Superconvergent Relations in Pion Photoproduction*

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We derive seven sum rules for pion photoproduction in the context of $SU(3)$ by assuming that the Regge trajectories $\alpha_{10}(0)$, $\alpha_{10^*}(0)$, $\alpha_{27}(0)<0$. We examine them for approximate saturation with N, $N^*(1236)$, and $N^{**}(1518)$.

I. INTRODUCTION

RECENTLY, Soloviev and de Alfaro *et al.*[1] have derived a class of superconvergent sum rules for strong-interaction scattering amplitudes on the basis of analyticity and reasonable arguments about high-energy behavior. Briefly the argument runs as follows. Suppose that an analytic function $f(\nu)$ satisfies a dispersion relation

$$f(\nu)=\frac{1}{\pi}\int_{-\infty}^{\infty} d\nu'\frac{\operatorname{Im}f(\nu')}{\nu'-\nu} \tag{1}$$

and is subject to the bound

$$|f(\nu)|<\nu^{\beta},\quad (\beta<-1)\quad \text{for}\quad \nu\rightarrow\infty\,. \tag{2}$$

Then it must satisfy the so-called "superconvergence" relation

$$\int_{-\infty}^{\infty} d\nu\, \operatorname{Im}f(\nu)=0\,. \tag{3}$$

The possibility that some of the strong-interaction scattering amplitudes may satisfy relations of the type (3) has attracted considerable attention.[2] In order to verify the relations and the assumptions involved, several authors[3] have considered the case of meson-baryon scattering and analyzed the one superconvergent sum rule assuming the high-energy behavior given by the Regge-pole model. It is important to provide further confirmation of these ideas by investigating superconvergence in other processes as well. We have analyzed the superconvergence relations for the process of photoproduction of mesons from baryons, a brief account of which was given earlier.[4] Here we present the details and extension of this work. We obtain seven nontrivial sum rules by assuming that the Regge trajectories $\alpha_i(t)$ have $\alpha_{27}(0)$, $\alpha_{10}(0)$, $\alpha_{10^*}(0)<0$. This assumption about $\alpha_i(t)$ is motivated by the observation that there is no experimental evidence for the existence of any low-lying mesons with $I=\frac{3}{2}$ or 2. If the sum rules are valid, we may regard this as strong evidence for the correctness of our assumption. Our results are summarized below.

(1) The sum rule due to **27** exchange holds reasonably well, which may imply that the assumption $\alpha_{27}(0)<0$ is correct and the sum rule is useful.

(2) In the approximation of keeping N, $N^*(1236)$, and $N^{**}(1518)$, the sum rules due to **10** and **10*** exchange are unsaturated and inconsistent. This casts doubt on the validity of the assumption $\alpha_{10}(0)$, $\alpha_{10^*}(0)<0$.

We present the sum rules in Sec. II and discuss their numerical evaluation in Sec. III. The Appendix lists contributions of N, N^*, and N^{**} to the various amplitudes.

II. SUM RULES

Let k, q, p_1, and p_2 be the 4-momenta of the photon, the meson, the initial baryon, and the final baryon, respectively.

We decompose the T matrix in terms of the four invariant amplitudes A, B, C, and D.[5] They are functions of the invariants

$$\nu=-(p_1+p_2)\cdot k/2M=-(p_1+p_2)\cdot q/2M$$

and $t=-(p_1-p_2)^2$, where M is the baryon mass.

To see if $A,\cdots,D$ satisfy superconvergence relations, we must know their behavior as $\nu\rightarrow\infty$. We assume that this behavior is given by the Regge-pole model. We may now apply the standard technique of Reggeization[6] to our amplitudes. Accordingly, we write the partial wave decomposition in the t channel, viz., $\gamma+\bar{\pi}\rightarrow N+\bar{N}$. We have, in the notation of Ball,[7] the

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[1] L. D. Soloviev, JINR Report No. E-2343, Dubna, 1965 (unpublished); V. de Alfaro, S. Fubini, G. Furlan, and G. Rossetti, Phys. Letters **21**, 576 (1966).

[2] J. Bronzan, I. Gerstein, B. Lee, and F. Low. Phys. Rev. Letters **18**, 32 (1967); V. Singh, *ibid.* **18**, 39 (1967); R. Oehme Phys. Rev. **154**, 1358 (1967); M. Kugler, Phys. Rev. Letters **17**, 1166 (1966); H. Goldberg, Phys. Letters **24B**, 71 (1967); D. Amati and R. Jengo, *ibid.* **24B**, 108 (1967); H. Harari, Phys. Rev. Letters **17**, 1303 (1966); **18**, 319 (1967); H. Pagels, *ibid.* **18**, 316 (1967).

[3] B. Sakita and K. C. Wali, Phys. Rev. Letters **18**, 29 (1967); P. Babu, F. J. Gilman, and M. Suzuki, Phys. Letters **24B**, 65 (1967); G. Altarelli, F. Buccella, and R. Gatto, *ibid.* **24B**, 57 (1967).

[4] M. S. K. Razmi and Y. Ueda, Phys. Rev. Letters **18**, 719 (1967). Unfortunately, this paper contains an error about crossing properties of the amplitudes $A,\cdots,D$, which is corrected in the erratum [*ibid.* **18**, 938 (1967)] and in the present paper.

[5] G. F. Chew, M. Goldberger, F. Low, and Y. Nambu (CGLN), Phys. Rev. **106**, 1345 (1957). We follow the notations of CGLN in this paper.

[6] S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962); G. Zweig, Nuovo Cimento **32**, 689 (1964); see also Ref. 7.

[7] J. S. Ball, Phys. Rev. **124**, 2014 (1961). Some typographic errors in the expressions for G_i in Ball's paper are corrected here.

following angular-momentum decomposition:

$$G_1=-\sum(J+\tfrac{1}{2})\beta_J{}^-P_J{}'(x'),$$
$$G_2=-\sum\{\tfrac{1}{2}\alpha_J{}^-[JP_{J+1}{}''(x')+(J+1)P_{J-1}{}''(x')]-(J+\tfrac{1}{2})\alpha_J{}^+P_J{}''(x')\},$$
$$G_3=+\sum\{\tfrac{1}{2}\alpha_J{}^+[JP_{J+1}{}''(x')+(J+1)P_{J-1}{}''(x')]-(J+\tfrac{1}{2})\alpha_J{}^-P_J{}''(x')-(J+\tfrac{1}{2})\beta_J{}^+P_J{}'(x')\},$$
$$G_4=-\sum\{\tfrac{1}{2}\alpha_J{}^+[JP_{J+1}{}''(x')+(J+1)P_{J-1}{}''(x')]-(J+\tfrac{1}{2})\alpha_J{}^-P_J{}''(x')\}.$$

The G_i's are connected to A, $\cdots$, D by the following relations:

$$G_1=-(1/8\pi)(k'p'/E')[A+Bt],$$
$$G_2=-(1/4\pi)k'p'C,$$
$$G_3=((E'-M)/8\pi E')k'(A+D\sqrt{t}),$$
$$G_4=(k'/16\pi E')(2MA-tD),$$

where

$$k'=(1/2\sqrt{t})(t-m_\pi{}^2),\quad E'=\tfrac{1}{2}\sqrt{t},$$
$$p'=\tfrac{1}{2}(t-4M^2)^{1/2},$$

and x' is the cosine of the scattering angle in the t channel.

Since $P_J(x')$ goes like $(x')^J$ as $x'\to\infty$, we see that G_i all behave at most like $\nu^{\alpha(t)-1}$ as $\nu\to\infty$, so that the invariant amplitudes A, $\cdots$, D are also expected to behave at most like $\nu^{\alpha(t)-1}$ as $\nu\to\infty$, where $\alpha(t)$ refers to the dominant Regge trajectory in the t channel, $\gamma+\bar\pi\to N+\bar N$. Thus condition (2) is satisfied by A, $\cdots$, D, provided that $\alpha(t)<0$. Notice that Eq. (3) is nontrivially satisfied only if $\text{Im}f(-\nu)=\text{Im}f(\nu)$. Keeping this and the crossing properties of $\text{Im}A$, $\cdots$, $\text{Im}D$ in mind, we are led to seven nontrivial relations:

$$\int_0^\infty d\nu\,\text{Im}f(\nu)=0,\qquad(4)$$

where $f=A^{(10)}$, $A^{(10^*)}$, $B^{(10)}$, $B^{(10^*)}$, $C^{(27)}$, $D^{(10)}$, and $D^{(10^*)}$, $A^{(R)}$, etc., being the $SU(3)$ eigenamplitudes in the t channel.

Using the $SU(3)$ crossing matrix,[8] we arrive at the following sum rules:

10:

$$\int_0^\infty d\nu\Big[-\frac{9}{40}\text{Im}\,A^{(27)}(\nu,t)+\tfrac{1}{4}\,\text{Im}A^{(10)}(\nu,t)+\tfrac{1}{4}\,\text{Im}A^{(10^*)}(\nu,t)-\tfrac{2}{5}\,\text{Im}A^{(8ss)}(\nu,t)-(\tfrac{1}{5}\sqrt{5})\,\text{Im}A^{(8sa)}(\nu,t)+(\tfrac{1}{5}\sqrt{5})\,\text{Im}A^{(8as)}(\nu,t)+\tfrac{1}{8}\,\text{Im}A^{(1)}(\nu,t)\Big]=0,\qquad(5)$$

[8] J. J. De Swart, Nuovo Cimento **31**, 420 (1964).

10*:

$$\int_0^\infty d\nu\Big[-\frac{9}{40}\text{Im}A^{(27)}(\nu,t)+\tfrac{1}{4}\,\text{Im}A^{(10)}(\nu,t)+\tfrac{1}{4}\,\text{Im}A^{(10^*)}(\nu,t)-\tfrac{2}{5}\,\text{Im}A^{(8ss)}(\nu,t)+(\tfrac{1}{5}\sqrt{5})\,\text{Im}A^{(8sa)}(\nu,t)-(\tfrac{1}{5}\sqrt{5})\,\text{Im}A^{(8as)}(\nu,t)+\tfrac{1}{8}\,\text{Im}A^{(1)}(\nu,t)\Big]=0.\qquad(6)$$

The sum rules for B and D can be obtained by substituting B and D for A in (5) and (6).

27:

$$\int_0^\infty d\nu\Big[\frac{7}{40}\text{Im}C^{(27)}(\nu,t)-\tfrac{1}{12}\,\text{Im}C^{(10)}(\nu,t)-\tfrac{1}{12}\,\text{Im}C^{(10^*)}(\nu,t)+\tfrac{1}{5}\,\text{Im}C^{(8ss)}(\nu,t)-\tfrac{1}{3}\,\text{Im}C^{(8aa)}(\nu,t)+\tfrac{1}{8}\,\text{Im}C^{(1)}(\nu,t)\Big]=0.\qquad(7)$$

Equivalent to the relations (5) and (6) are the following sum rules:

10+10*:

$$\int_0^\infty d\nu\Big[-\frac{9}{20}\text{Im}A^{(27)}(\nu,t)+\tfrac{1}{2}\,\text{Im}A^{(10)}(\nu,t)+\tfrac{1}{2}\,\text{Im}A^{(10^*)}(\nu,t)-\tfrac{4}{5}\,\text{Im}A^{(8ss)}(\nu,t)+\tfrac{1}{4}\,\text{Im}A^{(1)}(\nu,t)\Big]=0.\qquad(8)$$

10−10*:

$$\int_0^\infty d\nu[-(\tfrac{2}{5}\sqrt{5})\,\text{Im}A^{(8sa)}(\nu,t)+(\tfrac{2}{5}\sqrt{5})\,\text{Im}A^{(8as)}(\nu,t)]=0.\qquad(9)$$

Similar relations hold true for B and D.

The amplitudes $A^{(R)}$, etc., in Eqs. (5)–(9) are the $SU(3)$ eigenamplitudes in the S channel $\gamma+N\to\pi+N$. The coefficients of $A^{(R)}$, etc., are elements of the relevant $SU(3)$ crossing matrix.[8]

We approximate the integrals by keeping $N(938)$, $N_{33}{}^*(1236)$, and $N^{**}(1518)$ only. The contributions due to higher intermediate states are expected to contribute much less to photoproduction than to scattering.[9,10]

[9] M. Gourdin and P. Salin, Nuovo Cimento **27**, 193 (1963); **27**, 309 (1963); P. Salin, *ibid.* **28**, 1294 (1963). The values of C and C_4 used in our paper are slightly different from the corresponding values given in M. Gourdin and P. Salin. This is because of our use of more recent data on the (3,3) resonance. See also Ref. 10.

[10] S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **53**, 161 (1966).

The matrix elements involving N_{33}^* are defined as follows:

$$\langle N_\gamma^*(p')|J_\pi^{(\alpha)}(0)|N_\beta(p)\rangle = \left(\frac{MM^*}{p_0'p_0}\right)^{1/2}\left(-i\sqrt{3}\frac{\lambda}{m_\pi}\right)\begin{pmatrix}8 & 8 & 10\\ \alpha & \beta & \gamma\end{pmatrix} \times(p'-p)_\lambda \bar{u}_\lambda(p')u(p) \quad (10)$$

and

$$\epsilon_\lambda\langle N_\gamma^*(p')|V_\lambda^{(\alpha)}(0)|N_\beta(p)\rangle = \left(\frac{MM^*}{p_0'p_0}\right)^{1/2}\begin{pmatrix}8 & 8 & 10\\ \alpha & \beta & \gamma\end{pmatrix} i\sqrt{3}\frac{eC}{m_\pi} \times\bar{u}_\mu(p')\gamma_\nu\gamma_5 u(p)F_{\mu\nu}(p'-p)\,, \quad (11)$$

where $F_{\mu\nu}(k)=k_\mu\epsilon_\nu-k_\nu\epsilon_\mu$, while J_π and V_λ are the pionic and electromagnetic currents, respectively. Our definitions of the coupling constants λ and C are the same as those in Gourdin and Salin.[9,10] We have $\lambda=1.81$ and $C=0.345$. To write down the corresponding matrix elements of N^{**}, we assume that it is a member of an $SU(3)$ octet. We have

$$\langle N_\gamma^{**}(p')|J_\pi^{(\alpha)}(0)|N_\beta(p)\rangle = -(MM^{**}/p_0'p_0)^{1/2}(2\lambda'/m_\pi)i \times\left[-\sqrt{3}\begin{pmatrix}8 & 8 & 8_a\\ \alpha & \beta & \gamma\end{pmatrix}f'+\left(\frac{5}{3}\right)^{1/2}\begin{pmatrix}8 & 8 & 8_s\\ \alpha & \beta & \gamma\end{pmatrix}(1-f')\right] \times\bar{u}_\lambda(p')\gamma_5 u(p)(p'-p)_\lambda \quad (12)$$

and

$$\epsilon_\lambda\langle N_\gamma^{**}(p')|V_\lambda^{(\alpha)}(0)|N_\beta(p)\rangle = (MM^{**}/p_0'p_0)^{1/2}e/m_\pi^2 \times\left[-\sqrt{3}\begin{pmatrix}8 & 8 & 8_a\\ \alpha & \beta & \gamma\end{pmatrix}D^{(f)}+\left(\frac{5}{3}\right)^{1/2}\begin{pmatrix}8 & 8 & 8_s\\ \alpha & \beta & \gamma\end{pmatrix}D^{(d)}\right] \times\bar{u}_\mu(p')p_\nu' u(p)F_{\mu\nu}(p'-p)\,, \quad (13)$$

where

$$D^{(f)}+\tfrac{1}{3}D^{(d)}=D\,.$$

Again the coupling constants are given by Gourdin and Salin[9,10] as $\lambda'=1.97$ and $D=0.0177$.

The contributions of N, $N^*(1236)$, and $N^{**}(1518)$ to various amplitudes are listed in the Appendix.

For the time being we keep in the sum rules (5)–(9) contributions from N and N^* only. We then arrive at the following relations at $t=0$.

From C with **27** exchange:

$$\tfrac{1}{3}(1-f)geF_2^{(d)}(0)-gefF_2^{(f)}(0) = \tfrac{1}{4}eC\lambda MR_3(0)/m_\pi^2\,. \quad (14a)$$

From **10**+**10*** exchange:

$$A:\quad -\tfrac{3}{2}eC\lambda R_1(0)/m_\pi^2=0\,, \quad (15a)$$

$$B:\quad -\tfrac{3}{2}eC\lambda/m_\pi^2=0\,, \quad (16a)$$

$$D:\quad eg(1-f)F_2^{(d)}(0)=(9/8)eC\lambda MR_4(0)/m_\pi^2\,. \quad (17a)$$

From **10**−**10*** exchange:

$$A:\quad -e(1-f)g=0\,, \quad (18a)$$

$$B:\quad -(2/m_\pi^2)e(1-f)g=0\,, \quad (19a)$$

$$D:\quad (1-f)/f=F_2^{(d)}(0)/F_2^{(f)}(0)\,, \quad (20a)$$

where $F_2^{(f)}(0)=\mu_p'+\frac{1}{2}\mu_n'$, $F_2^{(d)}(0)=-\frac{3}{2}\mu_n'$, and $g^2/4\pi=14.5$. The quantity $(1-f)/f$ is the D/F ratio for the $\pi N\bar{N}$ vertex. μ_p' and μ_n' are the anomalous magnetic moments of the proton and neutron, respectively, with the values $\mu_p'=1.793$ and $\mu_n'=-1.913$. The $R_i(0)$ are functions of masses only and are listed in Table II.

III. NUMERICAL ANALYSIS OF THE SUM RULES AND DISCUSSION

We now turn to the numerical evaluation of these relations. To start with, we look at the relation obtained from **27** exchange, Eq. (14a). Using the values $F_2^{(f)}(0)=0.837$ and $F_2^{(d)}(0)=2.87$, we find that $f=0.30$.

One of the relations obtained from **10**−**10*** exchange, viz., Eq. (20a), expresses universality of the strong and electromagnetic D/F ratio. We note that it is independent of the N_{33}^* contribution.

The numerical analysis of Eqs. (15a)–(20a) corresponding to **10**+**10*** and **10**−**10*** exchange gives rather inconsistent results. In particular, we find that with **10**+**10*** exchange, D gives $f=2.2$, while with **10**−**10*** exchange, A, B, and D give $f=1$, $f=1$, and $f=0.226$, respectively. Equations (15a) and (16a) imply the absurd result that $C\lambda=0$. Whereas the result due to **27** exchange is reasonable, those from Eqs. (15a)–(20a) are really inconsistent. To see if this situation can be changed significantly, we have investigated various possibilities. Firstly, we know that there exists another independent gauge-invariant $(N^*N\gamma)$ coupling, namely,

$$\epsilon_\lambda\langle N_\gamma^*(p')|V_\lambda^{(\alpha)}(0)|N_\beta(p)\rangle = \left(\frac{MM^*}{p_0'p_0}\right)^{1/2}\begin{pmatrix}8 & 8 & 10\\ \alpha & \beta & \gamma\end{pmatrix}(-2\sqrt{3})\frac{eC_4}{m_\pi^2} \times p_\mu'\bar{u}_\nu(p')\gamma_5 u(p)F_{\mu\nu}(p'-p)\,. \quad (21)$$

Since the **10**−**10*** exchange sum rules do not contain an N^* contribution, they are unaffected by the inclusion of (21). However, the sum rules due to **27** and **10**+**10*** exchange are modified as follows for general t.

27 exchange:

$$eg\left[\tfrac{1}{3}(1-f)F_2^{(d)}(0)-fF_2^{(f)}(0)\right] = \tfrac{1}{4}e\lambda(M/m_\pi^3)\left[Cm_\pi R_3(t)+2C_4R_3'(t)\right].$$

10+10* exchange:

A: $\lambda[Cm_\pi R_1(t)+2C_4R_1'(t)]=0$,

B: $\lambda[Cm_\pi+2C_4R_2'(t)]=0$,

D: $eg(1-f)F_2^{(d)}(0)=(9/8)e\lambda(M/m_\pi^3)\times[Cm_\pi R_4(t)+2C_4R_4'(t)]$,

where $R_i'(t)$ are listed in Table III.

Differentiating with respect to t and setting $t=0$, we obtain the following results:

27 exchange sum rule gives $C_4=0$. With **10+10*** exchange, A and D give, respectively,

$$C_4/C=m_\pi/2M\approx 0.075\,,$$
$$C_4=0\,.$$

These results are to be compared with the experimental value of $C_4/C\approx -0.0116$. The result $C_4=0$ may be regarded as in good agreement with the experimental number $C_4=-0.004$.[9] Using the latter value of C_4 and comparing Table III with Table II at $t=0$, we see that the contributions due to (21) are 2%, 2%, 5%, and 1% of the corresponding contributions due to (11). Thus the relations obtained at $t=0$ are not affected appreciably. Since in the following we shall discuss relations obtained at $t=0$ only, we shall ignore contributions due to this small coupling from now on.

Secondly, we may look at the contribution of the next important intermediate state, namely, N^{**}. Again the contributions are listed in the Appendix. The sum rules (14a)–(20a) now read as

27 exchange:

$$eg[\tfrac{1}{3}(1-f)F_2^{(d)}(0)-fF_2^{(f)}(0)]=\tfrac{1}{4}e\lambda CMR_3(0)/m_\pi^2+2e\lambda'[\tfrac{1}{3}(1-f')D^{(d)}-f'D^{(f)}]ML_3(0)/m_\pi^3\,. \quad (14b)$$

10+10* exchange:

$$-\tfrac{3}{2}e\lambda CR_1(0)/m_\pi^2=(8/3)e\lambda'(1-f')D^{(d)}L_1(0)/m_\pi^3\,, \quad (15b)$$

$$-\tfrac{3}{2}e\lambda C/m_\pi^2=(8/3)e\lambda'(1-f')D^{(d)}L_2(0)/m_\pi^3\,, \quad (16b)$$

$$eg(1-f)F_2^{(d)}(0)=(9/8)e\lambda CMR_4(0)/m_\pi^2+2e\lambda'(1-f')D^{(d)}ML_4(0)/m_\pi^3\,. \quad (17b)$$

10−10* exchange:

$$-eg(1-f)=e\lambda'[(1-f')D^{(f)}-f'D^{(d)}]L_1(0)/m_\pi^3\,, \quad (18b)$$

$$-2eg(1-f)/m_\pi^2=e\lambda'[(1-f')D^{(f)}-f'D^{(d)}]L_2(0)/m_\pi^3\,, \quad (19b)$$

$$\frac{1-f}{f}=\frac{F_2^{(d)}(0)}{F_2^{(f)}(0)}+\frac{2\lambda'}{gfF_2^{(f)}(0)}\times[(1-f')D^{(f)}-f'D^{(d)}]\frac{ML_4(0)}{m_\pi^3}\,. \quad (20b)$$

The $L_i(0)$ are functions of masses only and are given in the Appendix. Inconsistencies still persist. To see this, let us divide Eq. (15b) by (16b). We then have

$$R_1(0)=L_1(0)/L_2(0)\,.$$

The left-hand side of this equation is $-7.7m_\pi^2$, while the right-hand side is $4.7\,m_\pi^2$. Similarly, on dividing (18b) by (19b) we have

$$\tfrac{1}{2}=L_1(0)/m_\pi^2L_2(0)\,.$$

The right-hand side of this equation is 4.7.

Note that these inconsistent results are independent of the actual numerical values of the $(N^*N\pi)$, $(N^*N\gamma)$, $(N^{**}N\pi)$, and $(N^{**}N\gamma)$ coupling constants.

We may now consider the effect of N^{**} inclusion from Eqs. (14b) and (20b). Of course, to give any precise numbers we have to know the values of f' and $D^{(d)}$. From the analysis of photoproduction sum rules, Fubini *et al.*[10] have suggested that $D^{(d)}=0$, so that $D^{(f)}=D$. We further assume that the D/F ratio for the $(N^{**}N\pi)$ vertex is the same as that for the $(NN\pi)$ vertex, i.e., $f'=f$. This assumption about the universality of the D/F ratio for the $(N^{**}N\pi)$ and $(NN\pi)$ vertices is not inconsistent with the present experimental data analyzed on the basis of a Regge-pole model. It has certain theoretical support, too, from the work of Gatto *et al.*[3] Equation (14b) then gives $f=0.21$. Equation (20b) yields approximately the same value for f. Since the assumption of pure F coupling for the $(N^{**}N\gamma)$ vertex is not definitely established, we may try other choices and see how our results are affected. Let us then consider a mixture of F and D coupling for the $(N^{**}N\gamma)$ vertex, and assume that the D/F ratios for the $(N^{**}N\gamma)$ and $(NN\gamma)$ vertices are the same, i.e., $D^{(d)}/D^{(f)}=F_2^{(d)}(0)/F_2^{(f)}(0)$. This gives $D^{(d)}=1.6\,D$ and $D^{(f)}=0.47\,D$. Using these values, we obtain $f=0.37$ from (14b) and $f=0.236$ from (20b). Irrespective of any assumptions regarding the $SU(3)$ structure of the $(N^{**}N\gamma)$ vertex, we see that the effect of the N^{**} contribution in Eqs. (17b) and (20b) is small (<4%). In the case of Eq. (14b), the contribution of N^{**} is not so small. However, the value of f obtained is not so far away from 0.30, a number obtained without the N^{**} inclusion.

If we take the common point of view, a good sum rule may be approximated by a small number of low-lying states. Conventional[9] and sum-rule-type calculations[10,11] on photoproduction show that N and N^* give the most important contributions. Intuitively, this situation may be understood by observing that in the case of photoproduction the high intermediate states excite high l-value multipoles whose contributions are expected to go down fast with increasing l and center-of-mass energy.[10] This is to be contrasted with the case of πN

[11] A class of sum rules different from the ones in Ref. 10 has been studied by various authors: N. Cabibbo and L. Radicati, Phys. Letters **19**, 697 (1966); N. Mukunda and T. K. Radha, Nuovo Cimento **44**, 726 (1966); S. Gasiorowicz, Phys. Rev. **146**, 1067 (1966).

scattering, where the higher intermediate states add up to a sizeable fraction of N and N^* contribution. In the present case, we have explicitly seen that the inclusion of N^{**} does not change the results (14a)–(20a) in any significant way. Thus, barring any dramatic change in the situation due to the contribution of still higher intermediate states, the sum rules due to **10** and **10*** are inconsistent and unsaturated. We may, therefore, conclude that $\alpha_{10}(0)$, $\alpha_{10^*}(0)<0$ is questionable. There is, of course, the possibility that the asymptotic behavior of the amplitudes A, B, and D is not as simple as the one given by the Regge-pole model. If, for instance, there are cuts present in the complex j plane, and they play an important role, then the whole superconvergence procedure may become questionable.[12] However, this is an open question. As far as the **27** exchange sum rule is concerned, it gives a value of the parameter f within the generally accepted range. This, together with the results obtained from the superconvergent relation for meson-baryon scattering, may be regarded as indicating correctness of the assumption $\alpha_{27}(0)<0$.

Note added in manuscript: After the completion of this work we received reports by L. K. Pande [Trieste Report No. IC/67/6 (unpublished)] and M. B. Halpern [Princeton report (unpublished)], who have discussed superconvergence in pion photoproduction by considering ρ and π exchange, respectively.

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APPENDIX

We write the T-matrix element for the process $\gamma(\alpha)+N(\beta)\rightarrow\pi(\lambda)+N(\mu)$ as[8]

$$\begin{pmatrix}8 & 8\\ \lambda & \mu\end{pmatrix}\Bigg|T\Bigg|\begin{pmatrix}8 & 8\\ \alpha & \beta\end{pmatrix}=\sum_{R,\xi,\eta}\begin{pmatrix}8 & 8 & R_\xi\\ \lambda & \mu & \gamma\end{pmatrix}\begin{pmatrix}8 & 8 & R_\eta\\ \alpha & \beta & \gamma\end{pmatrix}T^{(R\xi\eta)}.$$

Contributions of N and N^* to $\mathrm{Im}T^{(R\xi\eta)}$ are listed in Tables I–III.

Contribution of N^{**} to $\mathrm{Im}A,\cdots,\mathrm{Im}D$ is given below in the matrix form:

$$\begin{bmatrix}\mathrm{Im}A^{(8\xi\eta)}\\ \mathrm{Im}B^{(8\xi\eta)}\\ \mathrm{Im}C^{(8\xi\eta)}\\ \mathrm{Im}D^{(8\xi\eta)}\end{bmatrix}=\Delta_{\xi\eta}\begin{bmatrix}L_1(t)\\ L_2\\ L_3(t)\\ L_4(t)\end{bmatrix},$$

where

$$\Delta_{ss}=(5/3)(1-f')(2e\lambda'D^{(d)}/m_\pi^3)\pi\delta(S-M^{**2}),$$
$$\Delta_{sa}=-(\surd5)(1-f')(2e\lambda'D^{(f)}/m_\pi^3)\pi\delta(S-M^{**2}),$$
$$\Delta_{as}=-(\surd5)f'(2e\lambda'D^{(d)}/m_\pi^3)\pi\delta(S-M^{**2}),$$
$$\Delta_{aa}=3f'(2e\lambda'D^{(f)}/m_\pi^3)\pi\delta(S-M^{**2}),$$
$$L_1(t)=-(1/12M^{**})\{6MM^{**}t-(M^2-M^{**2})^2+m_\pi^2[(M-M^{**})^2-2MM^{**}]\},$$
$$L_2(t)=\tfrac{1}{2}(M+M^{**}),$$
$$L_3(t)=(1/12M^{**})[(M^2-M^{**2})(5M^{**}+M)-3M^{**}t+m_\pi^2(2M^{**}-M)],$$
$$L_4(t)=+(1/12M^{**})[-3M^{**}t+m_\pi^2(2M^{**}-M)+(M^{**}-M)^2(M+M^{**})].$$

TABLE I. N contribution to $\mathrm{Im}A,\cdots,\mathrm{Im}D$. The entries listed should be multiplied by $eg\,\pi\delta(S-M^2)$.

	8_{ss}	8_{sa}	8_{as}	8_{aa}
ImA	0	$-(2\surd5)(1-f)$	0	$6f$
ImB	0	$-(4\surd5)(1-f)/(m_\pi^2-t)$	0	$12f/(m_\pi^2-t)$
ImC	$-5(1-f)F_2(0)^{(d)}/3M$	$(\surd5)(1-f)F_2(0)^{(f)}/M$	$(\surd5)fF_2(0)^{(d)}/M$	$-3fF_2(0)^{(f)}/M$
ImD	$-5(1-f)F_2(0)^{(d)}/3M$	$(\surd5)(1-f)F_2(0)^{(f)}/M$	$(\surd5)fF_2(0)^{(d)}/M$	$-3fF_2(0)^{(f)}/M$

TABLE II. N^* contribution with coupling (11) to $\mathrm{Im}A,\cdots,\mathrm{Im}D$. Entries should be multiplied by $-(3eC\lambda/m_\pi^2)\pi\delta(S-M^{*2})$.

	10	
ImA	$(1/6M^{*2})[-3M^{*2}t-m_\pi^2(2M^*M+M^2)+M(M+M^*)^2(M-M^*)]$	$(\equiv R_1(t))$
ImB	1	
ImC	$(1/6M^{*2})[M^2(M-M^*)+M^{*2}(M+3M^*)-m_\pi^2M]$	$(\equiv R_3(t))$
ImD	$(1/6M^{*2})[M^2(M-M^*)-M^{*2}(3M^*+5M)-m_\pi^2M]$	$(\equiv R_4(t))$

TABLE III. N^* contribution with coupling (21) to $\mathrm{Im}A,\cdots,\mathrm{Im}D$. Entries should be multiplied by $-(6eC_4\lambda/m_\pi^3)\pi\delta(s-M^{*2})$.

ImA	$(1/12M^*)[(M^{*2}-M^2)^2+6MM^*t-m_\pi^2(M^2+M^{*2}+4MM^*)]$	$(\equiv R_1'(t))$
ImB	$(1/2)(M^*-M)$	$(\equiv R_2'(t))$
ImC	$(1/12M^*)[(5M^*-M)(M^{*2}-M^2)-m_\pi^2(2M^*+M)+3M^*t]$	$(\equiv R_3'(t))$
ImD	$(1/12M^*)[-(M^{*2}-M^2)(M+M^*)-m_\pi^2(2M^*+M)+3M^*t]$	$(\equiv R_4'(t))$

[12] I. J. Muzinich, Phys. Rev. Letters **18**, 381 (1967); R. J. N. Phillips, Phys. Letters **24B**, 342 (1967).