Electromagnetic Properties of Hadrons in a Triplet-Sextet Model*

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An investigation is made of the electromagnetic mass differences and magnetic moments of the baryons within the framework of a two-particle triplet-sextet model. The mass splittings are assumed to result from U-spin-invariant intrinsic mass splittings in the sextet and triplet, and from Coulomb and magnetic spin-spin interactions between the two particles. The two particles are taken to be in an $L=0$ orbital angular momentum state, and it is assumed that it is valid to use perturbation theory in the calculation of the electromagnetic mass differences. Sum rules and inequalities for the electromagnetic mass differences and magnetic moments are obtained, and the results compared with experiment. A calculation is also made of the electromagnetic mass differences of the 0^- and 1^- meson octets. Sum rules and inequalities are again obtained, and the results compared with experiment. All predictions of the triplet-sextet model except one are in agreement with the presently available experimental data. One prediction relating the meson and baryon mass splittings disagrees with the experimental result by $1\frac{1}{2}$ standard deviations. The analogous prediction of the quark model is considerably worse.

1. INTRODUCTION

A NUMBER of authors have considered models in which hadrons are composite objects composed of quarks or other fundamental particles. An especially simple class of such models comprises those in which baryons and mesons are assumed to be bound states of very heavy particles moving slowly in a deep potential well. The case in which the constituent particles are quarks has been summarized by Dalitz,[1] who gives references to many of the original papers. Another case, in which a baryon is a bound state of two fundamental particles, a sextet and a triplet, rather than a bound state of three quarks, has also been considered recently.[2] It is the purpose of this paper to investigate the electromagnetic mass splittings and magnetic moments of hadrons within the framework of the triplet-sextet model.

The problem of the electromagnetic properties of hadrons has already been considered by many authors.[3–12] In some of these papers, the predictions do not depend on any specific model of the hadrons, but on certain assumed transformation properties of the electromagnetic interaction.[3–6] In others, the quark model was assumed to be valid.[8–12] On the other hand, in the only work on electromagnetic properties within the framework of the triplet-sextet model, it was assumed that the baryon mass splitting arose entirely from intrinsic splittings among the members of the triplet and sextet.[13] However, as was pointed out by Veltman,[14] there is no inherent reason for the electromagnetic interactions between the triplet and sextet to contribute less than their intrinsic splittings. Furthermore, in Ref. 13, the magnetic moments of the baryons and the mass splittings of the mesons were not discussed at all. For these reasons, we choose to emphasize the triplet-sextet model. However, for purposes of comparison, we also discuss the quark model, making the same assumptions about the causes of the electromagnetic properties of the baryons.

Briefly, our assumptions are the following:

(1) Members of the baryon octet and decimet are bound states of heavy particles moving nonrelativistically in a potential well with zero orbital angular momentum. In the triplet-sextet model, a baryon is composed of two particles, one of which has spin 1 and the other spin $\frac{1}{2}$. In the quark model, of course, a baryon is composed of three spin-$\frac{1}{2}$ particles.

(2) The constituent particles are members of broken $SU(3)$ multiplets. The $SU(3)$ symmetry of the hadrons is broken both by intrinsic mass splittings among the members of the constituent multiplets, and also by symmetry-breaking interactions. All symmetry-breaking electromagnetic effects are assumed to be U-spin-invariant.[15]

(3) The electromagnetic interactions between the constituent particles are assumed to be of two kinds: the usual Coulomb interaction and a spin-dependent magnetic-contact interaction of the type responsible for atomic hyperfine structure.[12,16]

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[14] M. Veltman (private communication).
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[16] See, for example, A. Abragam, *Principles of Nuclear Magnetism* (Clarendon Press, Oxford, England, 1961), p. 170.

TABLE I. Experimental values of baryon electromagnetic mass splittings in MeV and magnetic moments.[a] The symbol for a baryon denotes its mass and μ denotes its magnetic moment in units of $e/(2p)$.

$n-p=1.3$	$\mu(p)=2.79$
$\Sigma^- - \Sigma^+ = 8.0\pm0.1$	$\mu(n)=-1.91$
$\Sigma^- - \Sigma^0 = 4.9\pm0.1$	$\mu(\Lambda)=-0.73\pm0.16$
$\Sigma^0 - \Sigma^+ = 3.1\pm0.1$	$\mu(\Sigma^+)=2.3\pm0.6$
$\Xi^- - \Xi^0 = 6.5\pm0.2$	
$N^{*0} - N^{*++} = 0.4\pm0.9$	
$N^{*-} - N^{*++} = 7.9\pm6.8$	
$Y^{*-} - Y^{*+} = 5.8\pm3.1$	
$\Xi^{*-} - \Xi^{*0} = 4.9\pm2.2$	

[a] From Ref. 19.

(4) The electromagnetic properties of the hadrons can be calculated with sufficient accuracy in first-order perturbation theory, using the unperturbed wave functions of the $SU(3)$ multiplets.

We now amplify these assumptions and make some comments about them.

In the quark model, the assumption that the relative orbital angular momentum of any two of the three quarks in a baryon is zero means that we assume the quarks are parafermions of order three.[17] We make this assumption primarily for simplicity in treating the electromagnetic interactions of the quarks. However, we note that the assumption that the quarks are in P states, as is required by Fermi statistics, may lead to difficulties with the electromagnetic form factors of the nucleon.[1,18]

One version of the triplet-sextet model is that the triplet is a quark and the sextet a bound S state of two quarks. In this version, the quarks again must satisfy parastatistics, and the spin-1 sextet is not a true boson. For this reason, we have adopted the name "triplet-sextet" model, rather than the name "fermion-boson" model used in Ref. 2.

We have several reasons for choosing a nonrelativistic model. First, a number of features of the strong interactions are adequately accounted for with this restriction.[1,2] Secondly, the Coulomb and magnetic contact interactions are reasonable in a nonrelativistic model, but more complicated electromagnetic effects would have to be included in a relativistic model. Third, if we make an assumption about the strong-interaction potential between the triplet and sextet, we can explicitly calculate the contribution to the electromagnetic energy of a baryon arising from the Coulomb and magnetic interactions.

Even in a nonrelativistic model, there might be expected to be a magnetic spin-orbit interaction and a magnetic dipole-dipole interaction between the particles. However, the former vanishes in an S state, while the latter, which has the structure of a tensor potential, vanishes in expectation value in an S state. Therefore, as we have remarked, the assumption of S states only leads to a considerable simplification.

Dolgov *et al.*[12] and Barton and Dare[12] have considered the mass splittings of hadrons with Coulomb and spin-spin magnetic interactions in the quark model. The latter two authors obtained an inconsistency between the mass splittings of the mesons and the mass splittings of the baryons.

Since in all the models considered we have assumed that electromagnetic effects are U-spin-invariant, we list the predictions of this symmetry for the baryon mass splittings and magnetic moments. These relations are automatic consequences of our models. The models, of course, are more restrictive, and therefore lead to additional predictions. Among the mass splittings of the octet, the well-known Coleman-Glashow[3] relation follows from U-spin invariance:

$$n-p+\Xi^- - \Xi^0 = \Sigma^- - \Sigma^+ . \qquad (1)$$

Our notation is that the particle symbol denotes its mass. Among the members of the decimet, the following three independent relations follow from U-spin invariance:

$$N^{*0} - N^{*+} = Y^{*0} - Y^{*+} , \qquad (2)$$

$$N^{*-} - N^{*0} = Y^{*-} - Y^{*0} = \Xi^{*-} - \Xi^{*0} . \qquad (3)$$

The experimental mass splittings, taken from the compilation of Rosenfeld *et al.*,[19] are given in Table I. The octet relation agrees rather well with experiment, while the decimet relations are consistent with the presently available data.

The relations among the magnetic moments of the baryon octet which follow from U-spin invariance are the following:

$$\mu(p)=\mu(\Sigma^+) , \qquad (4)$$

$$\mu(n)=\mu(\Xi^0) , \qquad (5)$$

$$\mu(\Sigma^-)=\mu(\Xi^-) . \qquad (6)$$

The experimental values from Rosenfeld *et al.*[19] are given in Table I. We do not list the relations among the magnetic moments of the decimet because measurement of these magnetic moments seems remote.

If it is assumed that the electromagnetic current transforms like the $T_1{}^1$ component of an octet, additional relations among them may be obtained.[3,6] These additional relations follow from some of our models, but not from all of them.

The plan of this paper is as follows: In Sec. 2 we discuss various versions of the triplet-sextet model, and the electromagnetic mass splittings which follow from them. In Sec. 3 we discuss relations among the magnetic moments of the octet as given by the triplet-sextet model. In Sec. 4 we assume that the properties of the sextet can be derived from a quark model, and consider the

[17] O. W. Greenberg, Phys. Rev. Letters 13, 598 (1964).
[18] A. Mitra, Phys. Rev. 151, 1168 (1966).
[19] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. 39, 1 (1967).

TABLE II. Masses and quantum numbers of triplet t and sextet s.

Symbol	Mass m	Isospin I	Third component I_3	Spin S	Single-field model Hypercharge Y	Single-field model Charge Q	Single-field model Baryon number B	Two-field model Hypercharge Y	Two-field model Charge Q	Two-field model Baryon number B
t_1	m_t	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$1-y$	$1-\frac{1}{2}y$	$1-b$
t_2	$m_t+\epsilon_1$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$1-y$	$-\frac{1}{2}y$	$1-b$
t_3	$m_t+\epsilon_1+\Delta_t$	0	0	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-y$	$-\frac{1}{2}y$	$1-b$
s_1	m_s	1	1	1	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	y	$1+\frac{1}{2}y$	b
s_2	$m_s+\epsilon_2$	1	0	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	y	$\frac{1}{2}y$	b
s_3	$m_s+\epsilon_3$	1	-1	1	$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	y	$\frac{1}{2}y-1$	b
s_4	$m_s+\epsilon_2+\Delta_s$	$\frac{1}{2}$	$\frac{1}{2}$	1	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$y-1$	$\frac{1}{2}y$	b
s_5	$m_s+\epsilon_3+\Delta_s$	$\frac{1}{2}$	$-\frac{1}{2}$	1	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	$y-1$	$\frac{1}{2}y-1$	b
s_6	$m_s+\epsilon_3+2\Delta_s$	0	0	1	$-\frac{4}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	$y-2$	$\frac{1}{2}y-1$	b

consequence of this assumption for the electromagnetic properties of baryons. In Sec. 5 we derive for comparison the predictions of the quark model with Coulomb and spin-spin magnetic interactions. In Sec. 6 the electromagnetic mass differences of the 0^- and 1^- octets are given, and a sum rule relating mass splittings of the pseudoscalar mesons to those of the baryons is obtained. Finally, in Sec. 7 we discuss our results.

2. ELECTROMAGNETIC MASS SPLITTINGS OF BARYONS

In this section we assume that each baryon of the octet and decimet may be described as a bound state of a particle of spin 1 and another particle of spin $\frac{1}{2}$. The spin-1 particle is assumed to be an $SU(3)$ sextet, consisting of an isospin triplet, doublet, and singlet with hypercharge y, $y-1$, and $y-2$, respectively. The spin-$\frac{1}{2}$ particle is assumed to be an $SU(3)$ triplet, consisting of an isospin doublet with hypercharge $-y+1$ and an isospin singlet with hypercharge $-y$.

The triplet and sextet are assumed to be bound in a very deep potential well, and to move nonrelativistically. If we assume that the well is a square well for definiteness and ease of calculation, we find disagreement with experiment, as will be shown later. In fact, the calculation provides slight evidence for a repulsive core in the interaction. We take the octet and decimet of baryons to be the $^2S_{1/2}$ and $^4S_{3/2}$ bound states of the system, in the usual spectroscopic notation.

Within this model, the triplet and sextet may be derived from a single quarklike field, with the sextet having the quantum numbers of a two-quark state and the triplet the quantum numbers of a single quark. The second possibility is that the triplet and sextet are quanta of two different fields. We call these possibilities the single-field and two-field models, respectively.

One distinction between these possibilities is that in the single-field model, the triplet and sextet must have fractional charge, whereas in the two-field model their charges may be integral (in units of the electronic charge). Since the electromagnetic properties of the triplet and sextet depend on their charges, the predictions of the two alternatives will in general be different. The two-field model contains the parameter y, which is related to the charges of the members of the triplet and sextet by the Gell-Mann–Nishijima formula. For one value of this parameter, $y=\frac{2}{3}$, a number of the predictions of the two-field model reduce to those of the single-field model. The baryon numbers of the triplet and sextet will also in general be different in the single-field and two-field models, but this fact is irrelevant for our considerations. In the single-field model, we may also try to calculate the electromagnetic properties of the sextet in terms of the properties of the triplet. However, we find that when we make such a calculation, we obtain a prediction which disagrees with experiment.

We list in Table II the masses and quantum numbers of the triplet and sextet as given by the single-field and two-field models. These expressions differ slightly from those given in Refs. 2 and 13, since we have incorporated U-spin invariance into the expressions and have for convenience defined the Δ's in a different way. The mass splittings are given in terms of five parameters: Δ_t, Δ_s, ϵ_1, ϵ_2, and ϵ_3, the first two of which are assumed to arise from a strong symmetry-breaking interaction, and the last three from electromagnetic interactions. The parameters Δ_t and Δ_s will not enter into our considerations further.

We assume that the strong potential binding the triplet and sextet is perturbed by an electromagnetic potential consisting of a Coulomb part V_c and a magnetic part V_m given by

$$V_c=e^2(1/r)Q_tQ_s\,,\quad V_m=-(8\pi/3)\boldsymbol{\mu}_t\cdot\boldsymbol{\mu}_s\delta(\mathbf{r})\,,\quad (7)$$

where e is the magnitude of the electron charge, Q_t and Q_s are the charge operators of the triplet and sextet, $\boldsymbol{\mu}_t$ and $\boldsymbol{\mu}_s$ are their magnetic moment operators, and r is the distance between the two particles. We assume that the operator δM, which is responsible for the electromagnetic contribution to the mass of a baryon, is the sum of four terms:

$$\delta M=\delta m_t+\delta m_s+V_c+V_m\,,\quad (8)$$

where δm_t and δm_s are electromagnetic mass shifts of the triplet and sextet, respectively. We set $\hbar=c=1$. To evaluate δM for a particular baryon, we must take its expectation value with respect to the unperturbed wave function $|B\rangle$. In the model, this function is a product of a spatial part ψ, a spin part χ_B, and a unitary spin part η_B. The function χ_B is simply the wave function of a particle of spin 1 and a particle of spin $\frac{1}{2}$ combined to form a total spin of $\frac{3}{2}$ or $\frac{1}{2}$. The unitary spin wave functions are given in an appendix to Ref. 2. For the present, we do not specify the form of the spatial wave function. However, in omitting the subscript B on ψ, we are explicitly assuming that it is the same for all members of the baryon octet and decimet. We then can write (8) as follows:

$$\begin{aligned}\delta B=&\langle\eta_B|\delta m_t|\eta_B\rangle+\langle\eta_B|\delta m_s|\eta_B\rangle\\&+e^2\langle\psi|1/r|\psi\rangle\langle\eta_B|Q_tQ_s|\eta_B\rangle\\&-(8\pi/3)|\psi(0)|^2\langle\chi_B\eta_B|\boldsymbol{\mu}_t\cdot\boldsymbol{\mu}_s|\chi_B\eta_B\rangle,\quad(9)\end{aligned}$$

where $\delta B=\langle B|\delta M|B\rangle$ and $|\psi(0)|^2=\langle\psi|\delta(\mathbf{r})|\psi\rangle$.

Equation (9) as it stands contains too many parameters to be of much use. The first three terms contain five parameters: The quantity $\langle\eta_B|\delta m_t|\eta_B\rangle$ depends on the parameter ϵ_1, $\langle\eta_B|\delta m_s|\eta_B\rangle$ depends on ϵ_2 and ϵ_3, the quantity $\langle\psi|1/r|\psi\rangle$ is unknown, and $\langle\eta_B|Q_tQ_s|\eta_B\rangle$ contains the parameter y. The fourth term in (9), which arises from the magnetic interaction, contains four additional parameters.

This can be seen as follows. From U-spin invariance any electromagnetic property of a member of an $SU(3)$ multiplet depends only on its charge, provided no other member of the multiplet has the same Y and I_3. Thus, the magnetic moments of the members of the triplet are given in terms of two parameters, while the magnetic moments of the members of the sextet are given in terms of three parameters. These parameters, together with $|\psi(0)|^2$, make a total of six. However, the last term in (9) contains only the product $|\psi(0)|^2\boldsymbol{\mu}_t\cdot\boldsymbol{\mu}_s$, and thus the number of independent parameters is reduced to four.

To reduce the number of parameters in the magnetic interaction, we employ a simplifying assumption about the transformation properties of the magnetic moment discussed by several authors.[3–6,20,21] We shall assume that the electromagnetic current associated with the triplet and sextet transforms like the component $T_1{}^1$ of an octet. This implies that their magnetic moments are proportional to their charges. Furthermore, as we shall see in Sec. 3, the proportionality constant g_t of the triplet and g_s of the sextet, are positive. Then their magnetic moments can be written

$$\boldsymbol{\mu}_t=g_t\mu_0Q_t\mathbf{S}_t,\quad\boldsymbol{\mu}_s=g_s\mu_0Q_s\mathbf{S}_s,\qquad(10)$$

where $\mu_0=e/2p$ is the proton Bohr magneton, p being the proton mass. Putting (10) in (9), we obtain

$$\begin{aligned}\delta B=&\langle\eta_B|\delta m_t|\eta_B\rangle+\langle\eta_B|\delta m_s|\eta_B\rangle+\alpha\langle\eta_B|Q_tQ_s|\eta_B\rangle\\&-\beta\langle\eta_B|Q_tQ_s|\eta_B\rangle\langle\chi_B|\mathbf{S}_t\cdot\mathbf{S}_s|\chi_B\rangle,\quad(11)\end{aligned}$$

where α and β are parameters given by

$$\alpha=e^2\langle\psi|1/r|\psi\rangle,\quad\beta=(8\pi/3)\mu_0{}^2g_tg_s|\psi(0)|^2.\quad(12)$$

It is clear from (12) that α is positive. Also, with the assumption that g_t and g_s are positive, β is positive or zero, depending on whether $|\psi(0)|^2$ is positive or zero. [If the potential has an infinite repulsive core $\psi(0)$ will be zero.] If we assume a definite form for the potential between the triplet and sextet, we can calculate the parameter α. To calculate β, we need in addition an estimate of the parameters g_t and g_s. We shall obtain these in the next section from a consideration of the magnetic moments of the baryons.

At present we evaluate the 10 electromagnetic mass splittings in terms of the six parameters ϵ_1, ϵ_2, ϵ_3, y, α, and β.

The spin expectation values are trivial to evaluate. They are

$$\langle\chi_B{}^{(10)}|\mathbf{S}_t\cdot\mathbf{S}_s|\chi_B{}^{(10)}\rangle=\tfrac{1}{2},\quad\langle\chi_B{}^{(8)}|\mathbf{S}_t\cdot\mathbf{S}_s|\chi_B{}^{(8)}\rangle=-1,\qquad(13)$$

where the superscript on χ_B denotes the $SU(3)$ representation. The remaining expectation values can be calculated with the aid of the $SU(3)$ wave functions of Ref. 2. We also note that for two baryons B_i and B_j belonging to the same isospin multiplet; we have

$$\delta B_i-\delta B_j=B_i-B_j.\qquad(14)$$

We then obtain the following results for the 10 electromagnetic mass differences:

$$n-p=-\tfrac{1}{3}\epsilon_1+\tfrac{2}{3}\epsilon_3+(\alpha+\beta)(\tfrac{5}{6}y-\tfrac{2}{3})\qquad(15)$$

$$\Sigma^0-\Sigma^+=\tfrac{1}{6}\epsilon_1+\tfrac{1}{2}\epsilon_2+\tfrac{1}{6}\epsilon_3+(\alpha+\beta)(\tfrac{1}{3}y-\tfrac{1}{6}),\qquad(16)$$

$$\Sigma^--\Sigma^+=\tfrac{1}{3}\epsilon_1-\tfrac{1}{3}\epsilon_2+\epsilon_3+(\alpha+\beta)\tfrac{2}{3}y,\qquad(17)$$

$$\Xi^--\Xi^0=\tfrac{2}{3}\epsilon_1-\tfrac{1}{3}\epsilon_2+\tfrac{1}{3}\epsilon_3+(\alpha+\beta)(-\tfrac{1}{6}y+\tfrac{2}{3}),\qquad(18)$$

$$N^{*+}-N^{*++}=\tfrac{1}{3}\epsilon_1+\tfrac{2}{3}\epsilon_2+(\alpha-\tfrac{1}{2}\beta)(\tfrac{1}{6}y-1),\qquad(19)$$

$$\begin{aligned}N^{*0}-N^{*+}&=Y^{*0}-Y^{*+}\\&=\tfrac{1}{3}\epsilon_1+\tfrac{1}{3}\epsilon_3+(\alpha-\tfrac{1}{2}\beta)(\tfrac{1}{6}y-\tfrac{1}{3}),\end{aligned}\qquad(20)$$

$$\begin{aligned}N^{*-}-N^{*0}&=Y^{*-}-Y^{*0}=\Xi^{*-}-\Xi^{*0}\\&=\tfrac{1}{3}\epsilon_1-\tfrac{2}{3}\epsilon_2+\tfrac{2}{3}\epsilon_3+(\alpha-\tfrac{1}{2}\beta)(\tfrac{1}{6}y+\tfrac{1}{3}).\end{aligned}\qquad(21)$$

Since we have ten baryon electromagnetic mass splittings given in terms of six parameters, it seems at first glance that we can obtain only four relations among these mass splittings. But U-spin invariance alone is sufficient to lead to four relations, those given in Eqs. (1), (2), and (3). Thus, superficially it seems as if we can obtain from our model only results which follow from U-spin invariance. However, because of the structure

[20] R. H. Dalitz, in *High Energy Physics: Les Houches Lectures*, edited by C. DeWitt and M. Jacob (Gordon and Breach Science Publishers, Inc., New York, 1965).

[21] L. Van Hove, CERN Report No. 65-24 1966 (unpublished).

of Eqs. (15)–(21), it turns out that we can obtain five relations among the baryon mass splittings. The additional relation is

$$N^{*-}-N^{*++}=3(N^{*0}-N^{*+}).$$

This relation has also been obtained by other authors using different assumptions, for example by Rubinstein.[8] At present $N^{*0}-N^{*+}$ is not known experimentally. However, using the relations which follow from U-spin invariance, this sum rule can be written

$$3(N^{*0}-N^{*+})=N^{*0}-N^{*++}+\Xi^{*-}-\Xi^{*0}. \tag{22}$$

This relation is consistent with the presently available data.

Eliminating ϵ_1, ϵ_2, and ϵ_3 from the five remaining mass differences, we can obtain two independent relations among α, β, and y. Two such are

$$(\Sigma^--\Sigma^+)+2(\Sigma^0-\Sigma^+)-(n-p)-\tfrac{1}{3}(\Xi^{*-}-\Xi^{*0})-\tfrac{4}{3}(N^{*0}-N^{*++})=2\alpha+\tfrac{1}{4}\beta(3y-2), \tag{23}$$

$$3(\Sigma^--\Sigma^+)-2(\Sigma^0-\Sigma^+)-(n-p)-3(\Xi^{*-}-\Xi^{*0})=\tfrac{3}{4}\beta(y+2). \tag{24}$$

Using the values of the experimental mass differences from Table I, (23) and (24) become

$$2\alpha+\beta(3y-2)/4=10.7\pm1.5\ \text{MeV}, \tag{25}$$

$$3\beta(y+2)/4=1.8\pm6.6\ \text{MeV}. \tag{26}$$

Unfortunately, the errors in Eq. (26) are so large that we cannot put any limits on the parameter y. We now specialize to the case $y=\frac{2}{3}$. Then, since $\alpha>0$ and $\beta\geq0$, Eqs. (23) and (24) become

$$\Sigma^--\Sigma^++2(\Sigma^0-\Sigma^+)-(n-p)-\tfrac{1}{3}(\Xi^{*-}-\Xi^{*0})-\tfrac{4}{3}(N^{*0}-N^{*++})>0, \tag{27}$$

$$3(\Sigma^--\Sigma^+)-2(\Sigma^0-\Sigma^+)-(n-p)-3(\Xi^{*-}-\Xi^{*0})\geq0. \tag{28}$$

The inequality (27) holds experimentally, while (28) is consistent with the present data.

3. MAGNETIC MOMENTS OF BARYONS

Since we have assumed that a baryon is a bound S state of a triplet and sextet, we can take the magnetic moment operator $\boldsymbol{\mu}$ of a baryon to be simply the sum of the magnetic moment operators of the triplet and sextet. We can write for $\boldsymbol{\mu}$ the following expression[22]:

$$\boldsymbol{\mu}=\mathbf{J}(\boldsymbol{\mu}_t+\boldsymbol{\mu}_s)\cdot\mathbf{J}/J^2, \tag{29}$$

where $\mathbf{J}$ is the spin operator for a baryon. Equivalently and more simply we can write

$$\mu_z=\mu_{tz}+\mu_{sz}. \tag{30}$$

Using Eq. (10) in Eq. (30) and taking the expectation value with respect to the state $|B\rangle$, we obtain

$$\mu(B)=g_t\langle\eta_B|Q_t|\eta_B\rangle\langle\chi_B|S_{tz}|\chi_B\rangle+g_s\langle\eta_B|Q_s|\eta_B\rangle\times\langle\chi_B|S_{sz}|\chi_B\rangle, \tag{31}$$

where $\mu(B)$ is the magnetic moment of a baryon in units of the proton magneton and χ_B is the spin wave function of a baryon with spin and z component both equal to J_B. For members of the octet, Eq. (31) becomes

$$\mu^{(8)}(B)=\tfrac{2}{3}g_s\langle\eta_B|Q_s|\eta_B\rangle-\tfrac{1}{6}g_t\langle\eta_B|Q_t|\eta_B\rangle, \tag{32}$$

while for the members of the decimet we have

$$\mu^{(10)}(B)=g_s\langle\eta_B|Q_s|\eta_B\rangle+\tfrac{1}{2}g_t\langle\eta_B|Q_t|\eta_B\rangle. \tag{33}$$

The expectation values of the charge operators are easily calculated using the unitary-spin wave functions from Ref. 2. Since nothing is known about the magnetic moments of the members of the decimet, we shall not write them down. We merely remark that unless $y=\frac{2}{3}$, the magnetic moments of the members of the decimet are not proportional to their charges. The magnetic moments of the members of the octet are

$$\mu(p)=[4g_s(3y+4)+g_t(3y-2)]/36=\mu(\Sigma^+), \tag{34}$$

$$\mu(n)=(4g_s+g_t)(3y-4)/36=\mu(\Xi^0), \tag{35}$$

$$\mu(\Lambda)=(4g_s+g_t)(y-1)/12, \tag{36}$$

$$\mu(\Sigma^0)=(4g_s+g_t)(3y-1)/36, \tag{37}$$

$$\mu(\Sigma^-)=[4g_s(y-2)+g_ty]/12=\mu(\Xi^-). \tag{38}$$

We have expressions for the eight baryon magnetic moments in terms of the three parameters g_t, g_s, and y. Therefore we can obtain five relations among the magnetic moments. Three of these, $\mu(p)=\mu(\Sigma^+)$, $\mu(n)=\mu(\Xi^0)$, and $\mu(\Sigma^-)=\mu(\Xi^-)$, follow simply from U-spin invariance. Two other independent relations are

$$\mu(\Sigma^0)=3\mu(\Lambda)-2\mu(n), \tag{39}$$

$$\mu(\Sigma^-)=-[\mu(n)+\mu(p)]+3[2\mu(\Lambda)-\mu(n)]. \tag{40}$$

The possibility of verifying the sum rule (39) seems remote, because of the difficulty of measuring the magnetic moment of the Σ^0. However, it should be possible to test the relation given by (40). This result is to be compared with the quark-model prediction for $\mu(\Sigma^-)$, which can be obtained from the above by setting $\mu(n)=2\mu(\Lambda)$. Then the quark model gives $\mu(\Sigma^-)=-0.9$, while the sextet-triplet model gives $\mu(\Sigma^-)=0.3\pm1.0$. Note that Eq. (40) is obtained from $\mu(\Sigma^+)+\mu(\Sigma^-)=2\mu(\Sigma^0)$ by using (39) and $\mu(p)=\mu(\Sigma^+)$.

Enough experimental information about the magnetic moments exists to obtain the values of the parameters y, g_t, and g_s. From (35) and (36) we obtain

$$\frac{\mu(\Lambda)}{\mu(n)}=\frac{3(y-1)}{3y-4} \tag{41}$$

independently of g_t and g_s. Using the experimental

[22] J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), pp. 30–35.

value for $\mu(\Lambda)/\mu(n)=0.38\pm0.08$, we can solve for the parameter y. We obtain $y=0.8\pm0.1$. Using $y=0.8$, we can use the experimental values of $\mu(p)$ and $\mu(n)$ to determine g_s and g_t. We obtain

$$y=0.8: \quad g_t=29\,,\; g_s=3.5\,,\; g_t/g_s=8.3\,. \qquad (42)$$

With the value of y given in (42), we have a two-field model with nonintegral charge. The best model with integral charge is the one with $y=0$. This gives $\mu(\Lambda)/\mu(n)=\frac{3}{4}$, in contradiction to the experimental value. For the single-field model ($y=\frac{2}{3}$), we obtain the predictions

$$\mu(n)=2\mu(\Lambda)=-2\mu(\Sigma^0)\,, \quad \mu(\Sigma^-)=-\mu(p)-\mu(n)\,. \qquad (43)$$

These are the same as the quark-model predictions. Again using the experimental values of $\mu(p)$ and $\mu(n)$, we obtain the parameters g_t and g_s. These become

$$y=\tfrac{2}{3}: \quad g_s=4.2\,,\; g_t=18\,,\; g_t/g_s=4.2\,. \qquad (44)$$

Equations (42) and (44) show that g_t and g_s are positive, as we have remarked in the previous section.

Now that we have estimates for g_t and g_s, we can calculate the parameter β as well as α from (12), provided we assume a definite form for the potential between the triplet and sextet. We choose an infinitely deep square well of radius a. Then $\psi(r)$ is given by

$$\psi(r)=\sin(\pi r/a)/[(2\pi a)^{1/2}r]\,. \qquad (45)$$

With this wave function, we obtain

$$\langle\psi|1/r|\psi\rangle=2.44/a\,, \quad |\psi(0)|^2=\pi/(2a^3)\,. \qquad (46)$$

Then, using the values of g_t and g_s from (44), we obtain

$$\alpha=0.018/a\,, \qquad (47)$$

$$\beta/\alpha=102/(pa)^2\,. \qquad (48)$$

Rather than guess at a value for the radius a, we estimate it by using the experimental values of the parameters α and β in the single-field model. Using the experimental mass differences in Eqs. (25) and (26) with $y=\frac{2}{3}$, we obtain

$$\alpha=5.3\pm0.8 \text{ MeV}\,, \quad \beta=0.9\pm3.3 \text{ MeV}\,. \qquad (49)$$

Then from (47), we get

$$a=0.67\pm0.10 \text{ F}\,, \qquad (50)$$

while from (48), using $\beta/\alpha\leq1$ from (49), we get

$$a\geq2.5 \text{ F}\,. \qquad (51)$$

From the inconsistency of (50) and (51), we conclude that a square-well potential is not adequate to represent the interaction between the triplet and sextet. In order to obtain a result in agreement with experiment, $|\psi(0)|^2$ must be appreciably smaller than the value given in (46). This suggests that there should be a soft repulsive core in the triplet-sextet interaction. At present, however, there is not nearly enough evidence in favor of the model to justify making calculations with such a complicated potential. Therefore we regard α and β as parameters to be determined from experiment. An improved measurement of β is clearly desirable, since if β should turn out to be negative, the model would have to be revised.

4. PROPERTIES OF THE SEXTET

We have seen that a two-field model with integral charges gives a prediction for the magnetic moment of the Λ which disagrees with experiment. For this reason, the two-field model loses much of its attractiveness. Turning to the single-field model, we may attempt to calculate some of the properties of the sextet, assuming it is a bound state of two triplets. In particular, we shall attempt to obtain relations between the parameters ϵ_2, ϵ_3, and g_s of the sextet and the parameters ϵ_1 and g_t of the triplet.

In analogy with the procedure we previously followed, we assume that the electromagnetic contribution to the mass of a sextet δm_s is given by a sum of four terms:

$$\delta m_s=\delta m_1+\delta m_2+V_c'+V_m'\,, \qquad (52)$$

where δm_1 and δm_2 are the electromagnetic mass shifts of the two triplets and V_c' and V_m' are the Coulomb and magnetic interactions. We are dropping the subscript t on quantities referring to the triplet, except for g_t. The Coulomb and magnetic terms are given by

$$V_c'=e^2Q_1Q_2/r_{12}\,, \quad V_m'=-(8\pi/3)\boldsymbol{\mu}_1\cdot\boldsymbol{\mu}_2\delta(r_{12})\,, \qquad (53)$$

where r_{12} is the distance between the two particles of the triplet which make up the sextet. To evaluate the expectation value of δm_s, we write the sextet wave function $|s_i\rangle$ as a product of space, spin, and unitary-spin parts. For simplicity we assume that the spatial part is symmetric under the interchange of the coordinates of the two triplets. (The spin and unitary-spin parts are automatically symmetric from the requirement that the bound state is a unitary sextet of spin one.) Thus the single field is assumed to be a parafermion field.

Using the notation $\langle s_i|\delta m_s|s_i\rangle=\delta s_i$, we obtain

$$\begin{aligned}\delta s_1&=(4/9)\xi\,,\\ \delta s_2&=\epsilon_1-(2/9)\xi\,,\\ \delta s_3&=2\epsilon_1+(1/9)\xi\,,\end{aligned} \qquad (54)$$

where

$$\xi=e^2\langle s|/r_{12}|s\rangle-\tfrac{2}{3}\pi g_t^2\mu_0^2|\psi_s(0)|^2\,. \qquad (55)$$

In (55), we have left the subscript i off s to indicate that the result is the same for all members of the sextet. We make no attempt to estimate the parameter ξ, which may be positive or negative. For a given isospin multiplet in the sextet, we have $\delta s_i-\delta s_j=s_i-s_j$. Then, using the fact that $s_2-s_1=\epsilon_2$, $s_3-s_1=\epsilon_3$, we obtain

$$\begin{aligned}\epsilon_2&=\epsilon_1-\tfrac{2}{3}\xi\,,\\ \epsilon_3&=2\epsilon_1-\tfrac{1}{3}\xi\,.\end{aligned} \qquad (56)$$

Eliminating ξ, we obtain the relation

$$\epsilon_2 = 2\epsilon_3 - 3\epsilon_1. \quad (57)$$

Using this relation in Eqs. (15)–(21) we obtain an additional independent relation among the baryon masses. One such relation is

$$\Sigma^- - \Sigma^+ - 2(\Sigma^0 - \Sigma^+) + 3(n-p) = (5/3)(\Xi^{*-} - \Xi^{*0}) + \tfrac{2}{3}(N^{*0} - N^{*++}). \quad (58)$$

Experimentally, the left-hand side of (58) is 5.7 MeV; the right-hand side is 8.5±3.7 MeV. Thus, within the experimental error, there is agreement.

We now turn to the magnetic moment of the sextet. We assume it is given by

$$\mu_{sz} = \mu_{1z} + \mu_{2z}. \quad (59)$$

Using $\boldsymbol{\mu}_i = g_t \mu_0 Q_i \mathbf{S}_i$ and taking expectation values, we obtain

$$2g_s = g_t. \quad (60)$$

This is to be compared with the value $4.2g_s = g_t$ which we obtained by fitting the experimental values of $\mu(p)$ and $\mu(n)$. If we use (60), we find

$$\mu(p)/\mu(n) = -2, \quad (61)$$

compared to the $SU(6)$ prediction of $\mu(p)/\mu(n) = -\frac{3}{2}$ and the experimental value $\mu(p)/\mu(n) = -1.46$.

Thus, the simple picture in which the magnetic moment of the sextet arises from the magnetic moment of two triplets does not give as good a value of $\mu(p)/\mu(n)$ as one might hope. In view of this disagreement with experiment, we shall consider the parameters of the sextet to be free.

5. ELECTROMAGNETIC PROPERTIES OF BARYONS IN THE QUARK MODEL

As we have stated previously, many authors have considered the problems of the baryon electromagnetic mass splittings and magnetic moments in the quark model. In particular, the authors of Ref. 12 have considered the mass splittings of hadrons, including a Coulomb and spin-spin magnetic interaction. However, we shall briefly consider the problem here, pointing out which of the results depend on the details of the baryon wave functions, and which do not.

We assume, following the authors of Ref. 12, that the contribution to the electromagnetic mass of a baryon is given by the sum of an intrinsic term, a Coulomb term, and a magnetic term[23]:

$$\delta M = \sum_i \delta m_i + \sum_{i<j} V_c^{ij} + \sum_{i<j} V_m^{ij}, \quad i,j = 1,2,3 \quad (62)$$

where

$$V_c^{ij} = e^2 Q_i Q_j / r_{ij},$$
$$V_m^{ij} = -(2\pi/3) g^2 \mu_0^2 \delta(\mathbf{r}_{ij}) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j Q_i Q_j.$$

Here $\boldsymbol{\sigma}_i$ and $\boldsymbol{\sigma}_j$ are the quark spin matrices of the ith and jth quark, and $\mathbf{r}_{ij}$ is the distance between them. We assume that the baryon wave function can be written as a product of two factors, one of which ψ depends only on the spatial coordinates of the quarks, and is symmetric under the interchange of any two spatial coordinates, and another φ_B which depends on the spin and unitary-spin coordinates.

Writing the baryon wave function $|B\rangle = \psi\varphi_B$ and taking the expectation value of δM, we obtain

$$\delta B = \sum_i \langle \varphi_B | \delta m_i | \varphi_B \rangle + \sum_{i<j} \langle \psi | e^2/r_{ij} | \psi \rangle \langle \varphi_B | Q_i Q_j | \varphi_B \rangle - (2\pi/3) g^2 \mu_0^2 \sum_{i<j} \langle \psi | \delta(\mathbf{r}_{ij}) | \psi \rangle \langle \varphi_B | Q_i Q_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j | \varphi_B \rangle, \quad (63)$$

where $\delta B = \langle B | \delta M | B \rangle$. Since by assumption ψ is completely symmetric under the interchange of any two quarks, the quantities $\langle \psi | e^2/r_{ij} | \psi \rangle$ and $\langle \psi | \delta(\mathbf{r}_{ij}) | \psi \rangle$ are independent of i and j, and can be taken outside the sums in Eq. (63). Then Eq. (63) becomes

$$\delta B = \sum_i \langle \varphi_B | \delta m_i | \varphi_B \rangle + u \sum_{i<j} \langle \varphi_B | Q_i Q_j | \varphi_B \rangle - v \sum_{i<j} \langle \varphi_B | Q_i Q_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j | \varphi_B \rangle, \quad (64)$$

where

$$u = \langle \psi | e^2/r_{ij} | \psi \rangle \quad \text{and} \quad v = (\tfrac{2}{3}\pi) g^2 \mu_0^2 \langle \psi | \delta(\mathbf{r}_{ij}) | \psi \rangle.$$

For members of the decimet, φ_B is unambiguously determined. There is only one way to form a decimet from the wave functions of three quarks: The baryon wave function must be totally symmetric in spin and unitary-spin indices separately. However, there are two linearly independent ways of constructing an octet wave function from the unitary-spin coordinates of three quarks. Similarly there are two independent ways of constructing a spin-$\frac{1}{2}$ wave function from three spin-$\frac{1}{2}$ wave functions. This means that in general the octet wave function must be written as a sum of four terms, each of which is a product of a spin and unitary-spin wave function:

$$\varphi_B^{(8)} = c_1 \chi^{(8)} \eta_B^{(8)} + c_2 \chi^{(8)} \bar{\eta}_B^{(8)} + c_3 \bar{\chi}^{(8)} \eta_B^{(8)} + c_4 \bar{\chi}^{(8)} \bar{\eta}_B^{(8)}, \quad B = 1, \cdots, 8, \quad (65)$$

where $\chi^{(8)}$ and $\bar{\chi}^{(8)}$ are two linearly independent spin wave functions with z component equal to $\frac{1}{2}$, and $\eta_B^{(8)}$ and $\bar{\eta}_B^{(8)}$ are the independent unitary-spin functions. The complex coefficients c_i satisfy

$$\sum |c_i|^2 = 1, \quad i = 1, \cdots, 4. \quad (66)$$

For definiteness we choose $\chi^{(8)}$ and $\eta_B^{(8)}$ each to be symmetric under the interchange of the relevant coordinates of quarks 1 and 2, while $\bar{\chi}^{(8)}$ and $\bar{\eta}^{(8)}$ are antisymmetric under this interchange. This specifies the

[23] We assume that the quarks have point charges and magnetic moments, while Barton and Dare used extended sources. Also, we assume parastatistics for the quarks while Barton and Dare assumed Fermi statistics.

octet wave functions uniquely in terms of the parameters c_i. The wave function $\varphi_B^{(8)}$ is more general than one would ordinarily choose. However, we shall see that many results are independent of the form of $\varphi_B^{(8)}$.

The expectation values $\sum_{i<j} \langle \varphi_B | \delta m_i | \varphi_B \rangle$ and $\sum_{i<j} \langle \varphi_B | Q_i Q_j | \varphi_B \rangle$ are independent of the parameters c_i arising in the octet wave function. However, since the expectation value $\sum_{i<j} \langle \varphi_B | Q_i Q_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j | \varphi_B \rangle$ contains correlations between spin and charge operators, its value does depend in general on the c_i. If we assume that the octet and decimet combine to form a 56-dimensional representation of $SU(6)$, the octet wave function is obtained from Eq. (65) with

$$c_1 = c_4 = 2^{-1/2}, \quad c_2 = c_3 = 0. \tag{67}$$

Although assuming the baryons form a 56-dimensional representation of $SU(6)$ implies Eq. (67), the converse is not true.

Substituting (65) and the decimet wave function in (64), we obtain

$$n - p = \epsilon_1 - \tfrac{1}{3}u + \tfrac{2}{3}v[\mathrm{Re}(c_1^* c_4 + c_2^* c_3) - |c_2|^2 - |c_3|^2], \tag{68}$$

$$\Sigma^0 - \Sigma^+ = \epsilon_1 - \tfrac{1}{3}u + v[\tfrac{1}{2}|c_1|^2 - (7/6)|c_2|^2 - (7/6)|c_3|^2 + \tfrac{1}{2}|c_4|^2 + (5/3)\,\mathrm{Re}(c_1^* c_4 + c_2^* c_3)], \tag{69}$$

$$\Sigma^- - \Sigma^+ = 2\epsilon_1 + \tfrac{1}{3}u + v[|c_1|^2 - \tfrac{1}{3}|c_2|^2 - \tfrac{1}{3}|c_3|^2 + |c_4|^2 + \tfrac{4}{3}\,\mathrm{Re}(c_1^* c_4 + c_2^* c_3)], \tag{70}$$

$$N^{*0} - N^{*+} = \epsilon_1 - \tfrac{1}{3}u + \tfrac{1}{3}v, \tag{71}$$

$$N^{*-} - N^{*++} = 3\epsilon_1 - u + v, \tag{72}$$

$$N^{*-} - N^{*0} = \epsilon_1 + \tfrac{2}{3}u - \tfrac{2}{3}v. \tag{73}$$

The other four independent electromagnetic mass differences are given by U-spin invariance. Despite all the parameters in these expressions, it turns out that we obtain the two sum rules given by Eqs. (22) and (57). The sum rule (22) was previously obtained with the triplet-sextet model, while (57) was obtained in that model only after requiring that the sextet be a bound state of two triplets. In addition, because $u > 0$ and $v \geq 0$, we get the following inequality:

$$\Sigma^- - \Sigma^0 - (n - p) > 0. \tag{74}$$

This result is consistent with the data.

If the wave functions $\varphi_B^{(8)}$ are given by the 56-dimensional representation of $SU(6)$, we obtain one additional sum rule and an additional inequality:

$$n - p = Y^{*-} - Y^{*+} - (\Xi^{*-} - \Xi^{*0}), \tag{75}$$

$$\Sigma^0 - \Sigma^+ - (n - p) \geq 0. \tag{76}$$

These relations are both satisfied by the present data.

We now turn to the magnetic moments of the baryons. Following Morpurgo,[24] we assume the magnetic-moment operator of a baryon is simply

$$\mu_z = \sum_{i=1}^{3} \mu_{iz} = \tfrac{1}{2} g \mu_0 \sum_i Q_i \sigma_{iz}. \tag{77}$$

Taking expectation values, we find that in units of $\frac{1}{2}g\mu_0$, the octet magnetic moments are

$$\mu(p) = \tfrac{2}{3}[|c_1|^2 + |c_4|^2 + \mathrm{Re}(c_1^* c_4 + c_2^* c_3)], \tag{78}$$

$$\mu(n) = -\tfrac{1}{3}[2|c_1|^2 + 2|c_4|^2 + 2\,\mathrm{Re}(c_1^* c_4 + c_2^* c_3) - 1], \tag{79}$$

$$\mu(\Lambda) = -\mu(\Sigma^0) = \tfrac{1}{2}\mu(n), \quad \mu(\Sigma^-) = -\mu(p) - \mu(n). \tag{43}$$

The other octet magnetic moments are given in terms of these by U-spin invariance. The relations (43), which have been obtained previously,[3–7] are the same as those given in the triplet-sextet model with $y = \frac{2}{3}$. They follow from the quark model independently of the values of the coefficients c_i. Using the values of the c_i from Eq. (67), we obtain the usual result,

$$\mu(p)/\mu(n) = -\tfrac{3}{2}. \tag{80}$$

It is interesting to note that despite the fact that $\mu(p)$ and $\mu(n)$ are each written in terms of four complex parameters c_i, it can be shown that if Eq. (80) is satisfied, then the c_i obey (67). This is seen as follows: Letting

$$\gamma \equiv |c_1|^2 + |c_4|^2 + \mathrm{Re}(c_1^* c_4 + c_2^* c_3),$$

we obtain from (78), (79), and (80) the condition $\gamma = \frac{3}{2}$. Now using

$$2\,\mathrm{Re}(c_1^* c_4 + c_2^* c_3) \leq \sum |c_i|^2 = 1,$$

we get $\gamma \leq |c_1|^2 + |c_4|^2 + \frac{1}{2}$. Then, since $\gamma = \frac{3}{2}$, we obtain the inequality

$$1 \leq |c_1|^2 + |c_4|^2. \tag{81}$$

But from $\sum |c_i|^2 = 1$ we have $|c_1|^2 + |c_4|^2 \leq 1$. Comparing with (81), we must have $|c_1|^2 + |c_4|^2 = 1$, which implies that $c_2 = c_3 = 0$. Then $\gamma - 1 = \frac{1}{2} = \mathrm{Re}\, c_1^* c_4$ or $\mathrm{Re}\, c_1^* c_4 = \frac{1}{2}$. But this can be satisfied only if $c_1 = c_4 = 1/\sqrt{2}$, where we have assumed without loss of generality that $c_1 \geq 0$. Thus, we have obtained the result given in (67).

6. ELECTROMAGNETIC MASS SPLITTINGS OF MESONS

The quark and triplet-sextet models with Coulomb and magnetic contact interactions may also be used to investigate the electromagnetic properties of the mesons. Both models allow the possibility of considering the 0^- and 1^- meson nonets to be the ${}^1S_0$ and ${}^3S_1$ bound states of a triplet and an antitriplet. With such an assumption the two models will only give different results if one assumes that the triplet-sextet model is a two-field model in which $y \neq \frac{2}{3}$. But as we saw previously, it is desirable to take $y = \frac{2}{3}$ in order to fit the magnetic-moment ratio $\mu(\Lambda)/\mu(n)$. (The value $y = 0.8$ gives a better fit, but we ignore this possibility.) Therefore the

[24] C. Becchi and G. Morpurgo, Phys. Rev. 140, B687 (1965); Phys. Letters 17, 352 (1965).

bound triplet-antitriplet states of both models are indistinguishable.

In the quark model, nonet states involving higher orbital angular momentum or more quark-antiquark pairs may also be used to represent objects with the quantum numbers of the meson nonets, but it is reasonable to assume that such objects lie higher in energy than the simpler structure described above. In the case of the sextet-triplet model however, the situation is somewhat different, since one cannot *a priori* rule out the possibility that the 0^- and 1^- meson nonets are the $^1S_0$ and $^3S_1$ bound states of a sextet-antisextet. However, there should also exist a 2^- nonet, corresponding to the $^5S_2$ state of the sextet and antisextet. This 2^- nonet must be assumed to lie somewhat higher in energy. Furthermore, the reducible representation $6\times\bar{6}$ contains in addition to an octet and singlet (which together we call a nonet) a 27-dimensional representation. We therefore have to make the additional assumption that the octet and singlet are more tightly bound than the **27**. We shall consider meson mass splittings in both the triplet-antitriplet and sextet-antisextet models, realizing of course that the former is preferable on grounds of simplicity.

At present there is not as much experimental information available concerning the electromagnetic properties of the mesons as of the baryons. Only two meson electromagnetic mass differences are at present known with any precision, namely, $\pi^+-\pi^0=4.6$ MeV and $K^0-K^+=4.1$ MeV. However, in the triplet-antitriplet model, the meson electromagnetic mass differences are described by three parameters, the quantity ϵ_1 and the effective strengths of the Coulomb and magnetic interactions. Therefore no presently verifiable sum rule, involving the meson mass splittings alone can be obtained. The situation with respect to magnetic moments is even worse, since the pseudoscalar mesons have no magnetic moment and the measurement of the magnetic moments of the vector mesons seems remote. We shall therefore not discuss the magnetic moments.

In the triplet-antitriplet or quark model we obtain one sum rule relating the mass splittings of the vector mesons to the mass splittings of the pseudoscalars. This is

$$\pi^+-\pi^0-(\rho^+-\rho^0)=\tfrac{3}{2}(K^{*0}-K^{*+})-\tfrac{3}{2}(K^0-K^+). \quad (82)$$

Furthermore, since we know the sign of the Coulomb and magnetic interactions, we can obtain three inequalities:

$$K^{*0}-K^{*+}\geq K^0-K^+, \quad (83)$$

$$\rho^+-\rho^0>\tfrac{1}{3}(\pi^0-\pi^+), \quad (84)$$

$$\pi^+-\pi^0>0. \quad (85)$$

The last of these inequalities is of course verified by experiment.

In the sextet-antisextet model we obtain the inequality (83) plus the inequality

$$\pi^+-\pi^0\geq\rho^+-\rho^0.$$

[This inequality of course also holds in the triplet-antitriplet model, as can be immediately seen from (82) and (83).] In the sextet-antisextet model, we obtain no equality relating the four mass differences, since the model contains four parameters: ϵ_2, ϵ_3, and the strengths of the Coulomb and magnetic interactions. Thus the sextet-antisextet model gives less information than the triplet-antitriplet model.

Thus far, we have obtained no contradiction with the available experimental data. However, each of the models allows us to obtain one additional sum rule relating the meson mass splittings to the baryon mass splittings. For the quark model this relation is

$$2(\pi^+-\pi^0)+3(K^0-K^+) \\ =3(n-p)+\Sigma^--\Sigma^+-2(\Sigma^0-\Sigma^+). \quad (86)$$

Equation (86), which holds independently of the values of the c_i in the octet wave function, is definitely in contradiction with experiment, the left side being 21.5 MeV and the right side being 5.9 MeV. Barton and Dare have already pointed out this contradiction, although they stated it in a different form.

For the triplet-sextet model of baryons, combined with the triplet-antitriplet model of mesons, we obtain the sum rule

$$2(\pi^+-\pi^0)+3(K^0-K^+) \\ =2(\Sigma^0-\Sigma^+)-3(n-p)-(\Sigma^--\Sigma^+) \\ +\tfrac{4}{3}(N^{*0}-N^{*++})+(10/3)(\Xi^{*-}-\Xi^{*0}). \quad (87)$$

Experimentally, the left-hand side is 21.5 MeV as before, while the right side is 11 ± 7 MeV. Thus, there is a $1\frac{1}{2}$ standard-deviation discrepancy. Again the difficulty is that the sum rule (87) involves the masses of the members of the decimet, which are not known to sufficient accuracy. Note however, that if we assume that the triplet-sextet model can be derived from the quark model, we obtain Eq. (57), which, when combined with (87) yields (86).

Finally, we remark that the sum rule relating the meson and baryon mass splittings in the sextet-antisextet model of mesons is

$$\pi^+-\pi^0+2.7(K^0-K^+) \\ =1.78(n-p)-0.60(\Sigma^0-\Sigma^+)+0.52(\Sigma^--\Sigma^+) \\ +1.04(N^{*0}-N^{*+})+0.52(\Xi^{*-}-\Xi^{*0}).$$

The numbers in this sum rule, although written in decimal form, are exact. Despite the fact that the sum rule involves decimet masses, it is definitely in contradiction with experiment, the left-hand side being 15.7 MeV and the right-hand side 9.1 ± 1.5 MeV. In evaluating this sum rule, we used Eq. (22).

7. CONCLUSIONS

We have seen that the triplet-sextet model of the baryons with Coulomb and magnetic interactions leads to one sum rule for the electromagnetic mass splittings of the baryons, in addition to the relations which follow from U-spin invariance. This sum rule is consistent with the presently available data. We have also seen that, using the experimental value of $\mu(\Lambda)/\mu(n)$, we can determine the hypercharge y which is a parameter of the model. The value obtained, $y=0.8\pm0.1$, is sufficiently close to that given by the single-field value, $y=\frac{2}{3}$, as to lead us to adopt this model over the two-field model. With the single-field model, we obtain in addition to the sum rules for the baryon masses, two inequalities which are also consistent with the experimental data.

For the mesons, the triplet-antitriplet model with $y=\frac{2}{3}$ predicts one sum rule and three inequalities, only one of which can be checked by the present experimental data. The predictions are indistinguishable from those of the quark model. Further, it is possible to obtain a sum rule relating the meson and baryon mass differences. While this result disagrees with experiment by $1\frac{1}{2}$ standard deviations, it is considerably better than the analogous result for the quark model. If we assume instead that the mesons are bound states of the sextet and antisextet, we obtain a much poorer agreement with experiment, and we discard this version of the model.

The single-field model contains nine parameters which enter into the expressions for the hadron electromagnetic mass differences and baryon magnetic moments. These are the intrinsic mass-splitting parameters ϵ_1, ϵ_2, and ϵ_3, the strengths of the Coulomb and magnetic interactions in the meson and baryon cases, and the g factors g_t and g_s of the triplet and sextet. To eliminate some of these parameters, we considered the sextet to be a bound state of two triplets. This assumption enabled us to eliminate the intrinsic mass parameter ϵ_3 and the sextet g factor g_s. We were then able to obtain one further sum rule for the baryon electromagnetic mass differences, but obtained also the prediction $\mu(\Lambda)/\mu(n)=-2.0$, which disagrees with experiment by 37%. We also obtained a relation between the meson and baryon mass splittings, which is the same as the one obtained with the quark model and which is in serious contradiction to experiment. Therefore, we must regard the mass splittings ϵ_2 and ϵ_3 and the sextet g factor g_s to be free parameters.

The quark model on the other hand, contains only six parameters ϵ_1, g_1, and the strengths of the Coulomb and magnetic interactions in the meson and baryon cases, provided it is assumed that the baryons form a 56-dimensional representation of $SU(6)$. In addition to the sum rule for the baryon mass differences which we obtained with the single-field version of the triplet-sextet model, the quark model, including SU_6, leads to two further sum rules and two inequalities among the mass differences, all of which are well satisfied. The two inequalities obtained with the quark model can be written so as to involve only the octet masses, whereas the inequalities predicted in the triplet-sextet model involve the decimet masses as well. The sum rules for magnetic moments obtained in the two models are quite similar. The triplet-sextet model with $y=\frac{2}{3}$ leads to one less sum rule than the quark model in that it does not predict a value for the ratio $\mu(p)/\mu(n)$.

The triplet-sextet model was originally introduced because of certain advantages in strong-interaction calculations. In particular, since a baryon is composed of two particles instead of three, it is easier to make detailed calculations which go beyond perturbation theory. But if the model is to be used for strong interactions, it should lead to correct predictions in the electromagnetic case. We have verified, at least within the framework of our assumptions, that there are no serious disagreements with present experiments. Therefore, as far as the present investigations go, the triplet-sextet model is a satisfactory alternative to the quark model.

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