radiation field of arbitrary spectral profile, when the counting time intervals are much shorter than the coherence time of the light. As mentioned previously, these generalized factorial moments are closely related to the high-order intensity correlation functions of the radiation field, and therefore contain information about the phase of the second-order complex degree of coherence γ_{ij} for the light field at two space-time points. As is well known, the knowledge of the phase of the degree of coherence is essential in determining the spectral profile of the light beam. Clearly, the N-fold joint photocount distributions provide useful and interesting information about the higher-order coherence properties of the radiation field.

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Lagrange Formulation for Systems with Higher Spin*

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Prescriptions for constructing the generalized Lagrange function for a system with an arbitrary spin S are presented. By the use of the spin projection operators introduced by Fronsdal, nonlocal field equations are constructed to describe these higher-spin systems. Then, auxiliary fields are introduced systematically to remove the nonlocalities appearing in these field equations. Lagrange functions describing systems with $S\leq 4$ are constructed explicitly according to this new prescription. For $S=0, \frac{1}{2}, 1$, they agree with the well-known local Lagrange functions. For $S=\frac{3}{2}$ and 2, they are equivalent to the results previously obtained by Rarita and Schwinger, and by Fierz and Pauli. With the help of the quantum action principle, canonical quantizations are carried out and Green's functions are constructed. Some physical positiveness requirements are also verified.

I. INTRODUCTION

THE problem of quantization for systems with higher spins has been studied extensively ever since the earlier development of quantum field theory.[1] The recent discovery of many higher-spin resonances arouses new interest in this problem. Roughly speaking, there are two different approaches to describe the field theory of higher spins. The first approach emphasizes the transformation properties of field variables under the homogeneous Lorentz group.[2] The physical interacting field operators are considered as the asymptotic field operators—the field operators before and after the interactions are taken place. This approach has the advantage that these asymptotic field variables satisfy very simple field equations, and that no complicated Lagrange function is required to describe them. These asymptotic field variables can be quantized easily by expanding them in terms of creation and annihilation operators, and their corresponding propagation functions can then be determined. This approach is very successful in perturbative applications as well as in the S-matrix theory. The simplicity in this approach originates from the fact that we have bypassed the detailed structures of the interactions. This advantage turns out to be its disadvantage when we try to describe the interaction. Neither the canonical commutator relations nor the stress tensor can be obtained without solving the full dynamics. The canonical quantization conditions as well as further consistency requirements in the presence of interactions are consequently ignored. The second approach follows that of Pauli and Fierz,[1] and demands that all field equations and subsidiary conditions should be derived from a generalized action principle. This classical approach has the advantages that the interaction can be introduced explicitly, and that the Green's function can be computed. The canonical quantization relations and the stress tensor of a system can be obtained directly from the action principle, even in the presence of interaction. The validity of all these consequences is not limited by perturbations. However, this approach has at least one defect. For a system with spin ≥ 2, even the construction of a free Lagrange function is very tedious and, in some sense, rather ambiguous. The introduction of auxiliary field variables is by itself quite arbitrary. In

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[1] M. Fierz, Helv. Phys. Acta **12**, 3 (1939); M. Fierz and W. Pauli, Proc. Roy. Soc. (London) **A173**, 211 (1939); A complete list of classical papers can be found in the bibliography of E. M. Corson, *Introduction to Tensors, Spinors, and Relativistic Wave Equations* (Blackie and Sons, Ltd., Glasgow, 1953).

[2] S. Weinberg, Phys. Rev. **133**, B1318 (1964); **134**, B882 (1964); D. L. Pursey, Ann. Phys. (N. Y.) **32**, 157 (1965); W. K. Tung, Phys. Rev. Letters **16**, 763 (1966); Phys. Rev. **156**, 1385 (1967).

the usual construction of a Lagrange function, such as for a system with spin-2, there are more than ten parameters to be determined[3]; and the number of parameters increases rapidly with increasing spin value. It is the purpose of this paper[4] to present a new method to construct the free Lagrange function for a system with an arbitrary spin. As we shall see, our method can be applied easily to systems with both integer and half-integer spins. The necessity of introducing the auxiliary field variables, and the physical meanings of these variables are also clarified in this new approach. For the purpose of simplicity, we concentrate our attention to a massive spin-S system described either by a totally symmetric tensor (S=integer), or by a totally symmetric tensor-spinor (S=half-integer). In Sec. II, the spin-projection operators are introduced and their properties are explored. In Secs. III and IV, the Lagrange functions for systems with integer spins, and with half-integer spins are constructed. In the last two sections, the canonical quantizations for these systems are carried out and the Green's functions are constructed.

II. SPIN-PROJECTION OPERATORS

The spin-projection operators were initially introduced by Fronsdal in an attempt to study the general properties of higher-spin systems.[5] These projection operators are nonlocal integral-differential operators through which fields transformed according to definite spins are projected out from an arbitrary field variable. Let us first consider a vector field $\varphi_\mu(x)$. As is well known, an arbitrary vector field provided a basis for a reducible representation of the Lorentz group. Designating this representation by D, we have in fact

$$D=D(1)\oplus D(0),$$

where $D(S)$ is the irreducible representation corresponding to the spin value S. The decomposition on the vector field itself is

$$\varphi_\mu=\varphi_\mu{}^{(0)}+\varphi_\mu{}^{(1)},$$

with

$$\varphi_\mu{}^{(0)}=\partial_\mu\varphi,\quad \partial_\mu\varphi^{\mu(1)}=0,$$

where $\varphi_\mu{}^{(0)}$ and $\varphi_\mu{}^{(1)}$ correspond to the spin-0 and spin-1 parts of the vector field. Based on this decomposition, we introduce the following projection operators:

$$\begin{aligned}\Theta_{\mu\nu}{}^{(0)}&=(\partial^2)^{-1}\partial_\mu\partial_\nu,\\ \Theta_{\mu\nu}{}^{(1)}&=g_{\mu\nu}-(\partial^2)^{-1}\partial_\mu\partial_\nu,\end{aligned}\tag{1}$$

which satisfy

$$\Theta_{\mu\lambda}(a)\Theta^\lambda{}_\nu(b)=\delta_{ab}\Theta_{\mu\nu}(a),\quad a,b=0,1\tag{2}$$

$$\partial_\mu\Theta^{\mu\nu}(1)=0.\tag{3}$$

Then, we have

$$\varphi_\mu(a)=[\Theta(a)\varphi]_\mu=\Theta_{\mu\nu}(a)\varphi^\nu$$

automatically. Irreducible representations corresponding to any other integral-spin value may be formed by taking direct products of D with itself and expanding the products into Clebsch-Gordan series. This general consideration actually gives us a practical method to construct the higher-spin projection operators. Now, let us study the properties of spin projection operators in general. It is well known that a system with an integer spin S can be represented by a totally symmetric tensor $\varphi_{\mu\nu\cdots\lambda}$, satisfying

$$(\partial^2-m^2)\varphi_{\mu\nu\cdots\lambda}=0,\tag{4}$$

$$\begin{aligned}\partial_\mu\varphi^{\mu\nu\cdots\lambda}&=0,\\ g^{\mu\nu}\varphi_{\mu\nu\cdots\lambda}&=0,\end{aligned}\tag{5}$$

while a system with a half-integer spin $S=n+\frac{1}{2}$ can be represented by a totally symmetric tensor-spinor $\psi_{\mu\nu\cdots\lambda}$, satisfying

$$(-i\gamma\partial+m)\psi_{\mu\nu\cdots\lambda}=0,\tag{6}$$

$$\begin{aligned}\gamma^\mu\psi_{\mu\nu\cdots\lambda}&=0,\\ \partial^\mu\psi_{\mu\nu\cdots\lambda}&=0.\end{aligned}\tag{7}$$

Descriptions corresponding to other representation of the Lorentz group can be obtained from these results through proper differentiations, and we shall not discuss them hereafter. It is convenient, temporarily, to express the subsidiary conditions (5) and (7) by the symbolic notations

$$\eta\varphi=0.\tag{8}$$

Now, we introduced an orthogonal projection operator,[5]

$$\Theta=\bar{\Theta}=\Theta^2,$$

with the following properties:

$$\eta(\Theta\varphi)\equiv0,\tag{9}$$

$$\eta\varphi=0\rightarrow\varphi=\Theta\varphi.\tag{10}$$

The projection operator $\Theta_{\mu\nu\cdots\lambda;\mu'\nu'\cdots\lambda'}(S)$, which projects the spin-S field out of an arbitrary totally symmetric S-rank tensor, is both simple and useful. It can be constructed in terms of $\Theta_{\mu\mu'}(1)$ through

$$\Theta_{\mu\nu\cdots\lambda;\mu'\nu'\cdots\lambda'}(S)=[\Theta_{\mu\mu'}\Theta_{\nu\nu'}\cdots\Theta_{\lambda\lambda'}]^T{}_{(\mu\nu\cdots\lambda);(\mu'\nu'\cdots\lambda')},\tag{11}$$

where

$$\Theta_{\mu\mu'}=\Theta_{\mu\mu'}(1),$$

and [ ]^T indicates that the traces of the expression in the square brackets are subtracted. Symmetrizations

[3] R. J. Rivers, Nuovo Cimento **34**, 386 (1964).

[4] We use the following notations: Greek indices run from 0 to 3 while Latin indices run from 1 to 3; $\{\gamma_\mu,\gamma_\nu\}=-2g_{\mu\nu}$ with $g_{\mu\nu}=(-1,1,1,1)$; γ^0=antisymmetric and imaginary, γ_k=symmetric and imaginary.

[5] C. Fronsdal, Nuovo Cimento Suppl. **9**, 416 (1958).

over indices $\mu\,\nu\cdots\lambda$ and over indices $\mu'\,\nu'\cdots\lambda'$ in the parentheses are understood. The projection operators corresponding to a half-integer spin can be derived analogously. The projection operator which projects the spin-S field ($S=n+\frac{1}{2}$, $n=$integer) out of a totally symmetric tensor-spinor of rank n was given by Fronsdal as[5]

$$\Theta_{\mu\nu\cdots\lambda,\eta;\mu'\nu'\cdots\lambda',\eta'}(S)$$
$$=-\frac{2S+1}{4(S+1)}(\gamma^{\sigma}\gamma^{\sigma'})_{\eta\eta'}\Theta_{\mu\nu\cdots\lambda\sigma;\mu'\nu'\cdots\lambda'\sigma'}(n+1)\,, \quad (12)$$

where η, η' stand for the spinor indices, and all other Greek letters stand for vector indices. One verifies easily that

$$\gamma\Theta(S)=\partial\Theta(S)=0\,.$$

In terms of these spin-projection operators, the field equations for system with an arbitrary spin S can be reduced to very simple forms, and they are

$$m^2\varphi_{\mu\nu\cdots\lambda}=\partial^2[\Theta(S)\varphi]_{\mu\nu\cdots\lambda}\,, \quad (13)$$

for an integer S, and

$$m\psi_{\mu\nu\cdots\lambda}=(i\gamma\partial)[\Theta(S)\psi]_{\mu\nu\cdots\lambda}\,, \quad (14)$$

for a half-integer S. These equations were initially constructed by Fronsdal, and are the starting point of our future construction. For $S=0$, $\frac{1}{2}$, and 1, these equations are identical to the usual field equations and are local. For $S>1$, however, these equations contain inverse D'Alembertian operators, $(\partial^2)^{-1}$, and consequently are no longer local equations. As we shall see, the purpose of introducing the auxiliary fields is simply to remove the nonlocality appearing in these field equations.

III. CONSTRUCTION OF LAGRANGE FUNCTIONS FOR SYSTEMS WITH INTEGER SPIN

We shall first construct the Lagrange function for a system with spin-2. A spin-2 field is described by the following nonlocal equation:

$$m^2\varphi_{\mu\nu}=\partial^2[\Theta(2)\varphi]_{\mu\nu}\,, \quad (15)$$

with

$$\varphi_{\mu\nu}=\varphi_{\nu\mu}\,,\quad \varphi_\mu{}^\mu=0\,,$$
$$\Theta_{\mu\nu,\lambda\sigma}(2)=\tfrac{1}{2}[\Theta_{\mu\lambda}\Theta_{\nu\sigma}+\Theta_{\mu\sigma}\Theta_{\nu\lambda}-\tfrac{2}{3}\Theta_{\mu\nu}\Theta_{\lambda\sigma}]\,.$$

Equation (15) can be written explicitly as

$$m^2\varphi_{\mu\nu}=\partial^2\varphi_{\mu\nu}-[\partial_\mu(\partial\varphi)_\nu+\partial_\nu(\partial\varphi)_\mu-\tfrac{1}{2}g_{\mu\nu}(\partial\partial\varphi)]+(\partial_\mu\partial_\nu-\tfrac{1}{4}g_{\mu\nu}\partial^2)\Psi\,, \quad (16)$$

with

$$(\partial\varphi)^\mu=\partial_\nu\varphi^{\mu\nu}\,,\quad (\partial\partial\varphi)=\partial_\mu\partial_\nu\varphi^{\mu\nu}\,,$$
$$\Psi=\tfrac{2}{3}(\partial^2)^{-1}(\partial\partial\varphi)\,. \quad (17)$$

Although it is local in appearance, Eq. (16) is in fact nonlocal. In order to transform Eq. (16) into a local equation, we have to interpret Ψ as a new auxiliary field variable rather than as the nonlocal expression defined in (17). Equation (16) will be equivalent to Eq. (15), if the constraint equations

$$\Psi=(\partial\partial\varphi)=0 \quad (18)$$

follow directly from the field equations. The special combination, $\partial_\mu\partial_\nu-\frac{1}{4}g_{\mu\nu}\partial^2$, guarantees that $\varphi_{\mu\nu}$ remains symmetric and traceless.[6] Now, our problem is reduced to a related but much simpler one: to construct the field equation satisfied by Ψ such that Eq. (18) follows automatically. Equation (16) implies

$$(\partial^2+2m^2)(\partial\partial\varphi)=\tfrac{3}{2}\partial^4\Psi\,, \quad (19)$$

and consequently,

$$(\partial\partial\varphi)=\tfrac{3}{2}(\partial^2-2m^2)\Psi+6m^2(\partial^2+2m^2)^{-1}\Psi\,. \quad (20)$$

Then, if the Ψ is chosen to satisfy

$$(\partial\partial\varphi)=\tfrac{3}{2}(\partial^2-2m^2)\Psi\,, \quad (21)$$

we are led to Eq. (18) automatically. Therefore, Eqs. (16) and (21) describe a spin-2 system. These equations can be derived from the following Lagrange function:

$$\mathcal{L}=\tfrac{1}{2}\varphi_{\mu\nu}(\partial^2-m^2)\varphi^{\mu\nu}+(\partial\varphi)_\mu(\partial\varphi)^\mu+\Psi(\partial\partial\varphi)-\tfrac{3}{4}\Psi(\partial^2-2m^2)\Psi\,.$$

In terms of a new symmetric tensor

$$h_{\mu\nu}=\varphi_{\mu\nu}+\tfrac{1}{4}g_{\mu\nu}\Psi\,,$$

which is no longer traceless, this Lagrange function can be expressed as

$$\mathcal{L}=\tfrac{1}{2}h_{\mu\nu}(\partial^2-m^2)h^{\mu\nu}+h(\partial\partial h)+(\partial h)_\mu(\partial h)^\mu-\tfrac{1}{2}h(\partial^2-m^2)h\,,$$
$$=-{}^\mu H^{\lambda\nu}\partial_\lambda h_{\mu\nu}+\tfrac{1}{4}({}_\mu H_{\nu\lambda}\,{}^\mu H^{\nu\lambda}-H_\lambda H^\lambda)-\tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu}-h^2)\,,$$

with

$${}_\mu H_{\nu\lambda}=\partial_\nu h_{\mu\lambda}-\partial_\lambda h_{\mu\nu}+g_{\mu\nu}[\partial_\lambda h-(\partial h)_\lambda]-g_{\mu\lambda}[\partial_\nu h-(\partial h)_\nu]\,,$$
$$h=h_\mu{}^\mu\,,\quad H_\lambda={}^\mu H_{\mu\lambda}\,.$$

The linearized Lagrange function is the Lagrange function we introduced in a previous publication.[7] If we define $h_{\mu\nu}$ as $\varphi_{\mu\nu}+ag_{\mu\nu}\Psi$, with an arbitrary real number $a\neq 0$, we obtain a Lagrange function which contains a free parameter.[3] We would like to point out that the existence of free parameters in the higher-spin Lagrange functions is due to the possibility of changing the scales of various auxiliary field variables—a fact with no physical content.

The technique developed above can be generalized to construct the Lagrange function for system with an

[6] The requirement that the expression on the right-hand side should have the same symmetry properties as that on the left-hand side is absolutely essential in constructing a Lagrange function. Conversely, the field equation derived from the variation of a field variable should possess the same symmetry properties as those of this particular field variable—a requirement which is sometimes overlooked.

[7] S. J. Chang, Phys. Rev. 148, 1259 (1966).

arbitrary spin. We shall use spin-3 as a further example. The nonlocal spin-3 field equation is

$$m^2\varphi_{\mu\nu\lambda}=\partial^2[\Theta(3)\varphi]_{\mu\nu\lambda}, \quad (22)$$

where $\varphi_{\mu\nu\lambda}$ is a totally symmetric and traceless tensor operators, and

$$\Theta_{\mu\nu\lambda,\mu'\nu'\lambda'}(3)=[\Theta_{\mu\mu'}\Theta_{\nu\nu'}\Theta_{\lambda\lambda'} -\tfrac{3}{5}\Theta_{\mu\nu}\Theta_{\mu'\nu'}\Theta_{\lambda\lambda'}]_{(\mu\nu\lambda);(\mu'\nu'\lambda')}. \quad (23)$$

This equation can be written as

$$\begin{aligned} m^2\varphi_{\mu\nu\lambda}=\partial^2\varphi_{\mu\nu\lambda}-\{&\partial_\mu(\partial\varphi)_{\nu\lambda}+\partial_\lambda(\partial\varphi)_{\nu\mu}+\partial_\nu(\partial\varphi)_{\lambda\mu} \\ &-\tfrac{1}{3}[g_{\mu\nu}(\partial\partial\varphi)_\lambda+g_{\nu\lambda}(\partial\partial\varphi)_\mu+g_{\lambda\mu}(\partial\partial\varphi)_\nu]\} \\ &+\tfrac{2}{3}[\partial_\mu\partial_\nu\Psi_\lambda+\partial_\nu\partial_\lambda\Psi_\mu+\partial_\lambda\partial_\mu\Psi_\nu \\ &-\tfrac{1}{6}\partial^2(g_{\mu\nu}\Psi_\lambda+g_{\nu\lambda}\Psi_\mu+g_{\lambda\mu}\Psi_\nu) \\ &-\tfrac{1}{3}(g_{\mu\nu}\partial_\lambda+g_{\nu\lambda}\partial_\mu+g_{\lambda\mu}\partial_\nu)(\partial\Psi)], \end{aligned} \quad (24)$$

with Ψ_μ being a nonlocal function of $(\partial\partial\varphi)_\mu$. Note that the right-hand side of Eq. (24) is constructed to be totally symmetric and traceless in accordance with the symmetry properties of $\varphi_{\mu\nu\lambda}$.[6] Following the previous argument, we then interpret Ψ_μ as an auxiliary field variable, and try to construct a field equation for Ψ_μ such that

$$\Psi_\mu=(\partial\partial\varphi)_\mu=0 \quad (25)$$

is satisfied automatically. Equation (24) implies

$$(\partial\partial\varphi)_\lambda=\tfrac{5}{6}(\partial^2-\tfrac{3}{2}m^2)\Psi_\lambda+\tfrac{1}{6}\partial_\lambda(\partial\Psi)+\tfrac{1}{2}m\partial_\lambda\chi +(15/8)m^4(\partial^2+\tfrac{3}{2}m^2)^{-1}\Psi_\lambda,$$

with

$$\chi=\tfrac{5}{2}m(\partial^2+\tfrac{3}{2}m^2)^{-1}(\partial\Psi).$$

Consequently, if Ψ_λ is chosen to satisfy

$$(\partial\partial\varphi)_\lambda=\tfrac{5}{6}(\partial^2-\tfrac{3}{2}m^2)\Psi_\lambda+\tfrac{1}{6}\partial_\lambda(\partial\Psi)+\tfrac{1}{2}m\partial_\lambda\chi, \quad (26)$$

then Eq. (25) will be satisfied automatically. One has to note that Eq. (26) is not yet a local equation. This indicates that we have to interpret χ as another auxiliary field variable. The equation satisfied by χ can be computed analogously as

$$(\partial^2-4m^2)\chi=\tfrac{3}{2}(\partial\Psi). \quad (27)$$

Equations (24), (26), and (27) describe a spin-3 system, and they can be derived from the following Lagrange function:

$$\mathcal{L}=\tfrac{1}{2}\varphi_{\mu\nu\lambda}(\partial^2-m^2)\varphi^{\mu\nu\lambda}+\tfrac{3}{2}(\partial\varphi)_{\lambda\nu}(\partial\varphi)^{\lambda\nu}+2\Psi_\lambda(\partial\partial\varphi)^\lambda -\tfrac{5}{6}\Psi_\mu(\partial^2-\tfrac{3}{2}m^2)\Psi^\mu+\tfrac{1}{6}(\partial\Psi)^2-m\Psi_\mu\partial^\mu\chi-\chi(\partial^2-4m^2)\chi. \quad (28)$$

The Lagrange functions for systems with spin 2, 3, and 4 are listed in the Appendix.

IV. CONSTRUCTION OF LAGRANGE FUNCTIONS FOR SYSTEMS WITH HALF-INTEGER SPIN

There is no intrinsic difficulty in applying our technique to systems with half-integer spin, although the algebra of γ matrices make the construction a little more complicated. Let us now construct the spin-$\frac{3}{2}$ Lagrange function explicitly. The nonlocal field equation for a spin-$\frac{3}{2}$ system is

$$\begin{aligned} m\psi_\mu&=(i\gamma\partial)[\Theta(\tfrac{3}{2})\psi]_\mu, \\ &=(i\gamma\partial)\psi_\mu-\tfrac{1}{3}i\gamma_\mu(\partial\psi)+\tfrac{2}{3}i(\gamma\partial)^{-1}\partial_\mu(\partial\psi), \end{aligned} \quad (29)$$

with

$$\gamma_\mu\psi^\mu=0.$$

Equation (29) can be rewritten as

$$m\psi_\mu=(i\gamma\partial)\psi_\mu-\tfrac{1}{2}i\gamma_\mu(\partial\psi)-i(\partial_\mu+\tfrac{1}{4}\gamma_\mu\gamma\partial)\Phi, \quad (30)$$

with

$$\Phi=-\tfrac{2}{3}(\gamma\partial)^{-1}(\partial\psi).$$

We then consider Eq. (30) as a local equation, and consequently interpret Φ as an auxiliary field. The equation for Φ must be chosen to imply

$$\Phi=(\partial\psi)=0.$$

Making use of Eq. (30), we have

$$i(-i\gamma\partial+2m)(\partial\psi)=\tfrac{3}{2}\partial^2\Phi,$$

which implies

$$i(\partial\psi)=-\tfrac{3}{2}(i\gamma\partial+2m)\Phi+6m^2(-i\gamma\partial+2m)^{-1}\Phi.$$

Then, the equation satisfied by Φ is

$$i(\partial\psi)=-\tfrac{3}{2}(i\gamma\partial+2m)\Phi. \quad (31)$$

Equations (30) and (31) are the required equations for describing a spin-$\frac{3}{2}$ system, and can be derived from

$$\mathcal{L}=-\bar\psi_\mu(-i\gamma\partial+m)\psi^\mu-i\bar\psi^\mu\partial_\mu\Phi-i\bar\Phi(\partial\psi) -\tfrac{3}{2}\bar\Phi(i\gamma\partial+2m)\Phi, \quad (32)$$

with

$$\bar\psi_\mu=\psi_\mu{}^\dagger\gamma^0, \text{ etc.}$$

In terms of a new vector spinor

$$\Psi_\mu=\psi_\mu+\tfrac{3}{2}\gamma_\mu\Phi,$$

which no longer satisfies the trace condition $\gamma_\mu\Psi^\mu=0$, the Lagrange function can be reduced to

$$\mathcal{L}=-[\bar\Psi_\mu(-i\gamma\partial+m)\Psi^\mu+\tfrac{1}{3}i\bar\Psi_\mu(\gamma^\mu\partial^\nu+\gamma^\nu\partial^\mu)\Psi_\nu +\tfrac{1}{3}(\bar\Psi\gamma)(i\gamma\partial+m)(\gamma\Psi)],$$

which is the Lagrange function introduced by Rarita and Schwinger.[8] If we define Ψ_μ as $\psi_\mu+a\gamma_\mu\Phi$, with an arbitrary nonvanishing complex number a, we obtain a Lagrange function which contains two arbitrary real parameters.[5,9]

The Lagrange function for systems with spin-$\frac{5}{2}$ as well as any other half-integer spins can be constructed analogously.[10] In the following, we just copy down the

[8] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).
[9] P. A. Moldauer and K. M. Case, Phys. Rev. **102**, 279 (1956).
[10] A. Kawakami and S. Kamefuchi, Nuovo Cimento **48**, 239 (1967). These authors constructed a quadratic Lagrange function for a Rarita-Schwinger field with spin-$\frac{5}{2}$.

field equations satisfied by a spin-$\frac{5}{2}$ system. The field equations are

$$-(-i\gamma\partial+m)\psi_{\mu\nu}-\tfrac{1}{3}i[\gamma_\mu(\partial\psi)_\nu+\gamma_\nu(\partial\psi)_\mu]-i[\partial_\mu\Phi_\nu+\partial_\nu\Phi_\mu +\tfrac{1}{6}\gamma_\mu(\gamma\partial)\Phi_\nu+\tfrac{1}{6}\gamma_\nu(\gamma\partial)\Phi_\mu-\tfrac{1}{3}g_{\mu\nu}(\partial\Phi)]=0\,,$$

$$2i(\partial\psi)_\mu+\tfrac{5}{2}[(i\gamma\partial+\tfrac{3}{2}m)\Phi_\mu-\tfrac{1}{2}i\gamma_\mu(\partial\Phi)] +3[i\partial_\mu+\tfrac{1}{4}i\gamma_\mu(\gamma\partial)]\chi=0\,,$$

$$-3i(\partial\Phi)+(-i\gamma\partial+3m)\chi+m\Omega=0\,,$$

and

$$m\chi-\tfrac{3}{5}(i\gamma\partial+3m)\Omega=0\,,$$

where

$$\psi_{\mu\nu}=\psi_{\nu\mu},\quad \gamma_\mu\psi^{\mu\nu}=\gamma_\mu\Phi^\mu=0\,,$$

and Φ_μ, χ, Ω are auxiliary fields. It is straightforward to verify that these field equations can be derived from the following Lagrange function:

$$\mathcal{L}=-\bar\psi_{\mu\nu}(-i\gamma\partial+m)\psi^{\mu\nu}-2i[\bar\psi^{\mu\nu}\partial_\mu\Phi_\nu+\bar\Phi_\mu(\partial\psi)^\mu] -\tfrac{5}{2}\bar\Phi_\mu(i\gamma\partial+\tfrac{3}{2}m)\Phi^\mu-3i[\bar\Phi^\mu\partial_\mu\chi+\bar\chi(\partial\Phi)] +\bar\chi(-i\gamma\partial+3m)\chi+m(\bar\chi\Omega+\bar\Omega\chi)-\tfrac{3}{5}\bar\Omega(i\gamma\partial+3m)\Omega.$$

The Lagrange functions for systems with spin $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ are also listed in the Appendix.

V. CANONICAL QUANTIZATION RELATIONS

Canonical commutator relations among the field variables of a higher-spin system can be carried out easily in the Lagrange formulation. Quantization follows simply from the identification of the generator G, which is associated with boundary variations, with the infinitesimal generator of unitary transformations on a quantum-mechanical system.[11] This is one of the most fundamental postulates of Schwinger's action principle. We shall illustrate how this principle can be applied to systems with both integer and half-integer spins. The quantization for a spin-2 system has already been studied in a previous publication.[7] In this paper, we shall develop a method which can be applied universally to a system with an arbitrary spin. For simplicity as well as for clarity, we shall use spin-2 and spin-$\frac{5}{2}$ as explicit examples.

In the Lagrange formulation, the generator of a quantum-mechanical system follows from the action principle through

$$\delta W=G_1-G_2\,,$$

with

$$W=\int_{\sigma 2}^{\sigma 1}\mathcal{L}d^4x.$$

For a spin-2 system, this leads to

$$G=\frac{1}{2}\int[\varphi_{\mu\nu}\delta\pi^{\mu\nu}-\delta\varphi_{\mu\nu}\pi^{\mu\nu}]d^3x\,, \tag{33}$$

with

$$\pi_{\mu\nu}=\partial^0\varphi_{\mu\nu}.$$

In this as well as in the future calculations, the variations of $\varphi_{\mu\nu}$ and $\pi_{\mu\nu}$ in the generator are assumed to satisfy the same equations obeyed by $\varphi_{\mu\nu}$ and $\pi_{\mu\nu}$. This is equivalent to assuming that only the independent components of $\varphi_{\mu\nu}$ and $\pi_{\mu\nu}$ can be varied arbitrarily. The variations of the dependent fields should be determined through the field equations. With the help of the relations

$$\pi^{0\nu}=\partial^0\varphi^{0\nu}=\partial_k\varphi_k{}^\nu\,,$$
$$\varphi^{0\nu}=-(m^2-\nabla^2)^{-1}\partial_k\pi_k{}^\nu\,,$$

the generator can be reduced to

$$\begin{aligned} G&=\frac{1}{2}\int[\varphi_{k\nu}\delta\pi^{k\nu}-\delta\varphi_{k\nu}\pi^{k\nu}+\pi^{0\nu}\delta\varphi^0{}_\nu-\delta\pi^{0\nu}\varphi^0{}_\nu]d^3x\\ &=\frac{1}{2}\int\{\varphi_{k\nu}[\delta_{kl}+(m^2-\nabla^2)^{-1}\partial_k\partial_l]\delta\pi_l{}^\nu\\ &\qquad-\delta\varphi_{k\nu}[\delta_{kl}+(m^2-\nabla^2)^{-1}\partial_k\partial_l]\pi_l{}^\nu\}d^3x\\ &=\frac{1}{2}\int[\varphi_{k\nu}\Lambda_{kl}\delta\pi_l{}^\nu-\delta\varphi_{k\nu}\Lambda_{kl}\pi_l{}^\nu]d^3x\,, \end{aligned} \tag{34}$$

with

$$\Lambda_{kl}=\delta_{kl}+(m^2-\nabla^2)^{-1}\partial_k\partial_l. \tag{35}$$

This procedure can be repeated, and leads to

$$G=\frac{1}{2}\int[\varphi_{kl}\Lambda_{kk'}\Lambda_{ll'}\delta\pi_{k'l'}-\delta\varphi_{kl}\Lambda_{kk'}\Lambda_{ll'}\pi_{k'l'}]d^3x. \tag{36}$$

Note that not all components of φ_{kl} and π_{kl} are independent. They satisfy the constraint equation

$$\Lambda_{kl}\varphi_{kl}=\Lambda_{kl}\pi_{kl}=0. \tag{37}$$

Now, the equal-time commutator relations among φ_{kl} and π_{kl} can be carried out easily as

$$[\varphi_{kl}(x),\varphi_{mn}(x')]=[\pi_{kl}(x),\pi_{mn}(x')]=0\,, \tag{38}$$

$$\begin{aligned} i[\varphi_{kl}(x),\pi_{mn}(x')]&=[\Lambda^{-1}{}_{km}\Lambda^{-1}{}_{ln}]^T{}_{(kl);(mn)}\delta(\mathbf{x}-\mathbf{x}')\\ &=\tfrac{1}{2}[\Lambda^{-1}{}_{km}\Lambda^{-1}{}_{ln}+\Lambda^{-1}{}_{kn}\Lambda^{-1}{}_{ml}\\ &\qquad-\tfrac{2}{3}\Lambda^{-1}{}_{kl}\Lambda^{-1}{}_{mn}]\delta(\mathbf{x}-\mathbf{x}')\,, \end{aligned} \tag{39}$$

where

$$\Lambda^{-1}{}_{kl}=\delta_{kl}-m^{-2}\partial_k\partial_l \tag{40}$$

is the inverse of Λ_{kl}. The last term in Eq. (39) is included to guarantee the vanishing of the constraint equation (37). The covariant commutator relations can be obtained easily as

$$\begin{aligned} i[\varphi_{\mu\nu}(x),\varphi_{\lambda\sigma}(x')]&=\tfrac{1}{2}[\Lambda^{-1}{}_{\mu\lambda}\Lambda^{-1}{}_{\nu\sigma}+\Lambda^{-1}{}_{\mu\sigma}\Lambda^{-1}{}_{\nu\lambda}\\ &\qquad-\tfrac{2}{3}\Lambda^{-1}{}_{\mu\nu}\Lambda^{-1}{}_{\lambda\sigma}]\Delta(x-x',m^2)\,, \end{aligned} \tag{41}$$

[11] J. Schwinger, Phys. Rev. **82**, 914 (1951); **91**, 713 (1953).

which is exactly

$$\theta_{\mu\nu,\lambda\sigma}(2)|_{\partial^2=m^2}\Delta(x-x',m^2), \tag{42}$$

where

$$\Delta(x-x',m^2)=i\int\frac{d^4p}{(2\pi)^3}e^{ip(x-x')}\epsilon(p)\delta(p^2+m^2)$$

is an invariant function introduced by Schwinger. Let us now generalize our result to a system with an arbitrary integer spin S. The generator for such a system is

$$G=\frac{1}{2}\int[\varphi_{\lambda\mu\cdots\nu}\delta\pi^{\lambda\mu\cdots\nu}-\delta\varphi_{\lambda\mu\cdots\nu}\pi^{\lambda\mu\cdots\nu}]d^3x$$
$$=\frac{1}{2}\int[\varphi_{lm\cdots n}\Lambda_{ll'}\Lambda_{mm'}\cdots\Lambda_{nn'}\delta\pi_{l'm'\cdots n'} \tag{43}$$
$$-\delta\varphi_{lm\cdots n}\Lambda_{ll'}\Lambda_{mm'}\cdots\Lambda_{nn'}\pi_{l'm'\cdots n'}]d^3x,$$

with

$$\pi_{\lambda\mu\cdots\nu}=\partial^0\varphi_{\lambda\mu\cdots\nu},$$
$$\Lambda_{kl}\varphi_{kl\cdots n}=\Lambda_{kl}\pi_{kl\cdots n}=0,\text{ etc.}$$

Then, we have the following equal-time commutator relations:

$$[\varphi_{kl\cdots n}(x),\varphi_{k'l'\cdots n'}(x')]=[\pi_{kl\cdots n}(x),\pi_{k'l'\cdots n'}(x')]=0, \tag{44}$$

$$i[\varphi_{kl\cdots n}(x),\pi_{k'l'\cdots n'}(x')]=[\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}\cdots\Lambda^{-1}{}_{nn'}]^T{}_{(kl\cdots n);(k'l'\cdots n')}\times\delta(\mathbf{x}-\mathbf{x}'), \tag{45}$$

which leads to the required covariant commutator relation

$$i[\varphi_{\lambda\mu\cdots\nu}(x),\varphi_{\lambda'\mu'\cdots\nu'}(x')]=\theta_{\lambda\mu\cdots\nu;\lambda'\mu'\cdots\nu'}(S)|_{\partial^2=m^2}\Delta(x-x',m^2). \tag{46}$$

The anticommutator relations for systems with half-integer spin can be obtained analogously. The generator for a spin-$\frac{5}{2}$ system is

$$G=i\int\psi_{\mu\nu}{}^\dagger\delta\psi^{\mu\nu}d^3x, \tag{47}$$

where $\delta\psi_{\mu\nu}$ are objects which anticommute among themselves as well as with ψ and $\psi^\dagger$ in accordance with the Fermi statistics. Making use of the field equations

$$(-i\gamma\partial+m)\psi_{\mu\nu}=\partial_\nu\psi^{\mu\nu}=0,$$

we have both

$$(m-i\gamma\nabla)\psi^{0\nu}=-i\gamma^0\partial^0\psi^{0\nu}=-i\gamma^0\partial_k\psi_k{}^\nu \tag{48}$$

and its adjoint equation

$$(m-i\gamma\nabla)\psi^{0\nu\dagger}=-i\gamma^0\partial_k\psi_k{}^{\nu\dagger}. \tag{49}$$

With the help of these equations, the generator can be reduced to

$$G=i\int[\psi_{k\nu}{}^\dagger\delta\psi^{k\nu}-\psi^{0\nu\dagger}\delta\psi^0{}_\nu]d^3x$$
$$=i\int\psi_{k\nu}{}^\dagger\Lambda_{kl}\delta\psi_l{}^\nu d^3x.$$

Repeating this process, we have

$$G=i\int\psi_{kl}{}^\dagger\Lambda_{kk'}\Lambda_{ll'}\delta\psi_{k'l'}d^3x. \tag{50}$$

This result is general, and can be applied to an arbitrary half-integer system. Analogous to systems with integer spin, one finds that not all components of ψ_{kl} are independent. It is straightforward to verify that ψ_{kl} satisfies the generalized trace condition

$$\Gamma_k\psi_{kl}=0, \tag{51}$$

where

$$\Gamma_k=\gamma_k-i(m-i\gamma\nabla)^{-1}\partial_k \tag{52}$$

obeys the simple anticommutation relations

$$\{\gamma^0\Gamma_k,\gamma^0\Gamma_l\}=2\Lambda_{kl}. \tag{53}$$

Relations such as

$$\Lambda_{kl}\psi_{kl}=0$$

follow from Eq. (51) through

$$(\gamma^0\Gamma_k)(\gamma^0\Gamma_l)\psi_{kl}=\Lambda_{kl}\psi_{kl},$$

and shall not be considered as independent restrictions. The equal-time anticommutator relation can be obtained easily by inverting the matrix $\Lambda_{kk'}\Lambda_{ll'}$ under the restriction (51), and we have

$$[\psi_{kl}(x),\psi_{k'l'}(x')]_+=[\psi_{kl}{}^\dagger(x),\psi_{k'l'}{}^\dagger(x')]_+=0,$$
$$[\psi_{kl}(x),\psi_{k'l'}{}^\dagger(x')]_+=[\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}]^T{}_{(kl);(k'l')}\delta(\mathbf{x}-\mathbf{x}') \tag{54}$$

$$=[\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}-\tfrac{1}{5}\Lambda^{-1}{}_{kl}\Lambda^{-1}{}_{k'l'}-\tfrac{2}{5}(\gamma^0\Gamma\Lambda^{-1})_k(\gamma^0\Gamma\Lambda^{-1})_{k'}\Lambda^{-1}{}_{ll'}]_{(kl);(k'l')}\delta(\mathbf{x}-\mathbf{x}'). \tag{55}$$

In Eq. (55), $[\ ]^T$ represents an expression whose traces are subtracted. In other words, this expression is constructed to satisfy Eq. (51) identically. We would like to point out that this inverting process can be carried out easily in the rest frame where $\Lambda^{-1}{}_{kl}$ reduces to δ_{kl}, and Γ_k to γ_k. The covariant commutator can be obtained through the relation

$$\psi_{\mu\nu}=(\delta_{\mu k}-g_{\mu0}\gamma^0\gamma_k)(\delta_{\nu l}-g_{\nu0}\gamma^0\gamma_l)\psi_{kl}.$$

It is straightforward, although quite tedious to verify that

$$[\psi_{\mu\nu}(x),\psi_{\lambda\sigma}(x')]_+=[\psi_{\mu\nu}{}^\dagger(x),\psi_{\lambda\sigma}{}^\dagger(x')]_+=0,$$
$$-i[\psi_{\mu\nu}(x),\bar\psi_{\lambda\sigma}(x')]_+=-(i\gamma\partial+m)\Theta_{\mu\nu,\lambda\sigma}(5/2)|_{\partial^2=m^2}\times\Delta(x-x',m^2). \tag{56}$$

These results are also general, and are applicable to an arbitrary half-integer system.

We now examine some positive-definiteness requirements. The requirement that the energy of a Bose system must be positive is indeed satisfied. For a spin-2 system, for instance, the energy can be computed easily through

$$\int \mathcal{L} d^3x = \frac{1}{2}\int [\varphi^{\mu\nu}\partial_0\pi_{\mu\nu} - \pi^{\mu\nu}\partial_0\varphi_{\mu\nu}]d^3x - P^0,$$

and is

$$P^0 = \frac{1}{2}\int [\varphi^{\mu\nu}(m^2-\nabla^2)\varphi_{\mu\nu} + \pi^{\mu\nu}\pi_{\mu\nu}]d^3x,$$

$$= \frac{1}{2}\int [\varphi_{kl}(m^2-\nabla^2)\Lambda_{kl}\Lambda_{k'l'}\varphi_{k'l'} + \pi_{kl}\Lambda_{kk'}\Lambda_{ll'}\pi_{k'l'}]d^3x$$

$$= \frac{1}{2}\int [q_{kl}(m^2-\nabla^2)\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}q_{k'l'} + p_{kl}\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}p_{k'l'}]d^3x > 0, \quad (57)$$

with

$$q_{kl} = \Lambda_{kk'}\Lambda_{ll'}\varphi_{k'l'}, \quad p_{kl} = \Lambda_{kk'}\Lambda_{ll'}\pi_{k'l'},$$

$$\int p_{kl}\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}p_{k'l'}d^3x = \int \{(p_{kl})^2 + (2/m^2)[(\partial p)_k]^2 + (1/m^4)(\partial\partial p)^2\}d^3x > 0, \quad (58)$$

where

$$(\partial p)_k = \partial_l p_{lk}, \quad (\partial\partial p) = \partial_k\partial_l p_{kl}.$$

We would like to emphasize that this technique can be generalized to an arbitrary Bose system with spin S as

$$P^0 = \frac{1}{2}\int [\varphi^{\lambda\mu\cdots\nu}(m^2-\nabla^2)\varphi_{\lambda\mu\cdots\nu} + \pi^{\lambda\mu\cdots\nu}\pi_{\lambda\mu\cdots\nu}]d^3x,$$

$$= \frac{1}{2}\int [q_{kl\cdots n}(m^2-\nabla^2)\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}\cdots\Lambda^{-1}{}_{nn'}q_{k'l'\cdots n'} + p_{kl\cdots n}\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}\cdots\Lambda^{-1}{}_{nn'}p_{k'l'\cdots n'}]d^3x > 0,$$

with

$$q_{kl\cdots n} = \Lambda_{kk'}\Lambda_{ll'}\cdots\Lambda_{nn'}\varphi_{k'l'\cdots n'},$$
$$p_{kl\cdots n} = \Lambda_{kk'}\Lambda_{ll'}\cdots\Lambda_{nn'}\partial^0\varphi_{k'l'\cdots n'},$$

and

$$\int p_{kl\cdots n}\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}\cdots\Lambda^{-1}{}_{nn'}p_{k'l'\cdots n'}d^3x$$
$$= \int \{(p_{kl\cdots n})^2 + C_1{}^S m^{-2}[(\partial p)_{kl\cdots m}]^2 + C_2{}^S m^{-4}[(\partial\partial p)_{k\cdots}]^2 + \cdots + m^{-2S}[(\partial\partial\cdots\partial p)]^2\}d^3x > 0. \quad (59)$$

The second requirement is that the vacuum expectation value of the anticommutator $[\psi,\psi^\dagger]_+$ for an arbitrary Fermi field variable ψ, must be positive.[12] We shall use the spin-$\frac{5}{2}$ system as an explicit example. To verify that a distribution function, such as

$$x^0 = x^{0\prime}: \quad \langle[\psi_{kl}(x),\psi_{k'l'}{}^\dagger(x')]_+\rangle = [\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}]^T{}_{(kl);(k'l')}\delta(\mathbf{x}-\mathbf{x}'), \quad (60)$$

is a positive matrix, we follow the usual trick of multiplying it by some well-behaved testing functions $f_{kl}(\mathbf{x})$ and $f_{k'l'}{}^*(\mathbf{x})$, and integrating the products over x and x'. This is equivalent to computing the anticommutator between the operator

$$\Psi = \int f_{kl}(\mathbf{x})\psi_{kl}(x)d^3x,$$

and its conjugate

$$\Psi^\dagger = \int f_{kl}{}^*(\mathbf{x})\psi_{kl}{}^\dagger(x)d^3x,$$

where the numerical function $f_{kl}(\mathbf{x})$ may be chosen to be symmetric, satisfying

$$(\gamma^0\Gamma\Lambda^{-1})_k f_{kl} = 0.$$

Then, the verification of $\langle[\psi_{kl}(x),\psi_{k'l'}{}^\dagger(x')]_+\rangle$ as a positive matrix is reduced to the verification of

$$\langle[\Psi,\Psi^\dagger]_+\rangle = \int f_{kl}(\mathbf{x})\Lambda^{-1}{}_{kk'}\Lambda^{-1}{}_{ll'}f_{k'l'}{}^*(\mathbf{x}')d^3x > 0,$$

which follows trivially from Eq. (58). The generalization of this result to other half-integer systems is straightforward.

VI. GREEN'S FUNCTIONS

One way to construct Green's functions for systems with integer spin is to replace the invariant function $\Delta(x-x', m^2)$ appearing in the covariant commutators, $i[\varphi_{\lambda\mu\cdots\nu}(x),\varphi_{\lambda'\mu'\cdots\nu'}(x')]$, by the proper invariant Green's function,[13] such as

$$\Delta_+(x-x', m^2) = \lim_{\epsilon\to 0}\int \frac{dp}{(2\pi)^4}\frac{e^{ip(x-x')}}{p^2+m^2-i\epsilon}.$$

Similarly, the Green's function for a half-integer system can be obtained by replacing the $(m+i\gamma\partial)\times\Delta(x-x', m^2)$ in the covariant anticommutators $i[\psi_{\lambda\mu\cdots\nu}(x),\bar\psi_{\lambda'\mu'\cdots\nu'}(x')]_+$ by

$$S_+(x-x') = (m+i\gamma\partial)\Delta_+(x-x').$$

[12] J. Schwinger, *Differential Equations of Quantum Field Theory* (Stanford University Press, Stanford, California, 1956); K. Johnson and E. C. G. Sudarshan, Ann. Phys. (N. Y.) **13**, 126 (1961).

[13] See, e.g., S. Weinberg's papers in Ref. 2.

Then, the Green's function can be expressed explicitly as

$$G_{\lambda\mu\cdots\nu;\lambda'\mu'\cdots\nu'}(x,x') = \Theta_{\lambda\mu\cdots\nu;\lambda'\mu'\cdots\nu'}|_{\partial^2=m^2}\Delta_+(x-x') \quad (61)$$

for systems with integer spin, and

$$G_{\lambda\mu\cdots\nu,\zeta;\lambda'\mu'\cdots\nu',\zeta'}(x,x') = \Theta_{\lambda\mu\cdots\nu,\zeta';\lambda'\mu'\cdots\nu',\zeta''}|_{\partial^2=m^2}S_+(x-x')_{\zeta''\zeta'} \quad (62)$$

for systems with half-integer spin. In this construction, however, there are always the ambiguities of adding some contact terms which are linear in $\delta(x-x')$. These extra terms correspond to the replacement of $(m^2-\partial^2)\times\Delta(x-x')$ in the covariant commutator, which is in fact zero, by $(m^2-\partial^2)\Delta_+(x-x')=\delta(x-x')$ in the Green's function. As for free fields, these terms do not propagate. They have no physical consequence other than to modify the transition amplitude by a phase factor which can never be detected.[14] In the interacting system, however, these terms do contribute; they lead to some contact interactions between the external currents.

An alternative definition of Green's function is introduced as the measurement of the response of the system to the variation of an external source, and is given by[15]

$$G_{\lambda\mu\cdots\nu;\lambda'\mu'\cdots\nu'}(x-x')=\delta\langle\varphi_{\lambda\mu\cdots\nu}(x)\rangle/\delta j^{\lambda'\mu'\cdots\nu'}(x')|_{j=0}.$$

It is straightforward to verify that this definition not only leads to the same nonlocal structures as given by Eqs. (61) and (62), but also gives definite prediction on these contact terms. We would like to emphasize that one should not take these predictions too seriously. We have to remind ourselves that there is always more than one way to describe a higher-spin system. For example, a spin-$\frac{3}{2}$ system can be described either by a vector spinor, or by a total symmetric third-rank multispinor. The contact terms computed from these two different formulations are in fact different.[16] The ambiguities associated with the Green's function still persist, and are now related to the possibility of introducing various descriptions. It is our opinion that these contact terms should be determined from the high-energy behavior of the Green's function. The extra information which can be deduced from experimental results may reveal to us which of the higher-spin descriptions should be used to describe the physical observed particles.

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[14] J. Schwinger, Phys. Rev. **152**, 1219 (1966).
[15] J. Schwinger, Proc. Natl. Acad. Sci. U.S. **37**, 452 (1951).
[16] G. S. Guralnik and T. W. B. Kibble, Phys. Rev. **139**, B712 (1965); S. J. Chang, thesis, Harvard University (unpublished).

APPENDIX: EXPLICIT EXPRESSION FOR THE HIGHER-SPIN LAGRANGE FUNCTIONS WITH SPIN≤4

A. The Lagrange Functions for Systems with an Integer Spin S

$S=2$

$$\mathfrak{L}=\tfrac{1}{2}\varphi^{\mu\nu}(\partial^2-m^2)\varphi_{\mu\nu}+(\partial\varphi)_\mu(\partial\varphi)^\mu +\Psi(\partial\partial\varphi)-\tfrac{3}{4}\Psi(\partial^2-2m^2)\Psi;$$

$S=3$

$$\mathfrak{L}=\tfrac{1}{2}\varphi^{\lambda\mu\nu}(\partial^2-m^2)\varphi_{\lambda\mu\nu}+\tfrac{3}{2}(\partial\varphi)_{\mu\nu}(\partial\varphi)^{\mu\nu}+2\Psi_\mu(\partial\partial\varphi)^\mu -\tfrac{5}{6}\Psi_\mu(\partial^2-\tfrac{3}{2}m^2)\Psi^\mu+\tfrac{1}{6}(\partial\Psi)^2+m\chi(\partial\Psi)-\chi(\partial^2-4m^2)\chi;$$

$S=4$

$$\mathfrak{L}=\tfrac{1}{2}\varphi^{\lambda\mu\nu\sigma}(\partial^2-m^2)\varphi_{\lambda\mu\nu\sigma}+2(\partial\varphi)_{\lambda\mu\nu}(\partial\varphi)^{\lambda\mu\nu}+3\Psi_{\mu\nu}(\partial\partial\varphi)^{\mu\nu} -\tfrac{7}{8}\Psi_{\mu\nu}(\partial^2-\tfrac{4}{3}m^2)\Psi^{\mu\nu}+\tfrac{1}{2}(\partial\Psi)_\lambda(\partial\Psi)^\lambda+4m\chi^\mu(\partial\Psi)_\mu -(15/2)\chi^\mu(\partial^2-27m^2/10)\chi_\mu+\tfrac{3}{2}(\partial\chi)^2+3m\Omega(\partial\chi) -(3/7)\Omega(\partial^2-20m^2/3)\Omega.$$

In these expressions, all field variables are chosen to be totally symmetric and traceless, and

$$(\partial\varphi)_\mu=\partial^\nu\varphi_{\nu\mu},\quad (\partial\partial\varphi)=\partial^\mu\partial^\nu\varphi_{\mu\nu},\quad \text{etc.}$$

B. The Lagrange Function for Systems with a Half-Integer Spin S

$S=\frac{3}{2}$

$$\mathfrak{L}=-\bar\psi_\mu(-i\gamma\partial+m)\psi^\mu-i[\bar\psi^\mu\partial_\mu\Phi+\bar\Phi(\partial\psi)] -\tfrac{3}{2}\bar\Phi(i\gamma\partial+2m)\Phi;$$

$S=\frac{5}{2}$

$$\mathfrak{L}=-\bar\psi_{\mu\nu}(-i\gamma\partial+m)\psi^{\mu\nu}-2i[\bar\psi^{\mu\nu}\partial_\mu\Phi_\nu+\bar\Phi_\mu(\partial\psi)^\mu] -\tfrac{5}{2}\bar\Phi_\mu(i\gamma\partial+\tfrac{3}{2}m)\Phi^\mu-3i[\bar\Phi^\mu\partial_\mu\chi+\bar\chi(\partial\Phi)] +\bar\chi(-i\gamma\partial+3m)\chi+m(\bar\chi\Omega+\bar\Omega\chi)-\tfrac{3}{5}\bar\Omega(i\gamma\partial+3m)\Omega;$$

$S=\frac{7}{2}$

$$\mathfrak{L}=-\bar\psi_{\lambda\mu\nu}(-i\gamma\partial+m)\psi^{\lambda\mu\nu}-3i[\bar\psi^{\lambda\mu\nu}\partial_\lambda\Phi_{\mu\nu}+\bar\Phi_{\mu\nu}(\partial\psi)^{\mu\nu}] -\tfrac{7}{2}\bar\Phi_{\mu\nu}(i\gamma\partial+4m/3)\Phi^{\mu\nu}-4i[\bar\Phi^{\mu\nu}\partial_\mu\chi_\nu+\bar\chi^\mu(\partial\Phi)_\mu] +\bar\chi^\mu(-i\gamma\partial+2m)\chi_\mu+m(\bar\chi^\mu\Omega_\mu+\bar\Omega^\mu\chi_\mu) -(5/7)\bar\Omega^\mu(i\gamma\partial+2m)\Omega_\mu-3i[\bar\Omega^\mu\partial_\mu\Theta+\bar\Theta(\partial\Omega)] +\tfrac{7}{2}\bar\Theta(-i\gamma\partial+4m)\Theta+2m(\bar\Theta\xi+\bar\xi\Theta) -(2/7)\bar\xi(i\gamma\partial+4m)\xi.$$

In these expressions, all field variables are totally symmetric in tensor indices and satisfy the trace conditions

$$\gamma^\mu\psi_{\mu\nu\lambda}=0,\ \cdots,$$

and

$$\bar\psi=\psi^\dagger\gamma^0,\text{etc.}$$