Dynamical Group $O(4,2)$ for Baryons and the Behavior of Form Factors*

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The spectrum of observed baryon levels with the same internal quantum numbers and the behavior of the form factors lead to the hypothesis that the group $O(4,2)$ is the dynamical group of the baryon levels with fixed internal quantum numbers. Indeed, the form factors in this theory have the correct anomalous singularity, as well as the double-pole behavior. The equality of $G_E(t)$ and $G_M(t)$ is discussed.

A CONSISTENT relativistic framework now exists that incorporates the discrete mass spectrum and the internal degeneracies of hadron states and allows one to perform practical calculations. In previous studies[1–3] we have discussed the scalar and vector-transition form factors of those hadron states that can be assigned to irreducible unitary representations of the group $O(3,1)$; i.e., baryon towers of increasing spin J with the same internal quantum numbers, each J value occurring once and with parity $(-1)^{J-1/2}$. Empirically, however, there are, for fixed internal quantum numbers, more states that can be accommodated in one, or with parity doubling, in two towers of $O(3,1)$. For example, there are *four* $J=\frac{1}{2}$, $I=\frac{1}{2}$, $N=1$ states. Thus, just from the point of view of the particle spectrum, a larger group for the space-time quantum numbers seems to be necessary. A more important point at present is the behavior of the form factors, which we want to discuss here. We will show that the problem of the spectrum, as well as the correct behavior of the form factors, can be explained by an extension of the previous formalism.

Although the $O(3,1)$ dynamics gives correct decay rates for baryons[3] (where we are concerned only with values of the form factors at small momentum transfer squared, t), the behavior of the electric and magnetic form factors for large t is not accurate enough: It is too slowly decreasing in t for the representation of $O(3,1)$ characterized by the value zero of the invariant parameter ν, and it is ocillatory for the representations with $\nu\neq 0$. (In the latter case one has also to specify the current operator Γ_μ among many conserved currents that one can construct.) Furthermore, the form factor has always a singularity at $t=4m^2$ (m=mass of the level), and behaves like $(1-t/4m^2)^{-3/2}$. Form factors of the type $(1-t/4m^2)^{-k}$ that also arise in $SL(6,C)$ theories[4] can never explain the experimental behavior for any value of the power k.

In a larger group than $O(3,1)$ there occurs a *new mixing effect*, which is not present in $O(3,1)$, and which results in form factors that decrease much faster and that have a singularity closer than at $t=4m^2$.

The mixing effect has been discovered in the case of the H atom, for which the dynamical group is $O(4,2)$,[5] and in the case of infinite-component wave equations with the dynamical group $O(4,2)$,[6] and has been used widely.[7,8] It says that the transition operators are group generators, not when applied to the states $|nlm\rangle$, but when applied to the new mixed states $|\tilde{n}lm\rangle = \exp(i\theta_n T)|nlm\rangle$, where T is a scalar [with respect to the rotation subgroup of $O(4,2)$] generator in the Lie algebra of $O(4,2)$. [There is no such T in the $O(3,1)$ group.] The result of this mixing effect is a characteristic form factor of the type $G(t)=(1-at)^{-2}$ for the ground state.

We propose therefore in this paper to use the unitary infinite-dimensional representations of the group $O(4,2)$, or more precisely of its covering group $SU(2,2)$, to represent the hadron levels. We have here the possibility of exactly fitting the transition form factors as well as the magnetic moments of the hadron levels.

The conformal group $SU(2,2)$ has been suggested before, but with the idea of incorporating the internal quantum numbers.[9] In the present work it is used entirely within the context of space-time quantum numbers. Also the finite-dimensional nonunitary representations of $SU(2,2)$ have been considered before.[10] The relation of this use to ours is analogous to that of the Dirac equation to the Majorana equation. The use of finite-dimensional representations gives no structure to the particles, whereas the infinite-dimensional representations result in considerable structure.

The group $O(4,2)$ also incorporates automatically an approximate $O(4)$ symmetry recently inferred from the

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[3] A. O. Barut and H. Kleinert, Phys. Rev. Letters **18**, 754 (1967). For an application of O(3,1) to mesons see W. Rühl, Nuovo Cimento **46**, 115 (1966).

[4] G. Cocho, C. Fronsdal, Harun Ar-Rashid, and R. White, Phys. Rev. Letters **17**, 275 (1966); H. Leuthwyler, *ibid.* **17**, 156 (1966); W. Rühl, Nuovo Cimento **44**, 572 (1966). The first-named authors assume phenomenologically, in addition to the $SL(6,C)$ result, the contribution of pole terms, which is outside the noncompact-group approach as we see it.

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[6] Y. Nambu, Progr. Theoret. Phys. Suppl (Kyoto) 37, 368 (1966); Phys. Rev. **160**, 1171 (1967).

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[8] A. O. Barut and H. Kleinert, Phys. Rev. **160**, 1149 (1967).

[9] D. Bohm, M. Flato, F. Halbwachs, P. Hillion, and J. P. Vigier, Nuovo Cimento **36**, 672 (1965). See also A. O. Barut, Phys. Rev. **135**, 5838 (1964), Sec. III.

[10] R. Delbourgo, M. A. Rashid, A. Salam, and J. Strathdee, in *High Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965).

properties of the S matrix in the scattering of unequal-mass particles.[11,12] In fact it clarifies the roles of the $O(3,1)$ [13] and $O(4)$ groups in these discussions, because as in the H atom, the $O(4,1)$ subgroup of the conformal group $O(4,2)$ is the dynamical group of the bound states, and the $O(3,2)$ subgroup, that of the scattering states. Moreover, these groups in our formalism are automatically approximate symmetry groups.[14]

The $O(4,2)$ description of the hadron levels is as follows: Let $L_{ab}=-L_{ba}$ $(a, b=1, 2, \cdots 6)$ be the 16 generators of $O(4,2)$. The subgroup $O(3)$ gives the usual angular momentum states $|j,m\rangle$, and the subgroup $O(4)$ the new quantum number n. We consider in this paper the simplest (so-called maximum degenerate) representation, the same as the type used in the H atom, but now for *fermions* (i.e., only half-integer values of j occur). The weight diagram is shown in Fig. 1 (where a possible spin doubling, as in the relativistic H atom, is also indicated). The explicit realization of this representation is exactly as in Ref. 5, but now the operators act on the ground state $(a^+|0\rangle$, etc.).

According to the prescriptions discussed in previous work, one has to choose for the Lie algebra of $O(4,2)$ the generators of the homogeneous Lorentz group, the generators of the mixing transformations (tilt), and finally the current operators. The homogeneous group is generated by L_{ij} ($\mathbf{L}$) and L_{i5} ($\mathbf{M}$), so that the states boosted to a momentum p are obtained by

$$|njm;p\rangle = e^{i\xi\cdot\mathbf{M}}|njm\rangle, \quad \text{with} \quad \tanh\xi = p/E.$$

With this choice of the homogenous Lorentz group, the generators of the mixing transformations can be a linear combination of L_{45}, L_{46}, and L_{56}, while the current Γ_μ can be a linear combination of the two currents

$$\Gamma_\mu^{(1)} = (L_{56}, L_{i6}), \quad \Gamma_\mu^{(2)} = (L_{45}, -L_{i4}).$$

The form factors (electric and magnetic) are then given by

$$\mathfrak{F}_\mu = \langle n'j'm'|e^{-i\theta_n T}\Gamma_\mu e^{i\xi\cdot\mathbf{M}}e^{i\theta_n T}|njm\rangle.$$

The calculation is the relativistic generalization to fermions of that done for the H atom.[5–8] All possible models of this type have been studied in detail else-

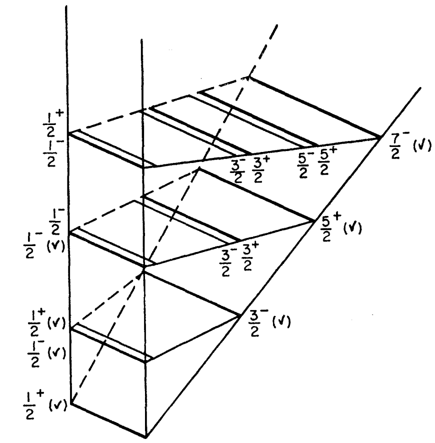

FIG. 1. Weight diagram of the maximum degenerate fermion representation of O (4,2). A possible spin doubling (or spin extension) as in the case of the relativistic H atom is also indicated. The check marks (√) indicate states that can be assigned to already known particles.

where.[15] The arbitrariness in the linear combinations in T that we have indicated above can be removed by the physical requirements of (i) charge normalization and (ii) current conservation. There remains a single parameter, which we fit to the position of the double pole in the ground state. The transition form factors of all other higher states are then predicted. The result is

$$G_M(t) = \mu(1-at)^{-2}$$

and

$$G_E(t) = \frac{1}{2}\left\{1+\left(1-\frac{1}{4m^2a}\right)(1-2\mu)\frac{at}{1-at}\right\}(1-at)^{-2}$$

with the normalization

$$G_E(0)=\tfrac{1}{2}, \quad G_M(0)=\mu, \quad a^{-1}\cong 0.7.$$

This theory gives uniquely $\mu=-\frac{1}{6}$. Because we have not yet taken into account the internal groups SU_2 or SU_3, we expect that the magnitude of μ will certainly change in such a larger group, but that the behavior in t will remain the same[16]. We note that for $\mu=\frac{1}{2}$ we have the absolute equality $G_E(t)=G_M(t)$. If, on the other hand, we take the empirical *isoscalar* value $\mu_S=0.44$, then the deviation of $G_E(t)$ from $G_M(t)$ at $t=2\ \text{BeV}^2$ is about 7%. Thus it would be important to measure $G_E(t)$ for large t values. We also note that the corresponding results in the $O(3,1)$ group (Majorana equation) were[1]

$$G_E(0)=\tfrac{1}{2}, \quad G_M(0)=-\tfrac{1}{4};$$

$$\frac{G_E(t)}{\frac{1}{2}} = \frac{G_M(t)}{\mu} = \left(1-\frac{t}{4m^2}\right)^{-3/2}.$$

The consequences of the theory are not restricted to form factors alone, and are not exhausted by far. The

[11] G. Domokos (unpublished). Note also that in nonrelativistic case the appearance of the $O(4)$ subgroup is a general property of all potentials.

[12] D. Freedman and J. Wang, Phys. Rev. Letters **17**, 569 (1966); Phys. Rev. **153**, 1596 (1967). The connection with these daughter trajectories will be evident if actual physical particles are assigned to these trajectories.

[13] H. Joos, in *Lectures in Theoretical Physics-Lorentz Group Symposium* (University of Colorado Press, Boulder, Colorado, 1964).

[14] In the dynamical-group approach the symmetry of the Hamiltonian *per se* does not play an important role. Whether or not the symmetry is exact, the dynamical noncompact group is the same. An exact and an approximate symmetry, as well as various mass spectra, can be built in a relativistically invariant way. Physically the dynamical group reflects the internal structure of the system; it is not a symmetry group of the S matrix.

[15] Hagen Kleinert, Ph.D. thesis, University of Colorado, 1967 (unpublished); Fortschr. Physik (in press); Phys. Rev. (in press).

parameter a in the form factors is expected to be related to the mass differences of the baryon levels.[16] We have not said anything about the mass spectrum in $O(4,2)$. This can be discussed with tensor operator methods or with the generalized Majorana equations. Also, the behavior of the mesons and decay properties are being investigated.

Finally, it should be remarked that in the present formalism the cross-channel amplitudes have to be evaluated separately because the boosters are different.[1,2]

[16] *Note added in proof.* Positive magnetic moments and correct mass spectra are obtained from a new current which contains besides Γ_μ a convective current P_μ, in a forthcoming paper by A. O. Barut, D. Corrigan, and H. Kleinert [Phys. Rev. (to be published)].

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Poles and Resonances in η Photoproduction

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Various combinations of poles and resonances have been tried to determine just which combinations yield reasonable fits to the total and differential cross-section data for the process $\gamma+p\rightarrow\eta+p$. In each case the best values of the parameters have been obtained by minimizing χ^2. Excellent fits ($\chi^2/N\approx0.8$) are obtained for certain combinations of poles and resonances. All solutions with the $S_{11}(1570)$ resonance omitted have rather poor values of χ^2, and the $P_{11}(1400)$ resonance cannot be used in lieu of the $S_{11}(1570)$ in this process. Evidence is obtained for classifying the $F_{15}(1688)$ as a member of an octet rather than a 27-plet. For all the models considered here, the value of the η-nucleon coupling constant $(g_\eta^2/4\pi)$ is less than 2, indicating a D/F ratio larger than $\frac{2}{3}$.

I. INTRODUCTION

THE differential cross section for the process $\gamma+p\rightarrow\eta+p$ has recently been measured by three different groups. Prepost *et al.*[1] (Stanford) made a measurement of $d\sigma/d\Omega$ at approximately 100° (center of mass) from threshold (~710 MeV) to around 960-MeV lab photon energy. Bacci *et al.*[2,3] (Frascati) measured $d\sigma/d\Omega$ over an energy range of 800–1000 MeV and η center-of-mass angle of 106° to 120°. Heusch *et al.*[4] (Caltech) made measurements at 45° from 940 to 1090 MeV. Examination of the data from these experiments shows that there is a rather sharp rise in the cross section just above threshold with a peak being reached in the general vicinity of 1570-MeV total center-of-mass energy. Following the rapid rise there is an almost equally rapid drop between the peak and around 1670 MeV with a hint of another rise beginning around 1710 MeV. A similar structure is also observed in η production by pions on nucleons.[5]

Several authors[6,7] have analyzed the process $\pi^-+p\rightarrow\eta+n$. Better agreement with the experimental data has been found by those authors who use an S-wave resonance in the neighborhood of 1500 MeV. However, the work by Minami[8] indicates that the effects of the D_{13} resonance are comparable to or larger than those of the S_{11} resonance in the region of the peak. It has been pointed out by Heusch[9] that both in pion and photoproauction of eta particles the rise above threshold appears to have a positive second derivative which suggest a P-wave behavior. Thus, the S-wave can not fit both the threshold and the first few cross section points. According to Heusch,[9] Bloom and Prescott[10] have found that both S_{11} and P_{11} resonances will match the data.

A less extensive analysis of the η photoproduction data has been made. Along with the work of Bloom and Prescott mentioned above there is the work of Logan and Uchiyama-Campbell,[11] who found that an S wave gave a good fit to the total cross section. However, they did not use the Caltech data and, of course, the angular dependence of the Frascati data had no effect on their analysis. Minami[8] has studied the process $\gamma+p\rightarrow\eta+p$ in order to obtain information concerning the partial widths Γ_η and Γ_π of the S_{11} resonance. In his analysis

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