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Baryon Mass Splittings in an $SU(6)\times O(3)$ Quark Model

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The positive- and negative-parity baryon mass shifts are investigated under the assumption that these particles belong, respectively, to the representations (**56**,1) and (**70**,3) of the group $SU(6)\times O(3)$, which are dynamically realizable from a 3-quark model with totally symmetric (S) functions, as found earlier by one of the authors. Two different types of $SU(2)$-invariant central forces $V^{(1)}$ and $V^{(2)}$, each of which is shown to be in conformity with the usual mass relations for the **56** states, are employed. One of these forces ($V^{(2)}$) is, however, found to violate the Gell-Mann–Okubo formula for certain negative-parity octets. It is also found that appreciable mixtures of both $V^{(1)}$ and $V^{(2)}$ are necessary even for a qualitative representation of the experimental masses. The effect of an $SU(2)$-invariant spin-orbit force of the type $\boldsymbol{\tau}_1\text{-}\boldsymbol{\tau}_2$ of modest strength ($\sim$25 MeV) is found to be very helpful in producing a reasonably good fit to the actual masses of the negative-parity baryons. Such a force has, however, no first-order effect on the 56 masses, on the assumption of orbital S functions, which can be constructed only with S-wave Q-Q pairs. The significance of this result is briefly discussed in connection with the question of quark statistics.

1. INTRODUCTION

ONE of the earliest concerns of the various symmetry groups (dynamical or otherwise) that have been proposed in recent years[1] has been the pattern of mass splittings in successive orders of hierarchy in symmetry breaking (strong, medium, and electromagnetic). Within $SU(3)$, the spectacular success of the Gell-Mann–Okubo (GMO) formula[2] for the mesons and baryons has given it the status of a convenient reference point with respect to which the mass formulas of the symmetry groups (at the appropriate stage of symmetry breaking) should be calibrated before their more detailed predictions come in for further scrutiny. These approaches may be broadly classified under two heads: (a) those based directly on general symmetry-breaking effects on a bigger symmetry group like $SU(6)$ [3] or $\tilde{U}(12)$,[4] and (b) more detailed dynamical models, of which the quark model[5–8] has received the greatest attention. The quark model, with its nonrelativistic features, seems to be capable of yielding in a very simple way a rich variety of results, many of which are in surprisingly good accord with experiment. In particular, it gives a number of interesting mass relations between hadrons, going far beyond the Gell-Mann–Okubo formula. For example, the simple assumption of two-body isospin-invariant Q-Q and Q-$\bar{Q}$ forces leads to the Schwinger mass formula[9] for the mesons; the equal mass difference between the pseudoscalar and vector mesons of $Y=0$ and 1, exemplified by $K^{*2}-\rho^2=K^2-\pi^2$; and a corresponding result for the octet and decuplet of baryons.[6,8] Inclusion of electromagnetic effects in the Q-Q potentials leads in a similar way to the Coleman-Glashow formula[10] and other interesting results.[11]

Most of these investigations have so far been confined to the **56** of baryons and the nonets of mesons (vector and pseudoscalar). Since, on the other hand, the results seem to provide a good deal of confidence in the *predictions* of the quark model (as distinct from the *existence* of the quarks), we feel that it is not too early to extend such investigations to the higher-mass baryons as well. This may be particularly interesting

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[1] For an exhaustive list of references on symmetry groups, see A. Pais, Rev. Mod. Phys. **38**, 215 (1966).
[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).
[3] M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 418 (1964); **13**, 509 (1964); K. Kawarabayashi, *ibid.* **14**, 86 (1965).
[4] D. J. Williams, Nuovo Cimento **44**, 330 (1966).
[5] G. Morpurgo, Physics **2**, 93 (1965).
[6] R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).
[7] R. H. Dalitz, in *Proceedings of the International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967).
[8] A. N. Tavkhelidze, in *Proceedings of the Seminar on High Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1967).
[9] J. Schwinger, Phys. Rev. Letters **12**, 237 (1964).
[10] S. Coleman and G. Glashow, Phys. Rev. Letters **6**, 423 (1960).
[11] H. R. Rubinstein, Phys. Rev. Letters **17**, 41 (1966).

for the negative-parity baryons, of which a good number have already been identified and their spins determined.

While the $SU(3)$ classification of these resonances still leaves much to be desired, it is perhaps a good working hypothesis to assume the Dalitz classification of these states,[6,7] which have been recognized either in terms of phase-shift analyses, or through direct observation of peaks in the appropriate mass distributions. The group appropriate to this classification is, of course, $SU(6)\times O(3)$, with the representation (**56**,1) for the usual octet and decuplet[12] and (**70**,3) for the negative-parity baryons. While a few (as yet missing) particles could still prove a hindrance to the classification according to this group and in particular according to the representation (**70**,3), the latter representation appears to us as one of the most economical, predicting considerably fewer unobserved particles than, e.g., higher representations like **1134** of $SU(6)$, proposed by some authors.[13] Further evidence for the qualitative validity of (**70**,3) for the negative-parity baryons is provided by a recent exhaustive calculation of the strong decay widths of these resonances[14] into pseudoscalar mesons plus positive-parity baryons, which gave a pattern in rather good accord with the available data. Finally, the group $SU(6)\times O(3)$ seems to have a simple dynamical appeal[15,16] based on an extension of the Wigner supermultiplet potential[17] applied to Q-Q forces and analyzed into individual partial waves of low l values.

For these reasons we shall take the $SU(6)\times O(3)$ model for the low-lying baryon states quite seriously for the purpose of this investigation of their mass shifts due to $SU(3)$-breaking Q-Q forces. As we shall not be interested in the electromagnetic mass shifts,[11] these forces will be taken as isospin-invariant. Nor shall we consider the effect of direct three-body forces on the mass formulas.

One of our main concerns will be to examine in somewhat greater detail the types or combinations of Q-Q forces which give rise to the GMO and allied formulas in the **56** of baryons, and the extent to which the latter may be satisfied for the negative-parity baryons. In a somewhat more quantitative way, we shall try to see if suitable combinations of such forces can be constructed so as to produce a mass pattern which is at least roughly in accord with the experimental levels for the negative-parity baryons.

In Sec. 2 we consider two independent sets of central potentials, termed $V^{(1)}$ and $V^{(2)}$, each of which is capable of reproducing the $SU(6)$ results for the **56** of baryons. In Sec. 3, the general mass pattern produced by each of $V^{(1)}$ and $V^{(2)}$ on the negative parity baryons is investigated, along with the effect of a spin-orbit force operative in the p states of Q-Q pairs. Section 4 is concerned with a semiquantitative fit to the actual masses of the negative-parity baryons to the extent that their J^P and $SU(3)$ assignments have been identified. The main features of the results are summarized, with particular emphasis on the essential roles played by both the potentials $V^{(1)}$ and $V^{(2)}$ in producing the negative-parity masses. The role of the spin-orbit force is also discussed in connection with the question of Fermi statistics for the **56** of the baryons.

2. SYMMETRY-BREAKING EFFECTS AND THE 56 MASS RELATIONS

The $3Q$ structures of the positive- and negative-parity baryons were given in a recent paper[16] by one of us (to be referred to as PDBR) on the basis of symmetric (S) wave functions which are consistent with parastatistics.[18] The reasons for the choice of symmetric functions rather than the antisymmetric (A) ones demanded by Fermi statistics are discussed in PDBR and elsewhere[19]; it is probably sufficient to mention here that the mass relations to be derived are to a large extent independent of this assumption[20] as long as the radial integrals are parametrized as such, rather than evaluated with the help of further dynamical assumptions.[21] For the sake of convenience, we reproduce the structure of the relevant wave functions. For the **56** of baryons which we discuss in this section, these functions are

$$\Psi_{(10)}=\psi^S\chi^S\phi^S\,, \tag{2.1}$$

$$\Psi_{(8)}=\psi^S(\chi'\phi'+\chi''\phi'')/\sqrt{2}\,, \tag{2.2}$$

where ψ, χ, and ϕ are, respectively, the spatial, spin, and $SU(3)$ wave functions of the $3Q$ system, and the various superscripts stand for symmetric and mixed symmetric states, in conformity with Verde's notation and phase convention.[22] For further details, we refer to PDBR. The $SU(6)$ symmetry for the **56** of baryons necessarily implies the same spatial wave function ψ^S for both the **10** and **8** states.

The symmetry-breaking force between a pair (ij)

[12] K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters **14**, 163 (1965).
[13] J. J. Coyne, S. Meshkov, and G. B. Yodh, Phys. Rev. Letters **17**, 666 (1966).
[14] A. N. Mitra and M. H. Ross, Phys. Rev. **158**, 1630 (1967).
[15] A. N. Mitra, Phys. Rev. **151**, 1168 (1966).
[16] A. N. Mitra, Ann. Phys. (N. Y.) **43**, 126 (1967).
[17] E. P. Wigner, Phys. Rev. **51**, 106 (1937).

[18] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).
[19] A. N. Mitra and R. Majumdar, Phys. Rev. **150**, 1194 (1966).
[20] The transition from a symmetric wave function to an antisymmetric one is effected by the following replacements in the symmetry structure of the orbital part of the wave function: $S\to A$, $M''\to M'$, $M'\to -M''$, $A\to S$. See also Ref. 14.
[21] However, a spin-orbit Q-Q force affects A and S functions differently. This point is discussed in further detail below.
[22] M. Verde, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 170; χ^S is a quartet spin function. $\alpha_1\alpha_2\alpha_3$ for $J^P=\frac{3}{2}^+$. (χ',χ'') are the doublet spin functions, which for $J^P=\frac{1}{2}^+$ are

$$\chi'=(\alpha_2\beta_3-\alpha_3\beta_2)\alpha_1/\sqrt{2}\,,$$
$$\chi''=-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3)\chi'/\sqrt{3}\,;$$

similar results held for ϕ^S, ϕ', and ϕ''.

of quarks may be written as

$$V_{ij}=V_{ij}^{(1)}+V_{ij}^{(2)}+V_{ij}(\mathbf{L}\cdot\mathbf{S}), \qquad (2.3)$$

where the first two terms represent the central forces and the last a spin-orbit force. We write[23]

$$V_{ij}^{(1)}=(c/\sqrt{3})(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\lambda_8^{(i)}+\lambda_8^{(j)})+d\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j+(g/\sqrt{3})(\lambda_8^{(i)}+\lambda_8^{(j)}), \qquad (2.4)$$

$$V_{ij}^{(2)}=a(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)+b(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\lambda_8^{(i)}\lambda_8^{(j)}+e(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)+f\lambda_8^{(i)}\lambda_8^{(j)}, \qquad (2.5)$$

where the λ's are the usual Gell-Mann matrices for each quark.[24] This apparently arbitrary division of the central forces into two distinct sets is based on the *a posteriori* observation that each of these two sets is separately capable of reproducing the Gell-Mann–Okubo formulas for the **56** of baryons. The terms represented by $V^{(1)}$ were indeed used previously by other authors[5–8] to obtain the conventional mass formulas, but the structure of $V^{(2)}$ does not seem to have been investigated earlier. As the simplest assumption, we take the parameters multiplying the various terms in the potentials as constants (therefore independent of the ij labels as well). The calculation of the energy shifts due to the symmetry-breaking terms, which we do perturbatively, therefore involves essentially a normalization integral in ψ^S which we specify according to

$$\int|\psi^S|^2d\tau=1. \qquad (2.6)$$

The mass formulas so derived for the **56** of baryons are given in Table I, separately for the two schemes $V^{(1)}$ and $V^{(2)}$.

As for the spin-orbit term $V(\mathbf{L}\cdot\mathbf{S})$ in (2.3), our assumption that the radial wave function is ψ^S, rather than the conventional ψ^A implies no contribution from the former, at least in a perturbation theory. The reason is simply that the structure ψ^S of $L^P=0^+$ is built entirely out of s-wave Q-Q pairs (see PDBR) on which a spin-orbit force cannot possibly have a first-order effect. The result is in principle different from that of Fermi statistics, where the structure of the antisymmetric wave function ψ^A of $L^P=0^+$ requires at least two p-wave pairs,[19] which in turn could be affected by the spin-orbit force as well. This distinction between ψ^S and ψ^A is clearly of a dynamical nature, and its effect on the mass formulas may in principle provide an additional means[25] for probing into the validity of Fermi statistics for quarks. The sensitivity of this probe is of course dependent on the strength of the spin-orbit force required to fit the negative-parity masses, and this strength turns out to be moderately small (~25 MeV). We shall come back to this question in Sec. 4.

TABLE I. Mass shifts among the **56** particles due to the potentials $V^{(1)}$ and $V^{(2)}$.

Particle	$V^{(1)}$	$V^{(2)}$
N	$-\frac{2}{3}c-d+\frac{2}{3}g$	$5a-\frac{1}{3}b-e+\frac{1}{3}f$
Σ	$\frac{2}{3}c-d$	$\frac{1}{3}a+b+\frac{1}{3}e-\frac{1}{3}f$
Λ	$-\frac{2}{3}c-d$	$3a-\frac{1}{3}b-e-\frac{1}{3}f$
Ξ	$-\frac{2}{3}g+d$	$\frac{4}{3}b$
N^*	$\frac{2}{3}c+d+\frac{2}{3}g$	$a+\frac{1}{3}b+e+\frac{1}{3}f$
Σ^*	d	$\frac{1}{3}a-\frac{1}{3}b+\frac{1}{3}e-\frac{1}{3}f$
Ξ^*	$-\frac{2}{3}c+d-\frac{2}{3}g$	0
Ω	$-\frac{4}{3}c+d-\frac{4}{3}g$	$\frac{4}{3}b+\frac{4}{3}f$

To come back to the **56** mass formulas with pure central forces, we note from Table I that in the $V^{(1)}$ scheme, the combination $-\frac{2}{3}(c+g)$ of its parameters plays exactly the same role as the mass difference Δ between the singlet and doublet quarks. Since each gives the equal-spacing rule for the decuplet, one may avoid duplication by merely noting that the effect of Δ could be alternatively simulated by the combination $-\frac{2}{3}(c+g)$ of the coefficients. With this understanding, the Δ parameter may be dropped, so that we have effectively 3 independent parameters in the $V^{(1)}$ scheme. This scheme yields, without any extra assumptions, the GMO formula for the octet and the equal-spacing rule for the decuplet. The more specific $SU(6)$ results obtainable in this scheme are[26]

$$\Xi-\Lambda=\Xi^*-\Sigma^*, \qquad (2.7)$$

$$\Sigma-\Lambda+\tfrac{2}{3}(N-\Xi)=\tfrac{2}{3}(N^*-\Xi^*). \qquad (2.8)$$

A good fit to the various masses of the **56** representation is obtained with the following values (in MeV):

$$c=56.2\,,\quad d=121.0\,,\quad \tfrac{2}{3}g=-177.0. \qquad (2.9)$$

In the four-parameter $V^{(2)}$ scheme, the combination $\frac{4}{3}(b+f)$ plays the same role as Δ, viz., that this quantity by itself gives the equal-spacing rule. We may therefore again omit Δ with the understanding that its effect is already incorporated in this scheme.

[23] $\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j=\sum_{\alpha=1}^{3}\lambda_\alpha^{(i)}\lambda_\alpha^{(i)}$, which is the $SU(2)$-invariant product of two Gell-Mann matrices, covers merely the first three components of each.

[24] M. Gell-Mann, in *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (W. A. Benjamin, Inc., New York, 1964).

[25] It seems that the physically observable effects distinguishing between symmetric (S) and antisymmetric (A) orbital functions are not too many. For example, the sum rules for important processes like meson-baryon scattering and production [G.C. Joshi, V. S. Bhasin, and A. N. Mitra, Phys. Rev. **156**, 1572 (1967)], photoproduction [S. Das Gupta and A. N. Mitra, Phys. Rev. **156**, 1581 (1967)] or strong-decay widths of negative-parity baryons (see Ref. 14) do not depend on such distinctions. On the other hand, the structure of electromagnetic form factors (see Refs. 7 and 19) is perhaps one of the few cases which could throw light on this important question, apart from dynamical preferences for S rather than A functions (see Ref. 15).

[26] We use the notation (N,Σ,Λ,Ξ) for different members of the $SU(3)$ octet, and $(N^*,\Sigma^*,\Xi^*,\Omega)$ for those of the decuplet. An identical notation will be used for the $SU(3)$ structure of the negative-parity baryons, except for the additional notation Y^* for the $SU(3)$ singlets that would appear in this case.

As $V^{(2)}$ has an extra parameter over $V^{(1)}$, it gives merely the broader $SU(3)$ result

$$\Omega - N^* = 3(\Xi^* - \Sigma^*)\,, \tag{2.10}$$

rather than the GMO and the equal-spacing rule. It also gives the $SU(6)$ relations (2.7) and (2.8) connecting the members of the octet and the decuplet.[27] One now requires the additional assumption

$$a+e=-3(b+f) \tag{2.11}$$

to obtain the (stronger) $SU(3)$ results symbolized by the GMO and the equal-spacing rule. We also record the values (in MeV):

$$a=-171\,,\quad b=-151\,,\quad e=-141\,,\quad f=261\,, \tag{2.12}$$

which, in accordance with (2.11), give a good independent fit to the actual **56** masses.

We have thus found two independent but essentially equivalent schemes for fitting the masses of the **56** representation. One could also consider any arbitrary mixture of the potentials $V^{(1)}$ and $V^{(2)}$ to give an equally satisfactory representation of the **56** masses. The more interesting question now concerns the mass pattern which these schemes, singly or in combination, produce for the negative-parity baryons.

3. MASSES OF NEGATIVE-PARITY BARYONS

The experimentally established negative-parity baryons, with all quantum numbers properly identified, are indeed very few. However, as mentioned in the Introduction, we shall assume the (**70**,3) representation of the group $SU(6)\times O(3)$, and this will specify uniquely the $SU(3)$ assignments of these particles. The only source of ambiguity is in the duplication of the octet states of $J^P=\frac{1}{2}^-$ and of $J^P=\frac{3}{2}^-$. To start with, we specify these states in terms of their spin configurations, doublet and quartet (denoted by $\mathbf{8}^d$ and $\mathbf{8}^q$, respectively), a classification which is broadly in agreement with Dalitz's general analysis of these states.[7] Table II gives the wave functions of the various $L^P=1$ $SU(3)$ multiplets in the (LSJ) scheme. Here (ψ_μ',ψ_μ'') are the vector orbital functions of mixed symmetry which are contracted with appropriate spin and $SU(3)$ functions by Gerjuoy-Schwinger techniques,[28] as explained in PDBR[16] and another recent paper[29] by one of us (ANM). The normalization used for the orbital functions is

$$\int \psi_\mu'^* \psi_{\mu'}' d\tau = \int \psi_\mu''^* \psi_{\mu'}'' d\tau = \tfrac{1}{3}\delta_{\mu\mu'}\,, \tag{3.1}$$

[27] P. Federman, H. R. Rubinstein and I. Talmi [Phys. Letters **22**, 208 (1966)] have also obtained the relations (2.7), (2.8), and (2.10) on the basis of $SU(2)$ invariance and two-body forces.

[28] G. Gerjuoy and J. Schwinger, Phys. Rev. **61**, 138 (1942).

[29] A. N. Mitra, Phys. Rev. **150**, 839 (1966).

TABLE II. Structure of the (**70**,3) wave functions. (For notation, see text and PDBR.) Only the spin-orbital structure of the type $(2,1)_a$ is shown under the **8** representation.

L	S	J	(**10**)	(**8**)	(**1**)
1	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_\mu'\chi_\mu'+\psi_\mu''\chi_\mu''$	$\psi_\mu'\chi_\mu''+\psi_\mu''\chi_\mu'$	$\psi_\mu'\chi_\mu''-\psi_\mu''\chi_\mu'$
1	$\frac{3}{2}$	$\frac{1}{2}$	0	$\psi_\mu'\chi_\mu^S$	0
1	$\frac{1}{2}$	$\frac{3}{2}$	$\psi_\mu'\sigma_\mu'+\psi_\mu''\sigma_\mu''$	$\psi_\mu'\sigma_\mu''+\psi_\mu''\sigma_\mu'$	$\psi_\mu'\sigma_\mu''-\psi_\mu''\sigma_\mu'$
1	$\frac{3}{2}$	$\frac{3}{2}$	0	$\psi_\mu'\sigma_\mu^S$	0
1	$\frac{3}{2}$	$\frac{5}{2}$	0	$\psi_{+1}'\chi_{3/2}^S$	0

while for the spin functions we have

$$\begin{aligned} J&=\tfrac{1}{2}: \\ &\chi_\mu'=i\sigma_{1\mu}\chi'\,, \\ &\chi_\mu''=-(\sqrt{3})^{-1}(i\sigma_{3\mu}+\epsilon_{\mu\lambda\nu}\sigma_{3\lambda}\sigma_{1\nu})\chi'\,; \end{aligned} \tag{3.2}$$

$$\begin{aligned} J&=\tfrac{3}{2}: \\ &\sigma_\mu'=\tfrac{1}{2}\sqrt{3}(\sigma_{3\mu}-\sigma_{2\mu})\,, \\ &\sigma_\mu''=-\sigma_{1\mu}+\tfrac{1}{2}(\sigma_{3\mu}+\sigma_{2\mu})\,, \\ &\sigma_\mu^S=\sigma_{1\mu}+\sigma_{2\mu}+\sigma_{3\mu}\,. \end{aligned} \tag{3.3}$$

The latter operators operate on $\chi_{3/2}^S=\alpha_1\alpha_2\alpha_3$.

The calculation of the mass levels is straightforward and follows essentially on the lines of Sec. 2. Here again, the assumed constancy of the parameters in the two potentials $V^{(1)}$ and $V^{(2)}$ makes the results independent of the details of the orbital functions, since only the total normalization, governed by Eq. (3.1), is involved.[30] There is, however, an important point of difference from the **56** case, where the first-order perturbation treatment was enough to remove the mass degeneracy between different $SU(2)$ multiplets of the octet and decuplet states. In contrast, the mass degeneracy between $SU(2)$ multiplets of the classification in Table II necessitates recourse to degenerate-perturbation methods. Thus the quartet states of $J^P=\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ show one common mass for each $SU(2)$ multiplet type. While these masses are distinct from the corresponding masses of the doublet states of **1**, **8**, and **10**, the latter are badly degenerate among themselves. To remove the latter degeneracy, it is necessary to diagonalize the relevant parts of the first-order Hamiltonian expressed in terms of the matrix elements of $V^{(1)}$ and/or $V^{(2)}$. For the N-, N^*-, and Ω-type (doublet) states of a given J^P value, it is clear from

[30] Actually, it is of greater interest to consider the effect of p-wave central forces on the negative-parity masses. Such forces make the potentials V_{ij} dependent on the labels i, j. However, for *central* forces, in either of the two schemes, the expectation values $\langle V_{ij}\rangle$, which are independent of the labels i, j, depend only on *one* radial integral for each of the terms c, d, g in $V^{(1)}$ or a, b, e, f in $V^{(2)}$. Thus the number of independent parameters is still *three* in each scheme, though the relation of the parameters to the potentials (from the evaluation of the radial integrals) depends on the detailed structure of the wave functions. It is with this understanding that we may formally use the same notation for the parameters of the potentials and those appearing explicitly in the mass formulas. This freedom of inclusion of a spatial structure in the potential $V^{(1)}$ or $V^{(2)}$ separately will be of use in a consistent interpretation of the results of mass fits to the negative-parity baryons in addition to the positive-parity ones.

Table II that the matrix size is simply 1×1 while for the states of the Ξ (or Ξ^*), Λ (or Y^*), and Σ (or Σ^*) type, one must handle 2×2 matrices.

Unfortunately, the mixing of the above states caused by the diagonalization process is appreciable, being as much as 50:50 for all the 2×2 matrices considered. One must therefore work with doublet states like

$$\Lambda^{\mathrm{I}}=(\sqrt{2})^{-1}(\Lambda+Y^*)\,,\quad \Lambda^{\mathrm{II}}=(\sqrt{2})^{-1}(\Lambda-Y^*) \tag{3.4}$$

for $I=0$ and corresponding states for $I=\frac{1}{2}$ and 1, respectively, where the phase conventions for the mixtures represented by the superscripts I and II are uniformly defined by Eq. (3.4). The algebraic mass formulas obtained in this manner are listed in Table III, separately for the $V^{(1)}$ and $V^{(2)}$ schemes.

The central forces $V^{(1)}$ and $V^{(2)}$ still leave degenerate the states of different J^P values ($\frac{1}{2}^-$, $\frac{3}{2}^-$ for doublets, and $\frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$ for quartets), and one must invoke noncentral forces to remove this degeneracy. The simplest type is a spin-orbit force represented by the last term of (2.3) which, though found ineffective for the **56** states, is now expected to play a more important role on the vector orbital functions ψ_μ characterized by p-wave configurations.

One could consider the following charge-hypercharge structures for the spin-orbit force:

$$\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\,;\quad \lambda_8^{(i)}+\lambda_8^{(j)}\,;\quad \lambda_8^{(i)}\lambda_8^{(j)}\,;$$
$$\tfrac{1}{3}-\tfrac{1}{4}\sum_{\alpha=1}^{8}\lambda_\alpha^{(i)}\lambda_\alpha^{(j)}\,, \tag{3.5}$$

of which the first three are merely $SU(2)$-invariant interactions, and the last an $SU(3)$-invariant one. The last one, which was indeed considered in PDBR,[16] was found to produce certain geometrical-looking mixtures between the quartet and doublet states. However, its capacity to remove degeneracy among various masses seems to be rather limited. While its presence is by no means ruled out, it would be interesting also to consider the other types listed in (3.5), as they are likely to yield more structure in the masses [being merely $SU(2)$-invariant].

An estimate of the strength of the spin-orbit force can readily be obtained from a comparison of the mass differences between like $SU(2)$ multiplets. Thus, the mass difference between the quartet N states $N^*(1688)$ and $N^*(1540)$, of $J^P=\frac{5}{2}^-$ and $\frac{1}{2}^-$, respectively, provides a reliable estimate of the spin-orbit strength. Indeed, on the basis of such comparisons, the strength of this force was estimated to be about 25–30 MeV, which is appreciably less than the mass differences between $SU(2)$ multiplets of different hypercharges (100–150 MeV).

Assuming, therefore, that the spin-orbit force is weaker than the central force, it is reasonable to ignore the coupling between the various states while estimating its effect in a perturbative manner. Since in this case it is not possible to ignore the spatial structure of the force, the mass shifts for the various states would be proportional to radial integrals which in general would depend on the spatial structure of the ψ_μ's. As such a mechanism in this case would involve a number of independent radial integrals,[31] this would amount to as many parameters being used to estimate the effect of the spin-orbit force. To avoid bringing in so many parameters just for the sake of a small effect, one must make some additional assumptions. For this purpose we make the same assumption as in Ref. 15 or PDBR, viz., that the spatial part of the spin-orbit force has a p-wave separable structure of the form

$$\langle\mathbf{p}|V_{LS}|\mathbf{p}'\rangle=i\lambda_{LS}(\mathbf{p}\times\mathbf{p}')\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)v(p)v(p')\,, \tag{3.6}$$

where the shape factor $v(p)$ is the same as used for the (much stronger) central p-wave force required to generate the central mass of the (**70**,3) multiplet.[16] In other words, we use the same calculational technique for the $SU(2)$-invariant spin-orbit interaction as was done in PDBR for the corresponding $SU(3)$-invariant force. This procedure would result in a modification of the strength parameters of the kernels of the relevant spectator functions for the various $SU(2)$ states with the replacement

$$\lambda_0\rightarrow\lambda_0+x_{LS}\lambda_{LS}\,, \tag{3.7}$$

where λ_0 is the strength of the central interaction and is essentially a geometrical factor depending on the spin, charge, and hypercharge quantum numbers of the $SU(2)$ states. Since $\lambda_{LS}\ll\lambda_0$, one may then proceed as in PDBR[16] to deduce mass shifts of the form

$$\Delta m_{LS}=\epsilon x_{LS}\,, \tag{3.8}$$

where ϵ is a constant independent of the spin-$SU(3)$ assignments. This result is so simple, depending as it

TABLE III. Mass shifts among the various (**70**,3) states due to the potentials $V^{(1)}$ and $V^{(2)}$. (For notation, see text.)

Particle	$V^{(1)}$ scheme	$V^{(2)}$
$N_{5/2}=N_{3/2}{}^q=N_{1/2}{}^q$	$d+\frac{2}{3}c+\frac{2}{3}g$	$-a+\frac{1}{3}b-e+\frac{1}{3}f$
$\Lambda_{5/2}=\Lambda_{3/2}{}^q=\Lambda_{1/2}{}^q$	d	$-a-\frac{1}{3}b-e-\frac{1}{3}f$
$\Sigma_{5/2}=\Sigma_{3/2}{}^q=\Sigma_{1/2}{}^q$	d	$\frac{1}{3}a-\frac{1}{3}b+\frac{1}{3}e-\frac{1}{3}f$
$\Xi_{5/2}=\Xi_{3/2}{}^q=\Xi_{1/2}{}^q$	$d-\frac{2}{3}c-\frac{2}{3}g$	0
$N_{3/2}{}^d=N_{1/2}{}^d$	$-d-\frac{2}{3}c+\frac{2}{3}g$	$a-\frac{1}{3}b-e+\frac{1}{3}f$
$N_{3/2}{}^*=N_{1/2}{}^*$	$-d-\frac{2}{3}c+\frac{2}{3}g$	$-a-\frac{1}{3}b+e+\frac{1}{3}f$
$\Omega_{3/2}=\mathrm{A}_{1/2}$	$-d+\frac{4}{3}c-\frac{4}{3}g$	$-\frac{4}{3}b+\frac{4}{3}f$
$\Lambda_{3/2}{}^{\mathrm{I}}=\Lambda_{1/2}{}^{\mathrm{I}}$	$-d+\frac{2}{3}c$	$a+\frac{1}{3}b-e-\frac{1}{3}f-2(a-\frac{1}{3}b)$
$\Lambda_{3/2}{}^{\mathrm{II}}=\Lambda_{1/2}{}^{\mathrm{II}}$	$-d-\frac{2}{3}c$	$a+\frac{1}{3}b-e-\frac{1}{3}f+2(a-\frac{1}{3}b)$
$\Sigma_{3/2}{}^{\mathrm{I}}=\Sigma_{1/2}{}^{\mathrm{I}}$	$-d+\frac{2}{3}c$	$-\frac{1}{3}a+\frac{1}{3}b+\frac{1}{3}e-\frac{1}{3}f-\frac{2}{3}(a+b)$
$\Sigma_{3/2}{}^{\mathrm{II}}=\Sigma_{1/2}{}^{\mathrm{II}}$	$-d+\frac{2}{3}c$	$-\frac{1}{3}a+\frac{1}{3}b+\frac{1}{3}e-\frac{1}{3}f+\frac{2}{3}(a+b)$
$\Xi_{3/2}{}^{\mathrm{I}}=\Xi_{1/2}{}^{\mathrm{I}}$	$-d+\frac{4}{3}c-\frac{2}{3}g$	$-\frac{4}{3}b$
$\Xi_{3/2}{}^{\mathrm{II}}=\Xi_{1/2}{}^{\mathrm{II}}$	$-d-\frac{2}{3}g$	$+\frac{4}{3}b$

[31] The situation for spin-orbit forces is different from the case of central forces where the number of independent parameters in the mass formulas happens to be equal to the corresponding number in the potentials (see Ref. 30). Indeed, the noncentral structure of the spin-orbit force yields a much richer variety of radial integrals.

does on a single free parameter, as to make the advantages of a simple dynamical assumption like (3.6), made on a relatively small effect (like the spin-orbit force), very strong in comparison with the disadvantages that would be caused by the presence of several free parameters (in the form of radial integrals) in the mass formulas. Moreover, it may be noted that the assumption (3.6) hardly amounts to any detailed model, but is merely a convenient expression for a (short-range) spin-orbit force in an "effective-range" spirit.

The values of x_{LS} determined for the various $SU(2)$ states before the mixing of the $\mathbf{8}^d$ and $\mathbf{10}$ or $\mathbf{1}$ states are shown in Table IV for the scheme $\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$ of (3.5), which is fairly close to the predictions of the $SU(3)$-invariant interaction[16] (except for involving less degeneracy).[32] The corresponding calculations for the different $SU(3)$ mixtures I and II of the doublet states of $\mathbf{8}$, $\mathbf{10}$, and $\mathbf{1}$ are easily performed by merely adding the contributions of the spin-orbit effect to the relevant 2×2 matrices mentioned earlier in this section. The results of this spin-orbit modification for these mixed states, including terms of order $\epsilon^2/V^{(1)}$ or $\epsilon^2/V^{(2)}$, are shown in the second part of this table.

TABLE IV. Spin-orbit parameters x_{LS} with a $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$-type force for the various negative-parity states.

Particle	$J^P=\frac{1}{2}^-$	$J^P=\frac{3}{2}^-$	$J^P=\frac{5}{2}^-$
N^q	$-15/4$	$-3/2$	$9/4$
Λ^q	$-5/2$	-1	$3/2$
Σ^q	$5/24$	$1/12$	$-1/8$
Ξ^q	0	0	0
N^d	-3	$3/2$	
Λ^d	-2	1	
Σ^d	$1/6$	$-1/12$	
Ξ^d	0	0	
N^*	1	$-1/2$	
Σ^*	$1/3$	$-1/6$	
Ξ^*	0	0	
Ω	0	0	
Y^*	-1	$1/2$	
	$V^{(1)}$	$V^{(2)}$	
$\Sigma_{1/2}{}^{\mathrm{I}}$	$\frac{1}{4}\epsilon-\frac{1}{192}\frac{\epsilon^2}{c}$	$\frac{1}{4}\epsilon-\frac{1}{192}\frac{\epsilon^2}{(a+b)}$	
$\Sigma_{1/2}{}^{\mathrm{II}}$	$\frac{1}{4}\epsilon+\frac{1}{192}\frac{\epsilon^2}{c}$	$\frac{1}{4}\epsilon+\frac{1}{192}\frac{\epsilon^2}{(a+b)}$	
$\Sigma_{3/2}{}^{\mathrm{I}}$	$-\frac{1}{8}\epsilon-\frac{1}{768}\frac{\epsilon^2}{c}$	$-\frac{1}{8}\epsilon-\frac{1}{768}\frac{\epsilon^2}{(a+b)}$	
$\Sigma_{3/2}{}^{\mathrm{II}}$	$-\frac{1}{8}\epsilon+\frac{1}{768}\frac{\epsilon^2}{c}$	$-\frac{1}{8}\epsilon+\frac{1}{768}\frac{\epsilon^2}{(a+b)}$	
$\Lambda_{1/2}{}^{\mathrm{I}}$	$-\frac{3}{2}\epsilon+\frac{3}{16}\frac{\epsilon^2}{c}$	$-\frac{3}{2}\epsilon+\frac{1}{32}\frac{\epsilon^2}{(a-\frac{1}{3}b)}$	
$\Lambda_{1/2}{}^{\mathrm{II}}$	$-\frac{3}{2}\epsilon-\frac{3}{16}\frac{\epsilon^2}{c}$	$-\frac{3}{2}\epsilon-\frac{1}{32}\frac{\epsilon^2}{(a-\frac{1}{3}b)}$	
$\Lambda_{3/2}{}^{\mathrm{I}}$	$\frac{3}{4}\epsilon+\frac{3}{64}\frac{\epsilon^2}{c}$	$\frac{3}{4}\epsilon+\frac{1}{128}\frac{\epsilon^2}{(a-\frac{1}{3}b)}$	
$\Lambda_{3/2}{}^{\mathrm{II}}$	$\frac{3}{4}\epsilon-\frac{3}{64}\frac{\epsilon^2}{c}$	$\frac{3}{4}\epsilon-\frac{1}{128}\frac{\epsilon^2}{(a-\frac{1}{3}b)}$	
$\Xi_{1/2}{}^{\mathrm{I}}=\Xi_{1/2}{}^{\mathrm{II}}$	0	0	
$\Xi_{3/2}{}^{\mathrm{I}}=\Xi_{3/2}{}^{\mathrm{II}}$	0	0	

[32] The predictions of the schemes $(\lambda_8{}^{(i)}+\lambda_8{}^{(j)})$ or $\lambda_8{}^{(i)}\lambda_8{}^{(j)}$ are not shown, as these turn out to be in violent disagreement with the observed mass pattern of the established cases.

Before concluding this section, we mention certain general features of the mass relations predicted by the $V^{(1)}$ and $V^{(2)}$ potentials for the negative-parity states. With $V^{(1)}$, GMO and the equal-spacing rule are trivially satisfied for the individual octets and decuplets, respectively, in the absence of coupling between the $\mathbf{8}^d$, $\mathbf{1}$, and $\mathbf{10}$ states. However, the coupling between the $\mathbf{8}^d$, $\mathbf{1}$, and $\mathbf{10}$ states brought about by $V^{(1)}$ results merely in the more general GMO relations, separately for the superscripts I and II, for each J^P value; but the equal-spacing rule for the $\mathbf{10}$ states is lost. This is easily verified from an inspection of Table III. For $V^{(2)}$, together with condition (2.11), GMO is satisfied for each of the $\mathbf{8}$ states of $J^P=\frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$ which do not mix with the $\mathbf{8}^d$, $\mathbf{1}$, or $\mathbf{10}$ states in the absence of spin-orbit forces. However, even with the neglect of coupling between the $\mathbf{8}^d$, $\mathbf{1}$, and $\mathbf{10}$ states, $V^{(2)}$ together with (2.11) *does not* yield GMO for the $\mathbf{8}^d$ states, or the equal-spacing rule for the $\mathbf{10}$ states. This last feature of the $V^{(2)}$ scheme is particularly interesting in the context of certain conjectures[33] that GMO may not, after all, be valid for the negative-parity baryons. We have actually found a potential which, while yielding conventional results for the $\mathbf{56}$ of baryons, has a distinctly different role to play for the negative-parity baryons.

An $SU(6)$ mass formula for the negative-parity baryon masses, when both $V^{(1)}$ and $V^{(2)}$ are present and the corresponding mixing of the $\mathbf{8}^d$, $\mathbf{1}$, and $\mathbf{10}$ states is taken into account, is[3]

$$2(\Omega-N^*)=3(\Xi^{\mathrm{I}}+\Xi^{\mathrm{II}}-\Sigma^{\mathrm{I}}-\Sigma^{\mathrm{II}}), \tag{3.9}$$

which is degenerate with respect to $J^P=\frac{1}{2}^-$ or $\frac{3}{2}^-$. Two $SU(6)$ relations, connecting the $\mathbf{56}$ masses with those of $(\mathbf{70},3)$ are [34,35]

$$\Xi-\Sigma=\Xi_{1/2}{}^q-\Sigma_{1/2}{}^q=\Xi_{3/2}{}^q-\Sigma_{3/2}{}^q, \tag{3.10}$$

$$\begin{aligned}\Sigma^*+\Sigma-N-\Xi&=N_{1/2}{}^*+N_{1/2}{}^q-2N_{1/2}{}^d\\&=N_{3/2}{}^*+N_{3/2}{}^q-2N_{3/2}{}^d,\end{aligned} \tag{3.11}$$

where in the last two expressions in each relation the $\mathbf{8}^q$, $\mathbf{8}^d$, or $\mathbf{10}$ members of the $(\mathbf{70},3)$ particles are indicated, and the first member of (3.10) or (3.11) refers to the usual positive-parity baryons.

Inclusion of a spin-orbit force, as in Table IV, does not significantly affect these relations. For example, Eq. (3.09) picks up a term -2ϵ on the left and $-\frac{3}{2}\epsilon$ on the right, resulting in a net violation of the equality

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[34] K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee Phys. Letters **15**, 79 (1965).

[35] We indicate the J value by a subscript to the main symbol, according to Ref. 26.

by an amount $\frac{1}{2}\epsilon \sim 12$ MeV. Eq. (3.10) is even less affected by this modification, the violating being only $-(5/24)\epsilon$ for the middle member ($J^P=\frac{1}{2}^-$) and $-\frac{1}{12}\epsilon$ for the last member ($J^P=\frac{3}{2}^-$). Equation (3.11), on the other hand, is somewhat more violated by the spin-orbit effect, the net "corrections" being $-(13/4)\epsilon$ and $+\epsilon$ for $J^P=\frac{1}{2}^-$ and $\frac{3}{2}^-$, respectively. Unfortunately, in the absence of sufficiently clear experimental identifications of the various members of the above relations, any meaningful comparison with experiment is at present premature.

4. RESULTS AND DISCUSSIONS

We now look into the possibility of a semiquantitative fit to the masses of the negative-parity baryons to the extent that they have been recognized experimentally. At first view, one could try with $V^{(1)}$ and/or $V^{(2)}$, separately or in association with a spin-orbit force. Since a lot of trial and error is involved, we start by ruling out a few simple possibilities. For example, taking the parameters (2.9) of $V^{(1)}$, we find that these are utterly inadequate for even a crude representation of the negative-parity data. Thus, $V^{(1)}$ predicts a discrepancy of 320 MeV from the experimental mass difference of barely 20 MeV between $N^*(1670)$ of $J^P=\frac{1}{2}^-$ (known to be a member of **10**) and $N^*(1688)$ of $J^P=\frac{5}{2}^-$ (a member of $\mathbf{8}^q$). Similar discrepancies of large magnitudes are noticed for the mass difference between, say,[35] $N_{5/2}(1688)$ and $N_{3/2}(1518)$. In an even worse fashion, the parameters (2.12) for $V^{(2)}$ are at complete variance with the data for the established cases. Finally, we have not succeeded in finding any suitable combination of the two sets of parameters (2.9) and (2.12) to give even a qualitatively correct picture of the masses of the negative-parity baryons.

Next we look for an alternative possibility for fitting the masses by determining some of the potential parameters from a few negative-parity baryons as input. Since we have already seen in Sec. 3 that $V^{(2)}$ is likely to play a more interesting role for the negative-parity baryons (not being tied to the GMO relations), we first seek to determine the parameters of $V^{(2)}$, rather than $V^{(1)}$, from some of these masses. As a working hypothesis, we choose the following masses as input:

$$N_{5/2}{}^q(1688)\,,\quad \Sigma_{5/2}{}^q(1765)\,, \tag{4.1}$$

$$Y^*(1405)\equiv\Lambda_{1/2}{}^{\mathrm{I}}\,,\quad Y^*(1520)\equiv\Lambda_{3/2}{}^{\mathrm{II}}\,. \tag{4.2}$$

It is then possible to check the parameters from such a determination against the masses of the following particles:

$$J^P=\tfrac{1}{2}^-:\quad N^*(1670)\equiv N_{1/2}{}^*\,,\quad N^*(1540)\equiv N^q\,; \tag{4.3}$$

$$J^P=\tfrac{3}{2}^-:\quad N^*(1518)\equiv N_{3/2}{}^d\,,\quad Y_1{}^*(1660)\equiv\Sigma^{\mathrm{I}}\,,\quad \Xi^*(1816)\equiv\Xi^{\mathrm{I}}\,. \tag{4.4}$$

In this respect, the biggest problem lies in the $SU(3)$ assignments. According to our calculations outlined in Sec. 3, we find strong admixtures of $\mathbf{8}^d$ and **10**, or $\mathbf{8}^d$ and **1** states, all of which have spin-doublet structures. It is only the spin-quartet $\mathbf{8}^q$ states whose mixing with the doublet states may be neglected in the absence of a strong spin-orbit coupling, an assumption justified from the analysis of Sec. 3.

The only quartet states in (4.1)–(4.4) are $N^q(1688)$ and $N^q(1540)$, according to the analysis of Dalitz[6] and the results for strong-decay widths.[14] All other states listed therein are strongly mixed doublet states. Thus, we have a choice of identification of the experimental states listed above with the assignments (superscript I or II) discussed in Sec. 3. We have (hopefully) indicated these assignments in (4.1)–(4.4) with a view to minimizing the discrepancy between theory and experiment. To determine the parameters of $V^{(2)}$, we have also to consider the effect of the spin-orbit force, which, according to Sec. 3, is $\epsilon\approx 26$ MeV. It turns out that the $SU(2)$ variety $\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$ of this force gives by far the best results—the next best, the $SU(3)$ version, being appreciably worse. After correcting for the spin-orbit effect, a fit to the masses (4.1) and (4.2) with $V^{(2)}$ above, leads to the following values (in MeV):

$$a=-31.0\,,\quad b=-264\,,\quad e=118\,,\quad f=235\,. \tag{4.5}$$

Now it turns out that even these values give very bad results for the masses of the particles (4.3) and (4.4). We notice, however, the interesting result that the large discrepancies in several cases are roughly equal and opposite from the $V^{(1)}$ and $V^{(2)}$ contributions. This indicates that large components of both $V^{(1)}$ and $V^{(2)}$ are necessary even for a qualitative understanding of the mass pattern, through a cancellation of large terms of opposite signs. As the simplest possibility, therefore, we have considered the effect of 50% mixtures of $V^{(1)}$ and $V^{(2)}$ with parameters taken from (2.9) and (4.5), respectively, and this reduces the scatter in the mass differences from several hundred MeV to the modest range of 20–60 MeV. We mention in passing that the results obtained by interchanging the roles of $V^{(1)}$ and $V^{(2)}$ [viz., determining the $V^{(1)}$ parameters from (4.1) and (4.2) and taking the $V^{(2)}$ parameters from (2.12)] are nowhere near the qualitative accord achieved with the procedure just outlined.

We consider the above numerical result so significant that we venture to offer a rough physical explanation of the mixture $\frac{1}{2}(V^{(1)}+V^{(2)})$ required to fit the masses of the negative-parity baryons. It is an observational fact that for the negative-parity baryons the mass difference between $Y=0$ and $Y=1$ particles, which may be called the "equal-spacing parameter," is roughly half that among the corresponding positive baryons. Therefore, if we suppose that the "equal-spacing parameter" is contributed almost entirely by $V^{(1)}$ and little by $V^{(2)}$, the formulas $V^{(1)}$ and $\frac{1}{2}(V^{(1)}+V^{(2)})$, respectively, for the **56** and (**70**,3) of baryons, provide

TABLE V. Mass fits for negative-parity baryons (MeV).

S. No.	Mass difference	$V^{(1)}$	Central effect $V^{(2)}$	$\frac{1}{2}(V^{(1)}+V^{(2)})\equiv\bar{V}$	Spin-orbit force $V(LS)$	$\bar{V}+V(\mathbf{L}\cdot\mathbf{S})\equiv\eta$	Exp. δ	Diff. $\delta-\eta$
1	$N_{5/2}-N_{1/2}{}^{*}$	317	-342	-13	28	15	8	-7
2	$N_{5/2}-N_{1/2}{}^{q}$	0	0	0	144	144	148	$+4$
3	$N_{5/2}-N_{3/2}{}^{d}$	317	-48	134	18	152	170	$+18$
4	$N_{5/2}-\Lambda_{3/2}{}^{\mathrm{II}}$	140	181	160	36	196	168	-28
5	$N_{5/2}-\Lambda_{1/2}{}^{\mathrm{I}}$	65	267	165	90	254	283	$+29$
6	$N_{5/2}-\Sigma_{5/2}$	-140	-79	-110	57	-53	-77	-24
7	$N_{5/2}-\Xi_{3/2}{}^{\mathrm{I}}$	-75	-342	-209	54	-155	-138	$+17$
8	$N_{5/2}-\Sigma_{3/2}{}^{\mathrm{I}}$	140	-143	-2	57	55	28	-27

a very simple understanding of this phenomenon. To see this point somewhat more clearly, we recall from Sec. 2, that the "equal-spacing parameter" in $V^{(2)}$ is represented by $\frac{4}{3}(b+f)$, whence a zero value for this parameter requires

$$b=-f,\quad a=-e, \tag{4.6}$$

by virtue of (2.11). Actually, the condition $b=-f$ is almost satisfied by the values (4.5), considering the large magnitudes for b and f. Though the other condition $a=-e$ is not satisfied by (4.5), this could well be due to the failure of the GMO formula for $V^{(2)}$. In any case we seem to have found a rather simple dynamical mechanism to understand the smaller magnitude (by roughly half) of the "equal-spacing parameter" for the negative parity baryons. The 50:50 mixture of $V^{(1)}$ and $V^{(2)}$ which gave a mass pattern in qualitative accord with experiment also leads, without extra assumptions, to the requisite magnitude of the equal-spacing parameter.

A more quantitative determination of the $V^{(2)}$ can now be made directly by the method of least squares to fit all the masses (4.0)–(4.4) *simultaneously*, with the help of the potential

$$\tfrac{1}{2}(V^{(1)}+V^{(2)})+V(\mathbf{L}\cdot\mathbf{S}), \tag{4.7}$$

the last term being of the $SU(2)$ type $\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$. The least-squares values of the parameters (in MeV),

$$a=-48.0,\quad b=-216,\quad e=+99.0,\quad f=+199, \tag{4.8}$$

yield the masses shown in Table V, which are in reasonable accord with experiment, with an error ranging between 10 and 30 MeV.

Table VI lists several predicted masses according to the present analysis. While some of these lie rather low in mass, indeed *lower* than a few observed ones, this fact by itself need not be an embarrassment. For, as has been found from the analysis of strong decays,[14] most of these states would be extremely hard to detect, because their widths are either too large to too small. Their effect could, however, be felt indirectly, e.g., through careful phase-shift analyses in $\bar{K}N$ scattering. In this connection, we wish to record a point of discrepancy between the results of strong-decay widths, which had indicated very little mixing between the representation states listed in Table II, and the present calculations, which predict very large mixing between the (doublet) states of $\mathbf{8}^d$, **10**, and **1** multiplets. Perhaps more experimental evidence is needed to clear up the picture.

While the formula (4.7) for the potential to represent the (**70**,3) masses contrasts rather prominently with the simple formula $V^{(1)}$ for the **56** masses, it is formally just a question of suitable projection operators to accommodate the different varieties. A simple possibility is to use projection operators $P[0]$ for $V^{(1)}$ and $P[1]$ for (4.7), where $P[l]$ is the projection operator for interaction in a Q-Q partial wave l. Since $V(\mathbf{L}\cdot\mathbf{S})$ is necessarily associated with at least $l=1$, it cannot of course be operative on the **56** states, as was already stated in Sec. 2. Since the parameters of $V^{(1)}$ already give a good fit to the **56** masses (within 10 MeV), this fact leaves little scope for other forces to play any useful role. While in our model, the s-wave structure of all Q-Q pairs in the **56** states, effectively keeps out the spin-orbit force, the situation would be quite different with Fermi statistics, where the mutual p-wave Q-Q pairs associated with an A function of $L^P=0^+$ could be badly affected by the spin-orbit force. If now the strength of the latter were to be determined by fits to the (**70**,3) masses, viz., $\epsilon\approx 25$ MeV, one would expect the same force to produce mass shifts of like magnitude ($\sim$25 MeV) among the members of the **56** states. Since, on the other hand, the $V^{(1)}$ parameters, as determined from the **56** masses, do not leave much scope for adjustments, distortions of the order of 25 MeV in these values could provide at least some hindrance to the assumption of Fermi statistics for Gell-Mann–Zweig quarks.

To summarize, we have found two independent sets of potentials, $V^{(1)}$ and $V^{(2)}$, each of which gives con-

TABLE VI. Mass predictions (MeV).

$\Lambda_{5/2}$	1746	$\Lambda_{3/2}{}^{q}$	1686	$\Lambda_{1/2}{}^{q}$	1660
$\Lambda_{3/2}{}^{\mathrm{I}}$	1486	$\Lambda_{1/2}{}^{\mathrm{II}}$	1438	$\Sigma_{1/2}{}^{\mathrm{II}}$	1485
$\Sigma_{3/2}{}^{q}$	1746	$\Sigma_{1/2}{}^{q}$	1749	$\Sigma_{3/2}{}^{\mathrm{II}}$	1475
$\Sigma_{1/2}{}^{\mathrm{I}}$	1670	$\Xi_{5/2}$	1804	$\Xi_{3/2}{}^{q}$	1804
$\Xi_{1/2}{}^{q}$	1804	$\Xi_{3/2}{}^{\mathrm{II}}$	1591	$\Xi_{1/2}{}^{\mathrm{I}}$	1830
$\Xi_{1/2}{}^{\mathrm{II}}$	1591	$N_{3/2}{}^{q}$	1598	$N_{1/2}{}^{d}$	1443
$N_{3/2}{}^{*}$	1706	$\Omega_{3/2}$	2102	$\Omega_{1/2}$	2102

ventional results for the **56**$^+$ sets; but $V^{(2)}$ predicts a departure from the GMO for the negative-parity baryons. While $V^{(1)}$ is ideally suited for the **56** states, the negative-parity (**70**,3) particles require roughly equal mixtures of both, in addition to a spin-orbit force of the $\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$ type. This also provides a simple dynamical mechanism for the empirical result that the negative-parity baryons show a magnitude for the "equal-spacing parameter" of ≈ 77 MeV, only about half the value of 140 MeV for the positive-parity ones. Finally, while the spin-orbit force required to fit the negative-parity masses is of rather modest strength ($\sim$25 MeV), it is big enough to show up as a vexing perturbation on the otherwise beautiful fit to the **56** masses with the help of $V^{(1)}$, thus making an antisymmetric function (Fermi statistics) much less favored than a S function, again in conformity with the results on the baryon form factors[19] as well as dynamical appeal.[15,16]

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Superconvergence Sum Rules for 2^+-0^- Scattering*

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We develop the kinematics for 2^+–0^- elastic scattering and determine the asymptotic behavior of the invariant amplitudes assuming (a) Regge behavior and (b) a Froissart bound. The corresponding sum rules are written down and an approximate saturation with a few low-lying single-particle states is attempted. In general, such saturation is not obtained.

I. INTRODUCTION

It has been noted by several authors[1] that amplitudes falling off sufficiently fast as $s\rightarrow\infty$ satisfy sum rules, which have been called "superconvergence sum rules" by de Alfaro, Fubini, Furlan, and Rosetti.[2] Such sum rules have been derived by current algebraic and Regge-pole-theory techniques, and many authors[3] have attempted to saturate them approximately by a small number of low-lying single-particle states and get relations among coupling constants and masses. Such attempts have met with various degrees of success.

In this paper we study the elastic scattering of 2^+ and 0^- mesons and derive superconvergence sum rules for the corresponding invariant amplitudes, assuming that they have the usual Regge behavior. We feel that this process should be a particularly interesting one to study, because a large number of sum rules are obtained because of the high spins involved. Some of the sum rules are actually a consequence of the Froissart bound[4] alone. Next we attempt to saturate the sum rules so obtained by low-lying (<1.5 GeV) one-particle states and find that under any reasonable approximation the only consistent solution is the trivial one (all coupling constants vanish). The only exception are the sum

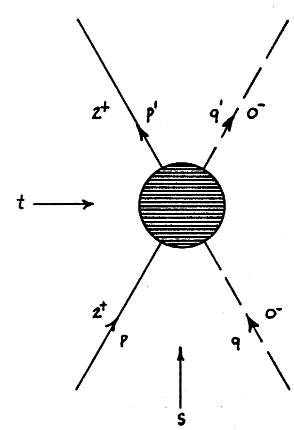

FIG. 1. 2^+-0^- elastic scattering.

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