A plausible assignment of the R_1, R_2, R_3 is that[12] the R_1 belongs to the 1^{--} nonet of the (0,2) supermultiplet and (a) R_2, R_3 belong to the 2^{--}, 2^{-+} nonets of the (0,2) supermultiplet, respectively, or (b) the R_2 contains an unresolved mixture of the 2^{--} and 2^{-+} isotriplets and R_3 belongs to the 3^{--} nonet. The $\pi_A(1640)$ and $K_A(1800)$ would conveniently fit into the 0^{-+} nonet of the (1,0) supermultiplet and the $g(1650)$ into its 1^{--} nonet.

Mesons of higher mass like the S, T, U would belong to supermultiplets with higher L values. The simple formula (2.10) would suggest that they belong to the (0,3) supermultiplet with mass

$$\bar{m}(0,3)\cong 2450\pm150 \text{ MeV}. \tag{2.13}$$

but they could well have higher L values.[12]

For our purposes the classification of individual resonances is not crucial, the broad outlines of the mass spectrum being more important than spin-parity assignments, especially for the higher mass resonance.

We now consider the contributions of the known and conjectured resonances to meson superconvergence sum rules.

Because of its relative simplicity we will first consider $\rho\pi$ elastic scattering.

3. ELASTIC $\rho\pi$ SCATTERING

For this and the following section we adopt the following notation. The pseudoscalar mesons are taken to have mass μ and incoming (outgoing) momentum q (q'), whereas the mesons with *nonzero* spin and mass m have incoming (outgoing) momentum p (p'). The Mandelstam variables are $s=(p+q)^2$, $t=(p'-p)^2$, $u=(p'-q)^2$ and the M functions are defined in terms of the combinations $P=\frac{1}{2}(p+p')$, $Q=\frac{1}{2}(q+q')$.

For $\rho\pi$ scattering we have

$$M_{\mu\nu}=A_{QQ}(s,t)Q_\mu Q_\nu+A_{PQ}(s,t)[Q_\mu P_\nu+Q_\nu P_\mu]+A_{PP}(s,t)P_\mu P_\nu+A_0(s,t)g_{\mu\nu}. \tag{3.1}$$

[11] E. H. Goldhaber, Ref. 7.

[12] R. H. Dalitz, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967).

In terms of the subamplitudes $A^{(T)}$, where T is the isospin in the cross channel, Regge pole and branch-point behavior implies[1,3]

$$s^2A_{QQ}^{(1)}(s,t)\to 0 \quad \text{as} \quad s\to\infty, \tag{3.2}$$

giving the superconvergence relation

$$\int \mathrm{Im}A_{QQ}^{(1)}(\nu,t)d\nu=0, \tag{3.3}$$

where $2\nu=s-(\mu^2+m^2)+\frac{1}{2}t$.

We shall approximate the integral (3.3) by a sum of s and u channel resonances at $t=0$. In the narrow-width approximation we have

$$\mathrm{Im}A_{QQ}(\nu,0)=\sum_{J\pm} A_{QQ}^{J\pm}\tfrac{1}{2}[\delta(\nu-\nu_{J\pm})+\delta(\nu+\nu_{J\pm})], \tag{3.4}$$

where A^{J+} $[A^{J-}]$ is the contribution of a resonance of spin J, parity $(-1)^J$ $[(-1)^{J+1}]$, and mass M_{J+} $[M_{J-}]$. The positions of the poles are given by $\nu_{J\pm}=\frac{1}{2}[M_{J\pm}^2-(m^2+\mu^2)]$.

For a spin-J field coupling to ρ and π fields let us take the effective interaction Lagrangians (omitting isospin for convenience)

$$\mathcal{L}^{J+}=[g^+/m^J]\chi_{\lambda_1\cdots\lambda_J}(p+q)\epsilon_{\lambda_1\mu\nu\sigma}\times\rho_\sigma^+(p)p_\mu q_\nu p_{\lambda_2}\cdots p_{\lambda_J} \tag{3.5}$$

and

$$\mathcal{L}^{J-}=[g^-/m^{J-2}]\chi_{\lambda_1\cdots\lambda_J}(p+q)\rho_\sigma^+(p)p_{\lambda_2}\cdots p_{\lambda_J}\times[g_{\lambda_1\sigma}+(C/m^2)p_{\lambda_1}q_\sigma] \tag{3.6}$$

for the normal and abnormal parity cases, respectively.[13]

It is not very difficult[14] to obtain $A_{QQ}^{J\pm}$ as

$$A_{QQ}^{J+}=-(g^+)^2\left(\frac{p_{J+}}{m}\right)^{2J-2}\frac{2^J(J!)^2(J+1)}{2J(2J)!} \tag{3.7}$$

and

$$A_{QQ}^{J-}=(g^-)^2\left(\frac{p_{J-}}{m}\right)^{2J-2}\frac{2^J(J!)^2}{(2J)!}\left\{\frac{p_{J-}^2C^2}{m^2}+\frac{2E_{J-}C}{M_{J-}}+\frac{m^2}{M_{J-}^2}\left[1+\frac{m^2}{2p_{J-}^2}\left(\frac{J-1}{J}\right)\right]\right\}, \tag{3.8}$$

[13] For the special case $\mathcal{L}^{0-}$ we take $g^-C\to g(\neq0)$ as $g^-\to 0$.

[14] R. C. King, thesis, University of London (unpublished); P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, Ann. Phys. (N. Y.) 40, 100 (1966); M. D. Scadron (to be published).

where

$$4M_{J\pm}^2 p_{J\pm}^2 = [M_{J\pm}^2 - (m+\mu)^2][M_{J\pm}^2 - (m-\mu)^2] \quad (3.9)$$

and $E_{J\pm}^2 = m^2 + p_{J\pm}^2$. We can re-express Eqs. (3.8) and (3.9) in terms of the $\pi(140)$ contribution $A_{QQ}(\pi)$, and the appropriate elastic widths as

$$\frac{A_{QQ}^{J+}(\chi)}{A_{QQ}(\pi)} = -\frac{(2J+1)}{6}\left(\frac{p_\rho}{p_{J+}}\right)^3 \frac{\Gamma_{\chi\to\rho\pi}}{\Gamma_{\rho\to 2\pi}} \quad (3.10)$$

for the normal parity case, and

$$\frac{A_{QQ}^{J-}(\chi)}{A_{QQ}(\pi)} = \frac{\alpha(J,C)}{\beta(J,C)} \frac{(2J+1)}{3} \frac{p_\rho^3}{m^2 p_{J-}} \frac{\Gamma_{\chi\to\rho\pi}}{\Gamma_{\rho\to\pi\pi}} \quad (3.11)$$

for the abnormal parity case, where α and β are defined as follows.

For $J\neq 0$,

$$Jm^4\alpha(J,C) = JM_{J-}^2 p_{J-}^2 C^2 + 2JEM_{J-}m^2C + [m^4J + m^2 p_{J-}^2(J-1)], \quad (3.12a)$$

$$Jm^6\beta(J,C) = m^6(J+1) + J(Em^2 + M_{J-}p_{J-}^2C)^2$$

and for $J=0$[13]

$$\alpha(0)/\beta(0) = m^2/p_{0-}^2. \quad (3.12b)$$

We note (for $J\neq 0$) that

$$0 \leq |\alpha(J,C)/\beta(J,C)| \leq m^2/p_{J-}^2, \quad (3.13)$$

if $(2J+1)m^4 \geq (J-1)p_{J-}^4$, and

$$m^2/p_{J-}^2 \leq |\alpha(J,C)/\beta(J,C)| < (p_{J-}^4(J-1) - m^4J)/p_{J-}^2 m^2(J+1), \quad (3.14)$$

if $p_{J-}^4(J-1) > (2J+1)m^4$.

In the absence of isospin quintuplets, $\mathrm{Im}A_{QQ}^{(1)}$ is the direct sum of all the $\mathrm{Im}A_{QQ}(\chi_{J\pm})$.[15]

Let us consider the (0,1) supermultiplet. The members of it that can couple to $\rho\pi$ are A_1, A_2, H, and H'. Using the masses given in Table I, $\Gamma_{\rho\to 2\pi} = 150\pm 10$ MeV, $\Gamma_{A_2\to\rho\pi} = 75\pm 8$ MeV, and $\Gamma_{A_1\to\rho\pi} = 130\pm 40$ MeV, we obtain

$$A_{QQ}^{2+}(A_2)/A_{QQ}(\pi) = 0.25\pm 0.03 \quad (3.15)$$

and

$$\frac{A_{QQ}^{1-}(A_1)}{A_{QQ}(\pi)} = (0.31\pm 0.05)\left(\frac{3.60+10.5C_A+0.70C_A^2}{11.2+1.05C_A+0.070C_A^2}\right) \lesssim 3.1\pm 0.05 \quad (3.16)$$

from (3.13). Without knowledge of decay correlations we cannot estimate the value of C_A directly. We have even less idea of the contribution from H since we have no knowledge of partial widths, and Eqs. (3.11) and (3.13) give

$$\frac{A_{QQ}^{1-}(H)}{A_{QQ}(\pi)} = \left(\frac{15.5+35.6C_H+7.3C_H^2}{110+4.4C_H+0.14C_H^2}\right)\frac{\Gamma_{H\to\rho\pi}}{\Gamma_{\rho\to\pi\pi}} \lesssim 52\frac{\Gamma_{H\to\rho\pi}}{\Gamma_{\rho\to\pi\pi}}. \quad (3.17)$$

We have no reason to assume that the A_1, H, H' contributions are significantly smaller than the π contribution, and indirect methods suggest that the contributions are of the same order of magnitude.[16,17]

Let us now attempt to calculate the contributions from the (0,2) and (1,0) supermultiplets. Of the isospin triplets, only the $\pi_A(1640)$ and the $\pi'(J^{PC}=2^{-+})$ couple to the $\rho\pi$. Taking $\Gamma_{\pi_A\to\rho\pi}<40$ MeV, Eq. (3.10) gives

$$\frac{A_{QQ}^{0-}(\pi(1640))}{A_{QQ}(\pi)} = 0.057\frac{\Gamma_{\pi_A\to\rho\pi}}{\Gamma_{\rho\to\pi\pi}} < 0.015. \quad (3.18)$$

Taking the π' mass as 1700 MeV gives [from (3.11) and (3.13)]

$$\frac{A_{QQ}^{2-}(\pi')}{A_{QQ}(\pi)} \lesssim 0.25\frac{\Gamma_{\pi'\to\rho\pi}}{\Gamma_{\rho\to\pi\pi}}. \quad (3.19)$$

Even with the very crude assumption that $\Gamma_{\pi'\to\rho\pi} \lesssim \Gamma_{R_i}^{\mathrm{tot}} \approx 30$ MeV ($i=1$, 2, or 3), Eq. (3.19) gives

$$\frac{A_{QQ}^{2-}(\pi')}{A_{QQ}(\pi)} \lesssim 0.07. \quad (3.20)$$

All the other resonances [from the (1.0 and (0.2) supermultiplets] contributing to the $\rho\pi$ sum rule will be isospin singlets. Taking the singlet masses as roughly degenerate with the triplet masses in the classification (b) of Sec. 2 gives

$$\frac{A_{QQ}^{3+}(\eta(\eta')(1750))}{A_{QQ}(\pi)} \cong -0.15\frac{\Gamma_{\eta(\eta')\to\rho\pi}}{\Gamma_{\rho\to\pi\pi}} \quad (3.21)$$

and

$$\frac{A_{QQ}^{1+}(\eta(\eta')(1630))}{A_{QQ}(\pi)} \cong -0.09\frac{\Gamma_{\eta(\eta')\to\rho\pi}}{\Gamma_{\rho\to\pi\pi}} \quad (3.22)$$

[15] Taking couplings $i\epsilon_{ijk}\chi^i\rho^j\pi^k$ and $\chi\rho^i\pi^i$ for χ triplet and singlet, respectively.

[16] Omitting the H H' contributions, Frampton and Taylor (Ref. 6) have shown that if the matrix elements of the current commutators that are equivalent to Eq. (3.2) are saturated with π, ω, A_1, A_2 then $C_A \cong 10$. If we include the H H' contributions we get the weaker result C_A, C_H, C_H, are not simultaneously approximately zero.

[17] The collinear group of $U(6)\times U(6)\times O_L(3)$ is $U(6)_W\times O(2)_W$. The $U(6)_W\times O(2)_W$ predictions (G. Costa *et al.*, Ref. 18) are $C_H = C_{H'} = \infty$ (see Ref. 13) and $C_A = m^2/2p^2 \approx 1$, where p is the center-of-mass momentum for $1^+ \to 1^- + 0^-$ decay. Although the comparative predictions between the different $SU(3)$ multiplets are often in strong disagreement with experiment it only needs a small s-wave symmetry-breaking term to eliminate the worst discrepancies.

for the (0,2) supermultiplet [the 2^{--} isosinglet give a contribution like (3.19)]. The $J^{PC}=1^{--}$ isosinglets of the (1,0) supermultiplet give a contribution like the $J^{PC}=1^{--}$ isosinglets of the (0,2) supermultiplet [Eq. (3.22)]. Equations (3.18) to (3.22) show that the contributions from the (1,0) and (0,2) supermultiplets are depressed by at least an order of magnitude relative to the $\pi(140)$ contribution.

The contributions from higher mass resonances belonging to supermultiplets with $n'>1$ or $L>2$ are even more strongly depressed. For example, for normal parity resonances of mass M_{J+} (in units of BeV) with $M_{J+}^2 \gg m^2$

$$\frac{A_{QQ}{}^{J+}(\chi)}{A_{QQ}(\pi)} \cong -\frac{(2J+1)}{30M_{J+}{}^3}\frac{\Gamma_{\chi\to\rho\pi}}{\Gamma_{\rho\to\pi\pi}}. \tag{3.23}$$

Assuming approximate constancy of the partial widths this M^{-3} behavior enables us to completely neglect the contributions of resonances with mass $m>2000$ MeV. Provided C is not approximately zero, we see from Eqs. (3.11) and (3.12a) that we have similar M^{-3} behavior for the other parity case.

Summing up this section, we have seen that the contributions to the sum rule (2.3) of *individual* resonances belonging to the (0,0) and (0,1) supermultiplets are of the same order of magnitude, whereas individual contributions from the (0,2) and (1,0) multiplets are depressed by at least an order of magnitude (and contributions from higher multiplets negligible).

As to the question of cancellation within supermultiplets the position is not clear, except that if the $U(6)_W \times O(2)_W$ predictions for the (0,1) supermultiplet are inserted in (2.3)[18] [in the limit of $(6,\bar{6},3)$ mass degeneracy], it is found that the (0,1) supermultiplet alone does *not* satisfy the sum rule.

4. SUPERCONVERGENCE RELATIONS FOR $2^{+}+0^{-}\to 2^{+}+0^{-}$ ELASTIC SCATTERING

The case of elastic $\rho\pi$ scattering is the simplest for which superconvergence sum rules exist. For elastic scattering of mesons with spins greater than unity the greater number of independent coupling constants makes it very much more difficult to assess the contribution to a sum rule of a resonance of arbitrary spin and parity. However, the last section suggests that only a few supermultiplets will contribute and the $\rho\pi$ amplitudes give some information on the form of the other meson scattering amplitudes.

Let us consider the elastic scattering of the $J^{PC}=2^{++}$ nonet with the pseudoscalar nonet, using the notation of the previous section. We write the scattering amplitude as

$$T(\nu,t)=\epsilon_{\mu'\nu'}(p')M_{\mu'\nu'\mu\nu}\epsilon_{\mu\nu}(p), \tag{4.1}$$

where

$$\begin{aligned}M_{\mu'\nu'\mu\nu} &= A_1Q_{\mu'}Q_{\nu'}Q_\mu Q_\nu + A_2(Q_{\mu'}Q_{\nu'}Q_\mu P_\nu + Q_{\mu'}P_{\nu'}Q_\mu Q_\nu)\\ &\quad + A_3Q_{\mu'}P_{\nu'}Q_\mu P_\nu + A_4(P_{\mu'}P_{\nu'}P_\mu Q_\nu + P_{\mu'}Q_{\nu'}P_\mu P_\nu)\\ &\quad + A_5P_{\mu'}P_{\nu'}P_\mu P_\nu + g_{\mu'\mu}[A_6g_{\nu'\nu} + A_7Q_{\nu'}Q_\nu\\ &\qquad + A_8(P_{\nu'}Q_\nu + Q_{\nu'}P_\nu) + A_9P_{\nu'}P_\nu].\end{aligned} \tag{4.2}$$

The amplitude proportional to $(P_{\mu'}P_{\nu'}Q_\mu Q_\nu + Q_{\mu'}Q_{\nu'}P_\mu P_\nu)$ has been excluded without introducing kinematic singularities since it can be expressed in terms of the others via the equivalence relation

$$\begin{aligned}m^2(P_{\mu'}P_{\nu'}Q_\mu Q_\nu &+ Q_{\mu'}Q_{\nu'}P_\mu P_\nu)\\ &= \tfrac{1}{4}t[(m^2-\tfrac{1}{4}t)(\mu^2-\tfrac{1}{4}t)-\nu^2]g_{\mu'\mu}g_{\nu'\nu}\\ &\quad + 2[(m^2-\tfrac{1}{2}t)(\mu^2-\tfrac{1}{4}t)-\nu^2]g_{\mu'\mu}P_{\nu'}P_\nu\\ &\quad - \tfrac{1}{2}t(m^2-\tfrac{1}{4}t)g_{\mu'\mu}Q_{\nu'}Q_\nu + \tfrac{1}{2}\nu t g_{\mu'\mu}(P_{\nu'}Q_\nu + Q_{\nu'}P_\nu)\\ &\quad - 4(\mu^2-\tfrac{1}{4}t)P_{\mu'}P_{\nu'}P_\mu P_\nu - 2(m^2-\tfrac{1}{2}t)P_{\mu'}Q_{\nu'}P_\mu Q_\nu\\ &\qquad + 2\nu(P_{\mu'}P_{\nu'}Q_\mu P_\nu + Q_{\mu'}P_{\nu'}P_\mu P_\nu).\end{aligned} \tag{4.3}$$

If we make the usual assumptions that the asymptotic behavior of the A_i is determined by Regge poles and branch points[1–3] [with $\alpha_0(0)=1$, $1>\alpha_1(0)$, $\alpha_2(0)>0$, where $\alpha_1(0)$ is the pole or branch point intercept for isospin I is the cross channel], we derive the nine sum rules

$$\int \nu^{S_I}\,\mathrm{Im}A_r{}^{(I)}(\nu,0)d\nu=0, \tag{4.4}$$

where

$$\begin{array}{lll} \text{(a) } r=1: & I=0, & S_I=1\\ & I=1, & S_I=0,2\\ & I=2, & S_I=1;\\ \text{(b) } r=2: & I=0, & S_I=0\\ & I=1, & S_I=1\\ & I=2, & S_I=0;\\ \text{(c) } r=3,7: & I=1, & S_I=0. \end{array} \tag{4.5}$$

We define $A_i{}^{J\pm}(\chi)$ in analogy with Eq. (3.4) for $I=1, 2, 3, 7$ as

$$\mathrm{Im}A_i{}^{(J)}(\nu,0) =\sum_{J\pm} A_i{}^{J\pm}(\chi)\tfrac{1}{2}[\delta(\nu-\nu_{J\pm})\pm\delta(\nu+\nu_{J\pm})] \tag{4.6}$$

taking upper and lower signs according as $\mathrm{Im}A_i{}^{(I)}(\nu,0)$ is even or odd in ν.

For the spin-J fields χ coupling to the $J^{PC}=2^{++}$ field F and π (omitting isospin labels for convenience), the effective interaction Lagrangians are

$$\begin{aligned}\mathfrak{L}^{J+} &= [g^+/m^{J-1}]\chi_{\lambda_1\cdots\lambda_J}(p+q)\\ &\quad\times\epsilon_{\lambda_2\mu\nu\sigma}F_{\sigma\rho}{}^+(p)p_\mu q_\nu p_{\lambda_3}\cdots p_{\lambda_J}\\ &\qquad\times(p_{\lambda_1}q_\rho/m^2+Dg_{\lambda_1\rho_1})\end{aligned} \tag{4.7}$$

[18] G. Costa, M. Tonin, and G. Sartori, Nuovo Cimento **39**, 352 (1965). This gives

$$A_{QQ}(A_2):A_{QQ}(A_1):A_{QQ}(H)+A_{QQ}(H') = -4p^2m^2:p^2(M^2+4EM+4p^2):8p^2m^2.$$

and

$$\mathcal{L}^{J-}=\frac{g^{-}}{(m)^{J-2}}\chi_{\lambda_1\cdots\lambda_J}(p+q)F_{\mu\nu}{}^{+}(p)p_{\lambda_3}\cdots p_{\lambda_J}$$
$$\times[(g_{\lambda_1\nu}+Cp_{\lambda_1}q_\nu/m^2)p_{\lambda_2}q_\nu+m^2C'g_{\lambda_1\nu}g_{\lambda_2\mu}] \quad (4.8)$$

for the normal and abnormal parity cases, respectively.

We note immediately that for the special case $D=C'=0$ the $A_i{}^{J\pm}(\chi)$ of Eq. (4.6) are numerically equal to linear combinations of the $A_{QQ}{}^{J\pm}$, $A_{PQ}{}^{J\pm}$, $\cdots$, etc. defined by Eq. (3.4) and similar equations for the remaining three amplitudes. For example,

$$A_1{}^{J+}=-g^{+2}\left(\frac{P_{J+}}{m}\right)^{2J-2}\frac{2^J(J!)^2(J+1)}{2J(2J)!}, \quad (4.9)$$

if $D=0$. (Note that m is now the mass of the 2^+ field.)

Now for resonances in the high-energy region with $M^2\gg m^2$, it is the terms proportional to C' and D that are relatively suppressed. Thus, for example, from (3.7) and (3.8) we see that (assuming a constancy of elastic widths) $A_1{}^{J\pm}$ has asymptotic behavior of M^{-7}. In the 1500–2000 MeV range we do not expect the contributions from the resonances of the (0,2) and (1,0) supermultiplets to be depressed as strongly as they were for the $\rho\pi$ case, but for the moment we assume that to a good approximation we can neglect them.

That the contributions of the mesons in the (0,1) supermultiplet cannot be neglected in general (and do *not* cancel) can be seen in the following way.

Consider the case of $f\pi$ elastic scattering. All resonances in the s and u channels must have $I=1$, $G=-1$. Thus only the $\pi(140)$ of the (0,0) supermultiplet can contribute. Since the $f\rightarrow\pi\pi$ coupling is certainly nonzero, the A_1, A_2, H, H' must give a large contribution to the $f\pi$ sum rules ($I=0$ in the cross channel). These contributions cannot be calculated, since we have no information on $2^+\rightarrow 2^++0^-$ coupling constants.

TABLE II. ν values for scattering processes.

Scattering process	Intermediate state χ		ν_χ (BeV^2)
$f'+K\rightarrow f'+K$	$K(495)$	$L=n'=0$	-1.13
	$K^*(890)$		-0.68
	$[K'(1230)]$	$L=1$, $n'=0$	-0.52
	$K_A(1320)$		-0.39
	$K_V(1420)$		-0.27
	$K_A(1800)$	$L>1$ or $n'>0$	0.38
$K_V+\pi\rightarrow K_V+\pi$	$K(495)$	$L=n'=0$	-0.88
	$K^*(890)$		-0.60
	$[K'(1230)]$	$L=1$, $n'=0$	-0.23
	$K(1320)$		-0.13
	$K(1420)$		-0.01
	$K_A(1800)$	$L>1$ or $n'>0$	0.60
$A_2+\pi\rightarrow A_2+\pi$	$\eta(550)$	$L=n'=0$	-0.71
	$\eta'(960)$		-0.39
	$\rho(760)$		-0.56
	$B(1210)$	$L=1$, $n'=0$	-0.13
	$f(1250)$		-0.07
	$D(1285)$		-0.03
	$E(1420)$		$+0.15$
	$f'(1500)$		$+0.29$
	$g(1650)$	$L>1$ or $n'>0$	0.48
	$R_1(1630)$		0.45
	$R_2(1700)$		0.60
	$R_3(1759)$		0.67

We now show that for some sum rules the contribution from the (0,1) supermultiplet is suppressed for kinematic reasons, and to a good approximation can be ignored. In Table II we exhibit the ν values of the intermediate states belonging to the (0,0) and (0,1) supermultiplets for those scattering processes of physical interest for which the $J^{PC}=2^{++}$ meson decays into both 0^-+1^- and 0^-+0^- final states are kinematically allowed. We notice that for a given process, with one or two exceptions we have

$$\left|\frac{\nu_{\chi'}(0,1)}{\nu_\chi(0,0)}\right|\lesssim\tfrac{1}{3}, \quad (4.10)$$

where χ, χ' are fields belonging to the (0,0) and (0,1) supermultiplets, respectively.

For $K_V(1420)\pi$ scattering the inequality (4.10) is fairly well satisfied. For $A_2\pi$ scattering the inequality is not so well satisfied, since the f' in particular has an exceptionally large ν value for the (0,1) supermultiplet. However, considering the f' as a member of an $SU(3)$ nonet, the absence of any significant $f'\rightarrow\pi\pi$ mode requires that f' transform predominantly like $T_3{}^3$.[19] To the extent that f' is pure $T_3{}^3$ it does not couple to $A_2\pi$. The suppression of the $f'A_2\pi$ coupling constant makes the violation of (4.10) not as bad as it superficially appears. For $f'K$ scattering (4.10) is roughly satisfied. It will be shown that for this latter case the validity of (4.10) happens to be irrelevant.

It follows that for any scattering process for which (4.10) is valid the contribution from the (0,1) supermultiplet to moment sum rules will be depressed (by a degree proportional to the order of the moment). It is not likely that the (0,1) supermultiplet can be neglected in the first moment sum rules, but for second moment and higher the suppression will be by at least an order of magnitude.

In the sum rules (4.5) there is only one sum rule containing second or higher moments. This is

$$\int\nu^2\,\mathrm{Im}A_1{}^{(1)}(\nu,0)d\nu=0. \quad (4.11)$$

[This is consistent with the $f\pi$ scattering dilemma, for which the sum rule (4.11) does not exist.]

Inserting the (0,0) supermultiplet alone in the sum rule (4.11), we obtain the following results:

(a) $K_V+\pi\rightarrow K_V+\pi$:

$$\nu_K{}^2g_{K_V{}^2K\pi}-\nu_{K^*}{}^2g_{K_VK^*\pi}{}^2=0, \quad (4.12)$$

[19] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965); R. J. Rivers, Phys. Rev. **150**, 1256 (1966).

where $g_{K_V K\pi}=g^+$ and $g_{K_V K^*\pi}=g^- C$ ($g^-=0$, see Ref. 13) in the terminology of Eqs. (4.7) and (4.8).

In the limit of $U(6)\times U(6)\times O(3)$ mass degeneracy ($\nu_K=\nu_{K^*}$), we reproduce the $U(6)_W$ predictions[20] [in which the 2^{++} mesons are assigned to the $(15,\overline{15})$ $U(6)\times U(6)$ multiplet, corresponding to two quarks and two antiquarks in a relative s wave] and not the $U(6)_W\times O(2)_W$ prediction[18] in which $g_{K_V K\pi}=2g_{K_V K^*\pi}$.

In fact both sets of predictions are strongly in disagreement with experiment. For example, taking $\nu_K=\nu_{K^*}$ in (4.12) and taking physical masses in the phase space gives

$$\Gamma_{K_V\to K^*\pi}/\Gamma_{K_V\to K\pi}=0.20\,, \tag{4.13}$$

in comparison[21] to the experimental ratio

$$\Gamma_{K_V\to K^*\pi}:\Gamma_{K_V\to K\pi}=(36\pm6):(52\pm5)\,. \tag{4.14}$$

Inserting the physical ν values in Eq. (4.12) improves the prediction, giving

$$\Gamma_{K_V\to K^*\pi}/\Gamma_{K_V\to K\pi}=0.44\,, \tag{4.15}$$

in much better agreement with experiment.

(b) $A_2+\pi\to A_2+\pi$:

$$\nu_\rho^2 g_{A_2\rho\pi}^2-\nu_\eta^2 g_{A_2\eta\pi}^2-\nu_{\eta'}^2 g_{A_2\eta'\pi}^2=0\,. \tag{4.16}$$

In the limit of $U(6)\times U(6)\times O_L(3)$ mass degeneracy ($\nu_\rho=\nu_\eta=\nu_{\eta'}$), we again reproduce the $U(6)_W$ results [and not the $U(6)_W\times O(2)_W$ prediction]. In this limit, Eq. (4.16) becomes

$$0.25\Gamma_{A_2\to\eta\pi}+6.25\Gamma_{A_2\to\eta'\pi}=0.18\Gamma_{A_2\to\rho\pi}\,, \tag{4.17}$$

giving $\Gamma_{A_2\to\eta'\pi}=2.1\pm0.2$ MeV, in reasonable agreement with the experimental value $\Gamma_{A_2\to\eta'\pi}<1.5$ MeV.

Inserting physical ν values in (4.16) gives $\Gamma_{A_2\to\eta'\pi}=4.2\pm0.6$ MeV, in rather worse agreement. The fact that the agreement with experiment is poorer for $A_2\pi$ scattering is probably due to the fact that (4.10) is less well satisfied, even when allowances have been made.

Since the f' is an isosinglet, we can make no predictions for $f'K$ scattering.

Let us finally attempt briefly to calculate the contributions of the $K_A(1800)$, the $g(1650)$, and the R_1, R_2, R_3 to the sum rule to see whether our neglect of the higher supermultiplets is justified.

In $K_V\pi$ scattering, $\nu_{K^*}\cong\nu_{K_A}$, so that the relative contribution of the K_A is given by

$$\frac{A_1^{0-}(K_A(1800))}{A_1(K^*(890))}=-0.3\left(\frac{m_{K_V}^2}{m_{K_A}^2}\right)\left(\frac{p_{K^*}}{p_{K_V}}\right)^5\frac{\Gamma_{K_A\to K_V\pi}}{\Gamma_{K_A\to K^*\pi}}=-0.13\pm0.08 \tag{4.18}$$

(taking $\Gamma_{K_A\to K_V\pi}=6.4\pm4.0$ MeV) if the $K_A(1800)$ has $J^P=0^-$.

In the $A_2\pi$ sum rule the $g(1650)$ gives a contribution relative to the $\rho(760)$ of

$$\frac{\nu_g^2 A_1^{1+}(g)}{\nu_\rho^2 A_1(\rho)}=0.6\frac{\nu_g^2}{\nu_\rho^2}\left(\frac{m_{A_2}^2}{m_g^2}\right)\frac{\Gamma_{g\to A_2\pi}}{\Gamma_{A_2\to\rho\pi}}\cong\frac{\Gamma_{g\to A_2\pi}}{44\text{ MeV}}\,. \tag{4.19}$$

Similarly,

$$\frac{\nu_{R_1}^2 A_1^{1+}(R_1)}{\nu_\rho^2 A_1(\rho)}\cong\frac{\Gamma_{R_1\to A_2\pi}}{44\text{ MeV}}\,. \tag{4.20}$$

For the R_2 (or at least the conjectured $J^{PC}=2^{--}$ isotriplet part of R_2), it is difficult to say anything, since the three independent coupling constants only enable a crude upper bound to be made. Inspection shows that the magnitude of the R_2 contribution is greatest for large C in Lagrangian (4.8) [as in the $\rho\pi$ case] and smallest for large C'. For $C'\approx0$, Eqs. (3.11) and (3.13) give

$$\left|\frac{\nu_{R_2}^2 A_1^{2-}(R_2)}{\nu_\rho^2 A_1(\rho)}\right|\lesssim\frac{\Gamma_{R_2\to A_2\pi}}{35\text{ MeV}}\,, \tag{4.21}$$

the maximum being attained for $C\to\infty$. We get a similar contribution to (4.20) and (4.21) for the R_3 contributions. In view of the small R_1, R_2, R_3 widths and the variety of decay modes the ratios (4.20) and (4.21) are very likely to be considerably less than unity, and it is plausible that the ratio (4.19) should be also.

The contributions of resonances (like S, T, U) belonging to higher supermultiplets are depressed very much more and can reasonably be neglected completely.

Thus after the possibility of partial cancellation has been taken into account, the assumption that the (1,0) and (0,2) supermultiplets can be neglected seems a fair approximation, and the assumption that all higher supermultiplets can be neglected a good one.

5. CONCLUSIONS

We have seen that the known meson resonances can be conveniently classified according to representations of $U(6)\times U(6)\times O(3)$ and a reasonable spectrum obtained by adopting a simple nonrelativistic quark model with suitable potentials.

Using this spectrum, we have examined the contributions of the mesons with masses $m\lesssim2000$ MeV to the $\rho\pi$ superconvergent sum rule. Two points emerge. Firstly, the resonances within the $L=0$, $n'=0$ and $L=1$, $n'=0$ supermultiplets give contributions to the sum rule of the same order of magnitude, whereas the contributions of the resonances within the higher supermultiplets are depressed by at least an order of magnitude. Secondly, although the $L=0$, $n'=0$ supermultiplet, if constrained to satisfy the sum rule alone, gives predictions consistent with $U(6)_W\times O(2)_W$, this is not the case for the $L=1$, $n'=0$ supermultiplet. For very heavy

[20] R. Delbourgo, M. A. Rashid, and J. Strathdee, Phys. Letters 14, 719 (1965).

[21] $U(6)_W\times O(2)_W$ gives the even worse result of

$$\Gamma_{K_V\to K^*\pi}/\Gamma_{K_V\to K\pi}=0.05.$$

An s-wave symmetry breaking term will have little effect on this ratio.

mesons of mass M, $M^2 \gg m_\rho^2$, the contributions to the sum rule are seen to be of the order M^{-3}, and can be completely neglected.

Because the contributions of neither the (0,0) or (0,1) supermultiplets can be ignored in meson-superconvergence sum rules we can only make predictions when one of the supermultiplets has its contributions depressed for reasons of kinematics. The scattering of $J^{PC}=2^{++}$ mesons with pseudoscalar mesons provides such a case because the low ν values of the resonances belonging to the (0,1) supermultiplet cause their contributions to be depressed in moment sum rules.

Predictions have been made which, in the limit of $U(6)\times U(6)\times O_L(3)$ mass degeneracy, are in agreement with the predictions of a $U(6)\times U(6)$ meson classification, rather than a classification by $U(6)\times U(6)\times O_L(3)$. In practice such higher-symmetry predictions are in strong disagreement with experiment. For the prediction where the kinematic suppression of the (0,1) supermultiplet is the stronger, the insertion of physical masses in the sum rule gives a prediction in much better agreement with experiment.

The assumption that the higher supermultiplets can be neglected for these sum rules has been examined insofar as there is experimental evidence and found to be reasonable.

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Errata

Bound States of a Relativistic Two-Body Hamiltonian: Comparison with the Bethe-Salpeter Equation, NGUYEN D. SON† AND J. SUCHER [Phys. Rev. **153**, 1496 (1967)]. There are typographical omissions in the expression for $V_0(k,k')$ after Eq. (12); it should read

$$V_0(k,k')=\frac{-g^2}{(2\pi)^3}\frac{1}{2kk'}\frac{(k+k')^2+\mu^2}{(k-k')^2+\mu^2}\times\frac{m}{E(k)}\times\frac{m}{E(k')}.$$

† Deceased.

Radiative ρ-Meson Decay, P. SINGER [Phys. Rev. **130**, 2441 (1963)].

(1) Eq. (12) should read

$$I(k)=k_m\left(\frac{m_\rho}{2}-\frac{m_\pi^2}{m_\rho}-k\right)\ln\frac{1+\xi}{1-\xi}-\xi\left[k_m\left(\frac{m_\rho}{2}-k\right)-k^2\right].$$

(2) The figures in Table I should read:

k (MeV)	15	30	45	60	105	165	225	285
R_k	2.3×10^{-2}	1.6×10^{-2}	1.2×10^{-2}	9.7×10^{-3}	5.3×10^{-3}	2.6×10^{-3}	1.1×10^{-3}	3.3×10^{-4}
R_k'	8.3×10^{-3}	5.1×10^{-3}	3.7×10^{-3}	3.0×10^{-3}	1.6×10^{-3}	7.4×10^{-4}	3.1×10^{-4}	8.7×10^{-5}

Figure 2 should also be appropriately corrected. The revised numbers are slightly higher than the original ones. The correction does not alter any of the conclusions of the paper and is given only to assure an error-free comparison with experiment.

(3) The heading of the fourth column of Table II should read

$$\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma)/\Gamma_{\rho^{+,-}}(2\pi).$$

I am grateful to M. Sapir for discovering the error in Eq. (12) and for re-evaluating Table I.