the experimental value for the lowest $T=\frac{3}{2}$ state, but the uncertaintly of the latter is rather high because of known difficulties in extracting spectroscopic factors from high excited states. The need for different optical-model parameters for the highly excited isobaric analog state is also indicated by the different shape of the two pure $j^\pi=\frac{5}{2}^+$ transitions to the ground state and to the 7.79-MeV state (Fig. 8). The (p,d) experiment yielded a somewhat smaller value ($S=0.94$) if the ground-state value is renormalized to 2.5, which is still too large to allow for other strongly excited $T=\frac{3}{2}$ states. A Mg^{26}-$(d,He^3)Na^{25}$ experiment would be desirable since it would yield a more reliable value for the strength of this transition and could also show whether or not other strongly excited $l=2$ transitions do occur.

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Mesonic Decays of Spallation Hyperfragments and the Λ-N Potential-Well Depth*

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The short-range ($\lesssim 10\,\mu$) mesonic hyperfragments produced by 1.1-BeV/c K^- mesons and 2.3-BeV/c K^- mesons in $K5$ nuclear emulsions have been studied in order to estimate the potential-well depth of the Λ-N interaction. Among more than 261 000 K^- interactions examined, we have observed 70 mesonic decays, which consist of 41 heavy hyperfragments, 28 light hyperfragments, and one π^+ decay. Based on eight π^--p-r decays (r indicates residual nucleus) in the lower region of a plot of binding energy versus mass, the upper limit of the potential-well depth of the Λ-N interaction was estimated to be 27.7±0.6 MeV, assuming a square-well potential. A further elaboration of a more accurate estimate of the binding energy and consequently derived potential-well depth is discussed. The nonmesonic-to-π^--mesonic decay ratio of the heavy hyperfragments ($A=65\pm40$) was estimated to be 153±27. This value agrees fairly well with the theoretical ratio 130 at $A=100$, estimated by Dalitz. The study of 28 light-hyperfragment events indicates that (75±22)% of the light hyperfragments originated in light nuclei (O, N, C).

I. INTRODUCTION

THE majority of secondary stars with short connecting tracks ($\lesssim 10\,\mu$) observed in high-energy K^--meson interactions in nuclear emulsions represent nonmesonic decays of heavy hyperfragments with a mass $A=65\pm40$. These heavy hyperfragments are residual nuclei containing trapped Λ^0 particles, and are produced by the spallation process (cascade and evaporation) during K^--meson interactions.[1-3]

It was first shown by Davis *et al.*[4] that the mesonic decays of heavy hyperfragments (HHF's) are useful in estimating the potential-well depth of the Λ-N interaction on the basis of a square-well potential. Subsequently, this problem was investigated by many authors. The potential-well depth of the Λ-N interaction was estimated as 25 to 30 MeV by Davis *et al.*[4] (5 events), 30 to 40 MeV by Cuevas *et al.*[5] (4 events), 27.2±1.3 MeV by Lemonne *et al.*[6] (22 events), 27±3 MeV by Lagnaux *et al.*[7] (11 events), 20 to 35 MeV by Key *et al.*[8] (10 events), and around 30 MeV by Prowse *et al.*[9] (6 events).

The difficulties in accurately estimating the potential-well depth were as follows.[4-9]

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(1) The mass of a HHF cannot be accurately estimated mainly due to inability to distinguish between Br and Ag interactions. The mass could be intermediate between light and heavy nuclei if a parent interaction contains a heavy-fragment track which is too short to be seen or is otherwise invisible.

(2) The residual nucleus resulting from the mesonic decay may be left in an excited state, either by direct transition, or by secondary collisions of the decay products.

(3) For the π^--r decay mode (r indicates residual nucleus), the binding energy of the least-bound proton cannot be known accurately because of uncertainty in the charge and mass of the hyperfragment. The semi-empirical binding energy[10] is 8.3 ± 1.6 MeV for the isotope region defined in Sec. IIIA.

(4) A neutron may be emitted.

(5) A decay from a long-lived isomeric state will give an underestimate of the binding energy. However, the decay from such a state is considered very rare.[11]

In this paper we attempt to estimate the potential-well depth of the Λ-N interaction with greater accuracy by overcoming the above difficulties. In earlier works[4,5,7] the authors estimated the average binding energy of certain HHF's and used an average mass derived statistically. Since the possible masses of HHF's are widely spread,[3] a better estimate requires better identification of masses. In our work, using methods discussed in Sec. IIIA, the choice is narrowed to Ag or Br interactions; the Ag-Br ambiguity is further discussed in Sec. IIIC. The possible occurrence of short or invisible fragment tracks is discussed in Sec. IIIA. If the residual nucleus resulting from the mesonic decay is left in an excited state, the binding-energy calculation results in an overestimate. Hence the lowest binding energies for a given mass region should be chosen to estimate the potential-well depth most accurately. Since the residual nuclei may be in excited states for even those HHF's whose binding energies are lowest, the binding energies should still be regarded as upper limits. However, the experimental data indicate that the excitation energies involved are very small for the events in the lowest region. It is unlikely that neutron emission occurs among those π^--p-r events which have the lowest binding energies (about 23 MeV), because its assumption would reduce the binding energies to less than 15 MeV (the light hyperfragment binding-energy region) due to the neutron separation energy (approximately 8 MeV) and the neutron kinetic energy. The π-r decay modes can not be used due to uncertainty in the binding energy of the least-bound proton.

From the above brief discussion we have chosen the following criteria in order to admit an event as a candidate for evaluation of the potential-well depth.

(1) The parent interaction should not contain any short-range prong less than 10 μ. (Fortunately we did not have such a case.)

(2) No π-r decay mode should be used regardless of its binding energy value.

(3) The binding energy in a given mass region should be among the lowest. Since there is a clustering of the lowest binding energy values at about 23 MeV, and since the error in individual binding energy values is about 0.30 MeV, we arbitrarily take 24 MeV as a cutoff for defining the lowest values. This is about three standard deviations above the mean at about 23 MeV.

According to the above criteria eight events were chosen to compute the potential-well depth. The values obtained were compared with the world survey data.

II. EXPERIMENTAL PROCEDURE

Two stacks of Ilford $K5$ emulsions 4 in.$\times$6 in.$\times$8 in. in size, each containing about 160 pellicles, were exposed, respectively, to the 1.1-BeV/c K^- beam at the Bevatron, Berkeley, and to the 2.3-BeV/c K^- beam at the Brookhaven A.G.S. The π contamination in the 1.1-BeV/c K^- beam was less than 5%. The emulsion densities were 3.745 ± 0.040 g/cc for the Berkeley stack and 3.827 ± 0.029 g/cc for the Brookhaven stack.

Both stacks were area scanned for double stars under low magnification (120$\times$), and each K^- interaction was examined under high magnification (1200$\times$) for the presence of a DS_{10} event. Here a DS_{10} event is defined as a double star with the length of the connecting track equal to or less than 10 μ.

The conventional method was used for the measurements of ranges, dip angles, and azimuths of hyperfragments; for the eight selected events the ranges were carefully measured twice. The measurement error was less than 2%. The errors on the binding energies include those contributed by range straggling, measurements, shrinkage factor, stopping power correction, and the uncertainty in the Q value of the Λ^0 decay. The errors of the binding energies are relatively small mainly due to the small straggling error associated with the low-energy pions emitted from HHF's.

In order to identify a mesonic decay of a DS_{10} event, all its secondary prongs were followed until pion and proton tracks could be distinguished by observation of scattering and ionization. Mesonic hyperfragments (MHF) thus found were measured and kinematically analyzed by a computer program. When no good fit to a light hyperfragment (LHF:$Z<8$) could be found, the hyperfragment was classified as heavy if the binding energy B_Λ was greater than 16 MeV. B_Λ was found from the formulas:

$$B_\Lambda=37.60-(E_\pi+E_p) \quad \text{for } \pi^-\text{-}p\text{-}r \text{ decay},$$
$$B_\Lambda=37.6+B_p-E_\pi \quad \text{for } \pi^-\text{-}r \text{ decay},$$

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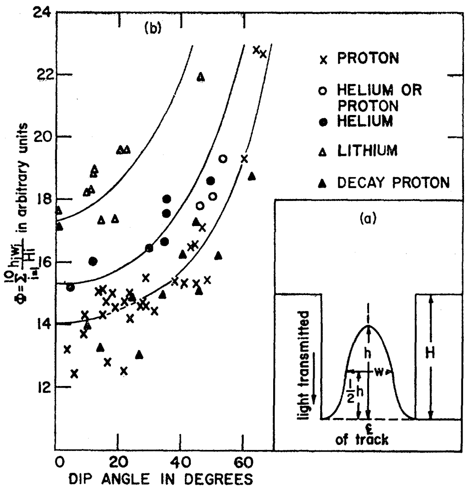

FIG. 1. (a) The sketch of the track profile indicating measured quantities in the track profile parameter $\Phi=\sum_{i=1}^{10}(h_iw_i/H_i)$, where H_i is the background light intensity, h_i/H_i is the fraction of light absorbed at the minimum of transmission, w_i is the full width at the half-height $\frac{1}{2}h_i$, and i refers to repetition of measurements (ten measurements for a track). (b) The profile measurements of all grey or dark prongs of the K^- parent interactions of the eight selected events, as a function of dip angle in the unprocessed emulsion. The lithium tracks found in the same stack were added for comparison.

where E_π and E_p are the kinetic energies of pion and proton, respectively, and B_p is the binding energy of the least-bound proton.[12]

There might be some possibility that a LHF was misinterpreted as heavy since the kinematic analysis alone can not always correctly identify a decay mode (e.g., in the two-neutron-emission case). However, the possibility was estimated to be very small from the fact that 197 long-range MHF's ($>10\,\mu$) which were found in the 2.3-BeV/c K^- stack were analyzed and could all be interpreted as light by kinematic analysis alone. Further evidence that the eight selected events are not light comes from a study of the relationship between the hyperfragment range and the nature of the parent interaction. In a sample of 28 short-range LHF's (see Sec. IIID), none of range $\leq 4\,\mu$ was emitted from a heavy nucleus. Of the eight selected HHF's all except one had a range less than $4.0\,\mu$ and were emitted from Ag or Br. Thus it would seem unlikely that there is any appreciable LHF contamination among these eight events.

TABLE I. Detailed statistics of observed mesonic hyperfragments.

K^--meson momentum	1.1 BeV/c	2.3 BeV/c
Number of K^- interactions observed	141 712	120 000
Total number of DS_{10} events observed	6028	1600
Total number of long-range mesonic hyperfragments ($>10\,\mu$)	474	197
Total number of short-range mesonic hyperfragments ($\lesssim 10\,\mu$)	62	8
Total number of mesonic heavy hyperfragments	33	8
Total number of short-range mesonic light hyperfragments ($\lesssim 10\,\mu$)	28	0
Number of π^+ decays of hyperfragments	1	0

The observed decay modes were π^--p-r, π^--r (possibly π^--n-r), and perhaps π^+-r. The detailed statistics of the observed hyperfragments are given in Table I.

In order to determine the charges of tracks coming from the parent interactions of the eight selected HHF's, a microdensitometer[13] incorporating an Enhancetron[14] was employed. The Enhancetron reduces the background noise by signal averaging, stores the resulting signals in a magnetic core memory and gives a pen recorder plot of the stored data.

As a parameter of the track profile we used the quantity[15]

$$\Phi=\sum_{i=1}^{10}\frac{h_iw_i}{H_i},$$

where H_i is the background light intensity, h_i/H_i is the fraction of light absorbed at the minimum of transmission, w_i is the full width at the half-height $\frac{1}{2}h_i$, and i refers to repetition of measurements (ten measurements for a track) (see Fig. 1a). We determined the charge of a track by comparing the profile of an unknown track with a known one in the neighborhood. The depth correction was obtained by observing a fast but dark He track which passed through one plate, and also by measuring flat Li^8 tracks, but the correction required was found to be negligible except near the emulsion surfaces. All tracks except light tracks ($>2000\,\mu$) and one steep track ($>70°$) were measured three times. In Fig. 1(b) the results of the profile measurements are given as a function of dip angle in the unprocessed emulsion for tracks produced in the eight selected events. The charges of tracks with ranges longer than $500\,\mu$ were also determined by delta-ray counting, and the two methods were found to give consistent results. The charges of the tracks thus determined are given in Table II. The ratio of the yield of charge 1 to charge 2 is 3.1 ± 1.2 if one assumes[16] $N_g/N_h=0.3$, where N_g is the number of grey tracks produced by knock-on particles and N_h is the number of heavily-ionized tracks ($g\geq 1.4g^*$, where g^* is the minimum grain density). This result is consistent with the value 2.5 for eight-prong stars calculated by evaporation theory.[16] No track with $Z>2$ was observed.

[12] The energy release in Λ^0 decay is $Q=37.60\pm0.12$ MeV. See: A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965). We assumed a proton binding energy of 8 MeV.

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TABLE II. The potential-well depths calculated for the binding energies and masses of the eight selected events.

No.	Event No.	Range (μ)	Pion energy (MeV)	Proton energy (MeV)	Binding energy (MeV)	Br or Ag inter-action	Number of prongs[a]	HHF charge	HHF mass (nucleon masses)	Well depth (MeV for $r_0=1.235 \times 10^{-13}$ cm
1	86J-HA25	2.0	5.09	8.70	23.81 ± 0.29	Br	$4(0\alpha)$	30	65_{-0}^{+6}	$29.1_{-0.3}^{+0.4}$
						Ag		42	98_{-5}^{+3}	$28.0_{-0.3}^{+0.3}$
2	73J-HA1	3.0	2.98	11.50	23.12 ± 0.31	Br	$3(0\alpha)+1\pi^{\pm}$	31 ± 1	74_{-9}^{+3}	$28.0_{-0.5}^{+0.3}$
						Ag[b]		43 ± 1	102_{-9}^{+3}	$27.2_{-0.4}^{+0.3}$
3	75J-HA18	1.6	12.75	1.86	22.99 ± 0.31	Br	$6(1\alpha)$	27	60_{-3}^{+1}	$28.5_{-0.4}^{+0.3}$
						Ag[b]		39	90_{-1}^{+2}	$27.3_{-0.3}^{+0.3}$
4	86J-HA10	2.4	12.40	2.22	22.98 ± 0.03	Br	$6(0\alpha)$	28	59_{-0}^{+6}	$28.6_{-0.3}^{+0.5}$
						Ag[b]		40	91_{-2}^{+6}	$27.3_{-0.3}^{+0.4}$
5	71J-FM18	5.8	4.87	9.80	22.93 ± 0.30	Br	$16(2\alpha)$	16	33_{-0}^{+4}	⋯
						Ag		28	59_{-0}^{+6}	$28.5_{-0.3}^{+0.5}$
6	77J-FB15	1.6	10.88	3.82	22.90 ± 0.30	Br	$6(1\alpha \text{ to } 3\alpha)$ $+1\pi^-$	27 ± 1	59_{-4}^{+6}	$28.5_{-0.4}^{+0.5}$
						Ag[b]		39 ± 1	91_{-6}^{+6}	$27.2_{-0.4}^{+0.4}$
7	73J-HA10	0.8	8.00	7.06	22.54 ± 0.30	Br	$6(3\alpha \text{ or } 2\alpha)$	25_{-0}^{+1}	56_{-2}^{+5}	$28.3_{-0.5}^{+0.3}$
						Ag[b]		37_{-0}^{+1}	86_{-2}^{+5}	$27.0_{-0.3}^{+0.4}$
8	86J-HA27	1.6	14.09	1.32	22.19 ± 0.31	Br	$7(1\alpha \text{ or } 2\alpha)$	26_{-1}^{+0}	57_{-3}^{+4}	$27.9_{-0.4}^{+0.4}$
						Ag[b]		38_{-1}^{+0}	88_{-4}^{+3}	$26.6_{-0.3}^{+0.3}$

[a] The interaction belongs to the second set of interaction assignments. (see IIIB).
[b] Prong number is that of parent interaction. The α particles were included in the number.

III. DATA ANALYSIS AND RESULTS

In the examination of 6028 DS_{10} events observed among 141 712 interactions of 1.1-BeV/c K^- mesons, and 1600 DS_{10} events observed among 120 000 interactions of 2.3-BeV/c K^- mesons, 41 mesonic HHF's, 28 LHF's and one π^+ decay have been found within 10 μ range from the K^- interactions. We first present the main object of our paper, the determination of the mass, binding energy, and potential-well depth of HHF's.

A. Mass Determination of Heavy Hyperfragments

The charge of a HHF was assumed to be the charge of Ag or Br (47 or 35) minus the total of the charges of the prongs of the K^- parent interaction, minus one. The core mass[17] for a given charge was assumed to be that of the most abundant natural isotope[18] or of the isotope with one less neutron,[19,20] and the errors were chosen so as to include isotopes of half-life seven days or more, the time between exposure to the K^- beam and emulsion development. The infrequency of observation of associated β decays at the decay points indicates that the above error assignment is reasonable. One associated β decay among 15 π^--p-r events and five associated β decays among 26 π^--r events were observed.

The charge and mass of a heavy hyperfragment were obtained by assuming that there is no invisible short-range prong emitted during K^- interactions, because the Coulomb barrier would suppress the emission of extremely slow particles despite lowered barrier height, and asymmetrical fission would occur only rarely. It should be added here that our assumption is not based on any empirically proven evidence of the absence of such prongs but on the rarity of their occurrence. Among 41 HHF's, only two K^- parent interactions contained second short-range prongs $(10\,\mu > R > 5\,\mu)$. According to the study by Beniston *et al.*,[21] the fraction of parent K^- interactions with second short-range prongs is 0.02 ± 0.01. For our sample of 28 short-range LHF's, among five short-range LHF's which were identified as originating in heavy nuclei only one event ($_{\Lambda}B^{11}$) has a range of less than 5 μ. Thus the observed range distribution is compatible with the prediction based on barrier penetration calculations, and the invisible short-range fragments would be rare ($<1\%$). Therefore, the above assumption seems to be reasonable for the statistical analysis.

None of the parent stars of our selected events showed a short ($\leq 10\,\mu$) prong which, because of uncertain charge and mass, would have made the mass estimate unreliable. [Only two out of forty one events involved a short-range prong and both events were π^--r decays. The charge of one prong was assumed to be $Z=4\pm2$ while that of the other was assumed to be a fission product because the hyperfragment range is unusually long (9.1 μ), and so the mass is likely to be intermediate ($B_{\Lambda}=20$ MeV).]

The number of α particles and heavier particles was small in our selected sample (See Sec. II); to each of the other events 1.3 α particles on the average were assigned by considering track appearance and prong numbers. Seven particles with $Z=3$ or 4 were identified by hammer head or track thickness. If one changes the number of α particles by ±1, the general picture of the binding-energy–versus–mass plot (Fig. 2) will not change much and the final results are not affected.

[17] This mass is $(A-1)$ for a HHF of mass A.
[18] Chart of the Nuclides, Knolls Atomic Power Laboratory.
[19] K. J. LeCouteur, Proc. Phys. Soc. (London) **63**, 259 (1950).
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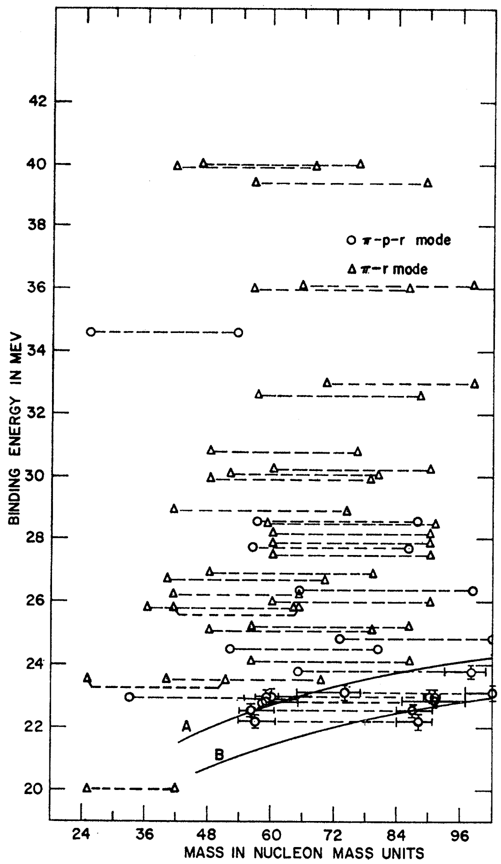

FIG. 2. The apparent binding-energy distribution of 41 heavy hyperfragments as a function of the estimated mass. The two possible masses of a hyperfragment are connected by a broken line. The curves A and B are binding energy plots as a function of the hyperfragment mass for the fixed values $D_\Lambda=28.3$ MeV and $D_\Lambda=27.1$ MeV, respectively.

B. Binding Energies of HHF's and the Potential-Well Depth (Square Well)

The apparent binding energies of all observed HHF's are given as a function of the estimated mass in Fig. 2. Mainly for the reasons described earlier, these energies have a wide spread, extending from 20 to 40 MeV and the possible masses of the HHF's are between about 25 to 102 nucleon masses. Two possible masses, corresponding to Br or Ag parents, are assigned to each event and are connected by a broken line.

Masses, except for the events in the lower part of the binding-energy–mass plot, were crudely estimated, and the error in charge of a HHF may be ± 1. This will not affect the estimate of the potential-well depth. The π^--r events were added to show a general trend of apparent binding energies.

The eight π^--p-r events in the lower region of the binding-energy–mass plot are the ones admitted as candidates for evaluation of the potential-well depth. Their binding energy, prong numbers of parent interactions, estimated masses, and charges are listed in Table II. Errors in the binding energy are standard deviations.

In the model we adopt, the nucleons provide for the Λ^0 particle a potential-well of a constant depth D_Λ, which is assumed to be independent of A. For a square-well potential, D_Λ is related to the binding energy B_Λ and to the nuclear radius $r_0(A-1)^{1/3}$ by

$$(D_\Lambda-B_\Lambda)^{1/2}\cot[(2\mu_\Lambda\hbar^{-2}(D_\Lambda-B_\Lambda))^{1/2}\times r_0(A-1)^{1/3}]=-B_\Lambda{}^{1/2}, \quad (1)$$

where μ_Λ is the reduced Λ^0 mass. The error of the potential-well depth was computed as a function of the errors of the binding energy and the mass from Eq. (1). Values of the potential-well depth for the binding energies and masses of the candidates in the lower region, calculated with $r_0=1.235\times10^{-13}$ cm,[22] are given in the last column of Table II for the two possible masses.

We notice that the potential-well depth values are clustered in two distinct groups near $D_\Lambda\cong28.3$ MeV and $D_\Lambda\cong27.1$ MeV. (The well depth is sensitive to the nuclear radius parameter. For example, the above values become $D_\Lambda=29.3$ MeV and $D_\Lambda=27.9$ MeV for $r_0=1.115\times10^{-13}$ cm and $D_\Lambda=27.5$ MeV and $D_\Lambda=26.5$ MeV for $r_0=1.350\times10^{-13}$ cm.) The curves A and B in Fig. 2 are binding energy plots as a function of the HHF mass for the fixed values $D_\Lambda=28.3$ MeV and $D_\Lambda=27.1$ MeV, respectively. Six events in the lower region, if attributed to Br, and events 1 and 5, if attributed to Ag, appear to lie on curve A. The first-mentioned six are on curve B, if attributed to Ag. The curves are separated by about 1.3 MeV, about four times the standard deviation of the binding-energy error. Thus this suggests an experimental possibility for determining the correct assignment. It should be noticed that the possible mass errors have been substantially reduced by determining the charges of prongs of the parent interactions accurately. If we include four events from the world survey data, which will be described below, the separation of our eight events into two sets of interaction assignments is further supported within the uncertainty in hyperfragment charges.

Thus we have two sets of interaction assignments; the first set consists of two possible Ag interactions and six possible Br interactions. The weighted average of depth is 28.3 ± 0.3 MeV (the weighting factor is 1 for each of six Br interactions, and $\frac{1}{2}$ for each of two Ag interactions) and the average mass is 63. The second set consists of six possible Ag interactions, the average depth value is 27.1 ± 0.3 MeV, and the average mass is 91. The second set members are indicated in Table II. It would be possible to choose a correct set by compiling more data, as discussed later. If we do not distinguish between Br or Ag interactions, a grand average of all

[22] V. Meyer-Berkhout, K. Ford, and A. Green, Ann. Phys. (N.Y.) 8, 119 (1959).

TABLE III. The world survey data (π-p-r decays of binding energy less than 24 MeV).

Event No.	Authors	K^- momentum, (BeV/c)	Range (μ)	Number of prongs	Pion energy (MeV)	Proton energy (MeV)	Binding energy (MeV)	Comments
1	Key *et al.*[a]	0.8	2.5	12	9.8	6.8	21.0	A visible recoil in the decay star. Intermediate in mass.
2	Key *et al.*[a]	0.8	10.0	11	16.7	0.7	20.0	A visible recoil in the decay star. Intermediate in mass.
3	Evan *et al.*[b]	At rest	4.0	2	6.9	9.5	21.2	The longer range indicates an intermediate mass.
4	Lemonne *et al.*[c]	At rest	0.9	5	11.5	3.3	22.8	Heavy in mass.
5	Lemonne *et al.*[c]	At rest	1.9	2	8.8	5.1	23.7	Heavy in mass.
6	Lagnaux *et al.*[d]	1.5	2.6	6	7.1	7.4	23.1	Heavy in mass.
7	Lagnaux *et al.*[d]	1.3	1.4	8	12.3	2.8	22.5	Heavy in mass.
8	Perlmutter[b]	0.8	2.3	4	11.1	9.3	17.2	A possible visible recoil of $<1\ \mu$ in the decay star, and 4.3 μ long recoil in the parent interaction. Intermediate in mass.

[a] Reference 8. [b] Reference 23. [c] Reference 6. [d] Reference 7.

the depth values is 27.7±0.6 MeV. The error was chosen so as to cover the Ag-Br ambiguity. This agrees with previous estimates[6,7] which were based on methods of mass estimation different from ours. In the next section we will discuss the above two sets further.

It is instructive to see how our eight events can be compared with the world survey data of π^--p-r decays.[4–9,23] In Table III we list those which have binding energy less than 24 MeV. For Events 1 and 2, the visible recoils in the hyperfragment decay have ranges of 1 to 2 μ. According to the range-momentum relation for heavy ions,[24] the $Z=20$ heavy ion of range 1 μ has a momentum of 300 MeV/c, while the sum of the pion and proton momenta opposing the recoil momentum is much smaller than 300 MeV/c. Thus the masses seem to be intermediate ($Z<20$). Larger prong numbers of parent stars also indicate possible intermediate masses for these two events. Event 2 involves a decay proton of range 9.0 μ which is also evidence that the mass of the event is intermediate.[1,25] The decay star of event 8 has a possible visible recoil of range less than 1 μ. The parent interaction contains a short-range prong (4.3 μ) and so the mass estimate is not accurate, but the small binding energy suggests that it also may be intermediate in mass. Events 4, 5, 6, and 7 are HHF's essentially similar to ours regarding the binding energies, prong numbers, and ranges. For event 3 the authors commented it might be of intermediate mass because of its longer range but the prong number agrees with this assumption only if the prongs include a heavy-fragment track, visible or invisible. Thus if we look at all the data, it is concluded that 12 HHF's have binding energies of about 23 MeV while four hyperfragments with binding energies of 21 MeV or less are indicated to be intermediate in mass. As far as HHF's are concerned, no event has been found with a binding energy lower than about 23 MeV, even if the world survey data are included.

C. Discussion—Ag-Br Ambiguity

It may be instructive to investigate the Ag-Br ambiguity to try to identify a correct set of interaction assignments. The following discussion is based on the experimental fact that for our events the observed binding energies and the estimated masses, under appropriate assumptions, are remarkably close to each other.

While the fraction of decays leading directly to an excited residual nucleus is difficult to estimate, excitation by collision processes in nuclear matter may be discussed in terms of mean free paths. The mean free paths of protons with energy 4 to 10 MeV and of pions with energy 10 to 20 MeV are estimated to be about 7.5×10^{-13} cm and 220×10^{-13} cm, respectively.[26,27] Therefore, when Λ^0 particles decay inside HHF's with $A=70$ in about 60% of π^--p-r modes both decay particles are estimated to escape from the nucleus without inelastic collision if there is no Coulomb barrier. Since the effective Coulomb barrier is 4 MeV[19] for a nucleus with $A=70$, a decay proton can leave the nucleus if its kinetic energy exceeds 4 MeV. It was crudely estimated by considering a HHF mass and the kinetic energy of its

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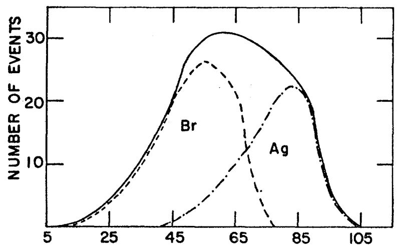

FIG. 3. The mass distribution (taken from Ref. 3) of heavy hyperfragments produced by 0.8 BeV/c K^- mesons as calculated from the observed range distribution.

decay proton that at least three events could be free from excitation due to collisions.

Six binding energies, all except the first and the last in Table II, agree within two standard errors. The estimated masses are close to each other in each set though generally different nuclear species are presumably involved. No clustering of binding energies at a higher value is observed. This suggests that unless the excitation energies in these different residual-nuclear species involved happened to be all equal (which seems unlikely), the excitation energies must have been too small to detect experimentally (or zero) for these events. This conclusion is further strengthened by the world survey data (events 4, 5, 6, and 7). If this is the case, the qualifications as "upper limit" of binding energy or potential-well depth could be omitted.

The mass distribution obtained by a Monte Carlo calculation[3] from the observed range distribution for a sample of nonmesonic HHF's produced by 0.8-BeV/c of K^--mesons is reproduced in Fig. 3. The masses are spread between $A=25$ to 95 and the mean mass is $A=70$. The mass distribution at 1.1-BeV/c should be similar to the above. In Fig. 3 we observe that HHF's with $A=65$ are twice as abundant as those with $A=90$. If one compares our two sets with Fig. 3, one is tempted to favor the first set.

The kinetic energies of three of our decay protons are 1.32, 1.86, and 2.22 MeV, and are much smaller than 4 MeV, the Coulomb barrier for $A=70$. (See also Table III.) Observation of a slow proton in HHF decay favors a relatively lower mass.[4,28] Therefore, it is unlikely that the group of six events is due to Ag interactions.

No events are located appreciably below the curve A, Fig. 2 unless they are interpreted as Ag interactions. This is likely to be true even if events 4, 5, 6, and 7 from the world survey data are included. If one assumes that all nine events (six from our data and three from the world survey) in the B_Λ region of 22.5 to 23.1 MeV are Ag interactions and so belong to the second set because of the close agreement in their binding energies, and if one also assumes that HHF's from Ag and Br interactions are equally abundant (see Fig. 3), the probability of seeing no Br interactions among nine events in the second set is only $2^{-9}=0.2\%$. This argument also favors the first set.

From the above discussion one may suggest that:

(1) The first set of interaction assignments is more likely to be correct, that is, (A) the assignment of $B_\Lambda=23.0\pm0.2$ MeV to $A\cong 60$ is favored; (B) the value D_Λ (square well)$=28.3\pm0.3$ MeV fits all eight events, with events 1 and 5 assigned to Ag interactions. The D_Λ error is due to the errors of B_Λ and A alone.

(2) The close agreements in the observed binding energies and the corresponding estimated masses suggest that the excitation energies involved may be negligibly small for these events. Hence the above values are very close to the actual binding energy and the actual potential-well depth (square well) rather than merely being upper limits.

It would be appropriate to point out, however, that the potential-well depth D_Λ is sensitive to the nuclear potential shape assumed and so the simple square-well depth does not represent a true value. A more realistic potential shape, with consideration of the reduced effective mass of the Λ^0 particle in nuclear matter and rearrangement energy, has been used in the theoretical calculations.[29] The present data are insufficient to consider possible effects due to specific nuclear structure.

An accumulation of more data would elucidate the above two points further.

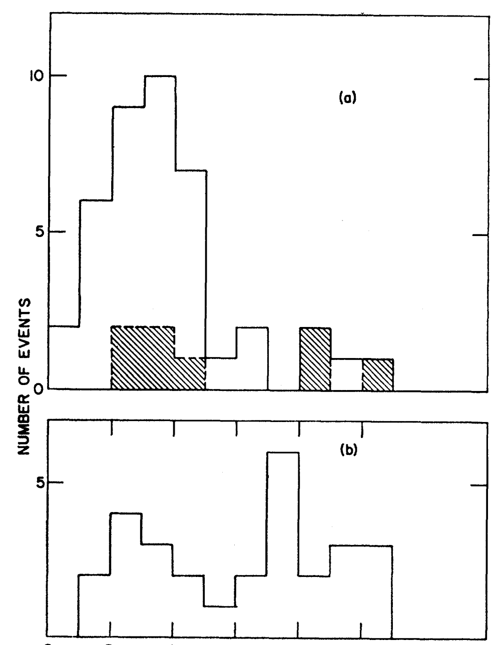

FIG. 4. The range distributions of (a) mesonic heavy hyperfragments (the shaded area indicates events from the 2.3-BeV/c K^- interactions), and (b) short-range mesonic light hyperfragments from the 1.1-BeV/c K^- interactions ($\lesssim 10\,\mu$).

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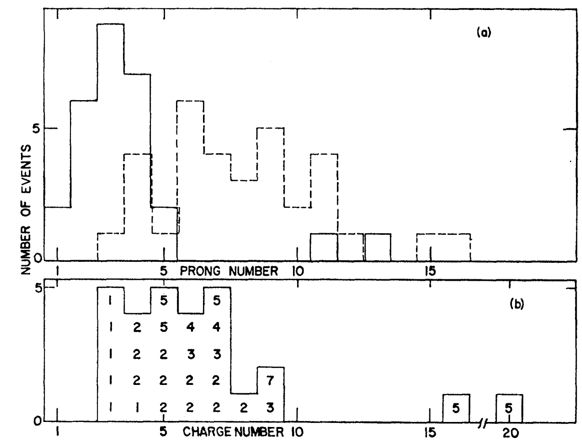

FIG. 5. (a) The prong-number distributions of the K^- parent interactions (from the 1.1-BeV/c K^- stack only) of 33 mesonic heavy hyperfragments (broken lines) and 28 mesonic light hyperfragments (solid lines). (b) Distribution of the sum of the prong charges of the 1.1-BeV/c K^- parent interactions containing mesonic light hyperfragments ($Z<8$). The number indicates the charge of a hyperfragment. (See Sec. IIID.)

D. Heavy Hyperfragments (HHF's) and Short-Range Light Hyperfragments (LHF's)

The range distributions of the mesonic HHF's and LHF's are given in Figs. 4(a) and 4(b). The HHF distribution is similar to that of nonmesonic HHF's from 0.8-BeV/c and 1.5-BeV/c K^- interactions.[3] In our restricted range interval, the average ranges of HHF's and LHF's are 3.6 and 6.0 μ, respectively. The angular distribution of HHF's is peaked forward, with a forward-to-backward ratio of 1.93 ± 0.62, while the angular distribution of LHF's is essentially isotropic.

In Fig. 5(a) the prong-number distributions[30] of the K^- parent interactions are given. The prong number distribution of parent interactions of LHF's is sharply peaked at small prong numbers while that of HHF's is broadly spread around the prong number eight. The average prong numbers of the HHF and LHF parent interactions are 8 ± 2 and 4 ± 3, respectively. Another difference of the two classes of hyperfragments is the emission rate of charged pions. Only three pions (in the 1.1-BeV/c K^- stack) of relatively low kinetic energy were emitted from the K^- parent interactions of HHF's while eleven pions of higher kinetic energy were emitted from K^- parent interactions of LHF's. These findings suggest that most LHF's have originated in light nuclei (O, N, C). In order to estimate the production rate in light nuclei, the prong number distributions of the parent interactions of LHF's and of HHF's were compared.[21] Since the HHF's originate entirely in heavy nuclei, the common region at small prong numbers can be attributed to production in heavy nuclei of LHF's if the two prong-number distributions are normalized. Thus 21 events, or $(75\pm22)\%$ of the LHF's, were found to originate in light nuclei. The distribution of the sum of the prong charges of K^- parent interactions containing LHF's is given in Fig. 5(b). Dark tracks of range less than 500 μ were assumed to have $Z=1$, while the charges of other tracks were determined by δ-ray counting. The pion charges were determined by following tracks to their ends and observing capture stars or characteristic decays, and were assumed to be negative for tracks which left the stack. For ambiguous hyperfragments the most likely charges were adopted. The sharp falloff at $Z=7$ indicates that most LHF's originated in light nuclei and also that the above production rate is reasonable. Although our statistics are still poor, we conclude that the majority of the LHF's are produced in light nuclei.

[30] The pion and hyperfragment tracks are excluded, because they are different from the other tracks in production and charge, and are not a measure of nuclear excitation.

E. Nonmesonic to π^- Mesonic Decay Ratio

This ratio was determined on the basis of 5958 nonmesonic HHF's in the mass range $A=65\pm40$ and 33 mesonic HHF's observed among 141 722 interactions of 1.1-BeV/c K^--mesons. The nonmesonic HHF's are contaminated with LHF's, scatterings, capture stars of negatively charged particles, and interactions in flight, but the contamination is rather small[1,31]; it was assumed to be 3%. The ratio of the zero-prong decays to charged-prong decays of the HHF's was estimated by Lagnaux *et al.*[7] to range from 9% to 45% for fractions of neutron-stimulated decays between 0.62 and 0.90. (The Λ^0 neutron-stimulated decay results in a zero-prong star unless the two emitted neutrons knock out at least one charged particle.) Since the neutron stimulation fraction 0.62 is reasonable from the study of LHF's,[25] the ratio was assumed to be 9%.

In general, the mesonic decay of a HHF is more difficult to observe than the nonmesonic decay because the mesonic decay gives a small star of one or two prongs close to the K^- parent interaction, and the connecting tracks are sometimes obscured by adjacent tracks. The scanning efficiency for mesonic HHF's relative to nonmesonic HHF's was estimated to be $(85\pm5)\%$ on the basis of double scanning, scanners' ability, and the degree of difficulty of observation. Furthermore, for the

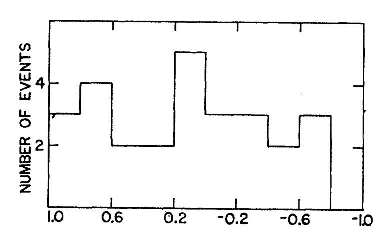

FIG. 6. The angular distribution of the decay-pion tracks from the π-r modes with respect to the direction of hyperfragment motion.

[31] E. R. Fletcher, J. Lemonne, P. Renard, J. Sacton, D. O'Sullivan, T. P. Shah, A. Thompson, P. Allen, Sr., A. Heerman, A. Montwill, J. F. Allen, M. J. Beniston, D. A. Garbutt, R. C. Kumar, P. V. March, T. Pniewski, and J. Zakrzewski, Phys. Letters 3, 280 (1963).

TABLE IV. Data on the π^+ decay of a hyperfragment.

Track	Range (μ)	Energy (MeV)
Hyperfragment	3.9	
π^+	9593.2	22.6
μ^+	561.0	

π-r decay mode, the identification is more difficult when the decay pion is emitted in a direction nearly opposite to the direction of hyperfragment motion, as is seen in Fig. 6. Correction for this loss amounted to two events. Thus, the nonmesonic to π^- mesonic decay ratio $R(\pi^-)_{\rm exp}$ of the HHF's with $A=65\pm40$ is 153 ± 27. This value agrees fairly well with $R(\pi^-)_{\rm theo}=130$ at $A=100$, predicted by Dalitz,[11] since one expects that the ratio $R(\pi^-)$ does not change rapidly with mass in this mass region.[32]

F. π^+ Decay of a Hyperfragment

One event interpreted as the π^+ decay of a hyperfragment has been found.[33] The data are given in Table IV. The π^+ meson was identified by its characteristic π-μ-e decay. An electron track (flat, at least 50 μ long) was associated with the hyperfragment decay point. No trace of a recoil was observed.

Because the hyperfragment charge can not be determined because of its short range, and the π^+ decay usually involves at least one neutron,[21] a unique interpretation is difficult. It is compatible with ${}_\Lambda\mathrm{Li}^7\rightarrow\pi^++\mathrm{He}^6+n$, ${}_\Lambda\mathrm{B}^{10}\rightarrow\pi^++\mathrm{Be}^9+n$, ${}_\Lambda N^{14}\rightarrow\pi^++\mathrm{C}^{13}+n$, and ${}_\Lambda\mathrm{N}^{15}\rightarrow\pi^++\mathrm{C}^{15}$. Furthermore, it could be interpreted as a π^+ decay of a HHF, because this hyperfragment emerged from a K^- interaction with 13 prongs. This indicated it originated in a heavy nucleus.

In the literature survey two events of similar appearance were reported. The first one was the π^+ decay of ${}_\Lambda\mathrm{H}^{3,4}\rightarrow\pi^++3n$, or $4n$.[34] The second one was the radiative decay of a Σ^+ particle.[35] The last interpretation is considered unlikely for our event because an electron track is associated with the decay point and because no normal decay of a Σ^+ particle has been observed so close to the K^- stars (less than $10\,\mu$). The empirical branching ratio for radiative decay is 0.18%.[36]

[32] J. Lemonne, C. Mayeur, J. Sacton, D. H. Davis, D. A. Garbutt, and J. Allen, Nuovo Cimento **34**, 529 (1964).

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IV. CONCLUSIONS

In the examination of over 261 000 K^- interactions, we have observed 41 mesonic HHF's, 28 LHF's, and 1 π^+ decay of a hyperfragment in the region very close to the K^- interactions ($\lesssim 10\,\mu$).

By the present method of analysis we have avoided some of the difficulties mentioned in Sec. I, and in our discussions we have shown that the remaining difficulties are not serious. In summary:

(1) We have shown in Sec. IIIA that the emission of invisible very short-range prongs should be negligible (<1%).

(2) The close clustering of the binding energies of eight selected events suggests that we are dealing with decay into the ground state or very low excited states. No such clustering is observed among HHF's with higher apparent binding energies.

(3) Since the selected events are independently shown to be HHF's, neutron emission in the decay can be excluded as it would give binding energies below the expected range for HHF's.

On the basis of eight π^--p-r decays in the lower region of a plot of binding energy versus mass, the upper limit of the potential-well depth D_Λ is estimated to be D_Λ (square well) $=27.7\pm0.6$ MeV.

If the experimental evidence is believed convincing enough to resolve the Ag-Br ambiguity, the assignment of $B_\Lambda=23.0\pm0.2$ MeV to $A\cong60$, and the value D_Λ (square well) $=28.3\pm0.3$ MeV is favored. The D_Λ error is due to the errors of B_Λ and A alone. The close agreements in the observed binding energies and the corresponding estimated masses suggest that the above data may be considered to be actual values, rather than merely upper limits.

The nonmesonic-to-π^--mesonic decay ratio is found to be about 153 ± 27 at $A=65\pm40$. This agrees fairly well with the theoretical prediction 130 at $A=100$ given by Dalitz.

The π^+ decay of a hyperfragment is ambiguous in interpretation, but probably represents an event with $Z\geq3$ and perhaps a HHF decay.

The study of 28 LHF events indicates that $75\pm22\%$ of these originated in light nuclei (O, N, C).

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