measure of the total energy release, corrected for recoil. For the cascade value, in addition to the value of E_{γ_2} determined in each of the four measurements, the mean value of E_{γ_1} obtained from averaging the two runs with the Co^{60} source in position was used. The final values quoted are the unweighted averages in all cases. No weighting was used, since it is felt that systematic errors may be as important as the statistical deviations. The error quoted for the individual measurements is that arising from the statistical uncertainties in peak positions only. The final quoted error results from convolution of the dispersion in the data ($\sigma=120$ eV) with the largest variance from the standards used (Mg^{24} $\sigma=120$ eV).

Since the cascade and cross-over values result from measurements on γ_0 and γ_2 in the same spectra, they are not completely independent, but are related through the common constant k. However, they are independent in regard to the determination of peak positions and nonlinearity corrections. The average result represents a significant improvement in precision with respect to previous measurements,[3,4] with which it is compared in the Table I.

It is a pleasure to acknowledge the technical assistance of J. R. Specht. The authors also wish to thank H. Mann and his co-workers who fabricated the germanium diode, and Miss J. P. Marion for her assistance in analyzing the data.

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Elastic Scattering of Deuterons from N^{14} between 1.8 and 5.5 MeV*†

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The elastic scattering of deuterons by N^{14} has been investigated for incident energies from 1.8 to 5.5 MeV. Excitation curves were measured at 90.0°, 125.3°, 140.8°, and 166.6° (c.m.), and all but the 90° curve showed broad fluctuations. Peaks which appeared at more than one angle were considered to be resonances, and energy levels in O^{16} were determined from them. Angular distributions from 18° to 166° (c.m.) were measured in 500-keV steps from 2 to 5.5 MeV, and optical-model analyses of these data were made both with volume absorption and with surface absorption. The surface-absorption calculations were best, and one such set of parameters was found which gave reasonable fits for the angular distributions at all energies. An optical-potential ambiguity appeared in the analysis of these data. The total reaction cross section at 3 MeV was found to be 947 mb±20% experimentally, and this value was compared with the theoretical value of 921 mb determined from the parameters referred to above.

1. INTRODUCTION

In recent years, scattering data of low-energy deuterons from light nuclei have slowly accumulated. There is sufficient data on C^{12}, for instance, that the application of a given model over the deuteron energy range 3 to 34 MeV can be attempted. Such a study with the optical model over this range has been done by Satchler[1] and over the range 2 to 13 MeV by Ploughe.[2] There is, however, a need for still more data on other light nuclei such that the question can be answered whether a given set of parameters in an optical-model analysis can be used for several light nuclei, as has been the case for higher-energy deuterons incident on more massive isotopes. See, for example, Perey and Perey[3] for several elements with $Z\geqslant 12$ and Halbert[4] for targets ranging from Ar to Sn.

Experimental results for elastic scattering of deuterons by N^{14} from 700 to 2100 keV reported by Seiler *et al.*[5] were fitted by means of an optical potential. Hodgson,[6] in a recent review article on the deuteron-nucleus optical potential, lists optical-model parameters communicated to him by Satchler for N^{14}-deuteron interactions at four incident deuteron energies from 10.9 to 27 MeV. This present work investigated the deuteron energy range of 1.8 to 5.5 MeV corresponding to excitation energies from 22.3 to 25.5 MeV in the compound nucleus O^{16}. Excitation curves were measured at 90.0°, 125.3°, 140.8°, and 166.6° (c.m.) at

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† The greater part of this work is taken from the dissertation presented by one of us (J.L.F.) in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the Ohio State University. Present address: Wittenberg University, Springfield, Ohio.

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which angles various Legendre polynomials are zero. Resonance level structure at these excitation energies has been detected by others[7] by observing reaction particles.

In addition to the fact that experimental excitation curves of elastically scattered deuterons from N^{14} are essential for the determination of energy levels in O^{16}, data from the reaction expressed as angular distributions may be interpreted directly with the optical model. The experiment was designed to provide this kind of information also.

2. EXPERIMENTAL MATTERS

The beam of deuterons obtained from The Ohio State University 5.5-MeV Van de Graaff accelerator was directed into a differentially pumped gas scattering chamber. The details of this arrangement have been published in another paper.[8] Briefly, the incident beam, after passing through the magnetic analyzer and the switching magnet, enters the differential pumping column where it is collimated. It crosses the scattering chamber, passes through a 0.000025-in. nickel foil, and is finally stopped in an evacuated collector cup. In the meantime, some of the scattered deuterons and charged reaction particles are detected by a Nuclear Diode, totally depleted 300-Ω-cm solid-state detector.

The target is a cylindrical region of gas of a length defined by two precisely machined circular apertures in front of the detector. The physical length of target subtended at the detector varies from 0.83 cm at 90° (lab) scattering angle to 3.19 cm at the forward or back angles of 15°40′ and 164°20′. Thus the target thickness for an average nitrogen gas pressure of 10 cm of oil (~7 mm Hg) ranged from 1.8 to 6.8 keV for 2-MeV deuterons and from 0.9 to 3.5 keV for 5-MeV deuterons. The pressure of the gas in the chamber was determined with an oil manometer filled with Convoil-20 and pressure regulation was achieved with a control system described elsewhere.[9] The measured drift in pressure during a run was usually negligible and in no case did it exceed 0.3%.

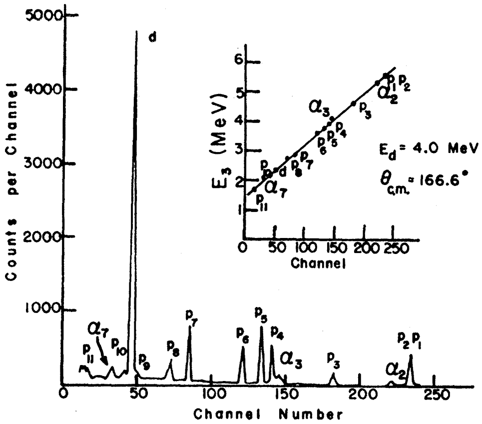

FIG. 1. Typical spectrum from the $N^{14}(d,d)N^{14}$ gas scattering experiment. The incident deuteron energy was 4.0 MeV. The inset shows the reaction particle energies as a function of the multichannel analyzer channel.

The energy of the deuterons determined from the field strength of the analyzing magnet was corrected for the energy loss of the deuterons as they passed through the gas of the differential pumping system to reach the target region. The gas was considered to traverse a path length $(D+L)$, where D is the distance from the center of the target to the exit of the collimating circular apertures of the differential pumping system and L an effective length of that column. The length L was found by detecting the 1.881-MeV $Li^7(p,n)$ threshold with a target at the center of the chamber and measuring the accelerator energy shift ΔE for an evacuated chamber compared with a gas-filled chamber. L is, then, the only unknown in the relationship $\Delta E = N\epsilon(D+L)$, where N, the number of molecules/cm^3, is calculated from the pressure of the chamber proper and ϵ is the stopping cross section at the threshold energy. The length $(D+L)$ was used throughout the energy range of the experiment to correct for energy losses by means of the empirical formula given by Whaling.[10]

As can be seen in a typical spectrum shown in Fig. 1, many proton and α-particle groups appear in this energy range which complicate the extraction of the pertinent deuteron information. Since these groups have a different rate of change of energy from the deuterons, as the beam energy changes there are incident energies where a reaction particle peak is coincident with the deuteron peak. In this cases the peaks were resolved by one of two methods. The α peak could be moved relative to the elastic deuteron peak by varying the pressure of the nitrogen gas in the chamber. The α particle would then lose a different amount of energy as it proceeded to the detector. The proton groups were separated by the use of a 0.75-mil Mylar film in front of the detector which shifted the deuteron peak more than the proton peak.

The major sources of error in our results are due to determination of the beam energy at the target, measurement of the detector lab angle and the geometrical factor G, given by Silverstein[11] for circular apertures, and errors involving the interaction of the beam with

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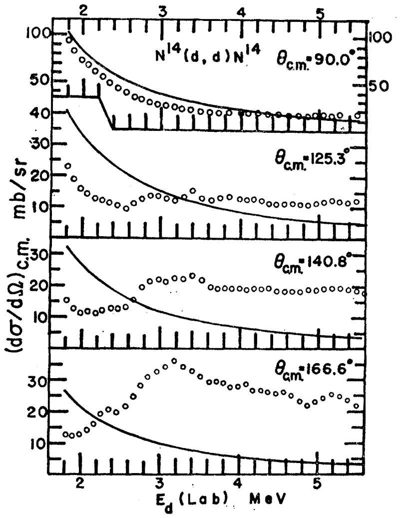

FIG. 2. Experimental excitation curves from the $N^{14}(d,d)N^{14}$, reaction at 90.0°, 125.3°, 140.8°, and 166.6° (c.m.). The solid line is the theoretical Rutherford differential cross section.

the scattering gas. Errors in the beam energy include a maximum relativistic correction of -0.15% at 5.5 MeV, a finite energy spread of the beam of $\pm 0.5\%$, and an uncertainty of the stopping-power values of the scattering gas of $\pm 5\%$. The maximum uncertainty in $\sin\theta_{\text{lab}}$ was $\pm 1.0\%$ and that in the G factor was $\pm 0.7\%$. A finite beam correction to the G factor due to Herring and Jones[12] was calculated to be -0.84% and was not included in our results. This is a systematic error in our work. The over-all accuracy of the alignment, measurement of the G factor, and charge collection, were checked by measuring p-p cross sections which are known to 0.2%.[13] The agreement was within 2% of Worthington's values. Errors due to measurement of the target density, scattering of the beam by the gas and the collector-cup foil, reliability of the current integrator, uncertainty in the number of incident deuterons, and the standard deviation in the yield were also calculated and the over-all error in the lab differential cross section due to these sources is $\pm 2.5\%$. The error in the values of the ratio of experimental cross section to the Rutherford cross section at large angles was set at a higher value of 10% because of large background corrections necessary at these angles. The error in the ratios from 20 to 120 deg (c.m.) is not larger than 5%.

The total reaction cross section was determined at 3 MeV by recording all of the protons, α, and deuteron counts as a function of the angle (lab) with a fast scaler and subtracting the counts due to the elastic deuterons determined from the analyzer. The amplifier was biased to pass pulses due to particles with energies of 1.33 MeV or larger. Eight α groups and ten proton groups would thus be included in addition to the deuteron at the extreme back angle, with other lower Q groups admitted at more forward angles. The differential cross section (lab) for the charged particles from all the reactions was determined from this difference and plotted versus the lab scattering angle. The total reaction cross section for these charged particles was found by numerical integration of this curve and the total neutron cross section for this reaction was taken

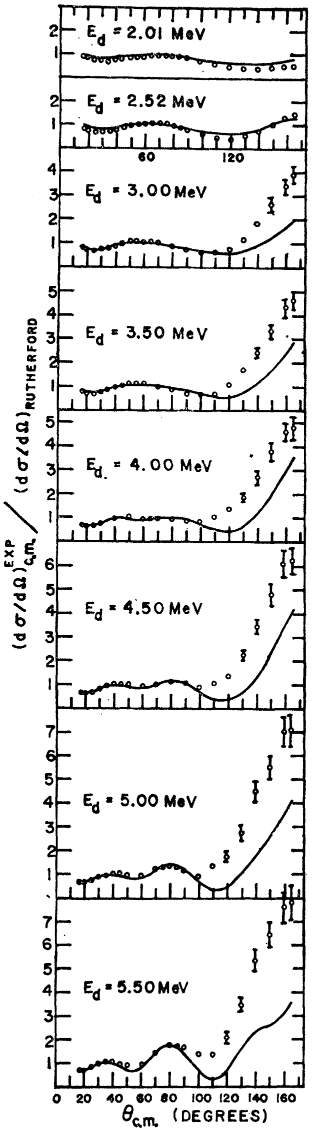

FIG. 3. Eight angular distributions of elastically scattered deuterons from N^{14} at the incident deuteron energies shown. The solid curves are theoretical surface-absorption, optical-model fits calculated with the use of Eqs. (1) to (5). The parameters for all of these curves are $V_{\text{Re}}=100$ MeV, $R_{01}=1.61$ F, $a_1=0.68$ F, $V_{\text{Im}}=20$ MeV, $R_{02}=1.9$ F, $a_2=0.9$ F, and $V_{s0}=0.0$ MeV. The data are shown as a ratio of the experimental to the Rutherford differential cross sections.

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from Jackson *et al.*[14] The sum of these two values is, then, the total reaction cross section.

3. EXPERIMENTAL RESULTS

To observe resonance effects most clearly the excitation curve was determined at the maximum attainable c.m. angle of 166.6° and then at 90.0° where all odd Legendre polynomials are zero, and at 125.3° and 140.8°, where P_2, and P_3 vanish, respectively. The data were taken in 80-keV steps between 1.8 and 5.6 MeV and are shown in Fig. 2 along with the calculated Rutherford cross section. Figure 3 shows eight angular distributions measured in 500-keV steps above 2 MeV. The data are plotted as the ratio of the experimental differential cross section (c.m.) to the Rutherford cross section. The uncertainty in the data points is between 5 and 10% as discussed above. The solid lines are theoretical fits and will be discussed below. An additional angular distribution was measured at 3.17 MeV but is not shown since these data were very similar to the 3-MeV data. Figure 4 shows the differential cross section (lab) for all the charged reaction particles as a function of the lab scattering angle. The total reaction cross section determined as explained earlier is 947 mb ±20%. This value is 2.5% different from the theoretical value of 921 mb calculated with the set of optical-model parameters below.

4. ANALYSIS OF DATA

The analysis of these data was pursued with two goals in mind. First, we desired to determine energy levels in the O^{16} compound nucleus from the excitation curves of Fig. 2. Second, we were interested in analyzing the angular distribution data in terms of an optical model.

A. O^{16} Energy Levels

Several energy levels in O^{16} were observed. By assuming that a resonance was indicated if a peak occurred at the same energy in two or more of the excitation curves an energy level could then be determined. Pronounced peaks in only one curve were considered to be indicative of possible energy levels. Energy levels were determined to exist at 22.8, 23.3, 23.5, 23.7, 24.3, 24.8, and 25.2 MeV. Less certain levels were determined at 22.6, 24.0, and 24.1 MeV. Errors due to energy calibration and energy increments of the incident beam produce an error no more than 0.1 MeV for all of these levels. Suffert[15] presents an updated table of levels in O^{16} in this energy range produced in other reactions and although all of the levels above agree with levels quoted in that paper within experimental error they are in general slightly higher.

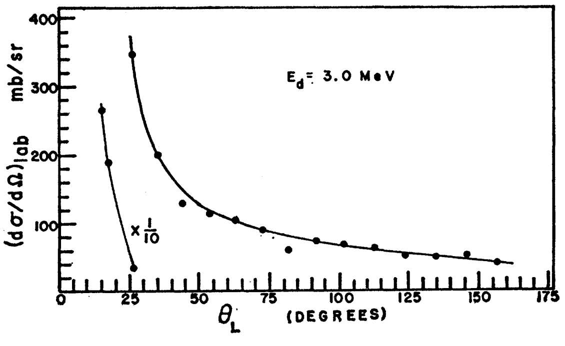

FIG. 4. The charged reaction particle, laboratory, and differential cross section as a function of the laboratory scattering angle for 3-MeV deuterons incident on N^{14} gas.

B. Optical-Model Analysis

Since a compound-nucleus, two-level analysis was not successful in Seiler's work (Ref. 5), from 0.7 to 2.1 MeV, we decided to concentrate on an optical-model approach alone. The major portion of the calculations was done on the IBM 7094 at The Ohio State University Computation Center with the ABACUS-2 computer program.[16] The scan procedure was employed.

The potential well, in general, comprises four terms:

$$V(r)=-V_{\mathrm{Re}}v_{\mathrm{Re}}(r)-iV_{\mathrm{Im}}v_{\mathrm{Im}}(r)-(V_{\mathrm{SR}}+iV_{\mathrm{SI}})\times v_{\mathrm{so}}(r)\mathbf{L}\cdot\boldsymbol{\sigma}+ZZ'e^2v_{\mathrm{Coul}}(r). \quad (1)$$

Since the spin of the deuteron is 1, and since only spins of zero or one-half can be used with this program, the spin-orbit term was set equal to zero. The inclusion of a spin-orbit term with another program will be discussed below. The Coulomb potential was considered to result from a uniform charge distribution inside a sphere of radius R_c. The simple $A^{1/3}$ dependence,

$$R_i=R_{0i}A^{1/3}, \quad (2)$$

was used for all of the radii involved. By fixing R_{0c} at 1.33 F, the number of parameters was reduced to six: the real and imaginary well depths, radii, and diffuseness parameters. The Saxon form for $v_{\mathrm{Re}}(r)$ was used in all cases:

$$v_{\mathrm{Re}}(r)=\frac{1}{1+\exp[(r-R_1)/a_1]}. \quad (3)$$

Two well forms were tried for $v_{\mathrm{Im}}(r)$, volume imaginary,

$$v_{\mathrm{Im}}(r)=\frac{1}{1+\exp[(r-R_2)/a_2]}, \quad (4)$$

and Gaussian-surface imaginary,

$$v_{\mathrm{Im}}(r)=\exp\{-[(r-R_2)/a_2]^2\}. \quad (5)$$

The first object of the analysis was to determine the best real well depth and radius for volume absorption.

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[14] J. E. Jackson, R. A. Blue, and T. R. Donoghue (to be published).

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[16] The ABACUS-2 optical-model program was written by Dr. Elliot H. Auerbach, currently at Brookhaven National Laboratory.

This was done by scanning over one of these parameters at a time and finding the minimum χ^2. The quantity χ^2 has the usual definition:

$$\chi^2=\sum_{n=1}^{N_E}\left[\frac{1}{N}\sum_{j=1}^{j=N}\omega_j(d\sigma_j^{\text{calc}}-d\sigma_j^{\text{expt}})^2\right], \qquad (6)$$

where N_E is the number of values of the energy in a given determination of χ^2, N is the number of experimental quantities at one energy, $d\sigma_j$ is the differential cross section, and ω_j, which was set equal to 1 for this analysis, is a weight factor.

One expects to find various pairs of V_{Re} (real depth) and R_1 (real radius) such that relatively small values of χ^2 occur for $V_{\text{Re}}R_1^2\approx$constant. One of the surprising results of these scans was that two such VR^2 curves were established. This seems to be the same effect noticed by Drisko *et al.*[17] Acceptable values of V_{Re} and R_1 were then used from both of these curves to determine the other parameters in order to achieve reasonable fits. When volume absorption scans continued to give poor results, surface absorption scans were attempted. These scans produced much more acceptable fits. The solid lines on Fig. 3 are drawn through the optical-model values, determined with an imaginary, Gaussian, surface potential with the following parameters: $V_{\text{Re}}=100$ MeV, $R_{01}=1.61$ F, $a_1=0.68$ F, $V_{\text{Im}}=20$ MeV, $R_{02}=1.9$ F, $a_2=0.9$ F, $V_{s0}=0.0$ MeV.

In order to investigate a possible spin-orbit effect, a final set of runs with these parameters was made with the Hunter optical-model program[18] at the Oak Ridge National Laboratory. A spin-orbit potential of 5 MeV and of the form

$$V_s(r)=\frac{\mathbf{L}\cdot\mathbf{s}}{r}\frac{d}{dr}\left\{\frac{V_{s0}}{1+\exp[(r-R_1)/a_1]}\right\} \qquad (7)$$

made little change in the fits and larger spin-orbit strengths made the fits worse. The positions of the maxima and minima on Fig. 3 were not changed and the theoretical values of the ratios at 120° were about 10% higher than those with no spin-orbit interaction.

5. CONCLUSIONS

The attempts to fit the experimental data within experimental error with one set of parameters in a surface-absorption optical-model analysis of the nucleus have succeeded with angles up to 100° (c.m.). Although the fits deviate at angles greater than 100° several significant conclusions can be drawn from the data and the analysis. First, it is interesting to note the large back-angle scattering. Shapiro[19] pointed out in 1962 that experimental discrepancies from the optical-model predictions for scattering at large angles shows that the effects of deformation of the deuteron during the scattering process are important. The increase in the ratio of the differential cross sections in our work as the beam energy was raised is consistent with his analysis.

Second, after we found that the analyses of these data indicated two VR^2 curves, we found that parameters from only one of these curves gave reasonably good fits to the data. The preferred curve had $VR^2=259$ MeV F^2. A 100-MeV real well depth is reasonable, as has been pointed out by Hodgson,[6] and the corresponding 1.61-F value for R_{01} (real radius) is not inconsistent when one considers the large radius of the deuteron. The facts that surface absorption gave the best fits and that the imaginary diffuseness parameter is rather large may indicate that the nitrogen-deuteron interaction starts to take place at distances considerably larger than the radii determined from the radius parameters. Our parameters are not inconsistent with those reported by Smith and Ivash[20] for O, Mg, and Al and hopefully more deuteron scattering data for nuclides in this mass and energy region will appear so that a more thorough optical-model analysis can be attempted.

ACKNOWLEDGMENTS

We wish to express special gratitude to Dr. K. W. Jones of Brookhaven National Laboratory who suggested the use of ABACUS-2 and who worked with one of us (J.L.F.) in the early application of this code. We are indebted to Dr. G. R. Satchler of Oak Ridge National Laboratory who made it possible for us to use the HUNTER code at the Oak Ridge National Laboratory. The support of the National Science Foundation through a science faculty fellowship for one of us (J.L.F.) for a portion of the time spent on this work is gratefully acknowledged and, finally, the help of Ronald Johnson in the machine shop and Frederick Riffle in taking the data is much appreciated.

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