Double Scattering of Mesons in the Quark Model for Baryons

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The effects of the double scattering of mesons have been evaluated in the quark model for baryons. The following relations among the meson-baryon scattering amplitudes hold in general: (i) $K^+p-K^-p=\pi^+p-\pi^-p+K^+n-K^-n$, (ii) $\langle K^+\Sigma^-|\pi^-p\rangle=\langle K^0\Xi^0|K^-p\rangle=\frac{1}{2}\langle K^+\Xi^-|K^-p\rangle=\langle K^0\Xi^-|K^-n\rangle=\langle\pi^+\Sigma^-|K^-p\rangle$. The strong Johnson-Treiman relations have also been obtained by taking into account the spin (apart from the unitary spin) wave function of the baryon within the quark model, so that there is an exchange of spins when two quarks are interchanged [a weaker assumption than $SU(6)$].

I. INTRODUCTION

RECENTLY Levin and Frankfurt,[1] and Lipkin and Scheck,[2] have proposed a very simple quark model for high-energy scattering of hadrons, based upon the assumption of additivity for two-body quark amplitudes, which has met with considerable success. Further consequences of this model have been discussed by Kokkedee and Van Hove,[3] Kokkedee,[4] and Alexander *et al.*[5]

The purpose of the present paper is to include the effect of the double scattering within the simple quark model of hadrons for the meson-baryon scattering cross sections at high energy. Thus information can be obtained about the production processes, in particular, where there are exchange of two (integral) units of charge, of hypercharge, or one unit of charge and one unit of hypercharge. It is needless to mention that in these cases the single scattering quark model is of no use. Further the results of the single-scattering quark model can also be examined with the double-scattering corrections.

The various assumptions of our model are as follows:

(i) The baryons form an octet of three quarks.[6]

(ii) The (pseudoscalar) mesons form an $SU(3)$ octet (no quark model).

(iii) The incident meson does not make more than two encounters with the quarks within the target baryon.

(iv) The first scattering of the incident meson is mainly in the forward direction.

In the next section the meson-quark scattering processes are considered in detail, so that the results can be used in Sec. III, where the meson-baryon scattering is taken up. The results are expressed in terms of the amplitudes only.

II. MESON-QUARK SCATTERING

We first consider the meson-quark scattering. In this process there are three independent parameters, which follow from the reduction, $\mathbf{3}\otimes\mathbf{8}=\mathbf{15}+\mathbf{6}^*+\mathbf{3}$. We use the notation Q^i for the quarks and $M^\alpha{}_\beta$ for the mesons, defined as

$$Q^i=\begin{bmatrix}\mathcal{P}\\ \mathcal{N}\\ \lambda\end{bmatrix},\quad M^\alpha{}_\beta=\begin{bmatrix}\frac{\pi^0}{\sqrt2}+\frac{\eta^0}{\sqrt6} & \pi^+ & K^+\\ \pi^- & -\frac{\pi^0}{\sqrt2}+\frac{\eta^0}{\sqrt6} & K^0\\ K^- & \bar K^0 & -\frac{2\eta^0}{\sqrt6}\end{bmatrix},\tag{2.1}$$

where $\mathcal{P}$ and $\mathcal{N}$ form the isodoublet and λ is the singlet. It is convenient to diagonalize the meson-quark scattering process in the t channel. Then the invariant amplitude for the scattering process, $Q^i+M\to Q^l+M'$, can be expressed in the following form:

$$\langle Q^lM'^\alpha{}_\beta|t_i{}^l|Q^iM^\mu{}_\nu\rangle=f(\bar Q_lQ^i)(\bar M^\alpha{}_\beta M^\beta{}_\nu)+g(\bar Q_lQ^i)(\bar M^\alpha{}_\beta M^\beta{}_\nu)+h(\bar Q_lQ^i)(\bar M^\alpha{}_\beta M^\mu{}_\alpha),\tag{2.2}$$

where f, g, and h are the three independent parameters. If we now use the notation, $(\pi^+\mathcal{P})$, etc. for the elastic scattering amplitudes and $\langle\pi^0\mathcal{N}|\pi^-\mathcal{P}\rangle$ etc. for the amplitudes of the production processes, then we have from (2.1) and (2.2):

$$(\pi^+\mathcal{P})=(\pi^-\mathcal{N})=(K^+\mathcal{P})=(K^0\mathcal{N})=f+h,\tag{2.3a}$$

$$(\pi^-\mathcal{P})=(\pi^+\mathcal{N})=(K^-\mathcal{P})=(K^0\mathcal{N})=f+g,\tag{2.3b}$$

$$(K^+\mathcal{N})=(K^-\mathcal{N})=(K^0\mathcal{P})=(\bar K^0\mathcal{P})=f,\tag{2.3c}$$

$$(\pi^0\mathcal{P})=(\pi^0\mathcal{N})=f+\tfrac12 g+\tfrac12 h,\tag{2.4a}$$

$$(\eta^0\mathcal{P})=(\eta^0\mathcal{N})=f+\tfrac16 g+\tfrac16 h,\tag{2.4b}$$

$$\langle\pi^0\mathcal{N}|\pi^-\mathcal{P}\rangle=\frac{1}{\sqrt2}(h-g),\tag{2.5a}$$

$$\langle\eta^0\mathcal{N}|\pi^-\mathcal{P}\rangle=\frac{1}{\sqrt6}(h+g),\tag{2.5b}$$

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[1] E. M. Lovin and L. L. Frankfurt, JETP Pis'ma v Redaktsiya 2, 105 (1965) [English transl.: Soviet Phys.—JETP Letters 2, 65 (1965)].
[2] H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966).
[3] J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento 42, 711 (1966).
[4] J. J. J. Kokkedee, Phys. Letters 22, 88 (1966).
[5] G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters 17, 412 (1962).
[6] G. Zweig, CERN reports TH 401 and 402, 1964 (unpublished).

$$\langle K^0\mathcal{P}|K^+\mathfrak{N}\rangle=\langle\pi^+\lambda|K^0\mathcal{P}\rangle=\langle\pi^-\lambda|K^-\mathfrak{N}\rangle=h\,, \quad (2.6a)$$

$$\langle K^-\mathcal{P}|K^0\mathfrak{N}\rangle=\langle K^0\lambda|\pi^-\mathcal{P}\rangle=\langle K^+\lambda|\pi^+\mathfrak{N}\rangle=g\,, \quad (2.6b)$$

$$\langle\pi^0\lambda|K^-\mathcal{P}\rangle=\frac{1}{\sqrt{2}}h\,,\quad \langle\eta^0\lambda|K^-\mathcal{P}\rangle=\frac{1}{\sqrt{6}}(h-2g)\,. \quad (2.6c)$$

Use of these relations, (2.3)–(2.6), will be made to evaluate the meson-baryon scattering amplitudes, in the next section.

III. MESON-BARYON SCATTERING

We now consider the meson-baryon scattering process

$$M+B\rightarrow M'+B\,. \quad (3.1)$$

The simple additivity assumption says that in the process (3.1) the incident meson scatters off a single quark Q^i in B to produce the outgoing meson and the Q^lth quark of B', while in the double-scattering corrections it interacts successively with two different quarks, Q^i and Q^j $(i\neq j)$ in B to bring them to the states, say the lth and mth quarks in B' and the final meson; the remaining quarks in B and B' are unaffected. Thus we can express the invariant amplitude for the process (3.1) as

$$\langle B'M'^{\alpha}{}_{\beta}|T|BM^{\mu}{}_{\nu}\rangle =\delta_4(q^{B'}+q^{M'}-q^B-q^M)\sum_{ij}\left[\langle Q^lM'^{\alpha}{}_{\beta}|t^l{}_i|Q^iM^{\mu}{}_{\nu}\rangle +\langle Q^lQ^mM'^{\alpha}{}_{\beta}|t^{lm}{}_{ij}|Q^iQ^jM^{\mu}{}_{\nu}\rangle\right], \quad (3.2)$$

where the summation in the last term is over distinct pairs of quarks in the wave function of B.

The first term on the right-hand side of (3.2) has already been expressed and evaluated in terms of three independent parameters in (2.2) and (2.3)–(2.6). The second term can be expressed in terms of single scattering amplitudes $t^l{}_i$:

$$\begin{aligned}\langle Q^lQ^mM'^{\alpha}{}_{\beta}|t^{lm}{}_{ij}|Q^iQ^jM^{\mu}{}_{\nu}\rangle =\sum_{\gamma\lambda}\big[&\langle Q^mM'^{\alpha}{}_{\beta}|t^m{}_j|Q^jM^{\gamma}{}_{\lambda}\rangle\langle Q^lM^{\gamma}{}_{\lambda}|t^l{}_i|Q^iM^{\mu}{}_{\nu}\rangle\\ &+\langle Q^lM'^{\alpha}{}_{\beta}|t^l{}_j|Q^jM^{\gamma}{}_{\lambda}\rangle\langle Q^mM^{\gamma}{}_{\lambda}|t^m{}_i|Q^iM^{\mu}{}_{\nu}\rangle\\ &+\langle Q^mM'^{\alpha}{}_{\beta}|t^m{}_i|Q^iM^{\gamma}{}_{\lambda}\rangle\langle Q^lM^{\gamma}{}_{\lambda}|t^l{}_j|Q^jM^{\mu}{}_{\nu}\rangle\\ &+\langle Q^lM'^{\alpha}{}_{\beta}|t^l{}_i|Q^iM^{\gamma}{}_{\lambda}\rangle\langle Q^mM^{\gamma}{}_{\lambda}|t^m{}_j|Q^jM^{\mu}{}_{\nu}\rangle\big], \quad (3.3)\end{aligned}$$

where the summation is over all possible $SU(3)$ indices γ and λ. When there are two identical quarks in the final state, the right-hand side of (3.3) must be multiplied by $\frac{1}{2}$.

We use the baryon-octet wave functions in terms of the three quarks as given by Zweig.[6] One can, of course, form the octet wave function from the quarks in two different ways. The results remain unchanged if one uses either of them or a linear (properly normalized) combination of them, because there are no transitions between these two octets. Thus using (3.2) and (3.3), and (2.3)–(2.6), we obtain the following results for the various meson-baryon scattering amplitudes:

$$\begin{aligned}(\pi^+p)=3f+2h+g+6f^2+2h^2+4gh+8fh+4fg\\ -[\tfrac{4}{3}h^2+\tfrac{4}{3}g^2-\tfrac{4}{3}gh]\,, \quad (3.4)\end{aligned}$$

$$\begin{aligned}(\pi^-p)=3f+2g+h+6f^2+2h^2+4gh+8fg+4fh\\ -[\tfrac{4}{3}h^2+\tfrac{4}{3}g^2-\tfrac{4}{3}gh]\,, \quad (3.5)\end{aligned}$$

$$(K^+p)=3f+2h+6f^2+2h^2+8fh-[2h^2]\,, \quad (3.6)$$

$$(K^-p)=3f+2g+6f^2+2g^2+8fg-[2g^2]\,, \quad (3.7)$$

$$(K^+n)=3f+h+6f^2+4fh-[2h^2]\,, \quad (3.8)$$

$$(K^-n)=3f+g+6f^2+4fg-[2g^2]\,, \quad (3.9)$$

$$\langle K^+\Sigma^+|\pi^+p\rangle=-g-4g(f+h)-[\tfrac{2}{3}h^2-\tfrac{4}{3}g^2+\tfrac{4}{3}gh]\,, \quad (3.10)$$

$$\langle\pi^-\Sigma^+|K^-p\rangle=-h-4h(f+g)-[\tfrac{2}{3}g^2-\tfrac{4}{3}h^2+\tfrac{4}{3}gh]\,. \quad (3.11)$$

In the following amplitudes there are no single-scattering terms:

$$\langle K^+\Xi^-|K^-p\rangle=\tfrac{2}{3}h^2+\tfrac{2}{3}g^2-(8/3)gh\,, \quad (3.12)$$

$$\langle K^0\Xi^0|K^-p\rangle=\tfrac{1}{3}h^2+\tfrac{1}{3}g^2-\tfrac{4}{3}gh\,, \quad (3.13)$$

$$\langle\pi^+\Sigma^-|K^-p\rangle=\tfrac{1}{3}h^2+\tfrac{1}{3}g^2-\tfrac{4}{3}gh\,, \quad (3.14)$$

$$\langle K^0\Xi^-|K^-n\rangle=\tfrac{1}{3}h^2+\tfrac{1}{3}g^2-\tfrac{4}{3}gh\,, \quad (3.15)$$

$$\langle K^+\Sigma^-|\pi^-p\rangle=\tfrac{1}{3}h^2+\tfrac{1}{3}g^2-\tfrac{4}{3}gh\,. \quad (3.16)$$

From (3.4)–(3.9) we get the weak Johnson-Treiman relation:

$$(K^+p)-(K^-p)=(\pi^+p)-(\pi^-p)+(K^+n)-(K^-n)\,. \quad (3.17)$$

From (3.12)–(3.16) we have

$$\begin{aligned}\tfrac{1}{2}\langle K^+\Xi^-|K^-p\rangle&=\langle K^0\Xi^0|K^-p\rangle\\ &=\langle\pi^+\Sigma^-|K^-p\rangle\\ &=\langle K^0\Xi^-|K^-n\rangle\\ &=\langle K^+\Sigma^-|\pi^-p\rangle\,. \quad (3.18)\end{aligned}$$

The $SU(3)$ relations are, of course, recovered.

$$\begin{aligned}\langle K^-p|K^-p\rangle=\langle\pi^-p|\pi^-p\rangle+\langle\pi^-\Sigma^+|K^-p\rangle\,,\\ \langle K^+p|K^+p\rangle=\langle\pi^+p|\pi^+p\rangle+\langle K^+\Sigma^+|K^+p\rangle\,.\end{aligned} \quad (3.19)$$

The relations (3.10) and (3.19) are the results obtained by Ruegg and Volkov[7] using $U_3\times U_3$ for the collinear processes. They can also be obtained from $SU(3)$ assuming that the contributions of the [10] and [10*] representations vanish.[8]

One other point that is worth mentioning is the negative sign in front of the last brackets of expressions (3.4)–(3.9). The negative sign comes from the exchange of two quarks, as for example, $\mathcal{P}$ and $\mathfrak{N}$ in p. We now take into consideration the spin as well as the unitary-spin wave functions of the baryons. In the following

[7] H. Ruegg and D. V. Volkov, Nuovo Cimento **43**, 84 (1966).
[8] A. Q. Sarker, Nuovo Cimento (to be published).

only the proton wave function will be discussed in detail. Let α denote the spin state up and β the spin state down for the quarks. Then the spin doublet wave functions for the three quarks are

$$\chi_1=\frac{1}{\sqrt{2}}(\alpha_1\alpha_2\beta_3-\alpha_1\beta_2\alpha_3)\,, \tag{3.20}$$

$$\chi_2=\frac{1}{\sqrt{6}}(\alpha_1\alpha_2\beta_3+\alpha_1\beta_2\alpha_3-2\beta_1\alpha_2\alpha_3)\,. \tag{3.21}$$

The $SU(3)$ wave functions in terms of quarks, which correspond to proton, are

$$\eta_1=\frac{1}{\sqrt{2}}(\mathcal{P}_1\mathcal{P}_2\mathcal{N}_3-\mathcal{P}_1\mathcal{N}_2\mathcal{P}_3)\,, \tag{3.22}$$

$$\eta_2=\frac{1}{\sqrt{6}}(\mathcal{P}_1\mathcal{P}_2\mathcal{N}_3+\mathcal{P}_1\mathcal{N}_2\mathcal{P}_3-2\mathcal{N}_1\mathcal{P}_2\mathcal{P}_3)\,. \tag{3.23}$$

Out of these four spin and unitary-spin wave functions [(3.20)–(3.23)] one can form the following four orthonormal wave functions for the proton[9]:

$$\phi_S=\frac{1}{\sqrt{2}}(\chi_1\eta_1+\chi_2\eta_2)\,, \tag{3.24}$$

$$\phi_A=\frac{1}{\sqrt{2}}(\chi_2\eta_1-\chi_1\eta_2)\,, \tag{3.25}$$

$$\phi_1=\frac{1}{\sqrt{2}}(\chi_2\eta_1+\chi_1\eta_2)\,, \tag{3.26}$$

$$\phi_2=\frac{1}{\sqrt{2}}(\chi_1\eta_1-\chi_2\eta_2)\,. \tag{3.27}$$

The wave function (3.24) is completely symmetric, (3.25) is antisymmetric, and (3.26) and (3.27) are mixed symmetric for the interchange of the three indices.[9,10]

We now assume the following:

(v) There is always an exchange of spin along with the exchange of the unitary spin.

Thus we have a negative sign from (3.22) or (3.23) and another negative sign due to exchange of spin from (3.20) or (3.21) as a result of the exchange scattering between $\mathcal{P}$ and $\mathcal{N}$. The above result is valid for all four types of combinations of wave functions [(3.24)–(3.27)] for the proton. Therefore, the contributions from the last brackets in (3.4)–(3.9) are added and we get the strong Johnson-Treiman relations[11] of $SU(6)$:

$$\tfrac{1}{2}[(K^+p)-(K^-p)]=(K^+n)-(K^-n)=(\pi^+p)-(\pi^-p)\,. \tag{3.28}$$

In the present quark model the relations (3.28) are valid subject to the assumption (v), but are independent of any statistics (Fermi or para) for the quarks.

For comparison of the relations (3.17), (3.28), (3.18), and (3.19) among the meson-baryon scattering amplitudes we refer to the already published literature.[2,7,12–14]

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[9] L. I. Schiff, Phys. Rev. **133**, B802 (1964).

[10] For the space part of the wave function we are assuming a symmetrical form, which is expected to give the tightest binding.

[11] K. Johnson and S. B. Treiman, Phys. Rev. Letters **14**, 189 (1965).

[12] H. Abarbanel and C. Callen, Phys. Letters **16**, 19 (1965); H. Harari and H. J. Lipkin, Phys. Rev. Letters **13**, 208 (1964); R. Good and N. Xuong, *ibid.* **14**, 191 (1965).

[13] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).

[14] The first half of (328) is reasonably well satisfied by the experimental data; however, the difference of $\pi^{\pm}$ cross sections is somewhat less consistent with the prediction of (3.28). Since these discrepancies seem to diminish towards high momenta (as reported in Ref. 13), they may be attributed to the π-K mass difference, or if the mechanism for the validity of assumption (v) is attributed to the exchange of vector mesons, then the discrepancies may also be due to the ρ-K^* mass difference.