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Electrodisintegration of the Deuteron. II. Final-State Interactions*

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The effects of interactions between the outgoing nucleons on the cross sections for the inelastic electron-deuteron scattering process $e+d\rightarrow e+p+n$ are examined in detail. Results are presented as corrections to the theoretical cross sections which were calculated in a previous paper with a relativistic theory but neglecting final-state interactions. The changes are significant for the determination of the electromagnetic form factors of the neutron from the experimental cross sections. The method of the paper follows the treatment of final-state interactions given by Durand, extended to include the D state of the deuteron and the coupling between final-state partial waves of the same total angular momentum and parity. The resulting expressions for the quasielastic peak cross sections are expected to be valid in the momentum-transfer range $0\lesssim q^2\lesssim 1.0$ (BeV/c)^2. An alternative approach is suggested for the analysis of experiments at larger q^2 using an absorption model of the type which has been applied successfully to high-energy particle reactions.

I. INTRODUCTION

THE best information on the electromagnetic form factors of the neutron has been obtained from experiments on the electrodisintegration of the deuteron, $e+d\rightarrow e+n+p$. However, the analysis of the experimental cross sections is complicated by problems related to the detailed structure of the deuteron and to the strong final-state interactions (FSI) between the outgoing nucleons. In a previous paper[1] (hereafter referred to as I), a treatment of the electrodisintegration was presented which considered higher-order corrections to the theory given by Durand.[2,3] We consider in this sequel the remaining important modifications to the theoretical cross sections—the final-state interactions (FSI) between the emergent nucleons.

Several authors[4–8] have calculated FSI corrections using basically the Durand theory and a variety of phenomenological nucleon-nucleon phase shifts. Their numerical results differ in detail because of different models for the final-state wave functions, approximations in the deuteron wave function, and a nonuniform treatment of the higher-order interaction terms. However, the predicted cross sections are qualitatively the same in the region of the large peak which corresponds to quasielastic scattering from a single nucleon: for $q^2\gtrsim 0.1$ (BeV/c)^2, the peak is decreased by -2% to -6% depending somewhat on the momentum transfer q^2. The FSI corrections to partial-wave amplitudes for transitions to final states of definite orbital angular momentum and parity tend to be much larger, but cancellation between the corrections to different partial-wave amplitudes leads to the rather small over-all correction to the cross sections.

Effects of FSI are found to be largest for q^2 less than $\sim$0.2 (BeV/c)^2 as expected because the relative momentum of the outgoing nucleons is low for this kinematical condition. Near the threshold for deuteron breakup, the effects of FSI are large enough to be distinguished experimentally.[9] However, theoretical uncertainties[10] make it undesirable to analyze cross sections in this region for information on the charge form factor of the neutron. In this paper, results for the cross sections $d^2\sigma/(d\Omega_e de_0')$ and $d^3\sigma/(d\Omega_e de_0' d\Omega_p)$ are tailored for use at the quasielastic peak of the scattered electron spectrum.

The calculation of these cross sections was considered in Sec. IV of I. The interaction of the electron with the two-nucleon system was calculated in first Born approximation, and the transition amplitude $\langle np|j_\mu|d\rangle$ was assumed to satisfy a Mandelstam representation. The leading contributions to the quasielastic peak cross section were shown to arise from the nucleon pole terms. These contributions are completely specified by the asymptotic properties of the deuteron wave function and the electromagnetic form factors of the free nucleons. The single dispersion integrals which correct the pole-term result have anomalous thresholds close to the physical region for quasielastic scattering. Furthermore, the spectral functions are practically identical in the anomalous region to those for the nonrelativistic deuteron wave function. Hence the replacement of the single-dispersion integrals by wave-function results is a convenient and accurate approximation provided the relativistic momentum variables are inserted properly.

The effects of FSI and meson-exchange corrections (which are ignored in this paper) are given by the Mandelstam double-dispersion relations. A close con-

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[8] K. Hölzl, G. Saller, and P. Urban, Phys. Letters **10**, 120 (1964); Acta Phys. Austriaca **19**, 168 (1964).

[9] M. R. Yearian and E. B. Hughes, Phys. Letters **10**, 234 (1964).

[10] The uncertainties are associated with off-mass-shell effects in the nucleon form factors and in the n-p-d vertex function. It may be that an analysis of this region is more amenable to Bosco's approach [see B. Bosco, Nuovo Cimento **23**, 1028 (1962)].

nection is also known to exist between the double spectral function in the anomalous region and the wave functions for the two-nucleon system in the initial and final state. This connection and the smallness of FSI effects suggests that these corrections can be computed in a semirelativistic approximation. Approximation of the dynamics should introduce no serious errors. Accordingly a semirelativistic Hamiltonian was developed in I for use with approximate nucleon-nucleon wave functions. It was shown that this wave-function approach, though ostensibly noncovariant, reproduces all interaction terms through $O(m^{-2})$ of the Mandelstam representation for the transition amplitude $\langle np|j_\mu|d\rangle$. The approach has the important advantage that it allows us to exploit in a simple manner the large amount of information available on the nucleon-nucleon interaction. We thereby sidestep the more complete and probably impractical calculation of the single and double dispersion relations by recasting their contribution to the amplitude into a form which is correct to $O(m^{-2})$ and can be computed using coordinate-space wave functions. A more refined calculation would simply add corrections to the corrections.

Our result for the cross section $d^2\sigma/(d\Omega_e de_0')$ differs from the Durand result mainly by the inclusion of terms arising from the D state of the deuteron and from coupling between final-state partial waves of the same total angular momentum and parity. The time-reversal symmetry of the partial-wave transition amplitudes gives a reliable method for inserting their correct relative phases in the cross sections. The method breaks down only at very high energies with the onset of important inelastic processes.

Significantly, more information is obtainable from experiments which measure the coincidence cross section $d^3\sigma/(d\Omega_p d\Omega_e de_0')$. General invariance arguments show that in the single-photon-exchange approximation, the azimuthal variation of this cross section is limited to terms in $\cos\phi$ and $\cos 2\phi$, with the angle ϕ measured between the electron-scattering plane and the plane containing the final nucleons. The $\cos\phi$ dependence appears directly as a result of interactions between the outgoing neutron and proton. Its observation is consequently a direct measurement of the presence of FSI. Possibilities for its detection are discussed in Sec. IIIC.

The usual method of calculating FSI corrections is extremely complicated at very high energies where inelastic processes occur involving the outgoing neutron and proton. The domain of applicability of the conventional analysis is limited thereby to values of q^2 less than ~ 1.0 $(\text{BeV}/c)^2$. However, it may be possible to extend the analysis of the electrodisintegration cross sections to higher momentum transfers using an absorption model of the type which has been applied so successfully to high-energy particle reactions.[11] In this paper, we explore the effects of a simple model of this type. The calculation indicates that inelastic channels in the final state reduce the cross section by $\sim 9\%$ relative to its unmodified value for $2\ (\text{BeV}/c)^2 \lesssim q^2 \lesssim 7\ (\text{BeV}/c)^2$.

In Sec. II, we use the symmetry of the transition amplitude to write the cross sections formally in terms of a minimum number of helicity amplitudes. The resulting expressions are used in Sec. III to obtain the effects of FSI using the semirelativistic Hamiltonian of I. In Sec. IV, we estimate the effect of absorptive modifications on the cross section $d^2\sigma/(d\Omega_e de_0')$ using a simple model.

II. FORMAL EXPRESSIONS FOR THE DIFFERENTIAL CROSS SECTIONS

A. Kinematics

We are concerned with the electrodisintegration of the deuteron in the region of the broad peak in the inelastic continuum. This peak results essentially from the quasielastic scattering from the individual nucleons in the deuteron but is spread out in energy by the momentum distribution of the nucleons in the bound state. In general, the four-momentum transfer q is connected to the electron energy variables by the relation

$$q^2 = 4e_0 e_0' \sin^2(\tfrac{1}{2}\vartheta)\,, \tag{1}$$

with e_0, e_0' the initial and final electron energies and ϑ the scattering angle. All the foregoing quantities are measured in the laboratory system. The momentum $p=|\mathbf{p}|$ of either nucleon in their final c.m. system is given by

$$p^2 = \tfrac{1}{4}q^2 + m(e_0-\epsilon) - me_0'\left(1+\frac{2e_0}{m}\sin^2(\tfrac{1}{2}\vartheta)\right), \tag{2}$$

while the electron 3-momentum transfer in that system is

$$\mathbf{q}^2 = q^2 + (p^2 - \tfrac{1}{4}q^2 + \alpha^2)^2/E^2\,, \quad E^2 = m^2 + p^2\,. \tag{3}$$

Here α^2 is related to the deuteron binding energy, $\epsilon = 2.226$ MeV, $\alpha^2 = m\epsilon$. Because of the binding energy of the deuteron, the maximum of the quasielastic peak occurs slightly below the final electron energy characteristic of elastic electron-nucleon scattering,

$$e_0'|_{\text{peak}} = (e_0-\epsilon)[1+(2e_0/m)\sin^2(\tfrac{1}{2}\vartheta)]^{-1}\,. \tag{4}$$

For this condition, Eqs. (1)–(3) show that the nucleon momentum is given by

$$p^2 = \tfrac{1}{4}q^2 \tag{5}$$

and the electron 3-momentum transfer in the c.m. system of the outgoing nucleons is essentially equal to the invariant 4-momentum transfer, $|\mathbf{q}| \to (q^2)^{1/2}$.

B. The Transition Amplitude

A first step in incorporating FSI corrections in the partial-wave transition amplitudes is the examination of

[11] See, for example, J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965); L. Durand, III, and Y. T. Chiu, Phys. Rev. **139**, B646 (1965); J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, *ibid.* **139**, B428 (1965).

their symmetry properties. Although the amplitudes are dynamical quantities, their number and symmetry properties are determined by purely kinematical requirements related to Lorentz invariance. Hence the cross sections can be written down formally in terms of a minimum number of amplitudes without first specifying the interaction Hamiltonian.

The transition amplitude may be written in a partial-wave expansion following Jacob and Wick[12]:

$$\langle np|j_\mu|d\rangle \equiv \langle p,\theta,\phi,\lambda_1,\lambda_2|j_\mu|d,\lambda_d\rangle = (4\pi)^{-1/2}\sum_{J,M}(2J+1)^{1/2}\mathfrak{D}_{M,\lambda}{}^{J*}(\phi,\theta,-\phi) \times\langle J,M,\lambda_1,\lambda_2|j_\mu|d,\lambda_d\rangle; \quad (6)$$

λ_1, λ_2, and λ_d are helicities for the proton, neutron, and deuteron, respectively, and λ is $\lambda_1-\lambda_2$. The helicity state $|d,\lambda_d\rangle$ describes a deuteron moving along the z axis with momentum $-\mathbf{q}$, and $|p,\theta,\phi,\lambda_1\lambda_2\rangle$ is a plane-wave helicity state for the final two-nucleon system. Both states are defined in the center-of-mass frame of the final nucleons. In this frame, the final proton has momentum p and moves in the θ, ϕ direction. The two-particle angular momentum state $|J,M,\lambda_1,\lambda_2\rangle$ is defined by Jacob and Wick [Eq. (18) of Ref. 12] as

$$|J,M,\lambda_1,\lambda_2\rangle = \frac{(2J+1)^{1/2}}{(16\pi^3)^{1/2}} \times\int d\Omega_p \mathfrak{D}_{M,\lambda}{}^{J*}(\phi,\theta,-\phi)|p,\theta,\phi,\lambda_1,\lambda_2\rangle. \quad (7)$$

Finally, the rotation coefficients $\mathfrak{D}_{m,n}{}^{j}(\alpha,\beta,\gamma)$ are defined with the convention of Rose.[13]

We consider the symmetry properties of the partial-wave transition amplitudes $\langle J,M,\lambda_1,\lambda_2|j_\mu|d,\lambda_d\rangle$ noting firstly the restrictions required by conservation of the electromagnetic current. In momentum space this restriction on the matrix elements of j_μ has the general form

$$(p'-p)_\mu\langle p'|j_\mu|p\rangle = 0. \quad (8)$$

In the reference frame used here, $q_\mu=(0,0,-q,q_0)$, this gives the condition that the matrix elements of j_3 are simple multiples of j_0,

$$\langle J,M,\lambda_1,\lambda_2|j_3|d,\lambda_d\rangle = -(q_0/q)\langle J,M,\lambda_1,\lambda_2|j_0|d,\lambda_d\rangle. \quad (9)$$

In particular, at the quasielastic peak (where $q_0=0$), we see that the longitudinal component of the current does not contribute to the electrodisintegration amplitude. More generally, this relation permits us to consider only the transverse components of the vector $\mathbf{j}$ and the scalar component j_0. It will be convenient in the following to use the spherical vector representation of the transverse currents,

$$j_{\pm1} = \mp\frac{1}{\sqrt{2}}(j_1 \pm ij_2). \quad (10)$$

We use the notation j_k ($k=\pm1,0$) to designate the current components of interest.[14]

Conservation of the angular-momentum operator J_3 provides further restrictions on the amplitudes. Recall the familiar eigenvalue equations and commutation relations for a vector and scalar operator with J_3:

$$J_3|d,\lambda_d\rangle = \lambda_d|d,\lambda_d\rangle, \quad J_3|J,M,\lambda_1,\lambda_2\rangle = M|J,M,\lambda_1,\lambda_2\rangle, \quad [J_3,j_k]=kj_k, \quad k=\pm1,0. \quad (11)$$

This gives the relation

$$\lambda_d\langle J,M,\lambda_1,\lambda_2|j_k|d,\lambda_d\rangle = \langle J,M,\lambda_1,\lambda_2|j_kJ_3|d,\lambda_d\rangle = (M-k)\langle J,M,\lambda_1,\lambda_2|j_k|d,\lambda_d\rangle, \quad k=1,0,-1. \quad (12)$$

Thus M is restricted to the values $M=\lambda_d+k$, and the sum over M in Eq. (6) can be eliminated.

The behavior of helicity states under the parity operation has been investigated by Jacob and Wick.[12] If the parity operator P is applied to a helicity state with momentum $\mathbf{p}$, the new state has momentum $-\mathbf{p}$ and opposite helicity. It is therefore more convenient to consider instead of P the operator Y which corresponds to a reflection in the x-z plane,

$$Y = e^{i\pi J_2}P = Pe^{i\pi J_2}. \quad (13)$$

This transformation leaves the momentum of a particle moving in the x-z plane unchanged, but changes the sign of its helicity. More generally, the transformed momentum of a helicity state differs from the initial momentum only in the sign of its y component, and

$$Y|p,\theta,\phi,\lambda_1,\lambda_2\rangle = (-1)^{s_1+s_2-\lambda}|p,\theta,-\phi,-\lambda_1,-\lambda_2\rangle. \quad (14)$$

Using this relation, we can deduce the effect of Y on the angular-momentum states. The result is

$$Y|J,M,\lambda_1,\lambda_2\rangle = (-1)^{M+s_1+s_2}|J,-M,-\lambda_1,-\lambda_2\rangle. \quad (15)$$

Under reflection in the x-z plane, only the component j_2 of the current operator changes sign,

$$Yj_kY^{-1} = (-1)^k j_{-k}, \quad k=1,0,-1. \quad (16)$$

Hence, one easily obtains the restrictions on the partial-wave matrix elements which result from conservation of Y parity.

$$\langle J,M,\lambda_1,\lambda_2|j_k|d,\lambda_d\rangle = \langle J,M,\lambda_1,\lambda_2|Y^{-1}(Yj_kY^{-1})Y|d,\lambda_d\rangle = (-1)^{M+s_1+s_2+k+s_d-\lambda_d}\langle J,-M,-\lambda_1,-\lambda_2|j_{-k}|d,-\lambda_d\rangle = \langle J,-M,-\lambda_1,-\lambda_2|j_{-k}|d,-\lambda_d\rangle, \quad k=1,0,-1. \quad (17)$$

[12] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

[13] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), p. 52.

[14] This notation therefore does not mean the usual spherical components of a 3-vector, i.e., j_0 does not equal j_3.

Equation (17) reduces the number of independent matrix elements of j_0, and permits the matrix elements of j_{-1} to be written in terms of those of j_{+1}.

The results of Eqs. (9), (12), and (17) can be combined to determine the number of distinct matrix elements. If the helicities are allowed to take all possible values, one has 48 possible partial-wave matrix elements of the current j, for each value of the total angular momentum J. However, the symmetry requirements restrict the number of independent amplitudes to 18, six associated with the scalar component of the current and 12 arising from the transverse component of the current.[15]

Unlike the previous symmetries, time-reversal symmetry does not reduce the number of independent amplitudes for the interaction. Instead, it can be used to exhibit the phase of the partial-wave transition amplitudes. The time-reversal transformation acting on a helicity state is equivalent to a rotation by π about the y axis.[16] For the partial-wave transition amplitudes, we obtain the result

$$\begin{aligned}\langle J,M,\lambda_1,\lambda_2|j_k|d,\lambda_d\rangle &= {}^T\langle J,M,\lambda_1,\lambda_2\text{ out}|j_k{}^T|d,\lambda_d\rangle^T\\ &=\langle J,M,\lambda_1\lambda_2,\text{ in}|e^{i\pi J_2}j_k{}^Te^{-i\pi J_2}|d,\lambda_d\rangle\\ &=\langle J,M,\lambda_1,\lambda_2,\text{ in}|j_k|d,\lambda_d\rangle\\ &=\textstyle\sum_\alpha\langle J,M,\lambda_1,\lambda_2,\text{ in}|\alpha,\text{ out}\rangle\langle\alpha,\text{ out}|j_k|d,\lambda\rangle.\end{aligned}\quad(18)$$

The arguments "out" and "in" refer to the spherical-wave modifications to the final states. We have used the fact that, under time-reversal, the spacelike parts of the electromagnetic current change sign, while the time-component is unchanged. This result and the rotation properties of the current were used in going from the second to the third lines in Eq. (18). The states denoted by $|\alpha,\text{ out}\rangle$, span all possible intermediate states, elastic and inelastic, which lead to a neutron and proton in the final state. However, most available experimental data are in a region of q^2 less than ~1.0 (BeV/c)^2. At the quasielastic peak ($p^2=\frac{1}{4}q^2$), this corresponds to n-p scattering at incident kinetic energies of ~0.5 BeV where inelastic cross sections are small. For this situation, Eq. (18) simplifies further:

$$\begin{aligned}\langle J,M,\lambda_1,\lambda_2|j_k|d,\lambda_d\rangle^* &=\sum_{\lambda_1'\lambda_2'}\langle J,M,\lambda_1,\lambda_2,\text{ in}|J,M,\lambda_1',\lambda_2',\text{ out}\rangle\\ &\qquad\times\langle J,M,\lambda_1',\lambda_2'\text{ out}|j_k|d,\lambda_d\rangle\\ &=\sum_{\lambda_1'\lambda_2'}(S_J)_{\lambda_1\lambda_2;\lambda_1'\lambda_2'}{}^{-1}\langle J,M,\lambda_1',\lambda_2',\text{ out}|j_k|d,\lambda_d\rangle.\end{aligned}\quad(19)$$

In the last line, we have used the definition of the S-matrix elements for elastic n-p scattering:

$$(S_J)_{\alpha,\beta}=\langle\alpha,\text{ out}|\beta,\text{ in}\rangle.$$

The above result for helicity states of the neutron-proton system carries through for states of definite parity, since the representations are related to each other by a real orthogonal transformation. In terms of Wigner 3-j symbols, the transformation matrix is

$$\begin{aligned}\langle J,M,L,S|J,M,\lambda_1,\lambda_2\rangle &=(-1)^{L-S}(2L+1)^{1/2}(2S+1)^{1/2}\\ &\quad\times\begin{pmatrix}L & S & J\\ 0 & \lambda & -\lambda\end{pmatrix}\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & S\\ \lambda_1 & -\lambda_2 & -\lambda\end{pmatrix}.\end{aligned}\quad(20)$$

For the present case this observation allows one to rewrite Eq. (18) in matrix form as[17]

$$\left\langle J,M\begin{bmatrix}J-1,1\\ J+1,0\\ J\quad 0\\ J\quad 0\end{bmatrix}\right|j_k|d,\lambda_d\rangle^*=\begin{bmatrix}S_{J-1,J-1}{}^{-1} & S_{J-1,J+1}{}^{-1} & 0 & 0\\ S_{J+1,J-1}{}^{-1} & S_{J+1,J+1}{}^{-1} & 0 & 0\\ 0 & 0 & S_{J,J,1}{}^{-1} & 0\\ 0 & 0 & 0 & S_{JJ0}{}^{-1}\end{bmatrix}\left\langle J,M\begin{bmatrix}J-1,1\\ J+1,1\\ J,\quad 1\\ J,\quad 0\end{bmatrix}\right|j_k|d,\lambda_d\rangle.\quad(21)$$

The remaining coupling in Eq. (21) between the parity states with L equal to $J\pm1$ can be eliminated by recasting the result in terms of eigenstates of the n-p system. For the uncoupled triplet and singlet states, the S-matrix element has the form $S_J=\exp(2i\delta_{J,J,S})$, where the quantity $\delta_{J,J,S}$ is the phase shift of the partial wave with total angular momentum J, orbital angular momentum, $L=J$ and spin, $S=0$, 1. For the remaining coupled triplet states, the S matrix can be written in the notation of Blatt and Biedenharn[18] as

$$S_J=U^{-1}DU\,,\quad(22)$$

where

$$U=\begin{pmatrix}\cos\epsilon_J & \sin\epsilon_J\\ \sin\epsilon_J & \cos\epsilon_J\end{pmatrix},\quad D=\begin{pmatrix}e^{2i\delta_{J,\alpha}} & 0\\ 0 & e^{2i\delta_{J,\beta}}\end{pmatrix}.$$

$\delta_{J,\alpha}$ and $\delta_{J,\beta}$ are the eigenphase shifts for a given J and

[15] The appearance of 12 distinct amplitudes for the transverse component is not unexpected, since this is the number required to describe the photodisintegration of the deuteron, there being no longitudinal component for this reaction.

[16] We use the identity for arbitrary states ψ, ϕ and their time-reversed states, ${}^T\langle\psi|\phi\rangle^T=\langle\psi|\phi\rangle^*$.

[17] A discussion of the n-p scattering matrix is given in the paper of Jacob and Wick (Ref. 12, p. 422), which is pertinent to the present treatment.

[18] J. M. Blatt and L. C. Biedenharn, Phys. Rev. **86**, 399 (1952).

ϵ_J is the coupling parameter. Recast in terms of eigenstates, Eq. (21) contains, by definition, a diagonal S matrix and therefore is expressible in the form

$$\langle J,M,\,\mathrm{eig}|\,j_k|\,d,\lambda_d\rangle^* = e^{-2i\delta_{J,\mathrm{eig}}}\langle J,M,\,\mathrm{eig}|\,j_k|\,d,\lambda_d\rangle\,,\quad k=0,\pm1\,,$$

or more succinctly as

$$\langle J,M,\,\mathrm{eig}|\,j_k|\,d,\lambda_d\rangle = e^{i\delta_{J,\mathrm{eig}}}\ (\text{real function})\,. \tag{23}$$

The form in Eq. (23) is the desired result, namely, that the partial-wave transition amplitudes when expressed as eigenstates of the final n-p system are the product of a real function and a known phase factor. This simple form depends crucially on the absence of inelastic processes in the final-state interaction. The inclusion of such effects in this formalism would introduce complex phase shifts, and a convenient separation of real and imaginary parts of the amplitude would no longer be possible. We are led to consider therefore an alternative scheme in Sec. IV which explores the effect of inelastic processes in the final state on the high-energy cross sections.

C. Formal Cross Sections

The amplitude decomposition in Sec. IIB is well-suited to the construction of the polarized cross sections for inelastic electron-deuteron scattering. However, we will consider only the unpolarized cross sections since accurate polarization experiments do not yet appear feasible.[19] In the first Born approximation, we require therefore the square of the full transition matrix element summed over final helicities and averaged over initial helicities. The differential cross section in which both the scattered electron and final proton are detected is given by

$$d^3\sigma/(d\Omega_e de_0' d\Omega_p) = (2\pi)^{-5}(e_0'/e_0)(mp/32E)\mathcal{T}_{\mu\nu}(e,e')T_{\mu\nu}(n,p,d)\,,\quad \mu,\,\nu=1,2,3,4. \tag{24}$$

As before p, E are the momentum and energy of either nucleon in their c.m. system. The tensors $\mathcal{T}_{\mu\nu}$, $T_{\mu\nu}$ describe the helicity sums for the electron vertex and strong interaction vertex, respectively,

$$\mathcal{T}_{\mu\nu}(e,e') = \tfrac{1}{2}\sum_{\lambda_e,\lambda_{e'}}\langle e',\lambda_{e'}|\,j_\mu|\,e,\lambda_e\rangle\langle e',\lambda_{e'}|\,j_\nu|\,e,\lambda_e\rangle^* = (2\alpha/q^4)(4\pi)^3[ee'\delta_{\mu\nu}-e_\mu e_\nu'-e_\mu' e_\nu].$$

$$T_{\mu\nu}(n,p,d) = \tfrac{1}{3}\sum_{\lambda_1,\lambda_2,\lambda_d}\langle np|\,j_\mu|\,d\rangle\langle np|\,j_\nu|\,d\rangle^*,\quad \mu,\,\nu=1,2,3,4.$$

Here α denotes the fine structure constant, $\alpha=e^2/(4\pi)$. The sum over the current components μ, ν may be simplified considerably using the symmetries of the transition amplitude given in Sec. IIB. The result is

$$\begin{aligned} d^3\sigma/(d\Omega_e de_0' d\Omega_p) &= (2\pi)^{-5}(e_0'/e_0)(mp/32E)[\mathcal{T}_{00}T_{00}(q^2/\mathbf{q}^2)^2+2\mathcal{T}_{++}T_{++} \\ &\quad +\mathcal{T}_{0+}(T_{0-}+T_{+0})(q^2/\mathbf{q}^2)+\mathcal{T}_{+0}(T_{-0}+T_{0+})(q^2/\mathbf{q}^2)+\mathcal{T}_{+-}(T_{+-}+T_{-+})] \\ &= \sigma_{\mathrm{Mott}}(mp/2\pi^2)(m/E)(1+\tau)^{-1}\Big\{T_{00}+2[1+2(1+\tau)\tan^2(\tfrac{1}{2}\vartheta)]T_{++} \\ &\quad +\frac{(e_0+e_0')}{(2e_0e_0')^{1/2}}\sec(\tfrac{1}{2}\vartheta)(T_{0+}+T_{+0})+(T_{+-}+T_{-+})\Big\}\,,\quad \tau=(q^2/4m^2). \end{aligned} \tag{25}$$

The subscripts 0, $\pm$ on the tensor components refer to the scalar and transverse components of the corresponding interaction currents, $k=0,\pm1$. In particular, using Eqs. (6) and (25) we have

$$\begin{aligned} T_{k,k'} &= \tfrac{1}{3}(4\pi)^{-1/2}\sum_j \hat{j}Y_{j,k'-k}(\theta,\phi)\sum_{J,J'}\sum_{\lambda_1,\lambda_2,\lambda_d}\hat{J}\hat{J}'\hat{L}\hat{L}'(-1)^{M'-\lambda}\begin{pmatrix} J & J' & j \\ -\lambda & \lambda & 0\end{pmatrix}\begin{pmatrix} J & J' & j \\ -M & M' & k-k'\end{pmatrix} \\ &\quad \times\langle J,M,\lambda_1\lambda_2|\,j_k|\,d,\lambda_d\rangle\langle J',M',\lambda_1,\lambda_2|\,j_{k'}|\,d,\lambda_d\rangle^*,\quad k,\,k'=0,\pm1\,, \end{aligned} \tag{26}$$

where we have introduced the compact notation,

$$\hat{l}=(2l+1)^{1/2}\,,$$

and included the ϕ dependence completely in the single spherical harmonic, $Y_{j,k'-k}(\theta,\phi)$. In the second line of Eq. (25), we have evaluated the electron trace in terms of electron variables in the laboratory. σ_{Mott} is the Mott cross section for the scattering of a relativistic electron by a Coulomb field,

$$\sigma_{\mathrm{Mott}}=(\alpha/2e_0)^2\cos^2(\tfrac{1}{2}\vartheta)\sin^{-4}(\tfrac{1}{2}\vartheta)\,.$$

The azimuthal dependence of the cross section is clear. In particular, the first two terms in the cross section, Eq. (25), are independent of the azimuthal angle ϕ.

[19] Durand, however (Ref. 3, Sec. III), has calculated the final proton polarization in the electrodisintegration of the deuteron.

However, the last two terms, which arise from interference of the scalar and transverse components of the interaction current, and the "+1" and "−1" components of the transverse current, are proportional, respectively, to $\cos\phi$ and $\cos 2\phi$.

The cross section $d^2\sigma/(d\Omega_e de_0')$ in which only the electron energy and angle are observed is obtained by integrating Eq. (25) over the direction of emission of the nucleons in their center-of-mass frame. The result is

$$d^2\sigma/(d\Omega_e de_0')$$
$$=\sigma_{Mott}(mp/\pi)(m/E)(1+\tau)^{-1}\left\{\frac{1}{2}\int_{-1}^{1}d(\cos\theta)T_{00}\right.$$
$$\left.+2[1+2(1+\tau)\tan^2(\tfrac{1}{2}\vartheta)]\frac{1}{2}\int_{-1}^{1}d(\cos\theta)T_{++}\right\}. \quad (27)$$

The calculation of the e-d scattering cross sections is now reduced to the evaluation of the components of the tensor $T_{k,k'}$, Eq. (26). The necessary partial-wave transition amplitudes will be evaluated in the following section using the semirelativistic interaction Hamiltonian of I.

III. DIFFERENTIAL SCATTERING CROSS SECTIONS

A. Method of Calculation

The basic ideas of our reduction of the Mandelstam dispersion relations for the transition amplitude were discussed in I. The nucleon pole terms and, to a lesser extent, the single dispersion integrals account for the dominant contribution to the cross section at the quasielastic peak. The pole terms can be calculated exactly in the relativistic theory. The main contribution of the single dispersion integrals were determined by the near equality between the spectral functions in the anomalous region and the spectral functions for the nonrelativistic deuteron wave function. The correspondence depends on interpreting the momenta of the nucleons in the deuteron as p, the relativistic momentum of the outgoing nucleons in their c.m. frame, and interpreting the 3-momentum transfer in this frame as the square root of the invariant 4-momentum transfer, $|\mathbf{q}|=(q^2)^{1/2}$. This kinematical condition is well approximated in the quasielastic peak region. Theoretical cross sections are consequently most reliable there.

Effects of FSI arise from the Mandelstam double dispersion relations in which anomalous thresholds are also present. The calculation of all diagrams which contribute to the double-dispersion relations would include not only corrections for final-state interactions but also the effects of meson-exchange currents on the scattering.[20] However, a simpler method is available for calculating approximately the corrections due to FSI. For the quasielastic peak condition, the double spectral function looks practically identical to that obtained by taking the matrix elements of the sum of electromagnetic interactions for a free neutron and proton between ostensibly nonrelativistic wave functions for the initial and final two-nucleon state. The identification of wave functions depends on the same interpretation of the momenta p and q as given above for the single dispersion relations. To implement this result, a semirelativistic interaction Hamiltonian was developed in I for use with approximate nucleon-nucleon wave functions. Matrix elements of the interaction Hamiltonian agree to $O(m^{-2})$ with the two-component reduction of the dispersion relation result. This method for calculating corrections for FSI is the same as that used by Durand.[2,3] Because the corrections are small, the approximate form of matrix elements used in their calculation should introduce little error.

The following expressions are obtained for the scalar and transverse components of the transition matrix elements based on the effective Hamiltonian derived in I:

$$\langle J,M,\lambda_1\lambda_2|j_0|d,\lambda_d\rangle$$
$$=\int d^3r\,\psi_{\lambda_1\lambda_2}^*(\mathbf{r})\{[F_{1p}-(q^2/4m^2)\kappa_pF_{2p}-F_{1p}(\partial_p{}^2/2m^2)$$
$$-(F_{1p}+2\kappa_pF_{2p})(\boldsymbol{\sigma}_p\cdot\mathbf{q}\times\boldsymbol{\partial}_p/4m^2)]e^{i\mathbf{q}\cdot\mathbf{r}/2}$$
$$+[\text{neutron terms}]e^{-i\mathbf{q}\cdot\mathbf{r}/2}\}\psi_{\lambda_d}(\mathbf{r}), \quad (28)$$

$$\langle J,M,\lambda_1\lambda_2|j_{\pm1}|d,\lambda_d\rangle$$
$$=\int d^3r\,\psi_{\lambda_1\lambda_2}^*(\mathbf{r})\{[-F_{1p}(i\mathbf{q}+2\boldsymbol{\partial}_p)_\pm/2m$$
$$+(F_{1p}+\kappa_pF_{2p})(\boldsymbol{\sigma}_p\times\mathbf{q})_\pm/2m]e^{i\mathbf{q}\cdot\mathbf{r}/2}$$
$$+[\text{neutron terms}]e^{-i\mathbf{q}\cdot\mathbf{r}/2}\}\psi_{\lambda_d}(\mathbf{r}). \quad (29)$$

$\psi_{\lambda_1\lambda_2}(\mathbf{r})$ and $\psi_{\lambda_d}(\mathbf{r})$ are nonrelativistic wave functions for the initial and final states and are defined below. The other notation is given in I.

We recall that the leading Born terms in the results of I are correct to all orders in m^{-1}, whereas the higher-order terms are correct only to order m^{-2}. In principle, one should include FSI corrections to all terms through order m^{-2}. However, it was argued that those higher-order terms involving derivatives in Eqs. (28) and (29) are inherently small. The calculation of FSI corrections to these corrective terms is therefore of dubious value, and we will omit such corrections in our results below. Note also that relativistic corrections are omitted in the FSI terms which we would argue again to be corrections to corrections. Hence, we use a subtraction method for giving these corrections to the unmodified cross sections of I, cf. Secs. IIIB, C.

[20] The effect of meson-exchange currents is difficult to estimate. See R. Blankenbecler, Phys. Rev. **111**, 1684 (1958). Durand has noted, however (Ref. 2, p. 1417), that configurations in which the nucleons are sufficiently close together that exchange of a meson is likely, yield only a relatively small fraction of the cross section.

The deuteron wave function was used in its standard form,

$$\psi_{\lambda_d}(\mathbf{r})=r^{-1}[u(r)\mathcal{Y}_{101}{}^{\lambda_d}+w(r)\mathcal{Y}_{121}{}^{\lambda_d}], \tag{30}$$

where the S- and D-state radial functions satisfy the normalization condition

$$\int_0^\infty [u^2(r)+w^2(r)]dr=1. \tag{31}$$

The functions, $\mathcal{Y}_{JLS}{}^M(\hat{r},s_1,s_2)$ are the usual orthonormal angular momentum eigenfunctions for definite J, L, and S. The wave functions for the final neutron-proton system are complicated by the two-nucleon interaction, particularly for the coupled triplet states. However, the time-reversal result of Sec. II provides a scheme for writing such wave functions in terms of eigenphase shifts. The helicity states for each angular-momentum J are rearranged into states of definite L, S using Eq. (20). The wave functions are then expressed as

$$\psi_{\lambda_1\lambda_2}(\mathbf{r})\rightarrow\psi_{J,M,\mathrm{eig}}(\mathbf{r})=(pr)^{-1}\sum_{L,S}u_{L,S,\mathrm{eig}}F_{J,L,\mathrm{eig}}(pr)e^{-i\delta_{J,\mathrm{eig}}}\mathcal{Y}_{JLS}{}^M,$$

that is,

$$\begin{bmatrix} e^{i\delta_{J,\alpha}}\psi_{J,M,\alpha} \\ e^{i\delta_{J,\beta}}\psi_{J,M,\beta} \\ e^{i\delta_{J,J,1}}\psi_{J,M,L=J,S=1} \\ e^{i\delta_{J,J,0}}\psi_{J,M,L=J,S=0} \end{bmatrix}=\begin{bmatrix} \cos\epsilon_J & \sin\epsilon_J & 0 & 0 \\ -\sin\epsilon_J & \sin\epsilon_J & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} (pr)^{-1}F_{J,J-1,\mathrm{eig}}(pr)\mathcal{Y}_{J,J-1,1}{}^M \\ (pr)^{-1}F_{J,J+1,\mathrm{eig}}(pr)\mathcal{Y}_{J,J+1,1}{}^M \\ (pr)^{-1}F_{J,J,1}(pr)\mathcal{Y}_{J,J,1}{}^M \\ (pr)^{-1}F_{J,J,0}(pr)\mathcal{Y}_{J,J,0}{}^M \end{bmatrix}. \tag{32}$$

The radial functions are subject to the asymptotic condition

$$F_{J,L,\mathrm{eig}}(pr)\rightarrow\sin(pr-L(\pi/2)+\delta_{J,\mathrm{eig}})\quad\text{for}\quad pr\gg L. \tag{33}$$

The transition matrix element is easily reduced to a form in which only radial integrals occur. These are an extension of Durand's definition for K_{JLS} integrals to include coupling in the final partial waves and the D state of the deuteron. Although the integrals contain complex factors, they are easily related to a set of real matrix elements $K_{JL\lambda}$ using the relations above. We define the radial matrix elements Q_{JLS} for the S state and $Q_{JLS}{}^l$ for the D state by the following matrix equation:

$$\begin{bmatrix} Q_{J,J-1,1} \\ Q_{J,J+1,1} \\ Q_{J,J,1} \\ Q_{J,J,0} \end{bmatrix}=\begin{bmatrix} \cos\epsilon_J & \sin\epsilon_J & 0 & 0 \\ -\sin\epsilon_J & \cos\epsilon_J & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} e^{-i\delta_{JL\alpha}}K_{JL\alpha} \\ e^{-i\delta_{JL\beta}}K_{JL\beta} \\ e^{-i\delta_{LL1}}K_{JJ1} \\ e^{-i\delta_{LL0}}K_{JJ0} \end{bmatrix},$$

where

$$K_{JL\lambda}=p^{-1}\int_0^\infty F_{JL\lambda}(pr)j_L(\tfrac{1}{2}qr)u(r)dr.$$

The same relation holds for the D-state integrals, $Q_{JLS}{}^l$ in terms of real integrals $K_{JL\lambda}{}^l$ where now we have the result

$$K_{JL\lambda}{}^l=p^{-1}\int_0^\infty F_{JL\lambda}(pr)j_l(\tfrac{1}{2}qr)w(r)dr.$$

In the absence of FSI, the phase shifts vanish and the radial functions $F_{JL\lambda}(pr)/pr$ reduce to spherical Bessel functions. The radial integrals are then independent of J and S so that Q_{JLS} reduces to K_L, where

$$K_L=\int_0^\infty j_L(pr)j_L(\tfrac{1}{2}qr)u(r)rdr. \tag{34}$$

B. The Cross Section $d^2\sigma/(d\Omega_e de_0')$

The cross sections can be written down immediately in terms of the foregoing matrix elements. For the integrated cross section, we obtain the following expression:

$$d^2\sigma/(d\Omega_e de_0')=\sigma_{\mathrm{Mott}}(mp/\pi)(m/E)I(\vartheta), \tag{35}$$

where

$$I(\vartheta)=I_0(\vartheta)+\sum_{JLS}C_{JLS}{}^{SS}\Delta_{JLS}{}^{SS}+\sum_{JLS}C_{JLS}{}^{SD}\Delta_{JLS}{}^{SD}+\sum_{JLS}C_{JLS}{}^{DD}\Delta_{JLS}{}^{DD}. \tag{36}$$

In Eq. (36), we have employed the usual subtraction procedure of extracting the Born term result, $I_0(\vartheta)$ for all partial waves from the relative corrections to the low partial waves. The form of $I_0(\vartheta)$ is given by Eq. (45) of I.

Quantities similar to $C_{JLS}{}^{SS}$ and $\Delta_{JLS}{}^{SS}$ have been used by Durand[2] and others[5,6] as a measure of the relative correction due to FSI on each partial wave. Here, to include coupling, we define $\Delta_{JLS}{}^{SS}$ as

$$\Delta_{JLS}{}^{SS}=|Q_{JLS}|^2-K_L{}^2. \quad (37)$$

The weighting coefficients $C_{JLS}{}^{SS}$ for each final state of the neutron-proton system are as follows:

$$\begin{aligned}
C_{L-1,L,1}{}^{SS}&=(2L-1)[a_1+(-1)^La_2]\\
&\quad+(3L-1)[a_3+(-1)^La_4],\\
C_{L+1,L,1}{}^{SS}&=(2L+3)[a_1+(-1)^La_2]\\
&\quad+(3L+4)[a_3+(-1)^La_4],\\
C_{L,L,1}{}^{SS}&=(2L+1)[a_1+a_3+(-1)^L(a_2+a_4)],\\
C_{L,L,0}{}^{SS}&=(4L+2)[a_3-(-1)^La_4],
\end{aligned} \quad (38)$$

where

$$\begin{aligned}
a_1&=\tfrac{1}{3}(1+\tau)^{-1}[(F_{1p}-\tau\kappa_pF_{2p})^2+(F_{1n}-\tau\kappa_nF_{2n})^2],\\
a_2&=\tfrac{2}{3}(1+\tau)^{-1}(F_{1p}-\tau\kappa_pF_{2p})(F_{1n}-\tau\kappa_nF_{2n}),\\
a_3&=\tfrac{1}{6}\tau[2\tan^2(\tfrac{1}{2}\vartheta)+(1+\tau)^{-1}]\\
&\quad\times[(F_{1p}+\kappa_pF_{2p})^2+(F_{1n}+\kappa_nF_{2n})^2],\\
a_4&=\tfrac{1}{3}\tau[2\tan^2(\tfrac{1}{2}\vartheta)+(1+\tau)^{-1}]\\
&\quad\times(F_{1p}+\kappa_pF_{2p})(F_{1n}+\kappa_nF_{2n}),\\
\tau&=(q^2/4m^2).
\end{aligned} \quad (39)$$

The quantities $\Delta_{JLS}{}^{SD}$, $\Delta_{JLS}{}^{DD}$ in Eq. (36) contain the FSI corrections which include the D state of the deuteron. However, because of the small D-state probability, as a practical matter, we need only include the contributions arising from the interference of amplitudes containing the S and D states of the deuteron. This gives the result

$$\begin{aligned}
C_{JL1}{}^{SD}\Delta_{JL1}{}^{SD}&=(2J+1)[a_1+(-1)^La_2](2\sqrt{15})(-1)^{J+1}\begin{pmatrix}L&L&2\\0&0&0\end{pmatrix}\begin{Bmatrix}L&1&J\\1&L&2\end{Bmatrix}\mathrm{Re}[Q_{JL1}Q_{JL1}{}^{L*}]+(2J+1)\\
&\quad\times[a_3+(-1)^La_4](2\sqrt{15})(2L+1)\sum_l(2l+1)\begin{pmatrix}L&l&2\\0&0&0\end{pmatrix}\sum_{\mu_1\mu_2\mu_3}(-1)^{J+\mu_3+\mu_1}\begin{pmatrix}L&1&J\\0&1+\mu_1&-1-\mu_1\end{pmatrix}\\
&\quad\times\begin{pmatrix}L&l&2\\-\mu_3&0&\mu_3\end{pmatrix}\begin{pmatrix}L&1&J\\\mu_3&\mu_2&-1-\mu_1\end{pmatrix}\begin{pmatrix}2&1&1\\\mu_3&\mu_2-1&-\mu_1\end{pmatrix}\mathrm{Re}[i^{-l}Q_{JL1}Q_{JL1}{}^{l*}],\\
C_{LL0}{}^{SD}\Delta_{LL0}{}^{SD}&=[a_3+(-1)^La_4]\sqrt{2}(-1)^{L-1}\sum_l(2l+1)\begin{pmatrix}L&l&2\\0&0&0\end{pmatrix}^2\mathrm{Re}[i^{-l}Q_{LL0}Q_{LL0}{}^{l*}].
\end{aligned} \quad (40)$$

In Eq. (40), and subsequent formulas, the symbols in curly brackets denote Wigner 6-j symbols.

C. The Cross Section $d^3\sigma/(d\Omega_e de_0' d\Omega_p)$

The FSI corrections to the cross section $d^3\sigma/(d\Omega_e de_0' d\Omega_p)$ are rather cumbersome to write down. Including the azimuthal dependence, we obtain the result

$$d^3\sigma/(d\Omega_e de_0' d\Omega_p)=\sigma_{\mathrm{Mott}}(m^2p/2\pi^2E)\Lambda(p,q), \quad (41)$$

where

$$\Lambda(p,q)=\Lambda_0(p,q)+\sum_{JLJ'L'S}[C_{JLJ'L'S}{}^{SS}\Delta_{JLJ'L'S}{}^{SS}+C_{JLJ'L'S}{}^{SD}\Delta_{JLJ'L'S}{}^{SD}]. \quad (42)$$

The Born term expression, $\Lambda_0(p,q)$ is given by Eq. (36) of I, extended to include the ϕ-dependent terms:

$$\Lambda_0(p,q)=(1+\tau)^{-1}\Big\{\Lambda_0{}^L(p,q)+[1+2(1+\tau)\tan^2(\tfrac{1}{2}\vartheta)]\times\Lambda_0{}^T(p,q)+\frac{e_0e_0'}{[8e_0e_0'\cos^2(\tfrac{1}{2}\vartheta)]^{1/2}}\times\Lambda_0{}^{LT}(p,q)+\Lambda_0{}^{TT}(p,q)\Big\}. \quad (43)$$

The superscripts on the angular distribution functions $\Lambda_0{}^j(p,q)$ are intended to indicate their origin as due to the longitudinal ($j=L$) and transverse ($j=T$), components of the interaction and also interference between these components, ($j=LT$, TT).

$$\begin{aligned}
\Lambda_0{}^L(p,q)&=G_{Ep}{}^2[\tilde{u}^2(k_p)+\tilde{w}^2(k_p)]+G_{En}{}^2[\tilde{u}^2(k_n)\\
&\quad+\tilde{w}^2(k_n)]+2G_{Ep}G_{En}[\tilde{u}(k_p)\tilde{u}(k_n)\\
&\quad+\tilde{w}(k_p)\tilde{w}(k_n)P_2(\hat{k}_p\cdot\hat{k}_n)],\\
\Lambda_0{}^T(p,q)&=\tau G_{Mp}{}^2[\tilde{u}^2(k_p)+\tilde{w}^2(k_p)]+\tau G_{Mn}{}^2[\tilde{u}^2(k_n)\\
&\quad+\tilde{w}^2(k_n)]+\tfrac{2}{3}\tau G_{Mp}G_{Mn}[\tilde{u}(k_p)\tilde{u}(k_n)\\
&\quad+\tilde{w}(k_p)\tilde{w}(k_n)(3P_2(\hat{k}_n\cdot\hat{k}_p)\\
&\quad+P_2(\hat{k}_n\cdot\hat{q})+P_2(\hat{k}_p\cdot\hat{q})-1)]\\
&\quad+\tfrac{1}{3}\sqrt{2}\tau G_{Mp}G_{Mn}[\tilde{u}(k_p)\tilde{w}(k_n)P_2(\hat{k}_n\cdot\hat{q})\\
&\quad+\tilde{u}(k_n)\tilde{w}(k_p)P_2(\hat{k}_p\cdot\hat{q})],\\
\Lambda_0{}^{TT}(p,q)&=\tfrac{1}{3}\tau G_{Mp}{}^2\cos2\phi[-\sqrt{2}\tilde{u}(k_p)\tilde{w}(k_p)P_2(\hat{k}_p\cdot\hat{q})\\
&\quad+\tilde{w}^2(k_p)P_2{}^2(\hat{k}_p\cdot\hat{q})]\\
&\quad+\tfrac{1}{3}\tau G_{Mn}{}^2\cos2\phi[-\sqrt{2}\tilde{u}(k_n)\tilde{w}(k_n)P_2(\hat{k}_n\cdot\hat{q})\\
&\quad+\tilde{w}^2(k_n)P_2{}^2(\hat{k}_n\cdot\hat{q})],\\
\Lambda_0{}^{LT}(p,q)&=0,
\end{aligned} \quad (44)$$

where

$$\begin{aligned}
\mathbf{k}_p&=\tfrac{1}{2}\mathbf{q}-\mathbf{p}, & k_p&=|\mathbf{k}_p|=(p^2+\tfrac{1}{4}q^2-pq\cos\theta)^{1/2},\\
\mathbf{k}_n&=\tfrac{1}{2}\mathbf{q}+\mathbf{p}, & k_n&=|\mathbf{k}_n|=(p^2+\tfrac{1}{4}q^2+pq\cos\theta)^{1/2},
\end{aligned} \quad (45)$$

and the nucleon form factors are regrouped into electric and charge form factors.

$$\begin{aligned}
G_{Ei}&=F_{1i}(q^2)-(q^2/4m^2)\kappa_iF_{2i}(q^2),\\
G_{Mi}&=F_{1i}(q^2)+\kappa_iF_{2i}(q^2), \quad i=p,n.
\end{aligned} \quad (46)$$

The S- and D-state functions $\tilde{u}(k_i)$, $\tilde{w}(k_i)$ are Fourier-

Bessel transforms of the coordinate space wave functions.

$$\tilde{u}(k_i)=\int_0^\infty j_0(k_ir)u(r)rdr\,,$$

$$\tilde{w}(k_i)=\int_0^\infty j_2(k_ir)w(r)rdr\,.$$

In a more complete treatment, $\tilde{u}(k_i)$, $\tilde{w}(k_i)$ would be replaced by dispersion integrals with anomalous thresholds obtained from the relativistic n-p-d vertex function (see comments Sec. IV or I).

Again note that the leading terms in Λ^L and Λ^T are correct to all orders in m^{-2} and reproduce the Rosenbluth scattering off the individual nucleons. However, the other, smaller contributions are correct only to terms of order (q^2/m^2). The $\cos\phi$ dependent term Λ^{LT} is zero in the absence of FSI, while the $\cos2\phi$ dependent term Λ^{TT} is present only when the D state of the deuteron is included in the calculation.

FSI corrections which involve only the S-state part of the deuteron are of course the largest. They are given by the quantities $\Delta_{JLJ'L'S}$, where

$$\Delta_{JLJ'L'S}{}^{SS}=\mathrm{Re}\{Q_{JLS}Q_{J'L'S}{}^*\}-K_LK_{L'}\,. \tag{47}$$

The relative weights $C_{JLJ'L'S}{}^{SS}$ of these corrections are given by

$$\begin{aligned}
C_{JLJ'L'1}{}^{SS}=&\tfrac{1}{3}\sum_j(2j+1)P_j(\cos\theta)\Bigg\{(2J+1)(2J'+1)(2L+1)(2L'+1)\begin{pmatrix}L&L'&j\\0&0&0\end{pmatrix}^2\begin{Bmatrix}L&L'&j\\J'&J&1\end{Bmatrix}^2\\
&\times(1+\tau)^{-1}[G_{Ep}+(-1)^LG_{En}][G_{Ep}+(-1)^{L'}G_{En}]+\tau[(1+\tau)^{-1}+2\tan^2(\tfrac{1}{2}\vartheta)]\\
&\times(2J+1)(2J'+1)(2L+1)(2L'+1)\begin{pmatrix}L&L'&j\\0&0&0\end{pmatrix}\begin{Bmatrix}L&L'&j\\J'&J&1\end{Bmatrix}\sum_{M=0,1}(-1)^{j+M+1}\begin{pmatrix}J'&J&j\\-M&M&0\end{pmatrix}\\
&\times\begin{pmatrix}L'&1&J'\\0&M&-M\end{pmatrix}\begin{pmatrix}L&1&J\\0&M&-M\end{pmatrix}[G_{Mp}+(-1)^LG_{Mn}][G_{Mp}+(-1)^{L'}G_{Mn}]\Bigg\}\,,\\
C_{LLL'L'0}{}^{SS}=&\tfrac{1}{3}\sum_j(2j+1)P_j(\cos\theta)(2L+1)(2L'+1)\begin{pmatrix}L&L'&j\\0&0&0\end{pmatrix}^2\tau[(1+\tau)^{-1}+2\tan^2(\tfrac{1}{2}\vartheta)]\\
&\times[G_{Mp}-(-1)^LG_{Mn}][G_{Mp}-(-1)^{L'}G_{Mn}]\,.
\end{aligned} \tag{48}$$

These corrections are the same as those given by Durand [Eq. (13) of Ref. 2] but generalized to include the form factors of the nucleons, and the coupling between final-nucleon states having the same total angular momentum and parity, but different orbital angular momenta.

The main effect of including the D state of the deuteron arises from its interference with the S state. The effect is to add the terms $C_{JLJ'L'S}{}^{SD}\Delta_{JLJ'L'S}{}^{SD}$, where

$$\begin{aligned}
&C_{JLJ'L'1}{}^{SD}\Delta_{JLJ'L'1}{}^{SD}\\
&=\tfrac{4}{3}(\sqrt{15})\sum_j(2j+1)P_j(\cos\theta)\Bigg[(2J+1)(2J'+1)(2L+1)(2L'+1)\\
&\times\begin{pmatrix}L&L'&j\\0&0&0\end{pmatrix}\begin{Bmatrix}L&L'&j\\J'&J&1\end{Bmatrix}(-1)^{J'+L+L'+1}\sum_l(2l+1)\begin{pmatrix}l&L'&j\\0&0&0\end{pmatrix}\begin{Bmatrix}l&L'&j\\J'&J&1\end{Bmatrix}\\
&\times\begin{pmatrix}L&l&2\\0&0&0\end{pmatrix}\begin{Bmatrix}l&1&J\\1&L&2\end{Bmatrix}(1+\tau)^{-1}[G_{Ep}+(-1)^{L'}G_{En}][G_{Ep}+(-1)^lG_{En}]\,\mathrm{Re}[i^lQ_{J'L'1}{}^*Q_{JL1}{}^l]\\
&+\tau[(1+\tau)^{-1}+2\tan^2(\tfrac{1}{2}\vartheta)](2J+1)(2J'+1)(2L+1)(2L'+1)(-1)^{J+J'}\begin{pmatrix}L&L'&j\\0&0&0\end{pmatrix}\begin{Bmatrix}L&L'&j\\J'&J&1\end{Bmatrix}\\
&\times\sum_l(2l+1)\begin{pmatrix}L&l&2\\0&0&0\end{pmatrix}\sum_{\mu_1\mu_2\mu_3}(-1)^{\mu_3}\begin{pmatrix}J&J'&j\\\mu_1&-\mu_2&0\end{pmatrix}\begin{pmatrix}L'&1&J'\\0&1+\mu_1&-1-\mu_1\end{pmatrix}\begin{pmatrix}L&l&2\\-\mu_3&0&\mu_3\end{pmatrix}\\
&\times\begin{pmatrix}L&1&J\\\mu_3&\mu_2&-1-\mu_1\end{pmatrix}\begin{pmatrix}2&1&1\\\mu_3&\mu_2-1&-\mu_1\end{pmatrix}[G_{Mp}+(-1)^{L'}G_{Mn}][G_{Mp}+(-1)^lG_{Mn}]\,\mathrm{Re}[i^lQ_{J'L'1}{}^*Q_{JL1}{}^l]\Bigg]\,,\\
&C_{LLL'L'0}{}^{SD}\Delta_{LLL'L'0}{}^{SD}\\
&=\tfrac{1}{3}\sqrt{2}\sum_j(2j+1)P_j(\cos\theta)(2L+1)(2L'+1)\begin{pmatrix}L&L'&j\\0&0&0\end{pmatrix}\begin{pmatrix}L'&L&j\\1&-1&0\end{pmatrix}\tau[(1+\tau)^{-1}+2\tan^2(\tfrac{1}{2}\vartheta)]\\
&\qquad\times[G_{Mp}-(-1)^LG_{Mn}][G_{Mp}-(-1)^{L'}G_{Mn}]\sum_l(2l+1)\begin{pmatrix}L&l&2\\0&0&0\end{pmatrix}^2\mathrm{Re}[i^lQ_{L'L'0}{}^*Q_{LL0}{}^l]\,.
\end{aligned} \tag{49}$$

We ignore corrections for FSI involving the square of the D-state wave function.

A cursory examination of the ϕ-dependent terms suggests that the $\cos\phi$ terms will be larger since they involve the factor $(q/2m)$ while the $\cos 2\phi$ terms contain the factor $q^2/4m^2$. In view of this and the small D-state probability, we have not calculated FSI corrections for the $\cos 2\phi$ terms. The corrected $\cos\phi$ terms appear in the cross section, Eq. (42), as $\Lambda^{LT}(p,q)$, with the same coefficient as $\Lambda_0{}^{LT}(p,q)$ in Eq. (43).

$$\Lambda^{LT}(p,q)=\cos\phi(q/2m)\tfrac{4}{3}\sum_j (2j+1)[j(j+1)]^{1/2}P_j{}^1(\cos\theta)(2J+1)(2J'+1)(2L+1)(2L'+1)$$
$$\times\begin{pmatrix} L & L' & j \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} L & L' & j \\ J' & J & 1\end{Bmatrix}\sum_{M=0,1}(-1)^M\begin{pmatrix} L & 1 & J \\ 0 & -M & M\end{pmatrix}\begin{pmatrix} L' & 1 & J' \\ 0 & 1-M & -1+M\end{pmatrix}\begin{pmatrix} J & J' & j \\ M & 1-M & -1\end{pmatrix}$$
$$\times[G_{Ep}G_{Mn}-G_{En}G_{Mp}]\left[\frac{(-1)^{L'}-(-1)^L}{2}\right]\mathrm{Im}[Q_{JL1}Q_{J'L'1}{}^*]. \quad (50)$$

Several points are worth noting for these terms. They contain only n-p final triplet states; the singlet terms are down by a factor $(q^2/4m^2)$ relative to the triplet terms. The nonzero contributions appear as the interference of amplitudes with n-p final states having odd relative parity, e.g., $(^3S_1,{}^3P_{2,1,0})$, $(^3P_{2,1,0},{}^3D_{2,1,0})$, etc. Hence, $\cos\phi$ terms will be appreciable only when there are significant amounts of higher partial waves. All $\cos\phi$ terms contain a factor $\sin\theta$ [in the factor $P_j{}^1(\cos\theta)=\sin\theta P_j'(\cos\theta)$], thereby making the region around $\theta\sim\pi/2$ most favorable for their detection. Since the usual ϕ-independent terms are largest near $\theta\sim 0$, π the flux of nucleons is reduced for intermediate values of θ, perhaps sufficiently so that the $\sin\theta$ behavior can be determined over the background flux produced by the tails of the ϕ-independent angular distribution.

IV. VERY-HIGH-ENERGY BEHAVIOR OF THE CROSS SECTIONS

We expect the foregoing formulas for the quasielastic peak cross sections to be applicable up to momentum transfers at which inelastic collisions between the outgoing nucleons becomes significant. A measure of the inelasticity is given by the nucleon-nucleon cross section data at c.m. momenta corresponding to that for e-d scattering at the quasielastic peak $(p^2=\frac{1}{4}q^2)$. For example, at $q^2=1.0$ (BeV/c)^2, the inelastic cross section is $\sim 20\%$ of the total n-p cross section while at $q^2=1.3$ (BeV/c)^2, it has risen to about 40%. On this basis, a reasonable limit to the conventional analysis of electrodisintegration cross sections would be $q^2\lesssim 1.0$ (BeV/c)^2 (although other considerations might impose a lower limit[21]). Higher values of q^2 are attainable in experiments with present-day electron accelerators. Hence, it is of interest to explore extensions of the present analysis to include, at least approximately, inelastic processes in the final state.

A description which extends the methods of Sec. II is hopelessly complicated since the inelastic channels give rise to complex phase shifts and complex wave functions as well as additional terms in the sum over intermediate states. However, an alternative procedure is suggested by the work on absorptive effects in peripheral reactions at high energies.[11] These calculations have had considerable success in describing qualitative and quantitative features of scattering processes in which single-particle exchange assumes a dominant role. The fundamental idea in these absorption models is that one may expect the contributions from a given exchange to be strongly suppressed in the low partial-wave amplitudes due to the strong competition from other open channels.

For the electrodisintegration process, the effect of absorption in the final state is easily demonstrated. To do so, we ignore the complications arising from the D state of the deuteron although the method can be extended to include it. The n-p interference terms which appear in the cross sections are also neglected since they are negligible as soon as q^2 is at all appreciable. We are ignoring the complications of particle spins to the extent that only diagonal elements of the S matrix are used in the calculation. However, the resulting modifications to the Born cross sections should still describe well the over-all high-energy behavior.

Absorptive effects which arise from close encounters of the particles become increasingly important as the momentum transfer becomes large. However the deuteron is a very diffuse structure, and the bulk of the contribution to the matrix elements K_L arises from large values of r, the internucleon separation. It is therefore reasonable, as a first approximation, to use the asymptotic form for the final n-p wave function in this outer region, and to neglect the contributions from values of r inside the angular-momentum barrier or the region of strong inelastic interactions. The final n-p radial wave function is then approximated as a sum of incoming spherical wave plus an outgoing spherical wave modified by a factor S_L, the partial-wave S-matrix elements. The integration over r is terminated for $r\lesssim L/p$. This yields

[21] For example, the onset of significant antiparticle contributions to the intermediate states in inelastic e-d scattering was estimated in I to occur at $q^2=4p^2\sim 0.8$ (BeV/c)^2.

a modified matrix element $K_L{}^A$, given by

$$K_L \to K_L{}^A = \int_{L/p}^{\infty} \left[\frac{h_L{}^{(2)}(pr)+S_L h^{(1)}(pr)}{2}\right] j_L(\tfrac{1}{2}qr)u(r)rdr$$
$$= \frac{1}{2}\int_{L/p}^{\infty} [j_L(pr)(1+S_L) - in_L(pr)(1-S_L)] j_L(\tfrac{1}{2}qr)u(r)rdr\,, \quad (51)$$

where j_L, n_L are the spherical Bessel and Neumann functions, and $h_L{}^{(1,2)}$ are spherical Hankel functions of the first and second kind.

The integral Eq. (51) is readily estimated by noting the Bessel and Neumann functions behave as trigonometric functions in the region, $pr \gg L$. At the quasielastic peak, $p \approx \frac{1}{2}q$, the functions $n_L(pr)$ and $j_L(\frac{1}{2}qr)$ essentially oscillate out of phase for large r. Hence, the integral containing their product is negligible compared to the integral containing the product, $j_L(pr)j_L(\frac{1}{2}qr)$. Furthermore, the functions $j_L(pr)$ are well behaved at small values of r, $\lim_{r\to 0} j_L(pr) \sim (pr)^L$, so there is little error incurred by extending the lower limit of integration to zero. In this approximation, Eq. (51) reduces simply to

$$K_L{}^A = \tfrac{1}{2}(1+S_L)\int_0^{\infty} j_L(pr)j_L(\tfrac{1}{2}qr)u(r)rdr = \tfrac{1}{2}(1+S_L)K_L\,. \quad (52)$$

The partial-wave matrix elements are thus reduced by a factor $\frac{1}{2}(1+S_L)$, where $|S_L|$ is less than unity in the absorptive region. This leads to the following modifications to the Born cross sections at very high energies, neglecting spin and the deuteron D state:

$$\frac{d^3\sigma}{(d\Omega_p de_0' d\Omega_e')} = \sigma_{\rm Mott}\frac{m^2 p}{4\pi^2 E}\{[\tilde{u}^A(k_p)]^2 G_p + [\tilde{u}^A(k_n)]^2 G_n\}\,,$$
$$\frac{d^2\sigma}{(d\Omega_e de_0')} = \sigma_{\rm Mott}\frac{m^2 p}{\pi E} M^A(p,q)[G_p+G_n]\,, \quad (53)$$

where

$$\tilde{u}^A(k_n) = \sum_L (2L+1)K_L{}^A P_L(\cos\theta) = \tilde{u}(k_p) + \sum_{L=0}^{L_{\rm Max}} (2L+1)K_L P_L(\cos\theta)\left(\frac{S_L-1}{2}\right),$$
$$M^A(p,q) = \frac{1}{2}\int_{-1}^{1} d(\cos\theta)[u^A(k_p)]^2 = M(p,q) + \sum_{L=0}^{L_{\rm Max}} (2L+1)K_L{}^2(S_L-1)\,. \quad (54)$$

G_p and G_n [defined by Eq. (61) of I], are the combinations of form factors which appear in the Rosenbluth cross section for electron-nucleon scattering.

In Eq. (54) the absorptive modifications are rewritten as additive corrections to the Born terms, $F(k_p)$, $M(p,q)$, and affect only the low partial waves. For a quantitative estimate of these corrections, we employed an opaque-disc model to describe the high-energy neutron-proton scattering. In the model, the scattering amplitude was assumed to be purely imaginary. The S-matrix elements were given by

$$S_L - 1 = -\gamma e^{-(\nu^2/p^2)L(L+1)}\,, \quad (55)$$

where γ is the opacity. γ was typically taken between 1.0 and 0.6, the value unity being favored at lower energies. The parameter ν^2 is fixed by the total n-p cross section

$$\nu^2 = 2\pi\gamma/\sigma_T\,. \quad (56)$$

Such a one-parameter model gives only fair agreement with available nucleon-nucleon cross sections in the range corresponding to center-of-mass momenta from 0.7 to 1.4 $(\mathrm{BeV}/c)^2$. The main defect of the model is the neglect of the real part of the scattering amplitude. Pertinent data was available mainly for p-p differential cross sections. Hence, these were used as an approximation to the n-p cross sections to obtain γ at various energies. ν^2 was still determined from n-p total cross-section values. With this model the following results were obtained:

The correction to the cross section $d^2\sigma/(d\Omega_e de_0')$ at the quasielastic peak was $-8.5\pm0.5\%$ for q^2 in the range 1.8 to 6.7 $(\mathrm{BeV}/c)^2$. The correction was remarkably uniform and was not changed significantly by including in the model an approximate correction for the real part of the scattering amplitude. The negative correction means that neutron form factors derived from uncorrected theoretical cross sections will be too small. This is particularly true for the charge form factor G_{En} which is rather sensitive to the subtraction analysis of nearly equal cross sections for e-p and e-d scattering.

The coincidence cross section $d^3\sigma/(d\Omega_e de_0' d\Omega_p)$ is more strongly peaked in the forward and backward directions when absorptive effects are included. The peaking arises from the suppression of contributions from the lower partial waves which are reduced by the absorption. The effect is illustrated in Fig. 1, which compares the corrected and Born-term values for the matrix element $\tilde{u}(k_p)$ at $q^2 = 3.89$ $(\mathrm{BeV}/c)^2$.

It is important to note that the absorptive correction given above applies to the peak cross section of coincidence experiments measuring quasielastic events only (i.e., no mesons produced). Another experiment is the measurement of the peak cross section when only the final electron is detected. For this case the corrections for absorption will be less than the above case with the "good" kinematics because some inelastic events will be counted along with the quasielastic events. Also, the average correction will be smaller for those experiments

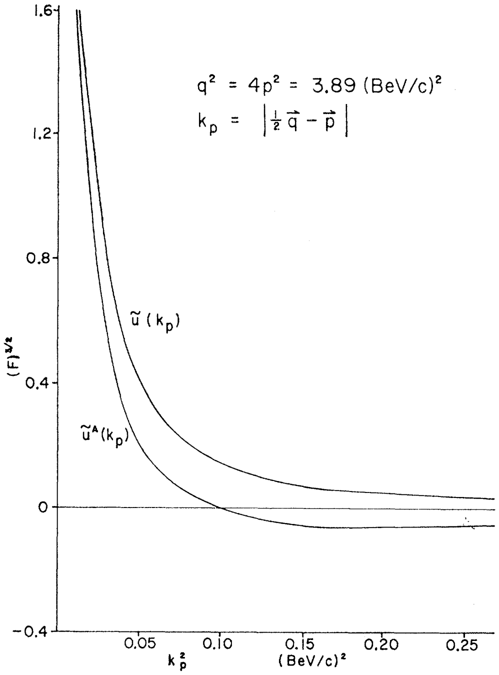

FIG. 1. Effect of absorption on the Fourier-Bessel transform function $\tilde{u}(k_p)$ at $p=0.981$ (BeV/c). Results were obtained using the opaque-disc model for absorption as described in the text and a Hulthén form for the deuteron wave function.

which integrate over the peak. This is because the absorption while reducing the peak may increase the number of events off the peak, and thereby affect the average number of events to a lesser degree. The averaging effect is accounted for by the sum rule which holds approximately for inelastic e-d scattering. The sum rule states[22] that if only the final electron is detected, the e-d cross section over the peak is equal to the sum of e-p and e-n cross sections. The result includes all the relativistic kinematic factors and to a first approximation, is independent of the aforementioned effects of strong interactions. However, it does assume that q^2 is constant over the quasielastic peak.

In summary, the purpose of the paper has been to present the corrections for final state interactions to inelastic electron-deuteron scattering in detail and indicate the region of momentum transfers over which the results are applicable. The changes are significant in the determination of the electromagnetic form factors of the neutron from the experimental cross sections. A major complication in the analysis appears at large momentum transfers [$q^2 \gtrsim 1.0$ (BeV/c)^2], where inelastic processes in the final n-p system require a more refined treatment of the theoretical cross sections. The exploratory calculation in Sec. IV suggests that these processes do change the cross section significantly ($\sim -9\%$). Derived estimates of the neutron charge form factor will consequently be even less certain at high q^2.

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[22] Reference 1, footnote 46.