Coherence Properties of Blackbody Radiation.* III. Cross-Spectral Tensors

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In a recent paper, cross-spectral tensors of quantized stationary electromagnetic fields were introduced and some of their properties were discussed. In the present paper, closed expressions for the cross-spectral tensors of blackbody radiation are derived and their behavior is illustrated by a number of diagrams. It is found that the correlation distance of a spectral component of wavelength λ is itself of the order of λ and is therefore independent of the temperature of the radiation.

1. INTRODUCTION

In two earlier papers, the coherence properties of blackbody radiation were studied on the basis of classical theory,[1] as well as on the basis of the theory of the quantized field.[2] In particular, closed expressions were obtained for the second-order coherence tensors in the space-time domain and their behavior was illustrated by a number of contour diagrams.

In the present investigation, coherence properties of blackbody radiation in the spectral domain are investigated. The analysis is carried out within the framework of the theory of the quantized field, but as is evident from the conclusions of Ref. 2, identical results would be obtained were the analysis carried out on the basis of classical theory.

2. THE SECOND-ORDER CROSS-SPECTRAL TENSORS OF A STATIONARY QUANTIZED ELECTROMAGNETIC FIELD

We begin by summarizing the definitions of second-order cross-spectral tensors that have been introduced into the general theory of stationary quantized electromagnetic fields in a recent publication.[3]

Let $\hat{E}(\mathbf{r},t)$ and $\hat{H}(\mathbf{r},t)$ be, respectively, the electric and magnetic field operators at the point $\mathbf{r}$ at time t, and let the superscripts $(+)$ and $(-)$ denote, respectively, their positive- and negative-frequency parts. We represent these operators as Fourier integrals with respect to the time variable:

$$\hat{E}^{(+)}(\mathbf{r},t)=\int_0^\infty \hat{e}^{(+)}(\mathbf{r},\nu)e^{-2\pi i\nu t}d\nu\,,\qquad \hat{E}^{(-)}(\mathbf{r},t)=\int_0^\infty \hat{e}^{(-)}(\mathbf{r},\nu)e^{+2\pi i\nu t}d\nu\,,\tag{2.1}$$

$$\hat{H}^{(+)}(\mathbf{r},t)=\int_0^\infty \hat{h}^{(+)}(\mathbf{r},\nu)e^{-2\pi i\nu t}d\nu\,,\qquad \hat{H}^{(-)}(\mathbf{r},t)=\int_0^\infty \hat{h}^{(-)}(\mathbf{r},\nu)e^{+2\pi i\nu t}d\nu\,.\tag{2.2}$$

If $\hat{\rho}$ is the density operator of the field, assumed to be stationary, the second-order coherence tensors in the space-time domain are defined by the expressions

$$\mathcal{E}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)=\mathrm{tr}\{\hat{\rho}\hat{E}_i^{(-)}(\mathbf{r}_1,t)\hat{E}_j^{(+)}(\mathbf{r}_2,t+\tau)\}\,,\tag{2.3a}$$

$$\mathcal{H}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)=\mathrm{tr}\{\hat{\rho}\hat{H}_i^{(-)}(\mathbf{r}_1,t)\hat{H}_j^{(+)}(\mathbf{r}_2,t+\tau)\}\,,\tag{2.3b}$$

$$\mathfrak{M}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)=\mathrm{tr}\{\hat{\rho}\hat{E}_i^{(-)}(\mathbf{r}_1,t)\hat{H}_j^{(+)}(\mathbf{r}_2,t+\tau)\}\,,\tag{2.3c}$$

$$\mathfrak{N}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)=\mathrm{tr}\{\hat{\rho}\hat{H}_i^{(-)}(\mathbf{r}_1,t)\hat{E}_j^{(+)}(\mathbf{r}_2,t+\tau)\}\,,\tag{2.3d}$$

where the subscripts i,j denote Cartesian components $(i,j=x,y,z)$.

The second-order cross-spectral tensors of the field may be defined by the formulas

$$W_{ij}^{(e)}(\mathbf{r}_1,\mathbf{r}_2,\nu)=\lim_{\Delta\nu\to0}\int_{\nu-\Delta\nu/2}^{\nu+\Delta\nu/2}\mathrm{tr}\{\hat{\rho}\hat{e}_i^{(-)}(\mathbf{r}_1,\nu)\times\hat{e}_j^{(+)}(\mathbf{r}_2,\nu')\}d\nu'\,,\tag{2.4a}$$

$$W_{ij}^{(h)}(\mathbf{r}_1,\mathbf{r}_2,\nu)=\lim_{\Delta\nu\to0}\int_{\nu-\Delta\nu/2}^{\nu+\Delta\nu/2}\mathrm{tr}\{\hat{\rho}\hat{h}_i^{(-)}(\mathbf{r}_1,\nu)\times\hat{h}_j^{(+)}(\mathbf{r}_2,\nu')\}d\nu'\,,\tag{2.4b}$$

$$W_{ij}^{(m)}(\mathbf{r}_1,\mathbf{r}_2,\nu)=\lim_{\Delta\nu\to0}\int_{\nu-\Delta\nu/2}^{\nu+\Delta\nu/2}\mathrm{tr}\{\hat{\rho}\hat{e}_i^{(-)}(\mathbf{r}_1,\nu)\times\hat{h}_j^{(+)}(\mathbf{r}_2,\nu')\}d\nu'\,,\tag{2.4c}$$

$$W_{ij}^{(n)}(\mathbf{r}_1,\mathbf{r}_2,\nu)=\lim_{\Delta\nu\to0}\int_{\nu-\Delta\nu/2}^{\nu+\Delta\nu/2}\mathrm{tr}\{\hat{\rho}\hat{h}_i^{(-)}(\mathbf{r}_1,\nu)\times\hat{e}_j^{(+)}(\mathbf{r}_2,\nu')\}d\nu'\,.\tag{2.4d}$$

It was shown in Ref. 3 that each of these cross-spectral tensors is the Fourier transform of the corresponding second-order coherence tensor defined by Eqs. (2.3), i.e.,

$$W_{ij}^{(e)}(\mathbf{r}_1,\mathbf{r}_2,\nu)=\int_{-\infty}^{\infty}\mathcal{E}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)e^{2\pi i\nu\tau}d\tau\,,\tag{2.5a}$$

$$W_{ij}^{(h)}(\mathbf{r}_1,\mathbf{r}_2,\nu)=\int_{-\infty}^{\infty}\mathcal{H}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)e^{2\pi i\nu\tau}d\tau\,,\tag{2.5b}$$

$$W_{ij}^{(m)}(\mathbf{r}_1,\mathbf{r}_2,\nu)=\int_{-\infty}^{\infty}\mathfrak{M}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)e^{2\pi i\nu\tau}d\tau\,,\tag{2.5c}$$

$$W_{ij}^{(n)}(\mathbf{r}_1,\mathbf{r}_2,\nu)=\int_{-\infty}^{\infty}\mathfrak{N}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)e^{2\pi i\nu\tau}d\tau\,.\tag{2.5d}$$

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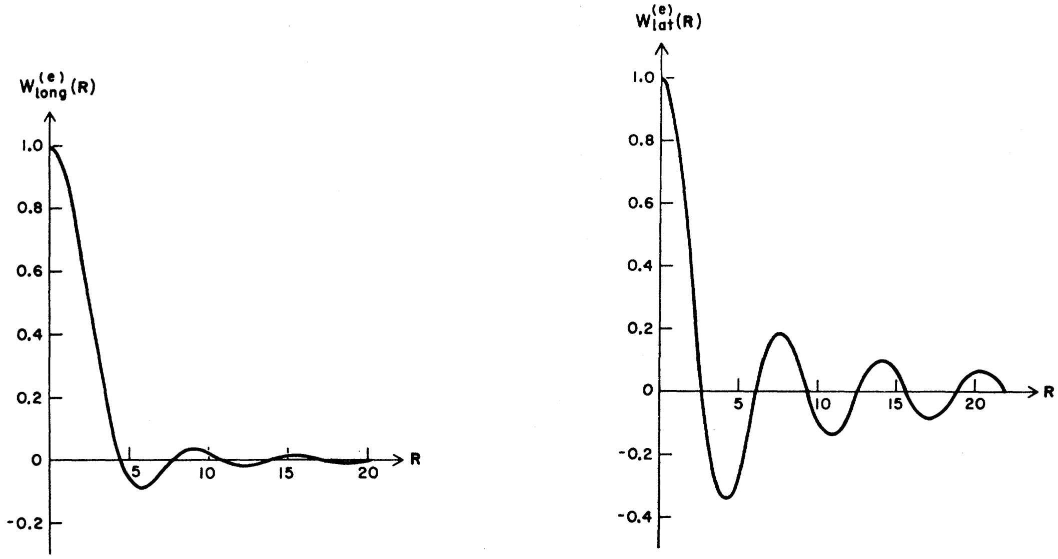
[1] C. L. Mehta and E. Wolf, Phys. Rev. **134**, A1143 (1964).

[2] C. L. Mehta and E. Wolf, Phys. Rev. **134**, A1149 (1964).

[3] C. L. Mehta and E. Wolf, Phys. Rev. **157**, 1188 (1967).



(a) (b)

FIG. 1. (a) Longitudinal normalized electric cross-spectral correlation. Variation of $w_{\rm long}^{(e)}(R)=w_{xx}^{(e)}(\mathbf{R})$ with R, when $\mathbf{R}$ is along the X axis. $w_{ij}^{(e)}(\mathbf{R})\equiv 3W_{ij}^{(e)}(\mathbf{r}_1,\mathbf{r}_2,\nu)/(2\pi A)$; $\mathbf{R}=\mathbf{r}_2-\mathbf{r}_1$, $A=8\pi h\nu^3/\{c^3[\exp(h\nu/KT)-1]\}$. (b) Lateral normailzed electric cross-spectral correlation. Variation of $w_{\rm lat}^{(e)}(R)=w_{xx}^{(e)}(\mathbf{R})$ with R, when $\mathbf{R}$ is perpendicular to the X axis.

We will now examine the cross-spectral tensors of blackbody radiation.

3. THE ELECTRIC AND THE MAGNETIC CROSS-SPECTRAL TENSORS OF BLACKBODY RADIATION

According to Eq. (3.10) of Ref. 2, the electric coherence tensor of blackbody radiation may be expressed in the form

$$\mathcal{E}_{ij}(\mathbf{r}_1,\mathbf{r}_2,\tau)\equiv\mathcal{E}_{ij}(\mathbf{r},\tau)=\frac{\hbar c}{4\pi^2}\int\frac{k^2\delta_{ij}-k_ik_j}{k(e^{\alpha k}-1)}\times\exp[i(\mathbf{k}\cdot\mathbf{r}-kc\tau)]d^3k,\quad(3.1)$$

where[4]

$$\mathbf{r}=\mathbf{r}_2-\mathbf{r}_1,\quad \alpha=\hbar c/KT,\quad(3.2)$$

$\hbar$ being Planck's constant divided by 2π, c the vacuum velocity of light, K the Boltzmann constant, and T the absolute temperature of the radiation.

Equation (3.1) may be rewritten in the form

$$\mathcal{E}_{ij}(\mathbf{r},\tau)=\frac{\hbar c}{4\pi^2}\left(\frac{\partial}{\partial r_i}\frac{\partial}{\partial r_j}-\delta_{ij}\frac{\partial^2}{\partial r_k\partial r_k}\right)\int_0^\infty k^2\,dk\int_0^\pi\sin\theta d\theta\times\int_0^{2\pi}\frac{\exp[i(kr\cos\theta-kc\tau)]}{k(e^{\alpha k}-1)}d\phi,\quad(3.3)$$

where we use spherical polar coordinates in the $\mathbf{k}$ domain, with the polar axis along the direction of $\mathbf{r}$. In (3.3) and elsewhere in this paper a summation is implied over repeated dummy indices unless otherwise stated. On carrying out the integration in (3.3) over the angular variables, we find that

$$\mathcal{E}_{ij}(\mathbf{r},\tau)=\frac{\hbar c}{\pi}\left(\frac{\partial}{\partial r_i}\frac{\partial}{\partial r_j}-\delta_{ij}\frac{\partial^2}{\partial r_k\partial r_k}\right)\times\int_0^\infty\frac{e^{-ikc\tau}}{e^{\alpha k}-1}\frac{\sin kr}{r}dk.\quad(3.4)$$

Next we substitute from (3.4) into (2.5a) and obtain the following expression for the electric cross-spectral tensor:

$$W_{ij}^{(e)}(\mathbf{r}_1,\mathbf{r}_2,\nu)\equiv W_{ij}^{(e)}(\mathbf{r},\nu)=\frac{\hbar c}{\pi}\left(\frac{\partial}{\partial r_i}\frac{\partial}{\partial r_j}-\delta_{ij}\frac{\partial^2}{\partial r_k\partial r_k}\right)\times\int_0^\infty\frac{\sin kr}{r(e^{\alpha k}-1)}dk\int_{-\infty}^\infty e^{i(2\pi\nu-kc)\tau}d\tau.\quad(3.5)$$

The integration over τ gives $(2\pi/c)\delta(k-2\pi\nu/c)$, where δ is the Dirac delta function. After integrating over k, the following expression for $W_{ij}^{(e)}(\mathbf{r},\nu)$ is obtained:

$$W_{ij}^{(e)}(\mathbf{r},\nu)=\frac{h}{\pi}\left(\frac{\partial}{\partial r_i}\frac{\partial}{\partial r_j}-\delta_{ij}\frac{\partial^2}{\partial r_k\partial r_k}\right)\frac{\sin(2\pi\nu r/c)}{r(e^{h\nu/KT}-1)}.\quad(3.6)$$

It is now convenient to introduce a dimensionless vector $\mathbf{R}$ (XYZ), defined by the equation

$$\mathbf{R}=(2\pi\nu/c)\mathbf{r}.\quad(3.7)$$

Evidently, $\mathbf{R}$ represents the separation of the points $\mathbf{r}_1$

[4] The variables $\mathbf{r}$ and τ appearing in Eq. (3.1) and elsewhere in this paper are the negatives of those appearing in Eq. (3.16) of Ref. 2.

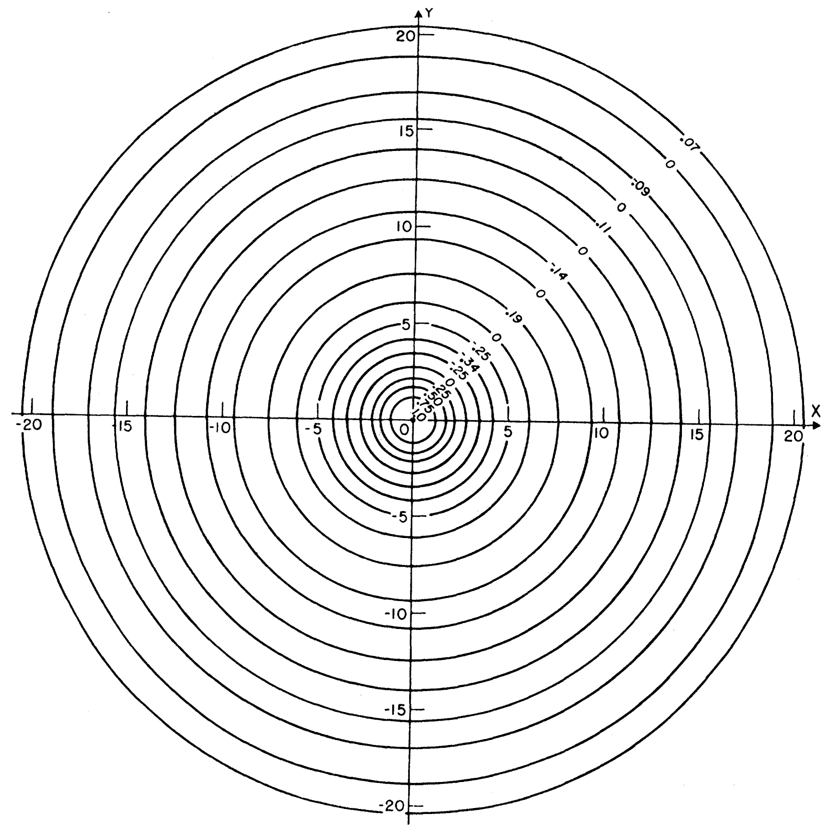


FIG. 2. Contours of $w_{zz}^{(e)}(\mathbf{R})$ in the XY plane.

and $\mathbf{r}_2$ in units of the reduced wavelength $\lambdabar=\lambda/2\pi=c/2\pi\nu$. From (3.6) we find that

$$W_{ij}^{(e)}(\mathbf{r},\nu)\equiv W_{ij}^{(e)}(\mathbf{R})=\pi A\left(\frac{\partial}{\partial R_i}\frac{\partial}{\partial R_j}-\delta_{ij}\frac{\partial^2}{\partial R_k\partial R_k}\right)\frac{\sin R}{R}, \quad (3.8)$$

where

$$A\equiv A=(\nu,T)\frac{8\pi h\nu^3}{c^3}\frac{1}{\exp(h\nu/KT)-1} \quad (3.9)$$

is the Planck distribution function. If we carry out the differentiation indicated in (3.8) we finally obtain the following expression for the electric cross-spectral tensor of blackbody radiation:

$$W_{ij}^{(e)}(\mathbf{R})=\pi A\{\delta_{ij}[j_0(R)-(1/R)j_1(R)]+(R_iR_j/R^2)j_2(R)\}, \quad (3.10)$$

where j_0, j_1, and j_2 are the spherical Bessel functions of order 0, 1, and 2, respectively, i.e.,

$$\begin{aligned} j_0(R)&=\sin R/R,\\ j_1(R)&=\sin R/R^2-\cos R/R, \quad (3.11)\\ j_2(R)&=(3/R^3-1/R)\sin R-(3/R^2)\cos R. \end{aligned}$$

In order to discuss the three-dimensional behavior of the spectral correlation it is convenient to normalize the cross-spectral tensor $W_{ij}^{(e)}(\mathbf{R})$. We define

$$w_{ij}^{(e)}(\mathbf{R})=W_{ij}^{(e)}(\mathbf{R})/[W_{ii}^{(e)}(0)W_{jj}^{(e)}(0)]^{1/2}, \quad (3.12)$$

(no summation in the denominator). Because of isotropy of the blackbody radiation field, $W_{ii}^{(e)}(0)=W_{jj}^{(e)}(0)$ and hence $w_{ij}^{(e)}$ is normalized, so that $w_{ii}^{(e)}(0)=1$. It also follows from the Schwarz inequality that $|w_{ij}^{(e)}(\mathbf{R})|$ is bounded between the values 0 and 1, i.e., that

$$0\leq|w_{ij}^{(e)}(\mathbf{R})|\leq 1. \quad (3.13)$$

From (3.10) we readily find that

$$W_{ii}^{(e)}(0)=W_{jj}^{(e)}(0)=\tfrac{2}{3}\pi A \quad (3.14)$$

and hence from (3.10) and (3.12) it follows that

$$w_{ij}^{(e)}(\mathbf{R})=\tfrac{3}{2}\{\delta_{ij}[j_0(R)-(1/R)j_1(R)]+(R_iR_j/R^2)j_2(R)\}. \quad (3.15)$$

If we set $i=j=x$ in (3.15) and take $\mathbf{R}$ in the direction of the X axis, we obtain the following expression for the *longitudinal* normalized electric cross-spectral correlation function $w_{\text{long}}^{(e)}$:

$$w_{\text{long}}^{(e)}(R)=3j_1(R)/R. \quad (3.16)$$

On the other hand, if we set $i=j=x$ in (3.15) and take $\mathbf{R}$ along a direction perpendicular to the X axis,

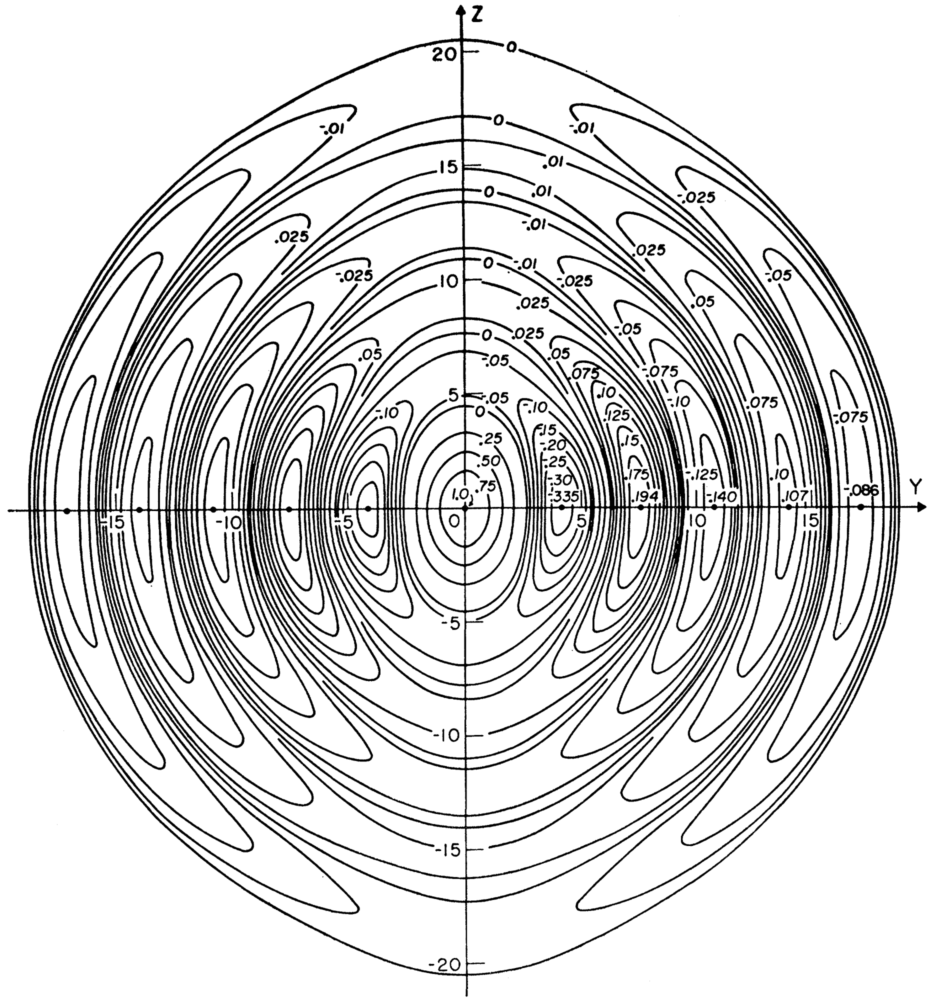


FIG. 3. Contours of $w_{zz}^{(e)}(\mathbf{R})$ in the YZ plane.

we obtain an expression for the *lateral* normalized electric cross-spectral correlation function $w_{\text{lat}}^{(e)}$:

$$w_{\text{lat}}^{(e)}(R)=\tfrac{3}{2}[j_0(R)-(1/R)j_1(R)]. \quad (3.17)$$

The behavior of the longitudinal and lateral coherence functions calculated from Eqs. (3.16) and (3.17), respectively, are shown in Figs. 1(a) and 1(b).

Let us now consider a diagonal component of $w_{ij}^{(e)}$, say the zz component. We have from (3.15)

$$w_{zz}^{(e)}(\mathbf{R})=\tfrac{3}{2}[j_0(R)-(1/R)j_1(R)+(Z^2/R^2)j_2(R)]. \quad (3.18)$$

Evidently, the contours of $w_{zz}^{(e)}$ in the XY plane ($Z=0$) are circles[5] (Fig. 2). The contours of $w_{zz}^{(e)}(\mathbf{R})$ in the YZ plane, computed from Eq. (3.18), are shown in Fig. 3. The surfaces $w_{zz}^{(e)}(\mathbf{R})=\text{const}$ are the surfaces of revolution, obtained by rotating these contours about the Z axis.

The behavior of the other two diagonal components, $w_{xx}^{(e)}$ and $w_{yy}^{(e)}$, is, of course, strictly similar.

Next, let us consider an off-diagonal component of $w_{ij}^{(e)}$, say the xy component. According to (3.15),

$$w_{xy}^{(e)}(\mathbf{R})=\tfrac{3}{2}(XY/R^2)j_2(R). \quad (3.19)$$

In Fig. 4, the variation of $w_{xy}^{(e)}(\mathbf{R})$ along the line $X=Y$, $Z=0$ is shown. Figures 5 and 6 show the

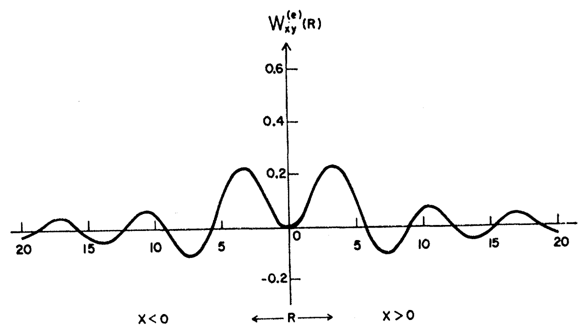


FIG. 4. The variation of $w_{xy}^{(e)}(\mathbf{R})$ along the line $X=Y$, $Z=0$.

[5] There is an error in the labeling of one of the contours in the corresponding diagram of Ref. 1. The label −0.1905 in Fig. 4 of Ref. 1 should read −0.0191.

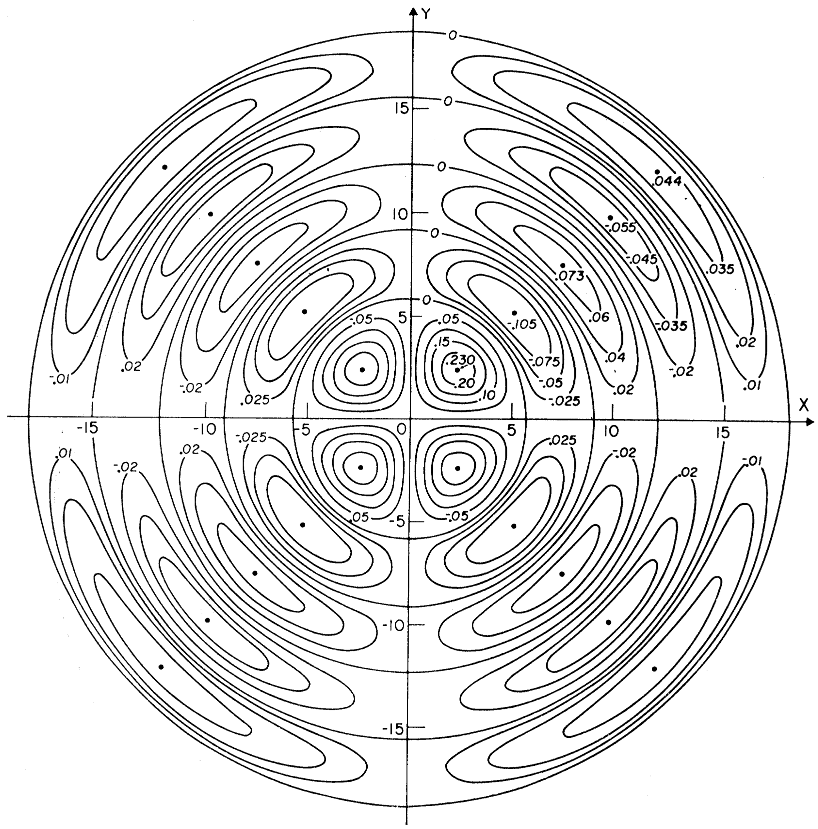


FIG. 5. Contours of $w_{xy}^{(e)}(\mathbf{R})$ in the XY plane.

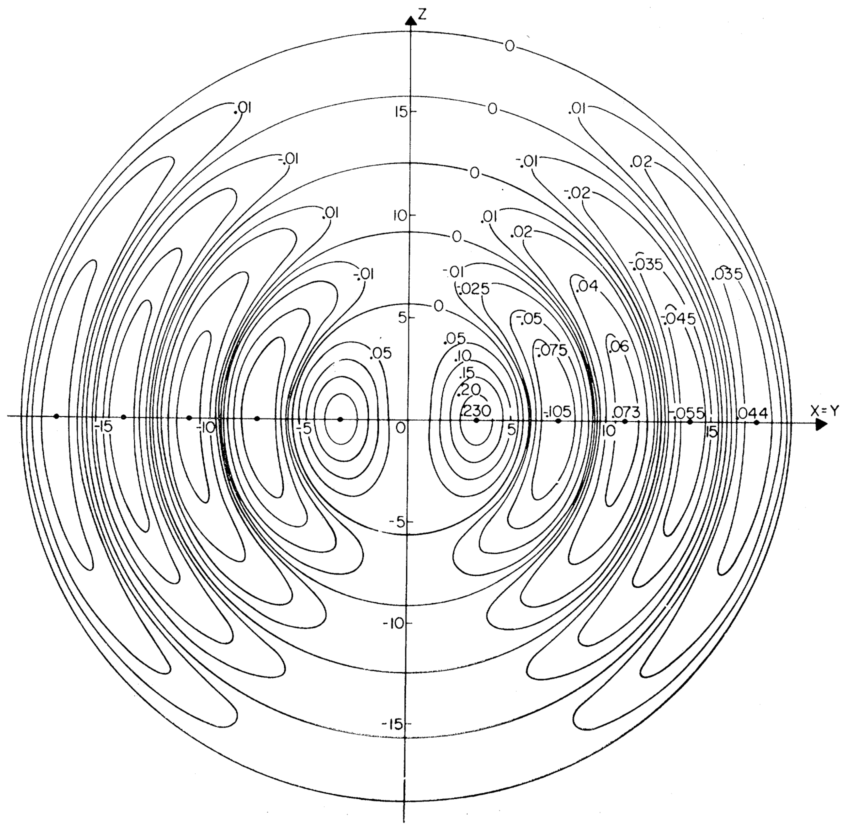


FIG. 6. Contours of $w_{xy}^{(e)}(\mathbf{R})$ in the plane $X=Y$.

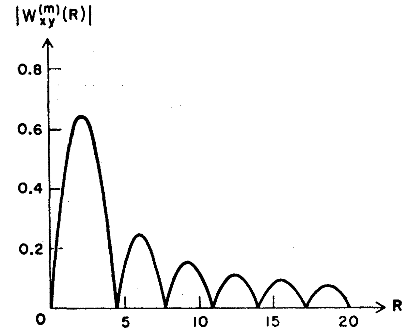


FIG. 7. The variation of $|w_{xy}^{(m)}(\mathbf{R})|$ along the Z axis. $w_{ij}^{(m)}(\mathbf{R}) \equiv 3W_{ij}^{(m)}(\mathbf{r}_1,\mathbf{r}_2,\nu)/(2\pi A)$; $\mathbf{R}=\mathbf{r}_2-\mathbf{r}_1$, $A=8\pi h\nu^3/\{c^3[\exp(h\nu/KT)-1]\}$.

contours of $w_{xy}^{(e)}(\mathbf{R})$ in the XY plane and in the plane $X=Y$, respectively.

The behavior of the other off-diagonal components of the tensor $w_{ij}^{(e)}$ is, of course, strictly similar.

It is seen from Figs. 1–6 that appreciable correlation of the electric field in the spectral domain extends for distances for which $R \lesssim 6$. Since according to Eq. (3.7) $R=2\pi\nu r/c=2\pi|r_2-r_1|/\lambda$, we see that the correlation distance of the spectral component of wavelength λ of the electric field is itself of the order of λ and is therefore independent of the temperature of the radiation.

It was shown in Ref. 2 that for blackbody radiation, the electric and the magnetic coherence tensors are equal to each other, i.e., that

$$\mathcal{H}_{ij}(\mathbf{r},\tau)=\mathcal{E}_{ij}(\mathbf{r},\tau), \tag{3.20}$$

where $\mathcal{H}_{ij}(\mathbf{r},\tau)\equiv\mathcal{H}_{ij}(r_1,r_2,\tau)$. Hence it follows from Eqs. (2.5a) and (2.5b) that the electric and magnetic cross-spectral tensors of blackbody are also equal to each other, i.e., that

$$W_{ij}^{(h)}(\mathbf{R})=W_{ij}^{(e)}(\mathbf{R}), \tag{3.21}$$

where $W_{ij}^{(h)}(\mathbf{R})\equiv W_{ij}^{(h)}(\mathbf{r}_1,\mathbf{r}_2,\nu)$. Hence all the results that we have established for the correlation properties of the electric field in the spectral domain apply without change to the magnetic field also.

4. THE MIXED CROSS-SPECTRAL TENSORS OF BLACKBODY RADIATION

According to Eq. (3.15) of Ref. 2 the mixed coherence tensors $\mathfrak{M}_{ij}$ and $\mathfrak{N}_{ij}$ [defined by Eq. (2.3c) and (2.3d) above] of blackbody radiation are given by[6]

$$\mathfrak{M}_{ij}(\mathbf{r},\tau)=-\mathfrak{N}_{ij}(\mathbf{r},\tau) \tag{4.1}$$

$$=\frac{hc}{4\pi^2}\epsilon_{ijl}\int\frac{k_l}{e^{\alpha k}-1}\exp[i(\mathbf{k}\cdot\mathbf{r}-kc\tau)]d^3k, \tag{4.2}$$

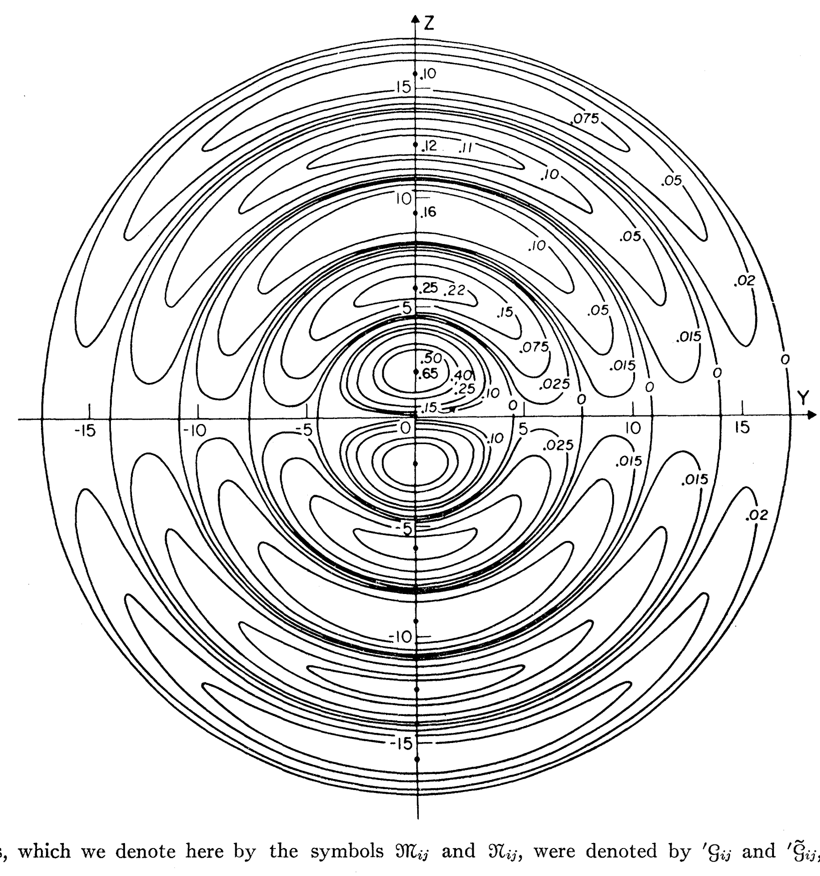


FIG. 8. Contours of $|w_{xy}^{(m)}(\mathbf{R})|$ in the YZ plane.

[6] In Ref. 2, the mixed coherence tensors, which we denote here by the symbols $\mathfrak{M}_{ij}$ and $\mathfrak{N}_{ij}$, were denoted by $'\mathcal{G}_{ij}$ and $'\tilde{\mathcal{G}}_{ij}$, respectively.

where the spatial arguments $\mathbf{r}_1$ and $\mathbf{r}_2$ enter again only in the combination $\mathbf{r}=\mathbf{r}_2-\mathbf{r}_1$ and ϵ_{jkl} is the completely antisymmetric unit tensor of Levi-Civita.

Equation (4.1) may be rewritten in the form

$$\mathfrak{M}_{ij}(\mathbf{r},\tau)=-\mathfrak{N}_{ij}(\mathbf{r},\tau)=\frac{\hbar c}{4\pi^2}\epsilon_{ijl}\left(-i\frac{\partial}{\partial r_l}\right)\times\int\frac{\exp[i(\mathbf{k}\cdot\mathbf{r}-kc\tau)]}{e^{\alpha k}-1}d^3k. \quad (4.3)$$

We now proceed in a similar way as in connection with Eq. (3.1). We change the variable of integration to spherical polar coordinates and integrate over the angular variables. We then obtain

$$\mathfrak{M}_{ij}(\mathbf{r},\tau)=-\mathfrak{N}_{ij}(\mathbf{r},\tau)=\frac{\hbar c}{\pi}\epsilon_{ijl}\left(-i\frac{\partial}{\partial r_l}\right)\times\int_0^\infty\frac{e^{-ikc\tau}}{e^{\alpha k}-1}\frac{\sin kr}{r}k\,dk. \quad (4.4)$$

Next we substitute from (4.4) into Eqs. (2.5c) and (2.5d) and obtain the following expressions for the two mixed cross-spectral tensors of blackbody radiation:

$$W_{ij}^{(m)}(\mathbf{R})=-W_{ij}^{(n)}(\mathbf{R})=i\pi A\,\epsilon_{ijl}(R_l/R)j_1(R), \quad (4.5)$$

where, as before, $\mathbf{R}$ is defined by Eq. (3.7) and A by (3.9). Again it is convenient to normalize the cross-spectral tensors. We define

$$w_{ij}^{(m)}(\mathbf{R})=-w_{ij}^{(n)}(\mathbf{R})=\frac{W_{ij}^{(m)}(\mathbf{R})}{[W_{ii}^{(e)}(0)W_{jj}^{(h)}(0)]^{1/2}}$$

(no summation). (4.6)

From (4.6), (3.14), and (3.21) it follows that

$$w_{ij}^{(m)}(\mathbf{R})=-w_{ij}^{(n)}(\mathbf{R})=\tfrac{3}{2}i\epsilon_{ijl}(R_l/R)j_1(R). \quad (4.7)$$

It is seen from (4.7) that $w_{ij}^{(m)}$ and $w_{ij}^{(n)}$ are antisymmetric tensors. Hence their diagonal components vanish and consequently there is now no longitudinal or lateral coherence. A typical off-diagonal component is,

$$w_{xy}^{(m)}(\mathbf{R})=-w_{xy}^{(n)}(\mathbf{R})=\tfrac{3}{2}i(Z/R)j_1(R). \quad (4.8)$$

It is seen that $w_{xy}^{(m)}$ and $w_{xy}^{(n)}$ identically vanish in the plane $Z=0$. In Fig. 7 the variation of $|w_{xy}^{(m)}(\mathbf{R})|$ along the Z axis is shown. Figure 8 shows the contours of $|w_{xy}^{(m)}(\mathbf{R})|$ in the YZ plane.

As in the case studied in Sec. 3, the spectral correlation distance is again seen to be of the order of the wavelength, and is therefore independent of the temperature of the radiation.

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