PHYSICAL REVIEW VOLUME 161, NUMBER 5 25 SEPTEMBER 1967

Compositeness Conditions for Particles with Identical Quantum Numbers

MICHAEL M. BROIDO*
Courant Institute of Mathematical Sciences, New York, New York

AND

JOHN G. TAYLOR*
Department of Physics, Rutgers, The State University, New Brunswick, New Jersey
(Received 24 June 1966)

We discuss field-theoretic mechanisms by which two particles having identical quantum numbers can be exhibited as composites of other particles. If this can be done, then, contrary to what other authors have asserted, it can be done by letting the composite wave-function renormalization constants tend to zero.

RECENTLY a good deal of attention has been given to the problem of describing composite particles in field theory. It was Jouvet who first suggested[1] that in order to turn an elementary particle into a composite, one should let its wave function renormalization constant Z_3 tend to zero. An extensive recent analysis[2] of the present authors has shown that this mechanism works, independently of model details and of special approximations, in an exceedingly large variety of contexts. However, a number of recent analyses[3–5] have suggested that special difficulties may arise in the case where there are two different particles with identical quantum numbers. In particular, Alexanian and Zimmermann[3] claim that "the vanishing of the wave function renormalization constant does not necessarily imply that the particle is composite."

In the present note we show in detail how the $Z_3 \to 0$ mechanism works for the case of two particles with identical quantum numbers, and that the claimed difficulties arising in Ref. 3 are spurious, i.e., are already covered by the analysis in Ref. 2. Our analysis also exposes a serious mistake in Ref. 4, and shows that in this case also nothing exceptional is going on.

We will consider an elementary particle and its conjugate described by fields ψ, $\bar{\psi}$. We will attempt to discuss two further fields $\phi_i(i=1, 2)$ which are to describe composites of ψ and $\bar{\psi}$ having *different* masses m_1. It seems necessary that these masses should be different in order that, in the case of identical quantum numbers, the notion of two different particles should make sense. Since we will be considering asymptotic states, we will assume that the two particles are stable, although most of our arguments can be carried through if they are unstable (with different 'complex masses').

In Ref. 2 we started with elementary particles and tried to make them both composite by putting $Z_i=0$ (throughout the present note we will deal explicitly with the wave-function renormalization only and will drop the suffiix 3). In the present situation, it is more convenient to consider what would happen if the problem could be solved, and to work backwards. Suppose then, that there are fields describing the particles ϕ_i as composites. The Haag-Ruelle scattering theory[6] is constructed in terms of fields $f_i(\psi)$ satisfying (by reason of the spectral assumptions)

$$\langle 0|[f_i(\psi)](x)[f_i(\psi)](y)|0\rangle = i\Delta_+(m_i^2; x-y), \quad (1)$$

which then give rise in the usual way to asymptotically free fields of mass m_i and to outgoing states $|i\rangle$ satisfying

$$\langle 1|2\rangle = 0 \quad (2)$$

by virtue of the differing masses. By repeating the construction we can retain continuum terms in (1) while maintaining (2). There may be difficulties associated with the quantization of nonlocal fields; we will ignore these for the present. This problem seems to arise in a similar context in connection with the description of bootstrapped symmetries by field theory.[7]

It is here that we run into difficulties associated with our attempt[2] to formulate a theory of composite particles sufficiently general to include both axiomatic field theories[6] and calculations in specific models[2–5]; for of course there is no known nontrivial example of an axiomatic field theory.

In an axiomatic field theory, we cannot assume that that the f_i are local, since it then seems that the S matrix will be trivial. In such a framework, then, one will have to abandon the use of dispersion relations for the composite scattering, and other methods depending directly on the use of local fields.

In the usual models this situation will not arise; however, for the very same reason, Haag-Ruelle theory

* Partly supported by the National Science Foundation at Rutgers University, New Brunswick, New Jersey.

1 B. Jouvet, Nuovo Cimento 5, 1 (1957).
2 M. M. Broido and J. G. Taylor, Phys. Rev. 147, 993 (1966).
3 M. Alexanian, report given at the American Physical Society Meeting in Washington, April, 1966 (unpublished); M. Alexanian and R. L. Zimmermann, Lawrence Radiation Laboratory Report No. UCRL-14808-T, 1966 (unpublished).
4 P. K. Srivastava and S. R. Choudhury, Nuovo Cimento 39, 650 (1965).
5 J. C. Houard and J. C. le Guillou, Nuovo Cimento 44, 484 (1966).
6 See, for instance, R. Jost, *General Theory of Quantized Fields* (American Mathematical Society, Providence, 1965), and references given there.
7 M. M. Broido and J. G. Taylor, Rutgers University Report (unpublished).

will not apply. We note that the purpose of Eq. (1) is to separate (in momentum space) the supports of the fields giving rise to the asymptotically free fields. In principle this can just as well be done by taking any suitable functions f_i such that the momentum-space supports of the two functions (1) (given by the Lehmann representation) are disjoint. This is of course a nonlinear process and is quite different from the linear separation process which was seen to fail in Ref. 1.

Furthermore, according to old arguments of Haag, Zimmermann, and others,[8] if two functions of ψ both have nonvanishing vacuum-to-one-particle matrix elements, they describe essentially the same composite. (The S matrices are the same.) However, our $f_i(\psi)$ avoid this pitfall because from (1) and (2) we get

$$\langle 0|f_i(\psi)|j\rangle=0 \quad \text{for} \quad i\neq j. \tag{3}$$

Thus we definitely do have two distinct composites.

Now we will construct a Lagrangian theory with elementary particles ϕ_i which gives in the $Z_i\to 0$ limit field equations

$$\phi_i=f_i(\psi), \tag{4}$$

which are the solutions of our problem. Such a Lagrangian is, very generally (Secs. 2–3 of Ref. 2),

$$\mathcal{L}=\mathcal{L}(\psi)+\sum_i Z_i\mathcal{L}_{\text{free}}(\phi_i)+\tfrac{1}{2}\sum_i \lambda_i[\phi_i-f_i(\psi)]^2,$$

leading to elementary field equations for the ϕ_i:

$$Z_i(\Box^2+m_i^2)\phi_i+\lambda_i\phi_i=\lambda_i f_i(\psi), \tag{5}$$

giving (4) in the limit $Z_i\to 0$. Of course one has to justify this by giving a Green's-function analysis, but this will be precisely identical to the one in[2], so we will not waste space by repeating it here. In particular, although the propagators for the ϕ_i

$$\langle 0|T(\phi_i(x)\phi_i(y))|0\rangle \tag{6}$$

may give rise to transitions when $Z_i\neq 0$, they *will not do so in the limit* $Z_i=0$, as is shown explicitly by Eqs. (1) and (2). The $Z_i=0$ propagators will have only one pole each, at the correct masses.

What we have shown then is that if composite fields exist at all, they can be obtained by our method. Of course, they may not exist; but this situation cannot be discussed in general—just as in Ref. 2 where certain restrictions on masses and coupling constants had to be obeyed if a composite particle was to arise at all, one cannot expect to get something for nothing.

Now let us see what happens if one naively tries to do a Green's-function analysis for two elementary particles with equal quantum numbers, coupled locally to other fields. Generalizing references[3–5], we can write down a completely model-independent equation for the clothed propagator-matrix:

$$D_{Fij}'=D_{Fij}+D_{Fik}'\Pi_{kl}D_{Flj}+D_{Fik}'\delta m_k^2 D_{Fkj}, \tag{7}$$

where i, j, k, $l=1$, 2 refer to the two particles. D_F represents the bare propagator:

$$D_{Fij}=\frac{\delta_{ij}}{p^2-m_i^2}. \tag{8}$$

D_{Fij}' is the clothed propagator, Π is the usual self-energy contribution and δm_i^2 are the mass shifts. Equation (7) is the definition of D_{Fij}' [compare Ref. 2, Fig. 13, and Ref. 9, Eq. (93)]; but model-dependent field equations have been used in these two] and is correct for any field theory with local interaction. Equation (7) can be conveniently rewritten

$$(D_F')_{ij}^{-1}=(D_F)_{ij}^{-1}-\Pi_{ij}+\delta m_i^2\delta_{ij}. \tag{9}$$

We wish to diagonalize D_{Fij}' so as to decouple the two particles. We need only consider the range of the variables below the production threshold for other particles, since both elementary and (later) composite-particle poles will be expected to occur below this threshold, if they are to be stable. In this region, Π is real and symmetric, so the diagonalization can always be carried out.

We wish the resulting propagators to have just one pole each at the correct renormalized masses M_i say. This can be achieved by suitable choices of the mass shifts δm_i^2.

So far, the discussion has referred to elementary particles. If now we try to make these composite, we find that this cannot in general be done, because the assignment of definite mass shifts has eliminated the degree of freedom required to fix a composite mass according to the discussion of Sec. 3 of Ref. 2. However, it may happen that the $Z=0$ contours do pass through exactly those points of the (g_i^2,m_i^2) planes which give the composite masses M_i [here g_i denotes the appropriate coupling constant, m_i the renormalized mass (Ref. 2)]. This is *exactly the Green's-function equivalent of the situation described by the fields* $f_i(\psi)$ introduced earlier in this note.

Suppose now that this accident does not occur: the desired bound-state masses are not those required to eliminate the redundant propagator poles. Then contributions of the form of Fig. 1 will no longer cancel to all orders [compare Eq. (6)] and the only way of eliminating them is to make sure that each vanishes identically, i.e., to decouple one of the composite fields from the elementary fields altogether. Instead of two composites, we obtain one composite and one free field, precisely as in the special cases considered in Ref. 3. We wish to underline that this is the general mechanism

[8] R. Haag, Phys. Rev. **112**, 669 (1958); W. Zimmermann, Nuovo Cimento **10**, 597 (1958).

[9] J. G. Taylor, Nuovo Cimento Suppl. **1**, 857 (1963).

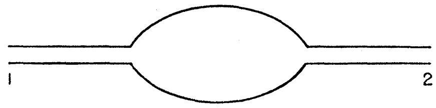

FIG. 1. A diagram which will contribute to the transition between certain composites unless one is completely decoupled. The double lines represent the two composites; the single lines represent other elementary particles to which they are coupled.

involved in such cases. In fact we see that the elaborate analytic apparatus of Refs. 3 and 4, referring as they do to specific models, have merely succeeded in obfuscating the real issues.

Reference 4 discusses an extended Lee model with two V particles, and purports to show that in order to obtain *one* composite V one must let *both* wave-function renormalization constants tend to zero. (The composite V is then undistinguishable from the elementary one.) The analysis is incorrect; it ignores a much simpler solution obtained by putting one Z zero and decoupling the other V particle in a suitable manner (Ref. 4, bottom of p.653: $\alpha_1 \rightarrow 0$, $g_2 \rightarrow 0$ with $g_2{}^2/\alpha_1$ finite in the limit). This is precisely the situation we have just described.

The only other work of which we are aware in which two particles with identical quantum numbers are considered in detail is Ref. 5. These authors do not attempt to make *both* particles composite, although there seems no reason why this should not be possible [cf. their Eq. (53)].

The analysis in Refs. 3–5 depends very much on the special models used. In particular, the effects of the vertex-function renormalization, which changes the general graph structure so much[2,7] is not clear. Furthermore, these authors do not specify their composite masses; the elementary masses have, of course,[2] vanished from the field equations. Thus it is not at all clear whether we can talk of two different particles. All these difficulties are avoided by the analysis given in the present paper.

Finally, one may briefly consider practical applications; a case in point is ϕ-ω mixing. One will almost certainly wish to consider also the effect of a symmetry group. In SU_3, the two particles are assigned to different representations, so that the special situation discussed in this paper does not arise. It could arise in SU_6; one might attempt to use the method of bootstrapping symmetries discussed in Sec. 7D of Ref. 2. These methods are applied to a completely SU_3-symmetric situation in Ref. 7. Now the most natural way of including symmetry breaking which will give increasing contributions at higher energies is by dynamical terms; but then the ϕ and ω are elementary again and the formal problems discussed in this paper again do not arise. One feels that the special case of particles of equal quantum numbers is more interesting from the point of view of the structure of field theories than in connection with direct practical applications.

Note added in proof. Since the present paper was written (early in 1966) a number of further articles have appeared.[10] We will add certain comments.

"Propagator renormalization constants Z_{ij} are frequently[10] defined as $s \rightarrow \infty$ limits rather than by $P^2 = m^2$ residue conditions. This will not give the same results, unless the Lehmann spectral functions are well behaved at infinity. To assume this is to beg the question, as is well known in two-particle unitarity models (these are classified by this asymptotic behavior).

Some authors[11] make much of the distinction between the Z_{ij} and "generalized wave-function renormalization constants" A_{ij} [essentially the objects of Eq. (3), including $i = j$]. In two-particle unitarity it is claimed[10] that the Z_{ij} must all vanish together, hence do not give rise to a viable compositeness criterion. This happens because of the divergence of a spectral integral for the Z_{ij}^{-1}.

But in all known exactly soluble models, including the Lee model[11] and the Lee model with relativistic kinematics,[12] the Z^{-1} goes to infinity not because an integral diverges, but because the composite limit is achieved by letting a bare coupling constant g tend to infinity, and $Z \sim g^2$. Thus the mechanism postulated in two-particle unitarity models is actually quite different from that observed in the Lee model.

These errors are compounded. Not only is Ref. 4 wrong in itself, but the analogy it suggests to the authors of Refs. 9 and 10 is also only apparent.[13]

One of us (M.M.B.) would like to thank the Courant Institute, where he was a visiting member while this work was carried out, for typing and duplicating the manuscript.

[10] For an extensive and up-to-date bibliography, see K. Hayashi, M. Hirayama, T. Muta, N. Seto, and T. Shirafuji [Fortschr. Physik (to be published)]. We disagree with many assertions in this article, however.

[11] For instance, T. Muta, Progr. Theoret. Phys. (Kyoto) **35**, 1099 (1966).

[12] F. J. Yndurâin, J. Math. Phys. **7**, 1133 (1966). For discussion of the composite limit of this model, see M. M. Broido (to be published). The relativistic Lee model with two composite V particles has recently been discussed by the authors of Ref. 4 [P. K. Srivastava and S. R. Choudhoury, Nuovo Cimento **43**, 239 (1966); our criticisms also apply to this paper].

[13] K. Kang [Nuovo Cimento **49**, 416 (1967)] has made criticisms of the work of Ref. 4 related to, but different from, those of the present article.