from Eq. (10a) by using the same γ but A_η, C_η, and Q_η. We find that C_η is negligible and that $A_\eta \simeq 0.194(\gamma=\pi)$, $A_\eta \simeq 0.271(\gamma=\frac{1}{2}\pi)$. The resulting branching ratios are also listed in Table I. It is evident that a sizeable suppression of the $\eta \to 3\pi_0$ mode can be obtained in this way. On the other hand, assuming the *usual* form of the electromagnetic interaction, it is very difficult to obtain the required suppression without a final-state interaction. If we set $\gamma=0$ then Eq. (10a) goes to[13]

$$\Gamma_\eta(000)/\Gamma_\eta(+-0)=\tfrac{3}{2}/[1+\tfrac{1}{4}a_\eta^2(\tfrac{1}{3}Q_\eta)^2]. \qquad (16)$$

It can be seen from Eq. (16) that a large suppression would require a gross distortion of the spectrum. A form of the η-decay matrix element similar to ours has been proposed on phenomenological grounds by Foster *et al.*[3]

Next let us briefly consider the branching ratios obtained by using Eq. (12) for $\delta_0(t)$. Taking $\tau=\frac{1}{6}M_\pi$, a value which does not give any final-state effects for $K\to 3\pi$, we calculate from Eqs. (10a) and (13) the value of $\Gamma_\eta(000)/\Gamma_\eta(+-0)$ to be 1.08, 1.26, 1.33, and 1.39, respectively, for $\phi=90°$, 60°, 45°, and 30°.

[13] K. Wali, Phys. Rev. Letters 9, 120 (1962).

Thus, we have shown how the current-algebra approach of Hara and Nambu can be simply modified to take account of final-state interactions in $K\to 3\pi$ and $\eta\to 3\pi$ decays. Because the Q value for $K\to 3\pi$ is so low, we expect little change of the previous results in this case. However, if the low energy $I=0$ π-π scattering is sufficiently strong, this mechanism may help to explain the anomalously low $\eta\to 3\pi^0/\eta\to\pi^+\pi^-\pi^0$ branching ratio. If it is definitely established that δ_0 is too small in the low-energy region and the experimental branching ratio remains low, alternative[14] and possibly radical explanations will have to be entertained.

We would like to thank Professor Y. Nambu for helpful discussions and encouragement. One of us (Y.T.C) would like to thank Professor S. P. Rosen for discussions.

[14] Since the first version of our manuscript, unpublished reports on this problem have appeared by T. Das, M. Grynberg, and K. Kikkawa (Rochester); R. H. Graham, L. O'Raifeartaigh, and S. Pakvasa (Syracuse); L. Clavelli (Chicago); J. Pasupathy and A. Vaidya (Rochester); S. L. Adler (Princeton).

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Bjorken Limit and Sum Rules for the π^0 Lifetime and the $\rho\omega\pi$ Coupling Constant*

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The Bjorken limit is applied to obtain sum rules for the lifetime of π^0 and for the vector-meson coupling constants. In particular we obtain predictions that $g_{\rho\omega\pi}^2/4\pi=0.53\pm0.1$ and $\tau_{\pi^0}\simeq(0.84\pm0.83)\times10^{-16}$ sec. The latter is in good agreement with the experimental value $(0.89\pm0.18)\times10^{-16}$ sec. From our knowledge of the approximate value of $g_{\rho\omega\pi}$, we conclude that the high-energy behavior of the $\pi^0\gamma\gamma$ vertex function is not important to the pion lifetime. The results of this calculation constitute a consistency condition on the plausibility of the Bjorken limit.

In a recent paper Bjorken[1] has used the chiral $U(6)\otimes U(6)$ algebra to obtain a high-energy behavior for the Fourier transform of the time-ordered product matrix element of two current densities:

$$M_{\mu\nu}=i\int d^4x\, e^{ik\cdot x}\langle A|T\{j_\mu^\alpha(x),j_\nu^\beta(0)\}|B\rangle, \qquad (1)$$

where k is an arbitrary four-vector. Bjorken shows that if the matrix element can be truncated, then, when $|k_0|\to\infty$ for $\mathrm{Im}k_0\geq 0$,

$$M_{\mu\nu}\to -\frac{1}{k_0}\int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\langle A|[j_\mu^\alpha(0,\mathbf{x}),j_\nu^\beta(0)]|B\rangle. \qquad (2)$$

We shall sketch briefly the derivation of (2). By inserting a complete set of states into the commutator of Eq. (1), we may write

$$\begin{aligned}M_{\mu\nu}=&\,i\int d^4x\, e^{ik\cdot x}\sum_n\{\theta(x_0)e^{i(P_A-P_n)\cdot x}\\&\times\langle A|j_\mu^\alpha(0)|n\rangle\langle n|j_\nu^\beta(0)|B\rangle+\theta(-x_0)\\&\times e^{-i(P_B-P_n)\cdot x}\langle A|j_\nu^\beta(0)|n\rangle\langle n|j_\mu^\alpha(0)|B\rangle\},\\=&-\int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\sum_n\bigg\{\frac{1}{k_0+P_{A0}-P_{n0}}\\&\times\langle A|j_\mu^\alpha(0,\mathbf{x})|n\rangle\langle n|j_\nu^\beta(0)|B\rangle-\frac{1}{k_0-P_{B0}+P_{n0}}\\&\times\langle A|j_\nu^\beta(0)|n\rangle\langle n|j_\mu^\alpha(0,\mathbf{x})|B\rangle\bigg\}.\end{aligned}$$

* Work supported by the National Science Foundation.

[1] J. D. Bjorken, Phys. Rev. 148, 1467 (1966). This application of the Bjorken limit arises out of private communications between Professor J. D. Bjorken and Professor D. A. Geffen.

If the matrix element can be truncated, we get Eq. (2).

We shall apply Eq. (2) to the electromagnetic current density and take matrix element between the vacuum and a π^0 state to obtain a relation between the coupling constants of the vector-meson–vector-meson–pion interaction and the vector-meson–pion–photon interaction. The requirement of the $1/k_0$ behavior of the matrix element also enables us to obtain a sum rule for the decay width of $\pi^0 \to 2\gamma$.

We let $j_\mu{}^\alpha = J_\mu$, $j_\nu{}^\beta = J_\nu$, $|A\rangle = |0\rangle$, and $|B\rangle = |\pi^0,q\rangle$, where J_μ is the electromagnetic current density of hadrons. Follow Ref. 1, we assume a quark structure for J_μ, then,

$$[J_\mu(0,\mathbf{x}),J_\nu(0)] = -2ie^2\varepsilon_{\mu\nu\lambda\tau}\xi^\lambda j_5{}^\tau(0)\delta^4(x) + \text{grad. term}, \quad (3)$$

where $\xi^\lambda = (1,0,0,0)$, $\varepsilon_{\mu\nu\lambda\tau}$ is the complete antisymmetric tensor of the Minkowski space, and $\varepsilon_{0123} = 1$, and the gradient term is the so-called Schwinger term which will be neglected in the following consideration.[2] The operator $j_{5\sigma}$ is defined as

$$j_{5\sigma} = (2/9)A_\sigma{}^{X^0} + (1/3)A_\sigma{}^{\pi^0} + (1/3\sqrt{3})A_\sigma{}^\eta,$$

where $A_\sigma{}^i$, $i = X^0, \pi^0, \eta$, is the axial-vector current which has the same $SU(3)$ transformation properties as the particle i.

If we substitute (3) into (2), we have

$$M_{\mu\nu} \to (2i/3k_0{}^2)e^2\varepsilon_{\mu\nu\lambda\tau}(k_0\xi)^\lambda\langle 0|A^{\pi^0\tau}|\pi^0,q\rangle + O(1/k_0{}^2).$$

The evaluation of the above matrix element is standard; we obtain

$$\langle 0|A_\tau{}^{\pi^0}|\pi^0,q\rangle = iC_{\pi^0}q_\tau,$$

where $C_{\pi^0} = M_NF_A/g_{NN\pi}$, $F_A = 1.18$, $g_{NN\pi}{}^2/4\pi \simeq 14.6$, and M_N is the mass of the nucleon. Then

$$M_{\mu\nu} \to \frac{2e^2}{3}C_{\pi^0}\frac{1}{k_0{}^2}\varepsilon_{\mu\nu\lambda\tau}(k_0\xi)^\lambda q^\tau + O(1/k_0{}^2). \quad (4)$$

On the other hand, Eq. (2) can be evaluated directly by inserting a complete set of states in the commutator. It can be shown, however, that the result depends on the value of **k**. This is because, as has been noted by Furlan *et al.*,[3] an expression like the one on the right-hand side of Eq. (2) is noncovariant. Consequently, the result of the limiting process of $|k_0| \to \infty$ depends on the frame of reference of k_0. We shall follow a covariant procedure to calculate (2) by calculating (1) in a dispersion relation and letting $|k_0| \to \infty$.[3]

We may write

$$M_{\mu\nu} = \varepsilon_{\mu\nu\lambda\tau}\frac{k^\lambda k'^\tau}{m_\pi}F(k^2,k'^2), \quad (5)$$

where $k' = q - k$. The quantity $F(k^2,k'^2)$ defined in the above expression is an invariant function in k^2 and k'^2, and is symmetric in the interchange of these two variables, i.e.,

$$F(k^2,k'^2) = F(k'^2,k^2).$$

The above equality follows Eqs. (1) and (5). We note that k^2 and k'^2 can be viewed as the masses of the virtual photons at the $\pi^0\gamma\gamma$ vertex, and $F(0,0)$ is related to the decay width of $\pi^0 \to 2\gamma$ by the formula

$$\Gamma_{\pi^0\to 2\gamma} = \frac{|F(0,0)|^2}{64\pi}m_{\pi^0}. \quad (6)$$

To evaluate the form factor $F(k^2,k'^2)$ we shall assume a once-subtracted dispersion relation[4]:

$$F(k^2,k'^2) = F(k^2,0) + \frac{k'^2}{\pi}\int\frac{\Delta F_x(k^2,x)dx}{x(x-k'^2)}, \quad (7)$$

where

$$\Delta F_x(k^2,x) = \pi\lambda_\rho f_{\rho\pi\gamma}(k^2)\delta(x-m_\rho{}^2) + \pi\lambda_\omega f_{\omega\pi\gamma}(k^2) \times\delta(x-m_\omega{}^2) + \pi\lambda_\varphi f_{\varphi\pi\gamma}(k^2)\delta(x-m_\varphi{}^2) + R(k^2,x)\theta(x-\Lambda_0{}^2). \quad (8)$$

In Eq. (8) we write explicitly the single-particle contribution from the vector mesons and include all the high-mass contributions in the function $R(k^2,x)\times\theta(x-\Lambda_0{}^2)$, where $\Lambda_0{}^2 \gg m_\rho{}^2$. The quantities λ_V, $V = \rho^0, \omega, \varphi$, stand for the $V-\gamma$ coupling constants of dimension (mass)^2. The function $f_{V\pi\gamma}(k^2)$ is defined through the following expression:

$$\langle V, q-k, \epsilon|J_\mu(0)|\pi^0, q\rangle = -\varepsilon_{\mu\nu\lambda\tau}\epsilon^\nu k^\lambda q^\tau\frac{1}{m_{\pi^0}}f_{V\pi\gamma}(k^2), \quad (9)$$

and $f_{V\pi\gamma}(0)$ is the dimensionless phenomenological coupling constant of the radiative decay $V \to \pi^0 + \gamma$.

By (8), Eq. (7) becomes

$$F(k^2,k'^2) = F(k^2,0) + k'^2\sum_{V=\rho,\omega,\varphi}\frac{\lambda_Vf_{V\pi\gamma}(k^2)}{m_V{}^2(m_V{}^2-k'^2)} + \frac{k'^2}{\pi}\int_{\Lambda_0{}^2}^\infty\frac{R(k^2,\Lambda^2)}{\Lambda^2(\Lambda^2-k'^2)}d\Lambda^2. \quad (10)$$

The evaluation of $F(k^2,0)$ is similar to that of Eq. (7). The result is

$$F(k^2,0) = F(0,0) + k^2\sum_{V=\rho,\omega,\varphi}\frac{\lambda_Vf_{V\pi\gamma}(0)}{m_V{}^2(m_V{}^2-k^2)} + \frac{k^2}{\pi}\int_{\Sigma_0{}^2}^\infty\frac{S(\Lambda^2)}{\Lambda^2(\Lambda^2-k^2)}d\Lambda^2, \quad (11)$$

where $S(\Lambda^2)$ represents the high-mass contribution to the absorptive part of $F(k^2,0)$ and $\Sigma_0{}^2 \gg m_\rho{}^2$.

[2] F. Buccella, G. Veneziano, R. Gatto, and S. Okubo, Phys. Rev. **149**, 1268 (1966).

[3] G. Furlan, F. Lanny, C. Rossetti, and G. Segrè, Nuovo Cimento **40**, 597 (1965).

[4] An unsubtracted dispersion relation does not predict correctly the π^0 lifetime. Nevertheless, we shall show later that the unsubtracted dispersion relation gives a rather close prediction to the experimental value of the π^0 lifetime. The difference is fixed up by the Bjorken limit. See also Eqs. (16) and (24).

To calculate $f_{V\pi\gamma}(k^2)$ we apply again a once-subtracted dispersion relation.[5] Using Eq. (9) to obtain the absorptive part of $f_{V\pi\gamma}(k^2)$ and separating the contributions of the vector mesons from those of the high masses, we find

$$f_{V\pi\gamma}(k^2)=f_{V\pi\gamma}(0)+k^2\sum_{V'=\rho,\omega,\varphi}\frac{\lambda_{V'}g_{VV'\pi}(m_{V'}^2)}{m_{V'}^2(m_{V'}^2-k^2)}+\frac{k^2}{\pi}\int_{\Omega_V^2}^{\infty}\frac{T_\nu(\Lambda^2)}{\Lambda^2(\Lambda^2-k^2)}d\Lambda^2, \quad (12)$$

where $g_{VV'\pi}(m_{V'}^2)$ is the "phenomenological" coupling constant of the V-V'-π interaction with all the three particles on the mass shell and $g_{VV'\pi}(m_{V'}^2)=g_{V'V\pi}(m_V^2)$. Again the function $T_V(\Lambda^2)$ represents the high-mass contributions and $\Omega_V^2\gg m_\rho^2$.

The form factor $F(k^2,k'^2)$ can now be obtained by substituting Eqs. (11) and (12) into (10) and symmetrizing in k^2 and k'^2. Abbreviating $f_{V\pi\gamma}(0)$ and $g_{VV'\pi}(m_{V'}^2)$ by $f_{V\pi\gamma}$ and $g_{VV'\pi}$, we obtain the expression for the form factor:

$$F(k^2,k'^2)=F(0,0)+\sum_{V=\rho,\omega,\varphi}\frac{\lambda_V f_{V\pi\gamma}}{m_V^2}\left(\frac{k^2}{m_V^2-k^2}+\frac{k'^2}{m_V^2-k'^2}\right)+\sum_{V,V'=\rho,\omega,\varphi}\frac{\lambda_V\lambda_{V'}g_{VV'\pi}k^2k'^2}{m_V^2m_{V'}^2(m_V^2-k^2)(m_{V'}^2-k'^2)}$$
$$+\frac{1}{2\pi}\int_{\Lambda_0^2}^{\infty}d\Lambda^2\left[\frac{k'^2R(k^2,\Lambda^2)}{\Lambda^2(\Lambda^2-k'^2)}+\frac{k^2R(\Lambda^2,k'^2)}{\Lambda^2(\Lambda^2-k^2)}\right]+\frac{1}{2\pi}\int_{\Sigma_0^2}^{\infty}d\Lambda^2\frac{S(\Lambda^2)}{\Lambda^2}\left(\frac{k^2}{\Lambda^2-k^2}+\frac{k'^2}{\Lambda^2-k'^2}\right)$$
$$+\frac{1}{2\pi}\sum_{V=\rho,\omega,\varphi}\frac{\lambda_V}{m_V^2}\int_{\Omega_V^2}^{\infty}d\Lambda^2\frac{T_V(\Lambda^2)}{\Lambda^2}\left[\frac{k^2k'^2}{(m_V^2-k^2)(\Lambda^2-k'^2)}+\frac{k^2k'^2}{(m_V^2-k'^2)(\Lambda^2-k^2)}\right]. \quad (13)$$

The asymptotic form of $F(k^2,k'^2)$ can now be easily derived. It reads

$$\lim_{|k_0|\to\infty}F(k^2,(q-k)^2)=F(0,0)-2\sum_{V=\rho,\omega,\varphi}\frac{\lambda_V}{m_V^2}f_{V\pi\gamma}+\sum_{V,V'=\rho,\omega,\varphi}\frac{\lambda_V\lambda_{V'}}{m_V^2m_{V'}^2}g_{VV'\pi}-B-\frac{1}{k_0^2}\left\{2\sum_{V=\rho,\omega,\varphi}\lambda_V f_{V\pi\gamma}-\sum_{V,V'=\rho,\omega,\varphi}\lambda_V\lambda_{V'}g_{VV'\pi}\left(\frac{1}{m_V^2}+\frac{1}{m_{V'}^2}\right)+m_\rho^2B'\right\}+O(1/k_0^2). \quad (14)$$

To obtain Eq. (14) we have used the identity

$$\frac{1}{x-y}=-\frac{1}{y}-\frac{x}{y^2}+\frac{x^2}{y^2}\frac{1}{x-y}$$

to expand the denominators of the various integrals in Eq. (13) and assumed that the resultant integrals so obtained are convergent. The quantities B and B' are defined as follows:

$$B=\frac{1}{2\pi}\int_{\Lambda_0^2}^{\infty}\frac{d\Lambda^2}{\Lambda^2}[R(\infty,\Lambda^2)+R(\Lambda^2,\infty)]+\frac{1}{\pi}\int_{\Sigma_0^2}^{\infty}\frac{d\Lambda^2}{\Lambda^2}S(\Lambda^2)-\frac{1}{\pi}\sum_{V=\rho,\omega,\varphi}\frac{\lambda_V}{m_V^2}\times\int_{\Omega_V^2}^{\infty}\frac{d\Lambda^2}{\Lambda^2}T_V(\Lambda^2),$$

$$m_\rho^2B'=\frac{1}{2\pi}\int_{\Lambda_0^2}^{\infty}d\Lambda^2[R(\infty,\Lambda^2)+R(\Lambda^2,\infty)]+\frac{1}{2\pi}\int_{\Lambda_0^2}^{\infty}\frac{d\Lambda^2}{\Lambda^2}\times\frac{d}{dt}[R(1/t,\Lambda^2)+R(\Lambda^2,1/t)]_{t=0}+\frac{1}{\pi}\int_{\Sigma_0^2}^{\infty}d\Lambda^2S(\Lambda^2)-\frac{1}{\pi}\sum_{V=\rho,\omega,\varphi}\lambda_V\int_{\Omega_V^2}^{\infty}d\Lambda^2\left(\frac{1}{m_V^2}+\frac{1}{\Lambda^2}\right)T_V(\Lambda^2).$$

We shall show later that our knowledge on the coupling constant enables us to conclude that B and B' are small. Comparing Eqs. (4) and (14), we obtain two sum rules:

$$F(0,0)-2\sum_{V=\rho,\omega,\varphi}\frac{\lambda_V f_{V\pi\gamma}}{m_V^2}+\sum_{V,V'=\rho,\omega,\varphi}\frac{\lambda_V\lambda_{V'}g_{VV'\pi}}{(m_V^2m_{V'}^2)}-B=0, \quad (15)$$

$$2\sum_{V=\rho,\omega,\varphi}\lambda_V f_{V\pi\gamma}/m_\rho^2-\sum_{V,V'=\rho,\omega,\varphi}\frac{\lambda_V\lambda_{V'}}{m_\rho^2}g_{VV'\pi}\times\left(\frac{1}{m_V^2}+\frac{1}{m_{V'}^2}\right)+B'=-\frac{2e^2}{3m_\rho^2}C_{\pi^0}. \quad (16)$$

To extract useful information from Eqs. (15) and (16), we must know the various vector-meson coupling constants. At the present time the vector coupling con-

[5] Here we also take a subtracted dispersion relation because an unsubtracted dispersion relation will give at low energies the result, for example, $f_{\rho\pi\gamma}=-(e/3\gamma_\rho)g_{\rho\omega\pi}$, where $g_{\rho\omega\pi}$ is the $\rho\omega\pi$ coupling constant. We want to avoid such a result. Also we are aware of the situation that $f_{\rho\pi\gamma}$ may be zero [see A. Donnachie and Graham Shaw, Ann. Phys. (N. Y.) **37**, 333 (1966)]; then the above result will reduce to $g_{\rho\omega\pi}=0$, which makes the decay $\omega\to3\pi$ hard to understand theoretically.

stants cannot all be evaluated phenomenologically; we must resort to models to evaluate some of them. We apply current algebra, taking symmetry-breaking effects to the first order, to obtain relations among the vector-meson coupling constants.[6] Incorporating those relations with the generally accepted information that[7,8]

$$f_{\varphi\pi\gamma}\simeq 0,\quad g_{\rho\varphi\pi}\simeq 0,\quad \sin\theta\simeq 1/\sqrt{3},$$

where θ is the ω-φ mixing angle, we have

$$\lambda_\rho = em_\rho^2/\gamma_\rho,\quad \lambda_\omega = -em_\rho^2/3\gamma_\rho,\quad \lambda_\varphi=\sqrt{2}em_\rho^2/3\gamma_\rho,$$
$$f_{\rho\pi\gamma}=-\tfrac{1}{3}f_{\omega\pi\gamma},$$

where $\gamma_\rho\simeq\gamma_{\rho\pi\pi}$ is the $\rho\pi\pi$ coupling constant. Since $m_\rho^2\simeq m_\omega^2$, Eqs. (15) and (16) are reduced to

$$F(0,0)+\frac{4e}{3\gamma_\rho}f_{\omega\pi\gamma}-\frac{2e^2}{3\gamma_\rho^2}g_{\rho\omega\pi}-B=0\,, \tag{17}$$

$$\frac{2e}{3\gamma_\rho}f_{\omega\pi\gamma}-\frac{2e^2}{3\gamma_\rho^2}g_{\rho\omega\pi}-\frac{e^2C_{\pi^0}}{3m_\rho^2}-B'=0\,. \tag{18}$$

From Eqs. (18) and (17) we can eliminate $g_{\rho\omega\pi}$ and obtain

$$F(0,0)=-\left(\frac{2e}{3\gamma_\rho}f_{\omega\pi\gamma}+\frac{e^2C_{\pi^0}}{3m_\rho^2}+B'-B\right). \tag{19}$$

Using the recent data of the decay width $\omega\to\pi+\gamma$ and $\rho\to 2\pi$,[9] we obtain that

$$f_{\omega\pi\gamma}^2/4\pi\simeq 0.15(1\pm0.2)\alpha\,,$$
$$\gamma_\rho^2/4\pi\simeq 2.7\,.$$

Then (19) gives the prediction

$$\begin{aligned}F(0,0)&\simeq -\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)(4.6\pm0.4)\times10^{-3}-B',\quad f_{\omega\pi\gamma}/(\gamma_\rho g_{NN\pi})>0\\ &\simeq -\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)(3.4\pm0.4)\times10^{-3}-B',\quad f_{\omega\pi\gamma}/(\gamma_\rho g_{NN\pi})<0.\end{aligned} \tag{20}$$

Since $B'-B\simeq B$ for $\Lambda_0^2, \Sigma_0^2, \Omega_V^2\gg m_\rho^2$, we have dropped the term B in the above expression. Comparing the predicted values in (20) with the experimental one

$$|F_{\exp}(0,0)|\simeq(3.3\pm0.4)\times10^{-3},$$

which is obtained by taking $\tau_{\pi^0}\simeq(0.89\pm0.18)\times10^{-16}$ sec[9] and applying Eq. (6), we can solve for the values of B': For $f_{\omega\pi\gamma}/(\gamma_\rho g_{NN\pi})>0$, we obtain

$$\begin{aligned}B'&\simeq -\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)1.2\times10^{-3},\quad \operatorname{sgn}F(0,0)=-\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)\\ &\simeq -\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)8\times10^{-3},\quad \operatorname{sgn}F(0,0)=\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)\,,\end{aligned} \tag{21}$$

and for $f_{\omega\pi\gamma}/(\gamma_\rho g_{NN\pi})<0$, we get

$$B'\simeq 0\,. \tag{22}$$

From Eqs. (18) and (21) which correspond to the case with appreciable contributions from the high-mass states, the following predictions on the sign and magnitude of $g_{\rho\omega\pi}$ result:

(i): For $B'\simeq -\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)1.2\times10^{-3}$,
$g_{\rho\omega\pi}^2/4\pi\simeq 0.14$, $\operatorname{sgn}g_{\rho\omega\pi}=\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)$.

(ii): For $B'\simeq -\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)8\times10^{-3}$,
$g_{\rho\omega\pi}^2/4\pi\simeq 0.08$, $\operatorname{sgn}g_{\rho\omega\pi}=-\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)$.

From Eqs. (18) and (22) which correspond to the case of negligible high-mass contributions, we have

(iii):
$$g_{\rho\omega\pi}=\operatorname{sgn}(f_{\omega\pi\gamma}/\gamma_\rho)\times\{|\gamma_\rho f_{\omega\pi\gamma}/e|+\gamma_\rho^2|C_{\pi^0}|/2m_\rho^2\}\,, \tag{23}$$
$$g_{\rho\omega\pi}^2/4\pi\simeq 0.53\pm0.11\,.$$

If we ignore completely the contribution of the high-mass states, i.e., $B'=0$, we also have

$$F(0,0)=\operatorname{sgn}\left(\frac{f_{\omega\pi\gamma}}{\gamma_\rho}\right)\frac{2}{3}\frac{\alpha^2}{\gamma_\rho^2/4\pi}\left\{\left|\frac{\gamma_\rho f_{\omega\pi\gamma}}{e}\right|-\frac{\gamma_\rho^2|C_{\pi^0}|}{2m_\rho^2}\right\} \tag{24}$$

which gives a π^0 mean life $\tau_{\pi^0}\simeq(0.84\pm0.18)\times10^{-16}$ sec.

The solutions (i) and (ii) of $g_{\rho\omega\pi}$, which are very small, do not agree with our knowledge of the approximate value of $g_{\rho\omega\pi}$.[10] The solution (iii) is, however, in agreement with what we know about the value of $g_{\rho\omega\pi}$. A comparison of the values of $g_{\rho\omega\pi}^2/4\pi$ and $F(0,0)$ of the solution (iii) with those obtained from other methods is shown in Table I.[11] An interesting feature of the present method is that it predicts also the relative signs of $g_{\rho\omega\pi}$,

[6] Bing-lin Young, Ph.D. thesis, University of Minnesota, 1966 (unpublished).

[7] Phenomenologically we find $g_{\rho\varphi\pi}^2/4\pi\simeq\alpha^2$. This is negligibly small in comparison with unity, which we expect to be the order of magnitude of strong-interaction coupling constants. Since $f_{\varphi\pi\gamma}$ is of the order of $(e/\gamma_\rho)g_{\rho\varphi\pi}$, then $f_{\varphi\pi\gamma}^2/4\pi\simeq\alpha^3$ which is also negligibly small in comparison with α, the strength characterizing the first-order electromagnetic interaction.

[8] The value of the ω-φ mixing angle is supported by the mass formula and the current-algebra calculations.

[9] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[10] Because of the approximate validity of the vector-meson-dominance model, we expect $g_{\rho\omega\pi}^2/4\pi$ to be the order of $(\gamma_\rho^2/4\pi\alpha)\times(f_{\omega\pi\gamma}^2/4\pi)$, which is about 0.4. The small values of $g_{\rho\omega\pi}^2/4\pi$ as obtained in solutions (i) and (ii) also make the decay $\omega\to 3\pi$ hard to understand theoretically.

[11] Because of the restriction on the mass values of ω, ρ, and π^0, $g_{\rho\omega\pi}$, which is defined for all three particle being on the mass shell in the present model, cannot be determined phenomenologically. The value determined from the single-vector-meson-dominance model is $g_{\rho\omega\pi}(0)$ while the value determined from the decay $\omega\to 3\pi$ by means of the model of M. Gell-Mann, D. Sharp, and W. G. Wagner [Phys. Rev. Letters **8**, 261 (1962)] is an average of the values of $g_{\rho\omega\pi}(k^2)$ range from $g_{\rho\omega\pi}(m_\pi^2)$ to $g_{\rho\omega\pi}((m_\omega-m_\pi)^2)$. In contradiction to the general expectations that $g_{\rho\omega\pi}(k^2)$ should be a smooth function of k^2 and the difference between these three values is small, the present calculation indicates that $g_{\rho\omega\pi}(k^2)$ may not be a slow varying function of k^2. This gives a possible explanation of the difference between the values of $g_{\rho\omega\pi}$ obtained from the Gell-Mann, Sharp, and Wagner model and the single-vector-meson-dominance model.

TABLE I. Values of $g_{\rho\omega\pi}^2/4\pi$ and $|F(0,0)|$ calculated in different models. The experimental value of $|F(0,0)|$ is $(3.3\pm0.4)\times10^{-3}$.

$g_{\rho\omega\pi}^2/4\pi$	$\lvert F(0,0)\rvert$	Sign of $f_{\omega\pi\gamma}/(\gamma_\rho g_{\rho\omega\pi})$	Models
0.45 ± 0.08	...	...	Gell-Mann, Sharp, and Wagner model[a]
0.41 ± 0.08	$(4.0\pm0.4)\times10^{-3}$	+	Single-vector-meson-dominance model, $\pi^0\to\gamma+\rho^0,\omega$, $\omega\to\pi^0+\rho$ ($\rho^0,\omega\searrow\gamma$; $\rho\searrow\gamma$)
0.64 ± 0.11	...	...	Two-vector-meson-dominance model of the decay $\pi^0\to\rho^0+\omega\to2\gamma$
0.32	...	...	Gasiorowicz and Geffen's Adler-Weisberger type sum rule[b]
...	$(5.4\pm0.3)\times10^{-3}[\tau_{\pi^0}\simeq(0.34\pm0.3)\times10^{-16}\text{ sec}]$	...	Lautrup and Olesen's two-baryon model[c]
0.54 ± 0.11	$(3.4+0.4)\times10^{-3}[\tau_{\pi^0}\simeq(0.84\pm0.18)\times10^{-16}\text{ sec}]$	+	The present model

[a] The formula of the decay width of $\omega\to3\pi$ in the model of M. Gell-Mann, D. Sharp, and W. G. Wagner [Phys. Rev. Letters 8, 26 (1962)] has been reconsidered in Ref. 6 by using the currently accepted mass values of ρ and ω.
[b] S. Gasiorowicz and D. A. Geffen, Phys. Letters 22, 344 (1966).
[c] B. Lautrup and P. Olesen, Phys. Letters 22, 342 (1966).

$f_{\omega\pi\gamma}/\gamma_\rho$, and $g_{NN\pi}$. These relative signs may provide an alternative way of checking the present model if the experiments to carry out such sign measurements are feasible.

The indication that $B\simeq0$ and $B'\simeq0$ give good predictions seems to suggest the following feature for high-mass contributions to the sum rules (17) and (18): The dynamics are oriented in such a way that the high-mass contributions from various individual terms that contribute to (17) and (18) cancel each other and the total effect of the high-mass contributions is negligible. This does not imply that the high-mass contribution of each individual term is also small. Let us consider the $\omega\pi^0\gamma$ vertex function defined by Eq. (9) as an example. If we apply the Bjorken limit to this vertex function and use the hypothesis of partially conserved axial-vector current (PCAC) in the infinite-momentum limit of π^0, we obtain the following sum rules:

$$f_{\omega\pi\gamma}(-\infty)=f_{\omega\pi\gamma}-\frac{\lambda_\rho}{m_\rho^2}g_{\rho\omega\pi}-\int d\Lambda^2\frac{T_\omega(\Lambda^2)}{\Lambda^2},$$

$$\frac{\lambda_\rho}{m_\rho^2}g_{\rho\omega\pi}+\int d\Lambda^2\frac{T_\omega(\Lambda^2)}{m_\rho^2}=0,$$

where the first and the second sum rules are obtained, respectively, from the zeroth- and first-order expansions of the inverse square of the π^0 energy. Note that the Bjorken limit is zero. These expressions indicate that the high-mass contribution $T_\omega(\Lambda^2)$ may not be small.

As shown in Eqs. (23) and (24), and Table I, the present model is close to the single-vector-meson model. The difference between the two is attributed to the Bjorken limit which may be considered as a high-energy correction to the low-energy vector-meson contributions. Because of the smallness of the Bjorken limit, the difference between these two models is also small. Therefore we may conclude that the single-vector-meson-dominance model might be expected to work reasonably well at low energies as well as high energies, although it does not give perfect predictions.

Finally we should like to remark that since there are ambiguities in defining the coupling constant $g_{\rho\omega\pi}$,[11] we shall not consider the present calculation as a strict test of the validity of the Bjorken limit. Instead, we consider it as a consistency condition on the plausibility of the Bjorken limit. A test of this limit has been considered by the author in connection with model studies of the decay process $\eta\to\mu^++\mu^-$.[12] It has been indicated that detailed experimental information is required in this connection, e.g., the decay width $\eta\to\mu^++\mu^-$, the lepton pair distribution in the Dalitz pair decay of η, $f_{\omega\pi\gamma}^2/4\pi$, $g_{\rho\omega\pi}^2/4\pi$, the width of $\eta\to2\gamma$, as well as the sign of $f_{\omega\pi\gamma}/(\gamma_\rho g_{\rho\omega\pi})$.

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[12] Reference 6; Bing-lin Young, this issue, Phys. Rev. 161, 1620 (1967). Applications of the Bjorken limit in these two articles arise also out of private communications between Professor D. A. Geffen and Professor J. D. Bjorken.