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# ($d$,He$^3$) Studies on Zr$^{90}$, Y$^{89}$, and Sr$^{88}$†

C. D. Kavaloski* J. S. Lilley,‡ D. C. Shreve, and Nelson Stein§
*Physics Department, University of Washington, Seattle, Washington*
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The ($d$,He$^3$) reaction on Zr$^{90}$, Y$^{89}$, and Sr$^{88}$ has been studied with 21-MeV deuterons from the University of Washington cyclotron. Angular distributions were obtained for transitions leading to the ground state and the 0.906-, 1.51-, and 1.75-MeV states of Y$^{89}$, the ground state and the 1.84-, 3.21-, 3.48-, and 3.64-MeV states of Sr$^{88}$, and to the ground state and the 0.403-, and 0.847-MeV states of Rb$^{87}$ over the laboratory angular interval 15°–65°. The angular distributions were compared with predictions from a distorted-wave Born-approximation (DWBA) calculation, and spectroscopic factors were obtained. The $l$ values of 1 and 3 were assigned to the transitions to the 3.48- and 3.64-MeV states of Sr$^{88}$, respectively, for which the spins and parities were not previously known. On the basis of the DWBA analysis, large fractions of the $(2p_{3/2})^{-1}$ and $(1f_{5/2})^{-1}$ proton hole strengths are assigned to the first few excited states of Y$^{89}$, Sr$^{88}$, and Rb$^{87}$, in qualitative agreement with simple shell-model predictions.

## I. INTRODUCTION

THE nuclei in the mass region $A=88$ have been the subject of several recent experimental and theoretical investigations.[1–5] The shell model predicts a neutron shell closure at $N=50$, so that many of the low-lying states of Rb$^{87}$, Sr$^{88}$, Y$^{89}$, and Zr$^{90}$ should be well represented by proton configurations. Proton subshell closures occur at $Z=38$ and 40. At $Z=40$ the last two protons should fill the $2p_{1/2}$ orbital, but because of differences in pairing energy, the configurations $(2p_{1/2})^2$ and $(1g_{9/2})^2$ are nearly degenerate, a situation which leads to configuration mixing in the Zr$^{90}$ ground state.[1,2]

The ground-state proton configuration of Sr$^{88}$ ($Z=38$) generally has been assumed to be closed.[1,2] If this is the case, low-lying states of Y$^{89}$ should arise from the 39th proton outside the filled $2p_{3/2}$ and $1f_{5/2}$ subshells. The ground state ($\frac{1}{2}^-$) and first excited state ($\frac{9}{2}^+$) are thought to be well described in this way, with the unpaired proton in the $2p_{1/2}$ and $1g_{9/2}$ orbitals, respectively.[6] Also, the ground state ($\frac{3}{2}^-$) and first excited state ($\frac{5}{2}^-$?) of Rb$^{87}$ may correspond to a Sr$^{88}$ core with proton holes in the $2p_{3/2}$ and $1f_{5/2}$ orbitals, respectively. The higher states of Y$^{89}$ are not as well understood, and those of Rb$^{87}$ are not known at all.

It has been noted[7] that the observed energies, spins, and parities of some of the states of Y$^{89}$ are consistent with the weak coupling of the unpaired $2p_{1/2}$ proton to excited states of the Sr$^{88}$ core.[8,9] For example, the states at 1.51-MeV ($\frac{3}{2}^-$) and 1.75-MeV ($\frac{5}{2}^-$) have the correct spins and parities to correspond to coupling the $2p_{1/2}$ proton to the 1.84-MeV (2$^+$) state of Sr$^{88}$. This weak-coupling core-excitation model has been useful in describing states in some even-odd nuclei where the presence of an extra-core particle or hole is expected to have little effect on the core state.[10–13] In the present case the model predicts two states of Y$^{89}$ for each core state of Sr$^{88}$, with a mean excitation energy close to that of the core state. The energy splitting of these states should be related to the dipole coupling strength between the core and the extra particle.[8]

A study of the inelastic scattering of $\alpha$ particles from Sr$^{88}$ and Y$^{89}$ has shown, however, that the relative differential cross sections predicted by the core-excitation model are not observed experimentally.[5] The sum of the cross sections for the excitation of the 1.51- and 1.75-MeV states of Y$^{89}$ is too small by a factor of 4 compared with that for the excitation of the 1.84-MeV (2$^+$) state of Sr$^{88}$. Also three, rather than two, neighboring positive-parity states of Y$^{89}$ were observed, for which the combined strength equals that of the 2.74-MeV (3$^-$) state of Sr$^{88}$. More recent inelastic-proton-scattering data seem to support these results.[14]

An alternative model of the 1.51-MeV ($\frac{3}{2}^-$) and 1.75-MeV ($\frac{5}{2}^-$) states of Y$^{89}$ describes them as proton holes in the $2p_{3/2}$ and $1f_{5/2}$ orbitals of Zr$^{90}$, respectively (see Fig. 1). The energies required to create these hole states[15] are approximately equal to the observed excitation energies, and the energy difference between the

† Work supported in part by the U. S. Atomic Energy Commission.
* Present address: Lowell Technological Institute, Lowell, Massachusetts.
‡ Present address: University of Minnesota, Minneapolis, Minnesota.
§ Present address: Physics Department, Yale University, New Haven, Connecticut.

[1] B. F. Bayman, A. S. Reiner, and R. K. Sheline, Phys. Rev. **115**, 1627 (1959).
[2] S. Cohen, R. D. Lawson, M. H. MacFarlane, and M. Soga, Phys. Letters **10**, 195 (1964).
[3] N. Auerbach and I. Talmi. Nucl. Phys. **64**, 458 (1965).
[4] H. N. Broek and J. L. Yntema, Phys. Rev. **138**, B334 (1965).
[5] J. Alster, D. C. Shreve, and R. J. Peterson, Phys. Rev. **144**, 999 (1966).
[6] J. S. Blair, Argonne National Laboratory Technical Report No. ANL-6878, 1964 (unpublished).
[7] S. M. Shafroth, P. N. Trehan, and D. M. Van Patter, Phys. Rev. **129**, 704 (1963).
[8] A. De-Shalit, Phys. Rev. **122**, 1530 (1961).
[9] A. Braunstein and A. De-Shalit, Phys. Letters **1**, 264 (1962).
[10] A. De-Shalit, Phys. Letters **15**, 170 (1965).
[11] A. Bussière, N. K. Glendenning, B. G. Harvey, J. Mahoney, J. R. Meriwether, and D. J. Horen, Phys. Letters **16**, 296 (1965).
[12] J. K. Kokame, K. Fukunaga, and H. Nakamura, Phys. Letters **14**, 234 (1965).
[13] J. Alster, Phys. Rev. **141**, 1138 (1966).
[14] M. M. Stautberg, J. J. Kraushaar, and B. W. Ridley, Bull. Am. Phys. Soc. **11**, 119 (1966); M. M. Stautberg, J. J. Kraushaar, and B. W. Ridley, Phys. Rev. **157**, 977 (1967).
[15] L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. **35**, 853 (1963).

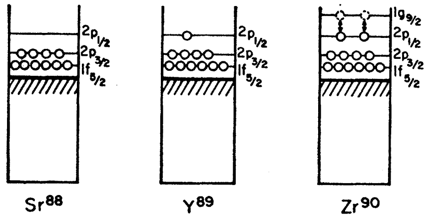


FIG. 1. Proton shell-model configurations for ground states of $Zr^{90}$, $Y^{89}$, and $Sr^{88}$. Levels below the $1f_{5/2}$ orbital are not shown. The known $Zr^{90}$ ground-state mixing, $(2p_{1/2})^2$-$(1g_{9/2})^2$, is indicated schematically.

states is similar to that between the ground and first excited states of $Rb^{87}$.

Similarly, applying a simple shell-model description to the states of $Sr^{88}$, one would expect from energy considerations that the lowest positive-parity states would correspond mainly to the proton configurations[5] $(2p_{1/2})(2p_{3/2})^{-1}$ and $(2p_{1/2})(1f_{5/2})^{-1}$. The first configuration can couple to give states of spin and parity $1^+$ and $2^+$, the second to give states of spin and parity $2^+$ and $3^+$. The model predicts, therefore, two $2^+$ states consisting of linear combinations of the two particle-hole configurations, and a $1^+$ and a $3^+$ state, each with one particle-hole configuration. Additional $2^+$ states would arise from the inclusion of higher-energy configurations such as $(2p_{3/2})^{-2}(2p_{1/2})^2$ and $(1f_{5/2})^{-2}(2p_{1/2})^2$, and small admixtures of these configurations could occur in the low-lying $2^+$ states as well. Experimentally, $2^+$ states have been found[16,17] in $Sr^{88}$ at 1.84 and 3.21 MeV, whereas neither a $1^+$ nor a $3^+$ state has been reported.

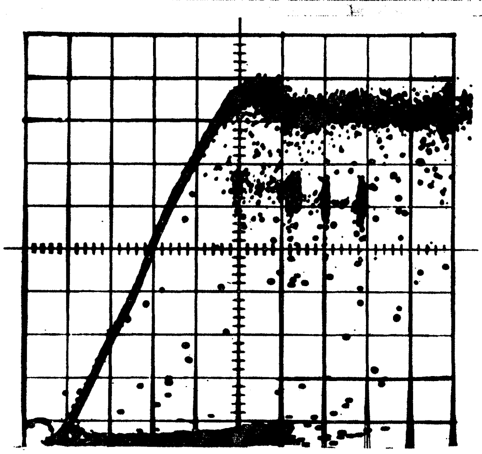

FIG. 2. An x-y oscilloscope display of multiplier output versus total energy for deuterons incident on $Y^{89}$. The multiplier output is proportional to the deflection along the vertical axis and the total energy $(E+\Delta)$ is displayed along the horizontal axis. Each point of the photograph represents a detected particle. The top bands of points corresponds to alpha particles, the lower to $He^3$ particles. Singly charged particles comprise the band of very small multiplier pulses near the bottom of the figure.

[16] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C., 1960), NRC 61-3-51.

[17] S. Shastry and R. Bhattacharyya, Nucl. Phys. 55, 397 (1964).

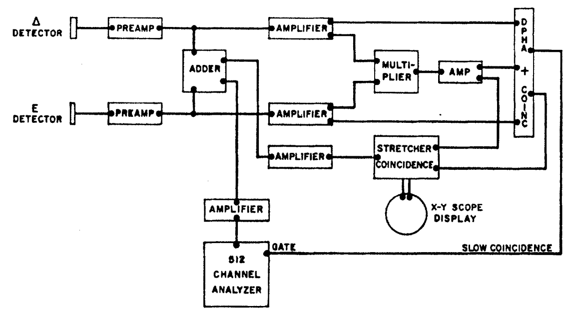


FIG. 3. Block diagram of the electronic system.

In the $(\alpha,\alpha')$ experiment[5] the 1.84-MeV state is excited much more strongly than the 3.21-MeV state. Its electromagnetic transition rate, however, is rather small compared with those for other first excited $2^+$ states,[15] indicating that, although the 1.84-MeV state exhibits some collective enhancement, it may consist mainly of the two particle-hole configurations mentioned above.

The present study was designed to test the usefulness of the simple shell-model description of the low-lying states in $Rb^{87}$, $Sr^{88}$, and $Y^{89}$ discussed above. The $(d,He^3)$ reaction was used because it is expected to excite preferentially simple proton-hole configurations.[18] Recent studies have shown that this reaction can be described as a direct proton pickup process,[18–20] and that spectroscopic information can be obtained from a DWBA analysis.

## II. EXPERIMENTAL PROCEDURE

The bombardments with deuteron beams of nominally 21 MeV were carried out in the 60-in. scattering chamber of the University of Washington 60-in. cyclotron. The external beam system has been described previously.[21]

The $Y^{89}$ and $Zr^{90}$ targets consisted of self-supporting metallic foils approximately 0.9 mg/cm² thick. The $Y^{89}$ foil was pure Y metal.[22] The Zr target was enriched[23] to 99% $Zr^{90}$. The $Sr^{88}$ target consisted of Sr metal isotopically enriched[24] to 99% $Sr^{88}$ evaporated on a thin gold backing. This target was prepared using a procedure described elsewhere.[25] After evaporation, the metallic $Sr^{88}$ target was transported to the scattering chamber in an inert atmosphere, thus inhibiting the buildup of $SrCO_3$. This was necessary to prevent background due to the intense $\alpha$-particle groups produced in

[18] J. L. Yntema and G. R. Satchler, Phys. Rev. 134, B976 (1964).

[19] J. L. Yntema, Phys. Letters 11, 140 (1964).

[20] J. C. Hiebert, E. Newman, and R. H. Bassel, Phys. Letters 15, 1960 (1965).

[21] See, for example, A. J. Liber, F. H. Schmidt, and J. B. Gerhart, Phys. Rev. 126, 1496 (1962).

[22] Obtained from F. Karasek, Argonne National Laboratory.

[23] Obtained from Oak Ridge National Laboratory, Isotopes Sales Division.

[24] Obtained from Electromagnetic Separation Group, Chemistry Division, A.E.R.E., Harwell, Berkshire, United Kingdom.

[25] J. Sauer, Rev. Sci. Instr. 36, 1374 (1965).

the reactions $C^{12}(d,\alpha)B^{10}$ and $O^{16}(d,\alpha)N^{14}$ from obscuring the $He^3$ peaks from the $Sr^{88}(d,He^3)Rb^{87}$ reaction.

Charged particles were detected with two silicon surface-barrier detectors arranged to form a $(dE/dx)$-$E$ particle identification system. A 100-$\mu$-thick transmission detector was backed by a 620-$\mu$ stopping detector. This system allowed $He^3$ particles between the energies of 12 and 36 MeV to penetrate the first detector and stop in the second detector. A field-effect transistor multiplier[26] provided particle identification. Figure 2 shows an $x$-$y$ oscilloscope display of the response of the multiplier circuit as a function of energy. Separation of the $He^3$ and $He^4$ particles is evident. Singly charged particles comprise the band of very small multiplier pulses at the bottom of the photograph.

A block diagram of the electronic circuitry is shown in Fig. 3. The $x$-$y$ oscilloscope displays were employed for testing and monitoring the particle identification system. The signals from the $E$ and $\Delta E$ detectors were linearly added, amplified, and then analyzed by a 512-channel pulse-height analyzer. Multiplier pulses corresponding to $He^3$ particles gated the 512 channel analyzer which recorded the total energy spectrum.

A third detector, set at a fixed scattering angle, monitored elastically scattered deuterons. It consisted of a 760-$\mu$-thick, diffused-junction silicon detector placed behind an $8.1\times10^{-2}$-cm Al foil which degraded the elastically scattered deuterons to about 12.5 MeV. This detector was useful for two purposes. First, it served as a check on the beam-integration system but, unlike the Faraday cup, was sensitive to beam wander over possible nonuniformities in the target. Second, by depositing a weak Th $C$ alpha source on the inside surface of the Al degrader and comparing its spectrum with the pulse height of the elastically scattered deuterons, the detector could monitor small changes (20 keV) in the energy of the beam. This was particularly useful for investigating the energy dependence of the $(d,He^3)$ reaction over a range of several hundred keV in the neighborhood of the nominal beam energy (see Sec. III).

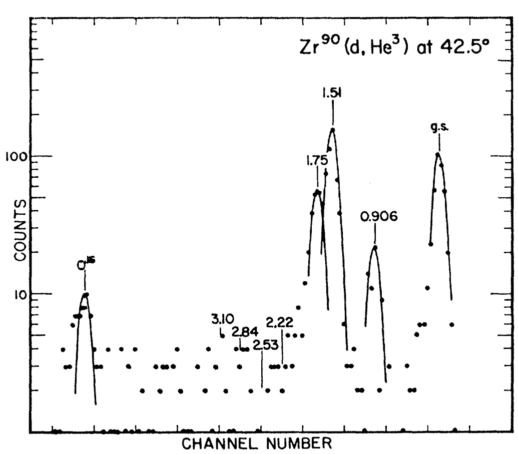


FIG. 4. Typical pulse-height spectrum for the reaction $Zr^{90}$-$(d,He^3)Y^{89}$. The excitation energies of the final states are listed above their respective peaks. The solid curves have been drawn in to guide the eye.

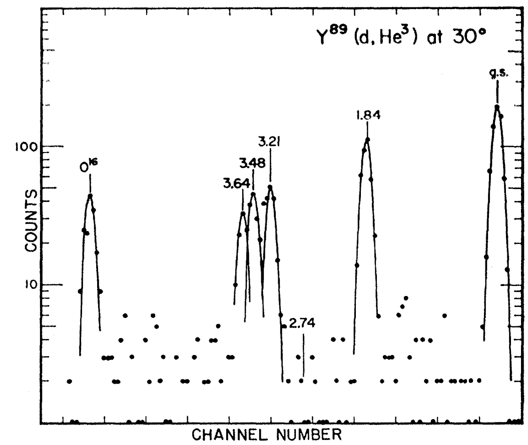


FIG. 5. Typical pulse-height spectrum for the reaction $Y^{89}$-$(d,He^3)Sr^{88}$. The excitation energies of the final states are listed above their respective peaks. The solid curves have been drawn in to guide the eye.

## III. RESULTS

Data were taken for the reactions $Zr^{90}(d,He^3)Y^{89}$ and $Y^{89}(d,He^3)Sr^{88}$ in 2.5° intervals from 15° to 50°, and in 5° intervals from 50° to 70°. Data for the $Sr^{88}(d,He^3)$-$Rb^{87}$ reaction were taken over the region 17.5° to 52.5° in 2.5° intervals. Representative energy spectra are shown in Figs. 4–6. The energy resolution was typically 130 keV and was due mainly to target-thickness effects. The small background noticeable in each of the three spectra (Figs. 4–6) was due mainly to $He^4$ particles that

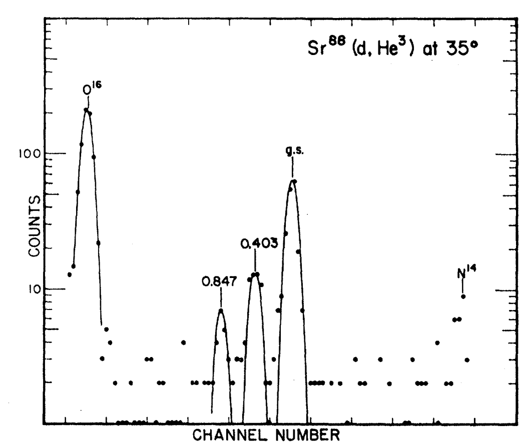


FIG. 6. Typical pulse-height spectrum for the reaction $Sr^{88}$-$(d,He^3)Rb^{87}$. The excitation energies of the final states are listed above their respective peaks. The solid curves have been drawn in to guide the eye.

[26] G. L. Miller and V. Radeka, in Proceedings of the Nuclear Conference on Instrument Techniques in Nuclear Pulse Analysis, Monterey, California, 1963 (to be published). Also see V. Radeka, IEEE Trans. Nucl. Sci. 11, 302 (1963).

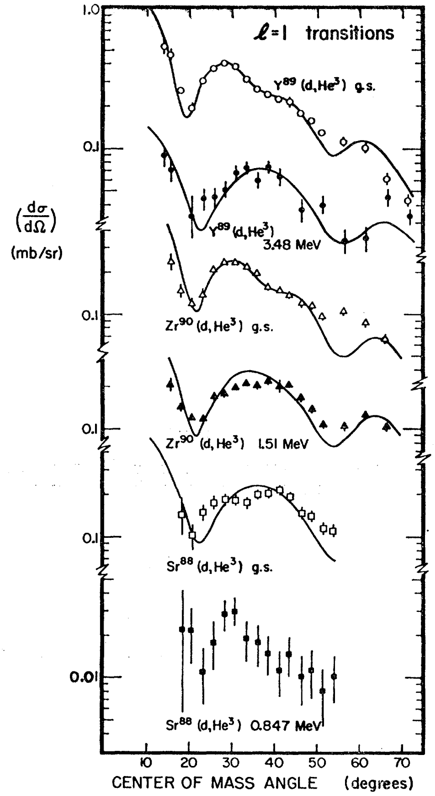


FIG. 7. Angular distributions for the $l=1$ transitions observed in this experiment. Statistical errors are shown wherever they exceed the point size. The solid curves are the results of the DWBA calculations.

lost much less than the normal amount of their energy in the $\Delta E$ detector and hence appeared in the $He^3$ window. It was measured that approximately 3% of the $He^4$ particles of a given total energy appeared in the $He^3$ spectra.

Angular distributions were obtained for the $Zr^{90}$-$(d,He^3)Y^{89}$ reaction leading to the ground state $(\frac{1}{2}^-)$ and the 0.906-MeV $(\frac{9}{2}^+)$, 1.51-MeV $(\frac{3}{2}^-)$, and 1.75-MeV $(\frac{5}{2}^-)$ states of $Y^{89}$, and for the $Y^{89}(d,He^3)Sr^{88}$ reaction leading to the ground state $(0^+)$ and the 1.84-MeV $(2^+)$, 3.21-MeV $(2^+)$, 3.48- and 3.64-MeV states of $Sr^{88}$. The energies of the 3.48-MeV and 3.64-MeV states were measured in this experiment to $\pm 20$ keV and may be the states listed in the *Nuclear Data Sheets*[16] at $3.52\pm 0.05$ and $3.68\pm 0.05$ MeV.[27] For the reaction $Sr^{88}(d,He^3)Rb^{87}$, angular distributions were measured for transitions to the ground state $(\frac{3}{2}^-)$, the 0.403-MeV $(\frac{5}{2}^-?)$, and 0.847-MeV states. No indication was seen of transitions to the 2.74-MeV $(3^-)$ state of $Sr^{88}$ or to the positive parity states at 2.22-, 2.53-, or 2.84-MeV of $Y^{89}$.

The energy dependence of the differential cross sections at a number of angles was studied over an energy interval of several hundred keV around the nominal incident energy. The cross sections were found to decrease by approximately 4% per 100 keV as the incident energy was lowered. Using the beam-energy monitor (see Sec. II), changes in the incident energy of this amount could be observed easily. Uncertainties in the relative cross sections arising from beam-energy variations therefore should be less than 4%.

The over-all uncertainties in the absolute cross section are due mainly to uncertainties in the target thickness. The $Zr^{90}$ and $Y^{89}$ target thicknesses were measured by weighing selected portions of the foils. It is estimated that the absolute cross sections for the $Zr^{90}(d,He^3)$ and $Y^{89}(d,He^3)$ reactions are uncertain to $\pm 8\%$. The thickness of the $Sr^{88}$ target could not be measured by direct weighing. Instead, the elastic scattering of 42-MeV $\alpha$ particles was measured with this target and compared with the previously measured cross section.[5] The uncertainty in the absolute cross section of the $Sr^{88}(d,He^3)$ reaction obtained in this way is estimated to be $\pm 14\%$.

Selection rules and the known spins and parities of the initial and final states restrict the angular-momentum transfer to a unique value for many of the observed transitions. Thus, the $Zr^{90}(d,He^3)Y^{89}$ transitions to both the ground state $(\frac{1}{2}^-)$ and the 1.51-MeV $(\frac{3}{2}^-)$ state, the $Y^{89}(d,He^3)Sr^{88}$ transition to the ground state $(0^+)$, and the $Sr^{88}(d,He^3)Rb^{87}$ transition to the ground state $(\frac{3}{2}^-)$ all proceed by the transfer of one unit of angular momentum. The angular distributions for these known $l=1$ transitions are shown in Fig. 7. Angular distributions for the transitions $Y^{89}(d,He^3)Sr^{88}$ to the 3.48-MeV state

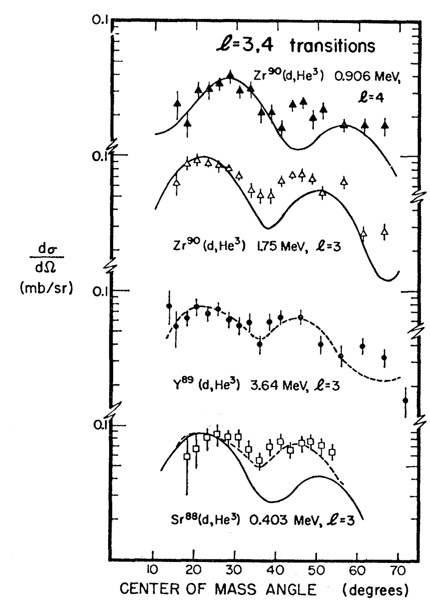


FIG. 8. Angular distributions for the $l=3$ and $l=4$ transitions observed in this experiment. Statistical errors are shown wherever they exceed the point size. The solid curves are the results of the DWBA calculations. The dashed curves correspond to the shape of the angular distribution for the $l=3$ transition to the 1.75-MeV state of $Y^{89}$.

[27] N. H. Lager, E. Eichler, and G. D. O'Kelley, Phys. Rev. **101**, 727 (1956).

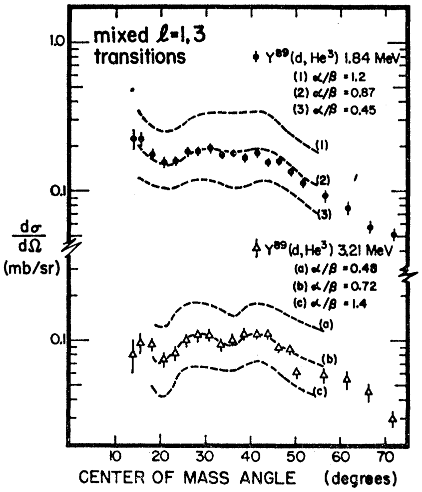


FIG. 9. Angular distributions for the mixed $l=1$, 3 transitions to the 1.84- and 3.21-Me V states of $Sr^{88}$. Statistical errors are shown wherever exceed the point size. The solid curves are the results of the DWBA calculations. The dashed curves for the 1.84-MeV state correspond to sums of the experimental shapes for the $l=1$ and $l=3$ transitions to the 1.51- and 1.75-MeV states of $Y^{89}$. The dashed curves for the 3.21-MeV state correspond to sums of the shapes of the $l=1$ and $l=3$ transitions to the ground and first excited states of $Rb^{87}$. The dashed curves have been arbitrarily displaced for clarity.

and $Sr^{88}(d,He^3)Rb^{87}$ to the 0.847-MeV state, for which spins are not known, are included in Fig. 7 because of their similarity to the known $l=1$ angular distributions.

The transition $Zr^{90}(d,He^3)Y^{89}$ to the 1.75-MeV $(\frac{5}{2}^-)$ state proceeds by the transfer of three units of angular momentum and the transition $Zr^{90}(d,He^3)Y^{89}$ to the 0.906-MeV $(\frac{9}{2}^+)$ state, by the transfer of four units of angular momentum. The angular distributions for these two transitions are shown in Fig. 8, along with the angular distributions for the transitions $Y^{89}(d,He^3)Sr^{88}$ to the 3.64-MeV state and $Sr^{88}(d,He^3)Rb^{87}$ to the 0.403-MeV state. The latter two are assigned $l=3$ because of their similarity to the $Zr^{90}(d,He^3)Y^{89}$ (1.75 MeV) angular distribution. The angular distributions alone do not rule out the possibility of $l=4$ for these two transitions. An $l=4$ assignment for the $Y^{89}(d,He^3)$-$Sr^{88}$ (3.64 MeV) transition is unreasonable because the ground state of $Y^{89}$ should not contain a significant amount of the $(1g_{9/2})^2$ proton configuration. The assignment of $l=4$ to the $Sr^{88}(d,He^3)Rb^{87}$ (0.403 MeV) transition is excluded because the parity of the 0.403-MeV state is known to be negative from Coulomb-excitation studies. A tentative spin assignment of $\frac{5}{2}$ has been made[16] based on shell-model arguments, which is consistent with the $l=3$ assignment made in the present study.

The transitions $Y^{89}(d,He^3)Sr^{88}$ to the 1.84-MeV $(2^+)$ and 3.21-MeV $(2^+)$ states can proceed by the transfer of both one and three units of angular momentum. The angular distributions for these mixed $l=1$, 3 transitions are shown in Fig. 9.

## IV. ANALYSIS

### A. Angular Distributions

The $(d,He^3)$ angular distributions were analyzed using the distorted-wave Born-approximation (DWBA) code TSALLY.[28] The nuclear optical potentials used were of the form

$$U=\frac{-V}{1+e^x}-i\left[W-W'\frac{d}{dx'}\right]\frac{1}{1+e^{x'}}, \qquad (1)$$

where $x=(r-R)/a$, $x'=(r-R')/a'$, $R=r_0A^{1/3}$, and $R'=r_0'A^{1/3}$. In addition, a Coulomb potential for a uniformly charged sphere of radius $R_c=r_cA^{1/3}$ was employed.

Optical-model parameters were obtained from published surveys[29,30] of analyses of deuteron and He³ elastic scattering. Since in both the work of Perey and Perey and Klingensmith *et al.*, several sets of parameters were found which fit the elastic-scattering data, the following procedure was adopted to choose the set which best represents the present data. First, an interpolation of the published parameters was made to obtain values appropriate to the mass region $A=90$. Then, using the shape of the $Zr^{90}(d,He^3)Y^{89}$ (g.s.) angular distribution as a standard for comparison, a choice was made among the various sets of "equivalent" parameters. This set of parameters, listed in Table I, was used for all transitions observed in this experiment.

In all the calculations, the binding energy of the proton was taken equal to its separation energy[31] and the bound-state wave functions were calculated using a Woods-Saxon well.[28] The parameters for the bound-state well are also included in Table I. A lower cutoff (LCO) on the radial integration was employed; the fits were appreciably worse if this was not done. The value chosen for the LCO (4.9 F) is similar to those used in related studies[18,32] and provided the best fit for the $l=1$ transitions. It was noticed, however, that the predicted magnitudes of the $l=3$ angular distributions were quite sensitive to the value of the LCO. Increasing the LCO from 4 to 6 F changed the magnitude of the $l=3$ angular

TABLE I. Optical-model parameters for the deuteron and He³ elastic channels and the bound-state proton. These parameters were used for all the transitions studied.

| | $V$ (MeV) | $W$ (MeV) | $r_0$ (F) | $r_c$ (F) | $a$ (F) | $r_0'$ (F) | $a'$ (F) | $W'$ (MeV) |
|---|---|---|---|---|---|---|---|---|
| $d$ | 95 | 0 | 1.15 | 1.15 | 0.81 | 1.34 | 0.65 | 72 |
| He³ | 64 | 14 | 1.54 | 1.30 | 0.55 | 0 | 0 | 0 |
| Bound-state proton | ··· | ··· | 1.25 | 1.25 | 0.65 | ··· | ··· | ··· |

[28] R. H. Bassell, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3420, 1962 (unpublished).
[29] C. M. Perey and F. G. Perey, Phys. Rev. **132**, 755 (1963).
[30] R. W. Klingensmith, H. J. Hausman, and W. D. Ploughe, Phys. Rev. **134**, B1220 (1964).
[31] A. G. Blair and D. D. Armstrong, Phys. Letters **16**, 57 (1965).
[32] A. G. Blair, Phys. Rev. **140**, B648 (1965).

distributions by a factor of 2, whereas the magnitude of the $l=1$ angular distributions changed by only 20%. Over this range of LCO no substantial $l=3$ shape changes occurred, and since the degree of agreement with the experimental shapes was generally poorer than in the $l=1$ cases, it was felt that the choice of LCO should be dictated by the $l=1$ data. Acceptable visual fits for the $l=1$ angular distributions were obtained for values of the LCO ranging from about 4.2 to 5.3 F. This ambiguity in the value of the LCO implies that there is an uncertainty in the absolute values of the $l=3$ spectroscopic factors of approximately ±30% from this source. The DWBA calculations imply, however, that the spectroscopic factors for all the $l=3$ transitions change in the same way as functions of the LCO and that the values relative to each other are much less uncertain then the absolute values.

Figures 7 and 8 show the DWBA fits obtained by the procedure just outlined. The over-all quality of the fits for the $l=1$ angular distributions (shown as solid lines in Fig. 7) is quite acceptable. The angular distributions for the transitions $Zr^{90}(d,He^3)Y^{89}$ (g.s.) and $Y^{89}(d,He^3)$-$Sr^{88}$ (g.s.) are fit quite well up to 60°. However, the fits to the angular distributions for the transitions $Sr^{88}$-$(d,He)Rb^{87}$ (g.s.) and $Zr^{90}(d,He^3)Y^{89}$ (1.51 MeV) are not as good. This may be due, in part, to the fact that in the latter transitions a $2p_{3/2}$ proton is picked up, whereas in the former transitions a $2p_{1/2}$ proton is picked up. (TSALLY has no provision for including a spin-orbit term.) Some evidence for a $j$ dependence in the $Zr^{90}(d,He^3)Y^{89}$ cross sections at a deuteron energy of 34.4 MeV has been reported.[33] However, the present angular distributions, obtained at 21 MeV, differ substantially from those results, precluding any meaningful extrapolation of the $j$ dependence observed there. Furthermore, the differences in shape for the two groups of transitions seem to be partially explained by the $Q$-value dependence predicted by the DWBA calculation. It is therefore not clear whether the differences are due solely to $Q$ dependence or to a $j$ dependence of the angular distributions as well.

The angular distribution for the transition $Y^{89}$-$(d,He^3)Sr^{88}$ (3.48 MeV) has approximately the same shape as that for the known $l=1$ transition $Zr^{90}(d,He^3)$-$Y^{89}$ (1.51 MeV) and thus appears to be pure $l=1$. Results from the $Sr^{88}(d,He^3)Rb^{87}$ (0.847-MeV) transition are included in Fig. 7 since they bear a resemblance to $l=1$ angular distributions, However, the large errors preclude any definite $l$ assignment.

The DWBA calculations for the $l=3$ and $l=4$ transitions (solid curves in Fig. 8) predict the shape and position of the first maximum reasonably well but fail to reproduce the relative magnitudes of the first and second maxima. The $l=3$ assignment for the transition $Y^{89}(d,He^3)Sr^{88}$ (3.64 MeV) is based on the similarity of its angular distribution to that of the known $l=3$ transition $Zr^{90}(d,He^3)Y^{89}$ (1.75 MeV) (shown as a dashed curve in Fig. 8).

The rather poor agreement of the $l=3$ DWBA predictions with the measured values was reflected in the inability of the theory to give good fits to the mixed $l=1$, 3 transitions $Y^{89}(d,He^3)Sr^{88}$ (1.84 MeV and 3.21 MeV). Reasonable fits were obtained, however, using the experimental $l=1$ and $l=3$ shapes for transitions with $Q$ values closest to the appropriate one. The shapes resulting from an appropriate adding procedure are quite insensitive to the relative amounts of $l=1$ and $l=3$ used, so that no reliable estimate of the separate components could be extracted.

The dashed curves (1,2,3) through the data points for the transition $Y^{89}(d,He^3)Sr^{88}$ (1.84 MeV) in Fig. 9 are weighted sums of the $l=1$ and $l=3$ shapes for the transitions to the 1.51-MeV and 1.75-MeV states of $Y^{89}$, respectively. The dashed curves $(a,b,c)$ through the data points for the transition $Y^{89}(d,He^3)Sr^{88}$ (3.21 MeV) are sums of the shapes of the angular distributions for the transitions to the ground state and 0.403-MeV state of $Rb^{87}$.

### B. Spectroscopic Factors

With the assumptions made in the TSALLY code (e.g., zero-range, no spin-orbit), the DWBA theory predicts a differential cross section for the pickup reaction $B(b,a)A$ of the form[28]

$$\frac{d\sigma}{d\Omega}=\frac{(2s_a+1)}{(2s_b+1)}N\sum_l S_l\sigma_l(\theta)\,, \tag{2}$$

where $s_a$ and $s_b$ are the spins of $a$ and $b$, respectively, $l$ is the orbital angular momentum transfer allowed by selection rules, $N$ is a normalization constant which includes the overlap $\langle a|b,x\rangle$, in which $x$ is the transferred particle, $\sigma_l(\theta)$ is a function generated by the DWBA calculation, and $S_l$ is the spectroscopic factor. It has been shown that if spin-orbit forces are included, $\sigma_l(\theta)$ depends not only on the orbital angular momentum $l$ but also on the total angular momentum of the transferred particle $j$. For calculations which neglect spin-orbit effects, account has been taken of the $j$ dependence via the normalization factor $N$.[18] In this work $N$ was taken to increase (decrease) by 10% for a $p_{3/2}$ ($p_{1/2}$) transition, and for higher $l$ transfer the effect was assumed to be proportional to $(2l+1)$.[32]

Normalizing $\sigma_l(\theta)$ to the experimental data results in a value for the product $N_jS_l$. It was assumed that $S_1=1$ for the transition $Y^{89}(d,He^3)Sr^{88}$ (g.s.). This assumption results in a value of $N_{1/2}=3.5$ which implies a spin-independent value of $N=3.9$. This value compares favorably with that obtained by Blair[33] from $(He^3,d)$ reactions in the Ni-Cu region ($N=3.8$), by Erskine *et al.*,[34] from the $Ca^{48}(He^3,d)$ reaction ($N=4.17$),

[33] B. M. Preedom, E. Newman, and J. C. Hiebert, Phys. Letters **22**, 657 (1966).

[34] J. R. Erskine, A. Marinov, and J. P. Schiffer, Phys. Rev. **142**, 633 (1966).

and with the theoretical values obtained recently by Bassel ($N=3.85$ or 4.42).[35] These values are all substantially larger than that used by Yntema in his analysis of limited data on the $Zr^{90}(d,He^3)Y^{89}$ reaction to the ground and first excited states.[19]

The spectroscopic factors obtained in this experiment are listed in Table II. For the mixed transitions ($l=1$, 3) the values shown correspond to the sum $(S_1+S_3)$, and were obtained as follows. DWBA predictions for $l=1$ and $l=3$ transitions of the appropriate $Q$ value were first obtained from the TSALLY code and a weighted average $\sigma_{1+3}(\theta)=[\alpha/(\alpha+\beta)]\sigma_1(\theta)+[\beta/(\alpha+\beta)]\sigma_3(\theta)$ was formed. The weights factors $\alpha$ and $\beta$ were determined by requiring an acceptable fit to the shape of the experimental angular distribution. For this purpose, experimental $l=1$ and $l=3$ shapes were employed because of the disagreement noted earlier between the theoretical and experimental $l=3$ shapes. Once the weights $\alpha$ and $\beta$ were determined, spectroscopic factors $(S_1+S_3)$ were obtained from the relation

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}}=\tfrac{2}{3}N\sigma_{1+3}(\theta)(S_1+S_3)\,. \tag{3}$$

As is evident from Fig. 9, the shapes of the experimental angular distributions did not provide a very sensitive test of the relative magnitude of $\alpha$ and $\beta$. However, the sum $(S_1+S_3)$ remained constant to within about $\pm 12\%$ over the range of the ratio $\alpha/\beta$ that resulted in acceptable visual fits (factor of 3). The values of $(S_1+S_3)$ listed in Table II for the transitions to the 1.84- and 3.21-MeV states of $Sr^{88}$ were obtained from the curves labeled 2 and $b$, respectively, in Fig. 9.

The model spectroscopic factors $S_{\text{model}}$ listed in Table II were calculated from the expression[36]

$$S_{lj}=n_{lj}|\langle\psi_{JM}{}^{A}|\Phi_{JM}(ls,J_f)\rangle|^2\,, \tag{4}$$

where $n_{lj}$ is the number of identical particles in the $lj$ level of the target nucleus, $\psi_{JM}{}^{A}$ is the antisymmetrized wave function of the target nucleus, and $\Phi_{JM}(ls,J_f)$ is the product wave function of the antisymmetrized final state (total angular momentum $J_f$) and a particle with orbital angular momentum $l$, spin $s$, coupled to the total angular momentum $J$ and projection $M$ of the target nucleus. In order to evaluate the model spectroscopic factors for the particular transitions studied, the following assumptions were made: (a) The ground state of $Sr^{88}$ has the proton configuration $(1f_{5/2})^6(2p_{3/2})^4$. (b) The observed excited states of $Sr^{88}$ contain only the proton configurations $(2p_{1/2})(2p_{3/2})^{-1}$ and $(2p_{1/2})(1f_{5/2})^{-1}$. (c) The 1.51- and 1.75-MeV states of $Y^{89}$ contain the same degree of $(2p_{1/2})^2-(1g_{9/2})^2$ configuration mixing as the ground state of $Zr^{90}$. (d) The other states studied are single-particle or single-hole states.

[35] R. H. Bassel, Phys. Rev. **149**, 791 (1966).
[36] M. H. MacFarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).

TABLE II. Spectroscopic factors for the $(d,He^3)$ reaction on $Zr^{90}$, $Y^{89}$, and $Sr^{88}$. Columns 1 through 7 refer to (1) reaction, (2) final-state excitation energy, (3) spin and parity, (4) reaction $Q$ value, (5) angular-momentum transfer, (6) model spectroscopic factor, and (7) experimental spectroscopic factor.

| (1) Reaction | (2) Final-state excitation energy (MeV) | (3) $J^\pi$ | (4) $Q$ value (MeV) | (5) $l$ | (6) $S_{\text{model}}$ | (7) $S_{\text{expt}}$ |
|---|---|---|---|---|---|---|
| $Zr^{90}(d,He^3)Y^{89}$ | g.s. | $1/2^-$ | $-2.83$ | 1 | $2a^2$ [a] | 1.14 |
| | 0.906 | $9/2^+$ | $-3.74$ | 4 | $2b^2$ [a] | 0.51 |
| | 1.51 | $3/2^-$ | $-4.34$ | 1 | 4 | 2.2 |
| | 1.75 | $5/2^-$ | $-4.58$ | 3 | 6 | 2.0 |
| $Y^{89}(d,He^3)Sr^{88}$ | g.s. | $0^+$ | $-1.95$ | 1 | 1 | 1.0 |
| | 1.84 | $2^+$ | $-3.80$ | 1, 3 | 2.5 | 2.2 |
| | 3.21 | $2^+$ | $-5.17$ | 1, 3 | 2.5 | 1.6 |
| | 3.48 | $(1^+)$ | $-5.44$ | 1 | 1.5 | 1.2 |
| | 3.64 | $(3^+)$ | $-5.60$ | 3 | 3.5 | 2.2 |
| $Sr^{88}(d,He^3)Rb^{87}$ | g.s. | $3/2^-$ | $-5.12$ | 1 | 4 | 2.6 |
| | 0.403 | $5/2^-$ | $-5.52$ | 3 | 6 | 2.2 |
| | 0.847 | | $-5.97$ | (1) | 0 | |

[a] For definitions of $a$ and $b$ see Eq. (6) of text.

## V. DISCUSSION AND CONCLUSIONS

A comparison of the experimental and model spectroscopic factors of Table II allows one to draw several conclusions. If it is accepted that the ground- and first-excited states of $Y^{89}$ correspond to a proton in the $2p_{1/2}$ and $1g_{9/2}$ orbitals, respectively,

$$\begin{aligned}\psi_{\text{g.s.}}(Y^{89})&=(2p_{1/2})\,,\\ \psi_{0.906}(Y^{89})&=(1g_{9/2})\,,\end{aligned} \tag{5}$$

and that the ground state of $Zr^{90}$ consists of a mixture of $(2p_{1/2})^2$ and $(1g_{9/2})^2$ proton configurations:

$$\psi_{\text{g.s.}}(Zr^{90})=[a(2p_{1/2})^2+b(1g_{9/2})^2]\,, \tag{6}$$

where $a^2+b^2=1$, then the spectroscopic factors $S_1$ and $S_4$ for the transitions to the ground and first excited states of $Y^{89}$ measure the quantities $2a^2$ and $2b^2$, respectively. The ratio of the experimental spectroscopic factors $S_1/S_4$ gives the result $(a/b)^2=2.2$, which agrees with several other experimental and theoretical determinations of this ratio.[1,2,19,37,38] The sum of the spectroscopic factors, $2a^2+2b^2$, is 1.65, which is in reasonable agreement, considering the uncertainties of DWBA analyses, with the expected value of 2, if the ground state is given by Eq. (6). In a recent report on the $Zr^{90}(d,He^3)$ reaction[33] a spectroscopic factor of 1.91 was obtained for the transition to the ground state of $Y^{89}$. This value is in substantial disagreement with the present value. The relative spectroscopic factors reported in Ref. 33 ($S_{1.51}/S_{\text{g.s.}}=2.1$), however, are in good agreement with the present results ($S_{1.51}/S_{\text{g.s.}}=1.9$). It seems likely, therefore, that the discrepancy in absolute values is due to some sort of dependence of

[37] R. B. Day, A. G. Blair, and D. D. Armstrong, Phys. Letters **9**, 327 (1964).
[38] J. Vervier, Nucl. Phys. **75**, 17 (1966).

the normalization constant $N$ on the optical-model parameters.[18]

The transitions to the 1.51- and 1.75-MeV states of $Y^{89}$ both have large spectroscopic factors, 2.2 and 2.0, respectively. As was mentioned previously, the model spectroscopic factors ($S=4$ and 6, respectively) were obtained by assuming that the degree of mixing between the $(2p_{1/2})^2$ and $(1g_{9/2})^2$ proton configurations in $Y^{89}$ is the same as that in the $Zr^{90}$ ground state. If the degree of mixing is different, then the matrix element of Eq. (3) will be reduced accordingly. For example, if in the 1.51- and 1.75-MeV states of $Y^{89}$ the paired protons were in the $(2p_{1/2})^2$ configuration only, i.e.,

$$\psi_{1.51}(Y^{89})=(p_{3/2})^{-1}(p_{1/2})^2 \quad \text{and} \quad \psi_{1.75}(Y^{89})=(f_{5/2})^{-1}(p_{1/2})^2, \tag{7}$$

then the model spectroscopic factors would be reduced to approximately 2.8 and 4.1, respectively. In any case, one is led to the conclusion that the wave function for the 1.51-MeV state of $Y^{89}$ contains at least one-half of the $(p_{3/2})^{-1}$ hole strength and the wave function for the 1.75-MeV state has at least one-third of the $(f_{5/2})^{-1}$ hole strength. The actual amounts are probably larger, since it is unlikely that the $(p_{1/2})^2$-$(g_{9/2})^2$ mixing is identical for $Y^{89}$ and $Zr^{90}$.

If the overlap of the wave functions for the states in question is not complete, then the remaining hole strength should occur in states of spin and parity $(\frac{3}{2}^-)$ and $(\frac{5}{2}^-)$ at higher excitation energies. Other negative parity states of $Y^{89}$ lie at excitation energies greater than 3 MeV.[5,14] It would be difficult to see transitions to these states in the present experiment because of Coulomb barrier considerations. The Coulomb barrier is about 14 MeV for $He^3$ particles incident on $Y^{89}$. Thus, although one cannot say conclusively from the analysis of this experiment that the 1.51- and 1.75-MeV states of $Y^{89}$ correspond to pure shell-model configurations, it is clear that for each state the single-hole strength is quite large.

The summed spectroscopic factors $S_1+S_3$ for the transitions leading to the 1.84- and 3.21-MeV states of $Sr^{88}$ are 2.2 and 1.6, respectively. In general, the wave functions for these $2^+$ states can be written as

$$\begin{aligned}\psi_{1.84}(Sr^{88})&=a_{11}(p_{1/2},p_{3/2}^{-1})_{J=2}+a_{12}(p_{1/2},f_{5/2}^{-1})_{J=2}+(\text{other terms}),\\ \psi_{3.21}(Sr^{88})&=a_{21}(p_{1/2},p_{3/2}^{-1})_{J=2}+a_{22}(p_{1/2},f_{5/2}^{-1})_{J=2}+(\text{other terms}),\end{aligned} \tag{8}$$

where "other terms" include proton configurations such as $(p_{3/2}^{-2})_{J=2}$ and $(f_{5/2}^{-2})_{J=2}$ (which one would not expect to reach by single-proton pickup from $Y^{89}$) and where the configurations that contribute to the matrix element of Eq. (3) have been explicitly written out. The spectroscopic factors for these transitions can be computed from Eq. (4) and are[36]

$$\begin{aligned}S_1(1.84)&=(\tfrac{5}{2})a_{11}^2,\\ S_3(1.84)&=(\tfrac{5}{2})a_{12}^2,\\ S_1(3.21)&=(\tfrac{5}{2})a_{21}^2, \quad \text{and}\\ S_3(3.21)&=(\tfrac{5}{2})a_{22}^2.\end{aligned} \tag{9}$$

The sums $(S_1+S_3)$ for the two transitions are then

$$S_1+S_3=(\tfrac{5}{2})(a_{i1}^2+a_{i2}^2), \tag{10}$$

where $i$ can be either 1 or 2. If the $2^+$ states contain only the configurations explicitly shown in Eq. (8), then $(a_{i1}^2+a_{i2}^2)=1$ and $(S_1+S_3)=\frac{5}{2}$. It is seen that the experimental sums $(S_1+S_3)$ account for 80% and 60%, respectively, of the wave functions of the two $2^+$ states and therefore that the dominant configurations in these states are $(p_{1/2},p_{3/2}^{-1})$ and $(p_{1/2},f_{5/2}^{-1})$.

The above results for the 1.51- and 1.75-MeV states of $Y^{89}$ and the 1.84- and 3.21-MeV states of $Sr^{88}$ argue against a weak-coupling-model description, for the former states whereby a $p_{1/2}$ proton couples to the $2^+$ core states to produce $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states in the odd nucleus. This is suggested because (1) the 1.51- and 1.75-MeV states each contain a dominant single-hole configuration and (2) *a* $p_{1/2}$ proton cannot couple weakly with the two dominant configurations of the $2^+$ states, since these already contain a $p_{1/2}$ proton. The exclusion principle would force the two $p_{1/2}$ protons to pair off to zero spin and hence to change significantly the nature of the core state in disagreement with the assumptions of the weak-coupling model.[7]

The transition to the 3.48-MeV state of $Sr^{88}$ exhibits an $l=1$ angular distribution. Since one expects a strongly excited $1^+$ state at approximately this excitation energy arising from the $(p_{1/2},p_{3/2}^{-1})_{J=1}$ configuration, it is attractive to regard the spin and parity of this state as $1^+$. The spectroscopic factor measured for the transition to this state is 1.2 compared with the value 1.5 expected for such a $1^+$ state.

The transition to the 3.64 MeV-state of $Sr^{88}$ displays an $l=3$ angular distribution. If this corresponds to the $3^+$ state expected from the $(2p_{1/2},1f_{5/2}^{-1})$ proton configuration, it would compare the quartet of states predicted by the shell model for $p_{3/2}$ and $f_{5/2}$ proton pickup from $Y^{89}$. The spectroscopic factor for this $l=3$ transition is 2.2, compared with the value 3.5 predicted by the model. In view of the uncertainty, mentioned earlier, in assigning absolute spectroscopic factors for $l=3$ transitions, the deviation from 3.5 may not be significant.

The absence of any observable strength to the 2.74-MeV $(3^-)$ state of $Sr^{88}$ and to the three positive-parity states at 2.22-, 2.53-, and 2.84-MeV of $Y^{89}$ is consistent with the results of the $(\alpha,\alpha')$ experiment[5] in which these states are the most strongly excited and hence, presumably collective.

The remaining spectroscopic factors in Table II involve the $Sr^{88}(d,He^3)Rb^{87}$ reaction in which only the ground state was appreciably excited. The value 2.6 for the g.s. transition is somewhat surprising since a value closer to 4 would be expected if the g.s. of $Rb^{87}$ were simply a $(p_{3/2})^{-1}$ hole in the $Sr^{88}$ core. The deviation from the simple shell-model prediction could be due to configuration mixing in the ground states of either $Rb^{87}$ or $Sr^{88}$, or both. Theoretically, one would not expect a large amount of configuration mixing in the ground state of $Sr^{88}$, and experimentally, no $0^+$ state of $Sr^{88}$ other than the ground state is known to exist below 3.5 MeV. Also, it should be recalled that the earlier assumption of unit spectroscopic factor for the $Y^{89}(d,He^3)Sr^{88}$ ground-state transition implies that no configuration mixing is present in the $Sr^{88}$ ground state, and this assumption leads to several consistent results. On the other hand, crude estimates indicate that $\frac{3}{2}^-$ levels of $Rb^{87}$, which could mix with the ground state, might occur at an excitation energy as low as 1.8 MeV. Unfortunately, these states could not have been detected in the present work because of Coulomb barrier effects and background due to contaminants in the target.

The transition to the first excited state of $Rb^{87}$ (0.403-MeV) yields a spectroscopic factor only $\frac{1}{3}$ of what would be expected for a pure $f_{5/2}^{-1}$ hole state. However, in view of the poor $l=3$ theoretical fit to the angular distribution and the general uncertainty for absolute $l=3$ spectroscopic factors, the actual amount of $f_{5/2}^{-1}$ hole strength in this state could be much greater.

The 0.847-MeV state of $Rb^{87}$ is excited weakly in this experiment. The shape of its angular distribution (Fig. 7) may suggest an $l=1$ transition, which would restrict its spin and parity to either $\frac{1}{2}^-$ or $\frac{3}{2}^-$. If it is $\frac{1}{2}^-$, some $(p_{1/2})^2$ admixture in the ground state of $Sr^{88}$ may be implied.

## VI. SUMMARY

Based on the simple shell model as shown in Fig. 1, excitation of the following states would be expected in a direct proton pickup process:

(1) $Zr^{90}(d,He^3)Y^{89}$: (a) two $l=1$ states ($\frac{1}{2}^-$ and $\frac{3}{2}^-$) corresponding to pickup of the $2p_{1/2}$ and $2p_{3/2}$ protons, (b) one $l=3$ state ($\frac{5}{2}^-$) from $1f_{5/2}$ pickup and (c) one $l=4$ state ($\frac{9}{2}^+$) from $1g_{9/2}$ pickup.

(2) $Y^{89}(d,He^3)Sr^{88}$: (a) two $l=1$ states ($0^+$ and $1^+$) from $2p_{1/2}$ and $2p_{3/2}$ pickup, respectively, (b) one $l=3$ state ($3^+$) from $1f_{5/2}$ pickup, and (c) two mixed $l=1$, 3 states ($2^+$) from $2p_{3/2}$ and $1f_{5/2}$ pickup.

(3) $Sr^{88}(d,He^3)Rb^{87}$: (a) one $l=1$ state from $2p_{3/2}$ pickup and (b) one $l=3$ state from $1f_{5/2}$ pickup.

An examination of Table II shows that the qualitative expectations are borne out very well. Eleven of the twelve transitions listed there are in accord with the eleven states given above. The twelfth transition (to the 0.847-MeV state of $Rb^{87}$) was quite weak. Moreover, since the strengths of the observed transitions are generally greater than 60% of the model values, it seems reasonable to conclude that the simple shell model provides a fairly adequate description of these states. In particular, the 1.51-MeV ($\frac{3}{2}^-$) and 1.75-MeV ($\frac{5}{2}^-$) states in $Y^{89}$ are better described as proton-hole states, rather than as arising from coupling of the $p_{1/2}$ proton to the first $2^+$ state of $Sr^{88}$ according to the weak-coupling core-excitation model.

There are several departures from the simple model. The spectroscopic factors for the transitions to the 1.51- and 1.75-MeV states of $Y^{89}$ are somewhat lower than calculated, suggesting that the amounts of $(p_{1/2})^2$-$(g_{9/2})^2$ mixing in the two nuclei may not be the same. Similarly, the spectroscopic factors to the ground and first excited states of $Rb^{87}$ are smaller than predicted, implying the possibility of some configuration mixing in either $Sr^{88}$ or $Rb^{87}$, or both, and once again that the mixing is different in the two nuclei.

Certain factors which tend to limit the strength of the quantitative conclusions drawn here should be mentioned. The limited agreement between the experiment and the DWBA calculations for the $l=3$ angular distributions creates a more than usual uncertainty in the resulting spectroscopic factors. In addition, the proximity of the $He^3$ energies to the Coulomb barrier precludes the search for "missing" hole strength in higher excited states which might serve to explain the differences between the model spectroscopic factors and the experimental values.

A higher deuteron-bombarding energy could, to some extent, remove the above limitations. Certainly the influence of the $He^3$ Coulomb barrier would be diminished. In addition, one might expect that less ambiguous DWBA analyses would result, since there is evidence that $(d,He^3)$ angular distributions become more "stripping-like" at higher energies.[33]

Several theoretical questions relating to the present work could be clarified by a study of the inverse process $(He^3,d)$. For example, the possibility of configurations such as $[(p_{3/2})^{-2}(p_{1/2})^2]$ and $[(f_{5/2})^{-2}(p_{1/2})^2]$ in the $Sr^{88}$ ground state could be tested by measuring the transitions $Sr^{88}(He^3,d)Y^{89}$ to the 1.51- and 1.75-MeV states. Similarly, the observation of $l=1$ transitions in the $Zr^{90}(He^3,d)Nb^{91}$ reaction would provide another indication of the $(p_{1/2})^2$-$(g_{9/2})^2$ mixture in the $Zr^{90}$ ground state.

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