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Study of Levels in ^{23}Mg†

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Levels in ^{23}Mg with excitation energies up to 6.8 MeV have been investigated by magnetic analysis of the α particles from the reaction ^{24}Mg(^{3}He,α)^{23}Mg at bombarding energies between 6 and 12 MeV. Between 4.4- and 6.8-MeV excitation energy, 16 new levels were found in ^{23}Mg. Angular distributions of the emitted α particles were measured and from distorted-wave Born-approximation analysis l values for the picked up neutrons were obtained. The particle-γ angular-correlation method of Litherland and Ferguson was used to obtain additional information on the spin assignments of the levels as well as branching and mixing ratios of the γ-ray transitions. The following spin and parity assignments have been established for the indicated levels: 0.451 MeV $\frac{5}{2}^+$, 2.048 MeV $\frac{7}{2}$ ($\frac{3}{2}$), 2.356 MeV $\frac{1}{2}^+$, 2.768 MeV $(\frac{1}{2},\frac{3}{2})^-$, 2.904 MeV $(\frac{3}{2},\frac{5}{2})^+$, 3.792 MeV $\frac{3}{2}^-$, 4.353 MeV $\frac{1}{2}^+$, 5.289 MeV $(\frac{3}{2},\frac{5}{2})^+$, and 5.992 MeV $(\frac{1}{2},\frac{3}{2})^-$. The experimental results are compared with those obtained for ^{23}Na and the predictions of the collective model.

I. INTRODUCTION

For many years the collective model has been reasonably successful in explaining the level properties of nuclei in the mass-25 region.[1] For some odd-mass nuclei having proton or neutron number equal to 11, both the calculation of minimum nuclear binding energy versus core deformation and measurements of the static properties suggest that in the ground state the odd nucleon is in the Nilsson orbit seven. Some of the excited states in these particular nuclear systems can then be interpreted as being members of the $K^\pi=\frac{3}{2}^+$ rotational band based on the ground state. The model parameters required to reproduce these static properties can be used to calculate the dynamic properties such as mixing parameters and branching ratios of the γ-ray transitions within the band. Howard *et al.*[2] carried out such calculations (neglecting band mixing) and obtained substantial agreement with experiment. Poletti and Start[3] further improved the agreement with their measurements on ^{23}Na and by using a consistent phase convention for the mixing parameters. On the basis of the over-all agreement obtained for ^{21}Ne, ^{21}Na, and ^{23}Na, they were encouraged to use the same average set of parameters to predict the properties of the first two excited states of ^{23}Mg (Ref. 3).

Energy levels up to 4.353-MeV excitation in ^{23}Mg have previously been observed using the ^{24}Mg(^{3}He,α)^{23}Mg reaction.[4] Angular distributions of the α particles to the ground and first-excited state were fitted, using a plane-wave approach, by a mixture of heavy-particle stripping and pickup with angular momentum transfer $l_n=2$. This leads to $J^\pi=(\frac{3}{2},\frac{5}{2})^+$ for both states.[5] Additional information from the positron decay of ^{23}Mg fixes the spin and parity of the ground state as $\frac{3}{2}^+$ (Ref. 6).

In the present investigation we have studied the levels in ^{23}Mg in some detail using the ^{24}Mg(^{3}He,$\alpha\gamma$)^{23}Mg reaction. The present study of the angular distribution of the α particles and measurements of the angular correlations between the α particles and the de-excitation γ rays has led to new information on spin and parity of the levels and determinations of branching and mixing ratios for the γ radiation. The experimental results have been compared with the theoretical predictions obtained from the collective model.

II. EXPERIMENTAL PROCEDURE

The targets were prepared by vacuum evaporation of metallic ^{24}Mg, enriched to 99.7%, from a small tantalum oven onto thin carbon foils. Self-supporting ^{24}Mg targets were also made by evaporation of the magnesium onto a glass slide, which had previously been coated with a thin layer of BaI. The BaI was then dissolved and the magnesium film floated off the glass slide and mounted on tantalum frames. These targets turned out to be very fragile and difficult to prepare with thicknesses less than about 100 μg/cm^2. They were only used when less than optimum resolution of the particle spectrum was needed and when it was essential to minimize the contribution of α particles from carbon.

The ^{3}He^{++} beam with energies between 6 and 12 MeV and intensity of 0.05–0.3 μA was obtained from the ONR-CIT tandem accelerator. The particles produced in the reaction were detected in a 61-cm double-focusing magnetic spectrometer which could be set at any angle from 0° to 150° to the beam direction. When measuring the α-particle spectra and their angular distributions, the particles were detected in an array of 16 solid-state

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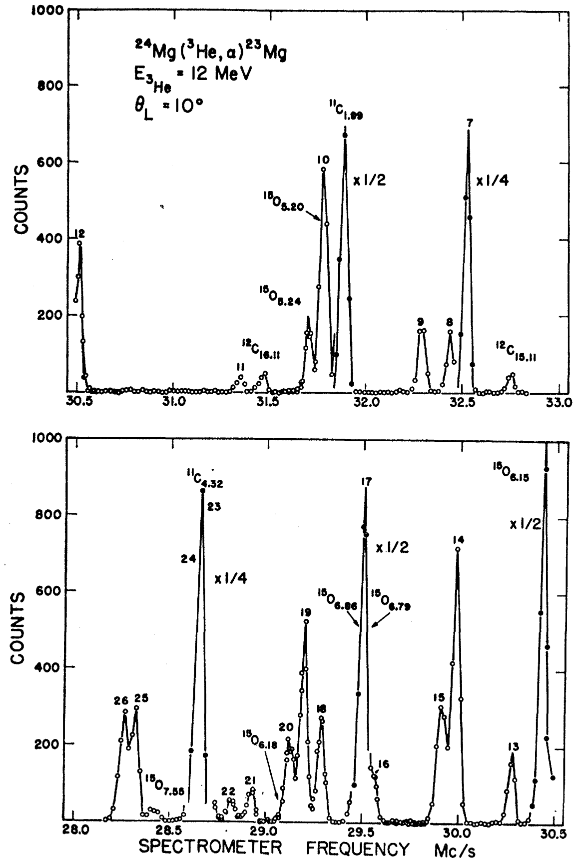

FIG. 1. Spectrum of α particles observed at 10° from the ^{24}Mg(^{3}He,α)^{23}Mg reaction at 12-MeV incident ^{3}He beam energy. Group number 7 corresponds to the 3.792±0.010-MeV level in ^{23}Mg and the groups corresponding to higher lying levels in ^{23}Mg are labeled 8 to 26. Groups due to carbon and oxygen contaminations were identified (for further details, see text).

counters placed in the focal plane of the magnetic spectrometer. The magnetic field was measured with an NMR device. The frequencies for the different α-particle groups were determined and the corresponding energies calculated using the spectrometer constant. The slight energy dependence of the spectrometer constant was taken into account in the calculations.

It has been pointed out by Litherland and Ferguson[7] that by detecting the outgoing particle in a nuclear reaction at 0° or 180° to the beam direction, one can often considerably restrict the number of magnetic substates which are populated. In the present particle-γ correlation measurements using the ^{24}Mg(^{3}He,$\alpha\gamma$)^{23}Mg reaction, the α particles were detected in a single solid-state counter at the focal point of the magnetic spectrometer, which was placed at 0° to the beam direction. Hence, as ^{24}Mg has a spin-zero ground state, only the ^{23}Mg magnetic substates $m=\pm\frac{1}{2}$ are populated. This fact will facilitate the interpretation of the α-γ correlation. The finite solid angle about 0° subtended by the spectrometer permits some small population of higher magnetic substates. However, because the acceptance half-angle was less than 2°, the effect was considered to be negligible.

The γ rays were detected in a 12.7×10.2 cm NaI(Tl) crystal 12.7 cm from the beam position on the target foil, and spectra were recorded at selected angles in a horizontal plane between 90° and 150° to the beam direction. The isotropy of the experimental arrangement was checked by performing an angular-distribution measurement with a radioactive source in the beam spot position on the target foil. The deviation from isotropy was found to be less than 0.5%. The steel wall of the scattering chamber contained a horizontal slot for the sliding seal of the magnetic spectrometer. Because this slot affected the absorption of the γ rays, an experimental absorption curve for the scattering chamber was measured using radioactive sources. The slot also introduced small corrections in the angular-correlation attenuation coefficients for the γ-ray detector. The ^{3}He beam was well collimated to a spot on the target less than 2 mm×2 mm, hence corrections due to the finite size of the γ-ray emitting source were not necessary.

The electronic equipment associated with the particle-γ correlations is described elsewhere.[8] Briefly, real and random α-γ coincidence events were recorded using two identical coincidence units in parallel. Both of the units were first adjusted to respond to real plus random events. Then an additional delay of 2 μsec in the particle pulses being fed into the second unit ensured that it recorded random events only. The resolving time of the two units was adjusted to be equal at about $2\tau=120$ nsec. The resulting coincidence spectra

TABLE I. Energy levels of ^{23}Mg from magnetic analysis of the α particles in the ^{24}Mg(^{3}He,α)^{23}Mg reaction. The energies were measured relative to the 3.792-MeV level.[4]

Group number (Fig. 1)	Excitation energy (MeV)	Error (keV)
7	3.792	±10
8	3.856	±15
9	3.968	±15
10	4.353	±15
11	4.675	±15
12	5.289	±15
13	5.451	±15
14	5.657	±15
15[a]	5.706	±20
16	5.935	±15
17	5.992	±15
18	6.137	±15
19	6.197	±15
20[a]	6.244	±20
21	6.381	±15
22	6.454	±15
23	6.520	±15
24	6.576	±20
25	6.781	±15
26	6.822	±15

[a] Possibly a doublet.

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were routed into different 200-channel sections of an RIDL 400-channel analyzer. The gain of the γ-ray system was kept constant by gain stabilization on the dominant 0.511-MeV peak in the single γ-ray spectrum.

In order to obtain the experimental α-γ angular correlations, the γ-ray spectra coincident with a fixed number of α particles emitted at 0° and populating the ^{23}Mg levels of interest were measured at 90°, 120°, 135°, and 150°. After correcting for the accidental coincidences, the γ-ray spectra were analyzed into their monoenergetic components, and the number of counts in the photopeaks were calculated. The method of analysis and presentation of results follows closely that outlined by Poletti and Warburton.[9] The experimental angular correlations were fitted to the theoretical formula

$$W(\theta)=\sum_k a_k P_k(\cos\theta)\,,$$

where θ is the angle between the direction of emission of the γ rays and the beam direction, and $P_k(\cos\theta)$ is the Legendre polynomial. The coefficients a_k are a function of the spins (J_1,J_2) of the states involved, the amplitude mixing ratio (δ) of the two competing multipole fields (L and L') of the γ radiation and the attenuation coefficients for the γ-ray detector. For an angular correlation of the second γ ray in a γ-ray cascade in which the first γ ray is unobserved, the spin of the first level and the mixing ratio of the unobserved radiation are introduced as additional parameters.

The a_k coefficients were calculated from the formulas and tables given in Ref. 9, except that the phase of the mixing ratio was as defined by Poletti and Start.[3]

For each set of possible spin values a normalized χ^2 test was applied over the range of possible values of the mixing ratio δ. Probability tables were then used to indicate ranges of δ which gave reasonable agreement with the data. Values of χ^2 below the 1% confidence limit were assumed to be in agreement while those between the 1% and 0.1% limits were considered to be a bad fit but could not be excluded. All values of χ^2 above the 0.1% limit were judged to be improbably large and the values were rejected.

III. ALPHA-PARTICLE SPECTRA AND LEVELS IN ^{23}Mg

Several α-particle spectra from the ^{24}Mg(^{3}He,α)^{23}Mg reaction were measured at different angles and ^{3}He beam energies. Particle groups to the 10 previously known levels[4] with excitation energies up to 4.353 MeV were confirmed and 16 additional higher-lying new levels in ^{23}Mg were established (Fig. 1 and Table I). The carbon and oxygen contaminants in the target material resulted in intense groups of α particles populating levels in ^{15}O and ^{11}C. These groups could, however, be easily identified since their energy variation with angle was different from that of the groups feeding the levels in ^{23}Mg. Transitions to the 15.11- and 16.11-MeV levels in ^{12}C from the ^{13}C(^{3}He,α)^{12}C reaction were also seen. In Fig. 1 the group labeled "7" corresponds to the 3.792 ±0.010-MeV level[4] in ^{23}Mg and the groups corresponding to higher-lying levels are labeled "8" to "26." Groups 10 and 17 could not be resolved from the weak groups to the 5.20, 6.79, and 6.86-MeV levels in ^{15}O at 10°. Furthermore, groups 23 and 24 are hidden under the strong group to the 4.32-MeV level in ^{11}C but they appear clearly at other angles. Because of the finite resolution of the experimental setup, some of the peaks might have more than one component. The somewhat variable shapes of groups 15 and 20 shown at some angles suggest that they may be doublets. The excitation energies quoted in Table I are the mean values of at least three different determinations. Levels corresponding to groups 11 to 26 have not been reported previously. Excitation functions for the α-particle groups to the low-lying levels in ^{23}Mg were also measured at a laboratory angle of 10° in the beam energy range from 8 to 11 MeV in order to find the optimum conditions for the particle-γ correlation measurements. The yield in some cases showed large fluctuations (up to a factor of 3) over a 200-keV change in beam energy, which suggests that in the beam-energy region covered here, there is considerable compound system formation.

IV. THE α-PARTICLE DISTRIBUTIONS

Angular distributions of the α-particle groups populating the levels in ^{23}Mg were measured from 0° to at least 45° in steps of 5°. The particle spectra at various angles were normalized to the integrated beam current. A beam intensity of 0.3 μA was used and no change in the target condition could be detected during the measurements.

In order to provide some further information on the spin and particularly the parity of some of the levels, a distorted-wave analysis was applied to extract the l values of the picked-up neutron (l_n). The computer program T-SALLY[10] was used for the calculations. Elastic-scattering potentials for the incoming and outgoing channels similar to those successfully applied to (^{3}He,α) reactions[11,12] on ^{39}K and ^{19}F were used. Slight variations were tried in order to obtain better fits. It was found that by changing the strengths of the potentials the amplitude of the oscillations in the calculated angular distributions could be altered. However, the positions of the maxima and minima could not be changed appreciably, and where good fits were obtained the l values were unambiguous.

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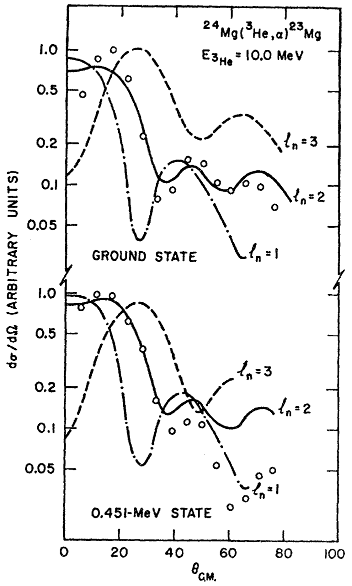

FIG. 2. Angular distributions of α particles from the $^{24}Mg(^3He,\alpha)^{23}Mg$ reaction leading to ground and first excited states in ^{23}Mg at an incident ^3He beam energy of 10 MeV. Open circles are experimental values and the curves are the DWBA predictions for different l_n values. The relative errors in the measured cross sections are about 10%.

A set of parameters was finally arrived at which gave reasonably good fits to most of the angular distributions, and these are listed in Table II. The real part of the ^3He well is appreciably deeper than that obtained[13] from the elastic scattering of 12-MeV ^3He on ^{24}Mg, while the real part of the well for the α particles is somewhat shallower than was found[14] from elastic scattering of 20-MeV α particles on ^{24}Mg.

It has been established[5] that the $^{24}Mg(^3He,\alpha)^{23}Mg$ reaction populates the ground and first-excited states by a pickup process. At 10-MeV bombarding energy, both of these states are strongly populated and the angular distributions are peaked in a forward direction (Fig. 2). Both curves are fitted reasonably well by $l_n = 2$ calculations. The distributions to these two levels differ slightly in shape, the difference being quite noticeable in our measurements at 12-MeV bombarding energy which have been carried back to 150° (Fig. 3). This may be due to a J-dependence of the pickup reaction.[15]

The angular distributions of the α-particle groups to the higher excited states are collected in Figs. 4, 5(a), and 5(b), together with the calculated fits. Some of the α-particle groups were comparatively weak (Fig. 1) and we were unable to obtain satisfactory angular distributions for them. All other groups except those to the 2.048- and 2.712-MeV levels were strongly populated. Relatively unambiguous assignments have been made for the levels at 2.356 MeV ($l_n=0$), 2.768 ($l_n=1$), 2.904 ($l_n=2$), 3.792 ($l_n=1$), 4.353 ($l_n=0$), 5.289 ($l_n=2$), and 5.992 ($l_n=1$).

TABLE II. Saxon-Woods potential wells for the ^3He and the α particles used in the DWBA analysis of the angular distributions of the α particles in the $^{24}Mg(^3He,\alpha)^{23}Mg$ reaction. V and W are the real and imaginary parts of the potentials with radius $r_0A^{1/3}$ ($r_0'A^{1/3}$) and diffuseness a (a'), respectively.

Particle	V (MeV)	W (MeV)	r_0 (fm)	a (fm)	r_0' (fm)	a' (fm)
^3He	181	16	1.1	0.85	1.8	0.6
α	40	10	1.8	0.65	1.8	0.6

V. THE α-γ CORRELATION MEASUREMENTS

A. The 0.451-MeV Level

The angular correlation for this level was measured at 6-MeV bombarding energy. The coincidence spectrum showed only the 0.45-MeV γ ray to the ground state. The annihilation radiation at 0.51 MeV, which is the dominant peak in the single spectrum, did not appear when the coincidence spectrum was corrected for the random coincidences. The angular distribution is shown in Fig. 6, where $d\sigma/d\Omega$ is plotted as a function of $\cos^2\theta$. In order to present the characteristics of the experimental data in a more compact form, the distribution was fitted by the least-squares method to an expansion of the form

$$Y(\theta)=a_0+a_2P_2(\cos\theta)+a_4P_4(\cos\theta).$$

The values for a_2/a_0 and a_4/a_0 are given in Table III.

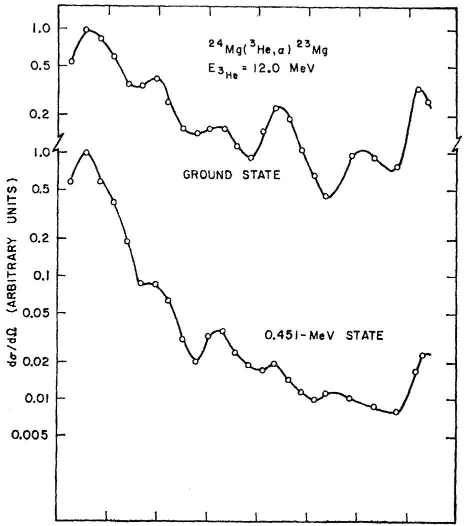

FIG. 3. Comparison of experimental angular distributions of α particles from the $^{24}Mg(^3He,\alpha)^{23}Mg$ reaction leading to ground and first excited states in ^{23}Mg at an incident ^3He beam energy of 12 MeV. The relative errors in the measured cross sections are about 10%.

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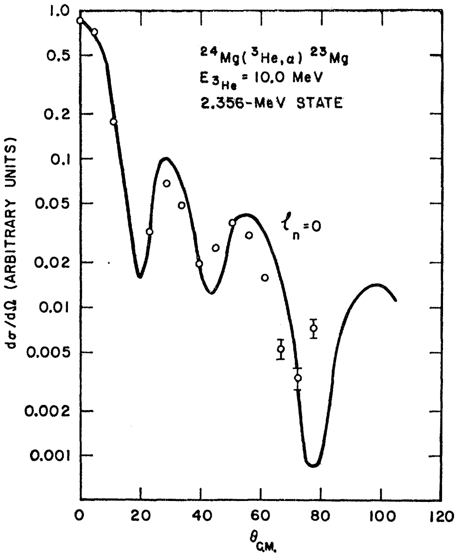


FIG. 4. Angular distributions of α particles from the ^{24}Mg(^{3}He,α)^{23}Mg reaction leading to the 2.356-MeV level in ^{23}Mg at an incident ^{3}He beam energy of 10 MeV. The curve is the DWBA prediction for a pickup with no angular momentum transfer ($l_n=0$). The relative errors in the measured cross sections are, when not indicated in the figure, about 10%.

When no higher terms than $P_2(\cos\theta)$ are present, the distribution should be a straight line when plotted against $\cos^2\theta$. No corrections have been made in Fig. 6 for the finite geometry of the γ-ray detector.

Fits to the experimental data were attempted for all possible values of spin up to $\frac{9}{2}$. Higher spin values were not considered, because for such values the lifetime limit set by the coincidence resolving time would require transition strengths of at least 1 000 Weisskopf units. The spin of the ground state was taken to be $\frac{3}{2}$. The results of the χ^2 fits given in Fig. 7 show that agreement with theory can only be obtained for spin assignments of $\frac{3}{2}$ and $\frac{5}{2}$. The minimum χ^2 values for the possible spin combinations and mixing ratios have been collected in Table IV.

TABLE III. Summary of the experimental angular correlations. The coefficients a_2/a_0 and a_4/a_0 obtained from the least-squares fit to the expansion $Y(\theta)=1+(a_2/a_0)P_2(\cos\theta)+(a_4/a_0)P_4(\cos\theta)$ have been corrected for finite geometry of the γ-ray detector.

Level (MeV)	Transition (MeV)	a_2/a_0	a_4/a_0
0.451	0.451–0	−0.57±0.07	−0.09±0.07
2.048	2.048–0.451	−0.71±0.05	0.01±0.07
	0.451–0	−0.50±0.11	−0.01±0.15
2.356	2.356–0	0.06±0.12	0.09±0.20
	2.356–0.451	0.01±0.09	0.00±0.12
	0.451–0	0.03±0.05	−0.01±0.08
2.768	2.768–0	−0.10±0.03	−0.17±0.04
2.904	2.904–0	0.07±0.04	0.02±0.05
	2.904–0.451	−0.11±0.05	0.02±0.07
	0.451–0	−0.25±0.04	0.02±0.06
3.792	3.792–0.451	−0.12±0.04	−0.02±0.06
	0.451–0	−0.51±0.09	−0.02±0.14

TABLE IV. Summary of the spin and mixing parameter assignments obtained in the present experiment together with their minimum χ^2 values. It has been assumed throughout that $J=\frac{5}{2}$ and $\delta_{10}=-0.075\pm0.024$ for the 0.451-MeV level. Confidence limits are $\chi^2=6.8$ for 0.1% and $\chi^2=4.7$ for 1%. J, J_1, and J_0 are the spins of the initial level, the first excited state, and the ground state, respectively. In cases where the 0.451-MeV γ ray is the second member of a cascade the minimum χ^2 values quoted are those within the range $\delta_{10}=-0.075\pm0.024$. The values of δ given in parentheses are those used for the unobserved cascade γ ray in the calculation of the theoretical correlation.

Level (MeV)	Transition (MeV)	Spin combination J	J_1	J_0	$(\chi^2)_{\min}$	Mixing ratio δ
0.451	0.451–0	$\frac{5}{2}$		$\frac{3}{2}$	0.35	−0.075±0.024
2.048	2.048–0.451	$\frac{7}{2}$	$\frac{5}{2}$		0.45	−0.182±0.025
		$\frac{3}{2}$	$\frac{5}{2}$		0.45	0.636±0.084
						or 2.72 ±0.50
	0.451–0	$\frac{7}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	0.15	(−0.182)
		$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	5.5	(0.636)
					21	(2.72)
2.356	2.356–0	$\frac{1}{2}$		$\frac{3}{2}$	0.46	undetermined
	2.356–0.451	$\frac{1}{2}$	$\frac{5}{2}$		0.20	undetermined
	0.451–0	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	0.30	(undetermined)
2.768–0	2.768–0	$\frac{1}{2}$		$\frac{3}{2}$	5.4	undetermined
		$\frac{3}{2}$		$\frac{3}{2}$	2.8	−0.306±0.022
					5.4	|δ|>50
2.904	2.904–0	$\frac{3}{2}$		$\frac{3}{2}$	0.91	−0.207±0.023
						or 20.7 ±9.5
		$\frac{5}{2}$		$\frac{3}{2}$	0.88	0.235±0.018
	2.904–0.451	$\frac{3}{2}$	$\frac{5}{2}$		2.4	0.010±0.039
		$\frac{5}{2}$	$\frac{5}{2}$		3.0	−0.517±0.053
	0.451–0	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	0.64	(0.010)
		$\frac{5}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	0.25	(−0.517)
3.792	3.792–0.451	$\frac{3}{2}$	$\frac{5}{2}$		0.13	0.011±0.037
						or −4.85 ±0.89
	0.451–0	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	0.3	(0.011)
					13	(−4.85)

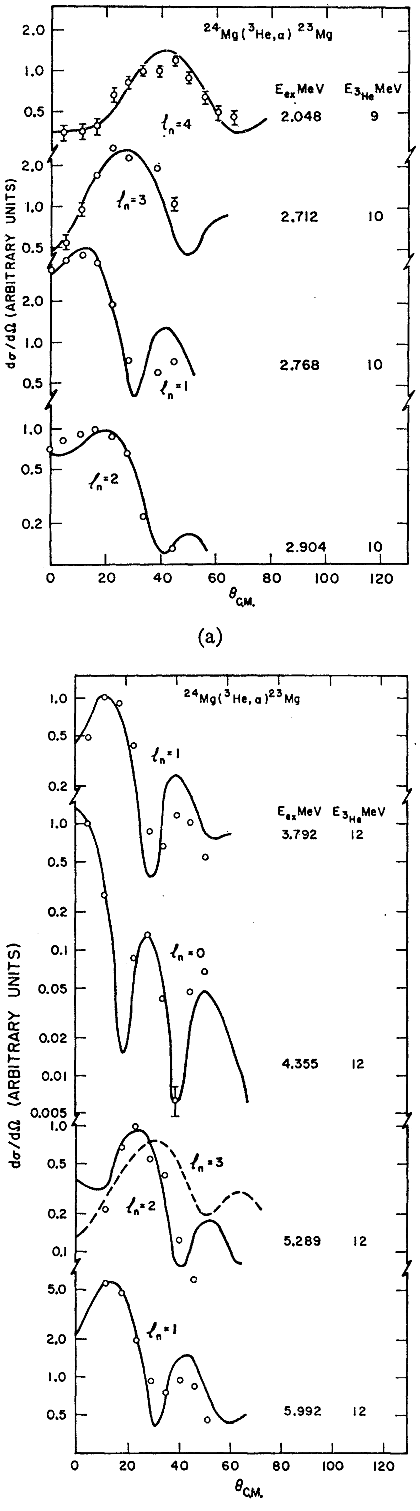
The angular distribution of the α particles to this level indicates $l_n=2$, which is consistent with the $J=\frac{3}{2}$ or $\frac{5}{2}$ assignment and also implies positive parity.

The choice between the $\frac{3}{2}$ and $\frac{5}{2}$ assignments cannot be made without additional information. We will see in the discussion of the 2.904-MeV level, however, that the cascade γ rays through the 0.451-MeV level can only be fitted satisfactorily for $J=\frac{5}{2}$ and $\delta_{10}=-0.075\pm0.024$.

B. The 2.048-MeV Level

It was found that the particle angular distribution for this level does not peak near 0° (Fig. 5) and that the level is weakly populated at all bombarding energies used in this experiment. It was therefore a time-consuming experiment to measure this α-γ correlation. Analysis of the sum of the spectra at the different angles (Fig. 8) showed that the dominant decay mode (85±5)% is through the first excited state with (15±5)% decaying to the ground state. The experimental data from the correlation measurements are collected in Table III. The ground-state transition was too weak to give a meaningful correlation.

In the analysis, the 0.451-MeV level was assumed to have a spin of $\frac{5}{2}$ and the ground state was assigned $J=\frac{3}{2}$. The χ^2 curves shown in Fig. 9 indicate that the only



(a)

(b)

FIG. 5. Angular distributions of α particles from the ^{24}Mg(^{3}He,α)^{23}Mg reaction leading to various excited states in ^{23}Mg. The curves are the DWBA predictions with the l_n values giving the best fits. The relative errors in the measured cross sections are, when not indicated in the figure, about 10%.

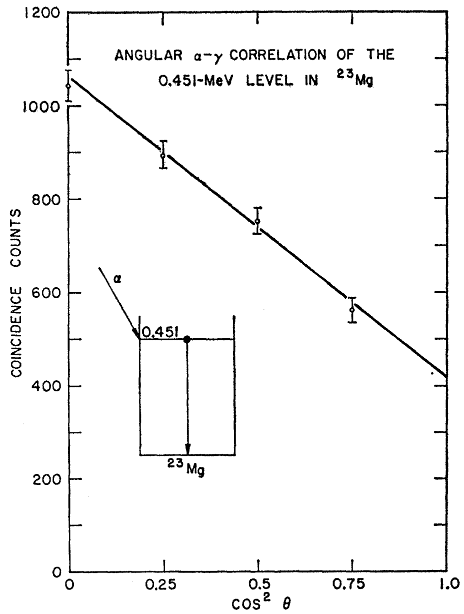


FIG. 6. Angular distribution of the 0.45-MeV γ rays in coincidence with the α particles leading to the 0.451-MeV level detected at 0°. The straight line fits well to the experimental points, which indicates that the a_4/a_0 coefficient must be small (see text and Table III).

possible solutions are $J=\frac{3}{2}$ with $\delta_{21}=0.636\pm0.084$ or 2.72 ± 0.50 and $J=\frac{7}{2}$ with $\delta_{21}=-0.182\pm0.025$. These results are summarized in Table IV. An analysis of the 0.451-MeV cascade γ-ray correlation is shown in Fig. 10. In this case we have used the measured values of δ_{21} and acceptable solutions must give values of δ_{10} in agreement with those measured previously (see Sec. V A). Only $J=\frac{7}{2}$ gives a good fit. The $J=\frac{3}{2}$ case with $\delta_{21}=2.72\pm0.50$ is ruled out, but the case with $\delta_{21}=0.636\pm0.084$ cannot be rigorously eliminated as $(\chi^2)_{\min}$ is just below the 0.1% confidence limit. We therefore assign the 2.048-MeV level $J=\frac{7}{2}$ but with $J=\frac{3}{2}$ still a possibility.

The second excited state is only weakly populated by the ^{24}Mg(^{3}He,α)^{23}Mg reaction and the possibility of processes other than neutron pickup is not small. The angular distribution was measured therefore at 9-MeV bombarding energy where the cross section has a minimum and it is possible that the influence of compound-nucleus formation was small. The angular distribution was fitted quite well by an $l_n=4$ pattern which would suggest that $J^\pi=\frac{7}{2}^+$ is the correct assignment. However, this evidence should not carry too much weight.

C. The 2.356-MeV Level

As was pointed out in a previous section, the angular distribution of the α particles showed an unmistakable $l_n=0$ transition to the 2.356-MeV level which unambiguously determines the spin and parity of that level to be $\frac{1}{2}^+$. At 0°, the energy of the α particles

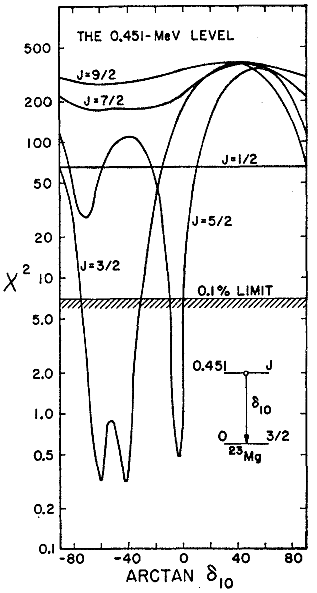


FIG. 7. χ^2 versus arctan δ_{10} for the γ-ray angular distribution observed in coincidence with α particles leading to the 0.451-MeV level in ^{23}Mg. Both $J=\frac{5}{2}$ and $\frac{3}{2}$ give acceptable fits.

populating the 2.356-MeV level was very close to the strong α-particle group to the ground state in ^{11}C. One could expect a small continuous buildup of carbon contamination on the target foil and thus an increase of that group. We were therefore forced to use very high spectrometer resolution to be sure that the pulses in the particle channel were due only to the 2.356-MeV level. From the summed coincident γ-ray spectrum, a ground-state transition $(30\pm5)\%$ and a transition $(70\pm5)\%$ to the first excited state were seen. The angular correlations were isotropic (Table III) within the statistical uncertainty; this result is consistent with a $J=\frac{1}{2}$ spin assignment to the 2.356-MeV level (Table IV).

D. The 2.712-MeV Level

At all bombarding energies used in the present investigation, this level was too weakly populated to allow us to obtain a meaningful correlation. Since the next level in ^{23}Mg is only 56-keV distant, thin targets and fairly high spectrometer resolution had to be used to resolve the groups. With the spectrometer at 10°, coincidence spectra were measured at 8.8-MeV beam energy at 100°, 130°, and 145° to the beam direction and the summed spectra indicated that $(65\pm10)\%$ of the decay goes via the 0.451-MeV state while the remaining $(35\pm10)\%$ decays to the 2.048-MeV state. Because of the low-particle yield and the high-singles γ-ray count rate, considerable pile-up occurred. However, an upper limit of 10% was placed on a possible ground-state transition. The large errors on the branching ratio are a result of poor statistics and the limited number of angles at which the spectra were measured.

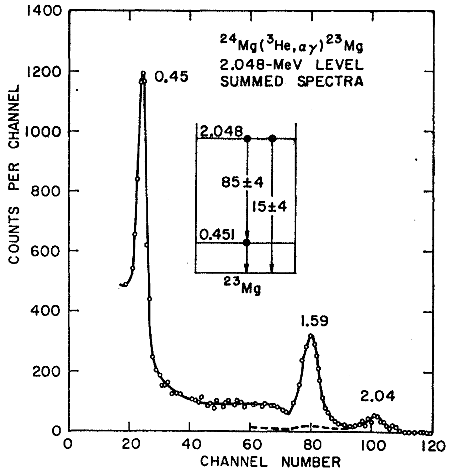


FIG. 8. The sum of the γ-ray spectra measured at 90°, 120°, 135°, and 150° in coincidence with α particles leading to the 2.048-MeV level in ^{23}Mg.

Analysis of the particle angular distribution showed an $l_n=3$ pickup pattern (Fig. 5). However, in view of the weakness of the reaction to this level, the fit may well be fortuitous. A comparison of the Weisskopf estimates of transition strengths and the measured branching ratio suggests that $J^\pi=\frac{9}{2}^+$ or $\frac{7}{2}^-$, the latter being in accordance with the l_n value.

E. The 2.768-MeV Level

The level was studied at 8-MeV bombarding energy. The coincidence spectra showed only a 2.77-MeV γ ray and a very weak peak at 0.511 MeV from coincident

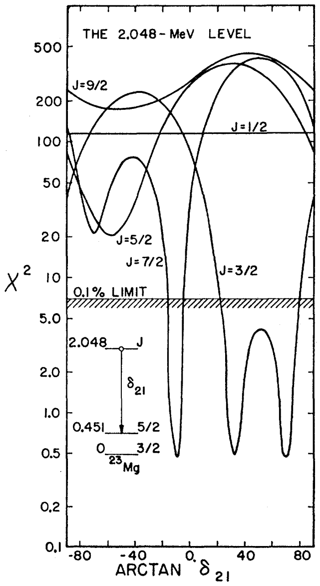


FIG. 9. χ^2 versus arctan δ_{21} for angular distribution of the 2.048–0.451-MeV γ-ray transition observed in coincidence with α particles leading to the 2.048-MeV level in ^{23}Mg. Both $J=\frac{7}{2}$ and $\frac{5}{2}$ give acceptable fits.

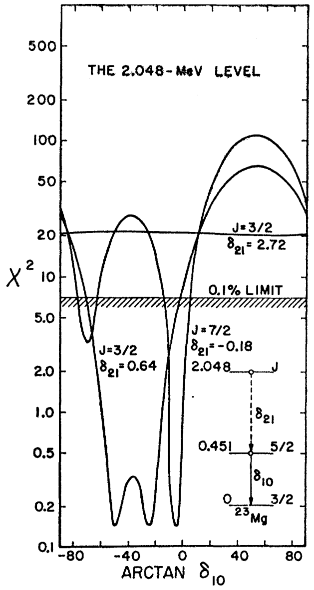


FIG. 10. χ^2 versus arctan δ_{10} for the angular distribution of the 0.451–0.0-MeV γ-ray transition observed in coincidence with α particles leading to the 2.048-MeV level in ^{23}Mg. Only a $\frac{7}{2}$ spin assignment for the 2.048-MeV level and a mixing ratio of the 2.048–0.451-MeV transition, $\delta_{21}=-0.18$, gives a value of δ_{10} consistent with that of $\delta_{10}=-0.075$ shown in Fig. 7. The $J=\frac{3}{2}$, $\delta_{21}=0.64$ values cannot, however, be ruled out rigorously since the corresponding χ^2 curve in this figure is slightly below the 0.1% limit for $\delta_{10}=-0.075$.

annihilation radiation. In order to confirm our γ-ray energy calibration, a measurement was made using α particles populating the 0.451-MeV level. It was concluded that the 2.768-MeV level decays 100% to the ground state; the branches to the 0.451- and 2.048-MeV levels each being less than 5%.

The X^2 analysis of the angular correlations eliminated $J=\frac{7}{2}$ and $\frac{9}{2}$ while giving fits for $J=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ with $(X^2)_{\min}$ between the 1% and 0.1% limits (Table IV). Although the correlation was nearly isotropic, the yields at 90° and 150° were both almost 10% lower than at 120° and 135°. In view of the moderate success in fitting these data, the correlation was repeated at a later date but with the same result.

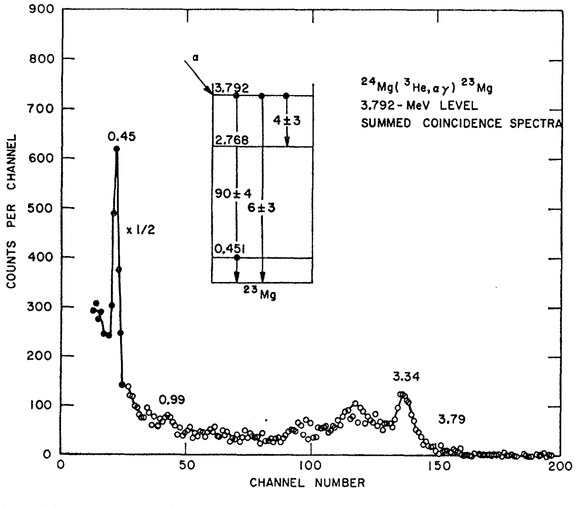


FIG. 11. The sum of the γ-ray spectra measured at 90°, 120°, 135° and 150° with α particles leading to the 3.792-MeV level.

An investigation of the particle spectrum was then made to see if the 2.768-MeV level was, in fact, a close doublet. The measurements at $\theta_L=45°$ showed a small broadening of the peak which might be interpreted as due to a very weakly populated level at about 2.80 MeV. However, this evidence is too weak to establish a new level beyond doubt.

If we assume a single level, the angular correlation is consistent with the assignments $J=\frac{1}{2}$ with δ undetermined, $J=\frac{3}{2}$ with $\delta=-0.306\pm0.022$ or $|\delta|>50$, and $J=\frac{5}{2}$ with $\delta=0.161\pm0.015$. The α-particle pickup pattern is consistent with $l_n=1$ which implies $J^\pi=\frac{1}{2}^-$ or $\frac{3}{2}^-$. In the latter case, $|\delta|>50$ can be ruled out as the transition must be predominantly $E1$. The value $\delta=-0.306\pm0.22$ is also suspect because this value implies a minimum component of 7.3% $M2$ radiation. This argument is, of course, not conclusive but suggests a $\frac{1}{2}^-$ assignment for the 2.768-MeV level.

F. The 2.904-MeV Level

The angular correlation was measured at 9-MeV bombarding energy. The summed coincidence spectra showed three γ rays which can be interpreted as a ground-state transition (63±5)% and a cascade (37±5)% through the 0.451-MeV level. In the analysis of this level we did not assume a spin for the 0.451-MeV

TABLE V. Summary of the angular-correlation analysis of the γ rays from the 0.451-MeV level and the cascade γ rays from the 2.904-MeV level. For the 0.451-MeV cascade γ rays the mixing parameter given in parentheses was used for the unobserved transition when calculating the theoretical angular correlation. The $(\chi^2)_{\min}$ value is the lowest point on the χ^2 curve throughout the range of δ_{10} indicated at the top of each section. A full explanation of the fitting procedure is given in the text.

Level (MeV)	Transition (MeV)	Spin combination J_6	J_1	J_0	$(\chi^2)_{\min}$	Mixing ratio δ
						δ_{10}
0.451	0.451–0		$\frac{5}{2}$	$\frac{3}{2}$	0.35	-0.075 ± 0.024
			$\frac{3}{2}$	$\frac{3}{2}$	0.32	-1.918 ± 0.380
						or -0.901 ± 0.147
	$J_1=\frac{5}{2}$	$\delta_{10}=-0.075\pm0.024$				δ_{61}
2.904	2.904–0.451	$\frac{3}{2}$	$\frac{5}{2}$		2.4	-4.85 ± 0.96
						or 0.010 ± 0.039
		$\frac{5}{2}$	$\frac{5}{2}$		3.0	-0.517 ± 0.053
	0.451–0	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	23	(-4.85)
					0.64	(0.010)
		$\frac{5}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	0.25	(-0.517)
	$J_1=\frac{3}{2}$	$\delta_{10}=-1.92\pm0.38$ and -0.901 ± 0.15				
	2.904–0.451	$\frac{3}{2}$	$\frac{3}{2}$		2.4	-13.4 ± 4.7
						or -0.339 ± 0.035
		$\frac{5}{2}$	$\frac{3}{2}$		2.4	0.141 ± 0.023
	0.451–0	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	75	(-13.4)
					8.5	(-0.339)
		$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	9.4	(-0.141)

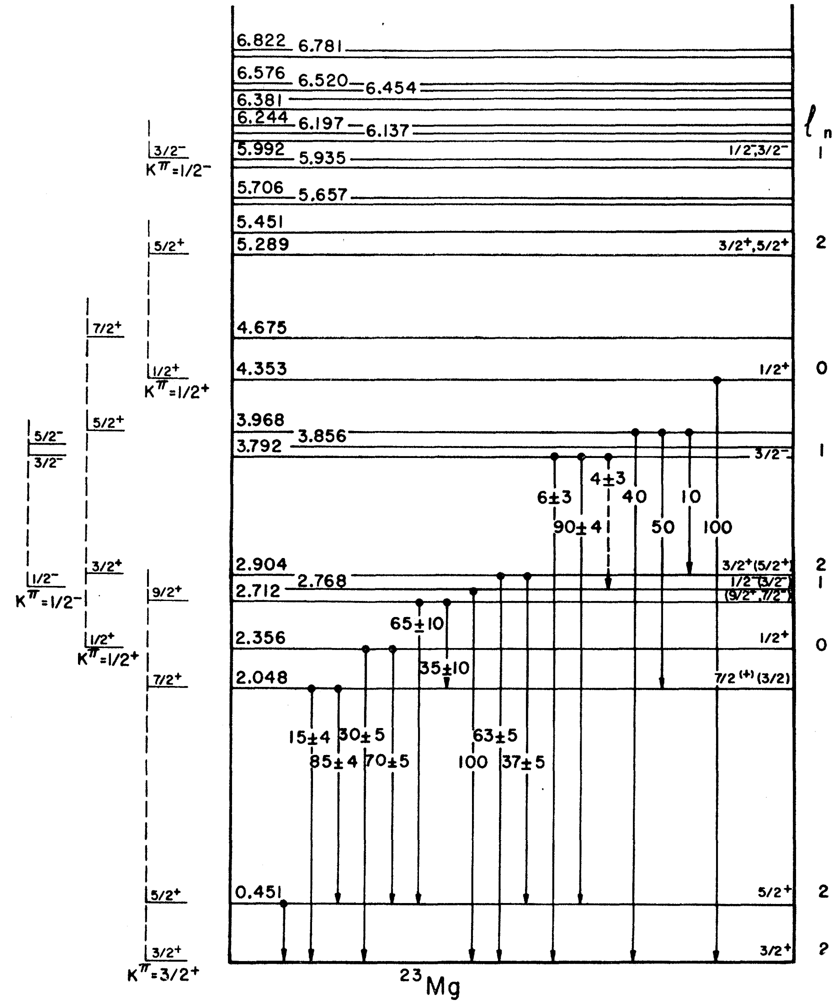


FIG. 12. A summary of the energy levels in ^{23}Mg and their γ-ray decay together with spin and parity assignments from the present investigation. For mixing values, see Table V and text. The rotational bands, as they are proposed in the discussion, are also shown.

level because we will use this case to show that the spin of the first excited state must be $\frac{5}{2}$.

If we first let $J=\frac{5}{2}$ for the 0.451-MeV level, the 2.904-MeV level can only be fitted for $J\leq\frac{5}{2}$. The spin $\frac{1}{2}$ assignment gives $(\chi^2)_{\min}$ slightly above the 1% confidence limit for the 2.904–0.451-MeV transition. However, the 0.451-MeV cascade γ ray is significantly anisotropic, and this rules against $J=\frac{1}{2}$. If we now assume a spin of $\frac{3}{2}$ for the 0.451-MeV level, we again find that $J=\frac{3}{2}$ or $\frac{5}{2}$ are the only possibilities for the 2.904-MeV level (see Table V).

Having established possible spins of the 2.904-MeV level and mixing parameters for the cascade transition to the first excited state, we can use these in an attempt to fit the angular correlation of the 0.451-MeV cascade γ ray. In this case we can only accept solutions which are consistent with the measured values of δ_{10} presented in Sec. V A. The results of this analysis, summarized in Table V, show conclusively that the spin of the 0.451-MeV level is $\frac{5}{2}$.

We conclude that both the $l_n=2$ pickup pattern and the angular correlation measurements indicate $J^\pi=\frac{3}{2}^+$ or $\frac{5}{2}^+$ for the 2.904-MeV level and that the spin of the 0.451-MeV level is $\frac{5}{2}$. These results, together with the appropriate mixing parameters, are summarized in Table IV.

G. The 3.792-MeV Level

The 3.792-MeV level was found to decay mainly to the 0.451-MeV level (90±4)% but a weak ground-state transition was also observed (6±3)% (Fig. 11). The small peak at 0.99 MeV can possibly be interpreted as a transition (4±3)% to the 2.768-MeV level and the ground-state transition from that level might be obscured by the 3.34-MeV γ ray. The angular correlations were anisotropic (Table III) excluding spin $\frac{1}{2}$. Only spin $\frac{9}{2}$ could be rejected by the χ^2 test (Table IV). Fits were obtained for $J=\frac{3}{2}$, with $\delta=-4.85\pm0.89$ or 0.011 ± 0.037, $J=\frac{5}{2}$ with $\delta=-0.522\pm0.051$ and $J=\frac{7}{2}$ with $\delta=0.100\pm0.018$. Since the 3.792-MeV level was strongly populated (Fig. 1), there is good reason to believe the $l_n=1$ value obtained from the fit to the particle distribution (Fig. 4). We claim, therefore, that $J^\pi=\frac{3}{2}^-$. The negative parity requires that the γ rays

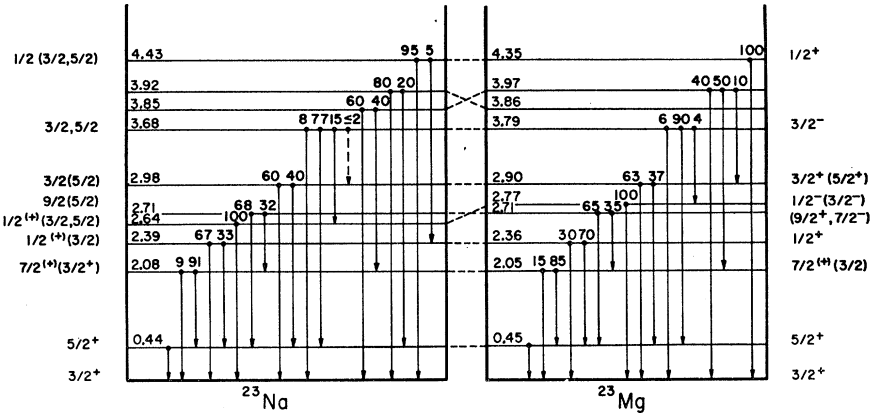


FIG. 13. Comparison of the level diagrams of ^{23}Na and ^{23}Mg. We have connected the probable analog states with dashed lines.

are an $M2/E1$ mixture and consequently we can reject the solution $\delta=-(4.85\pm0.89)$. This value can also be rejected on the grounds that it gives inconsistent results when fitting the 0.451-MeV cascade correlation (see Table IV).

H. γ Decay of the 3.968- and 4.353-MeV Levels

For some of the higher excited states, where the α-particle energy was close to the beam energy, sufficient ^{3}He ions were multiply scattered around the magnet to obscure the α-particle group. This effect could be overcome by moving the spectrometer away from 0°. However, the axial symmetry was then lost and only the γ-ray branching ratios could be determined.

The γ-ray spectrum from the 3.968-MeV level was measured at 9.4-MeV bombarding energy, with the magnetic spectrometer at 20° and the NaI crystal at 140° to the beam direction. A ground-state transition and a triple cascade consisting of 1.92-, 1.57-, and 0.45-MeV γ rays were clearly seen. There are two possible decay modes which can result in these energies; either through the 2.356- and 0.451-MeV levels or through the 2.048- and 0.451-MeV levels. We have already established, however, that the 2.356-MeV level has a 30% ground-state branch and since the number of counts around 2.35 MeV is very low, the former decay scheme can be eliminated. With the latter possibility, we would expect a small peak at 2.04 MeV, which does appear although it is somewhat masked by the large 1.92-MeV peak. Small peaks seen at 1.09, 2.40, and 2.90 MeV can be accounted for by a weak cascade through the 2.904-MeV level. A possible weak direct transition to the 0.451-MeV level would be hard to extract because it would be obscured by the strong ground-state transition. An upper limit of 10% of the ground-state transition could, however, be estimated. Since the γ-ray spectrum was studied at only one angle, the error in the branching ratios is likely to be substantial.

A comparison of the Weisskopf estimates of transition strengths and the measured branching ratios favors $\frac{5}{2}$ or $\frac{7}{2}^-$ assignments for the 3.968-MeV level.

The γ-ray spectrum of the 4.353-MeV level was measured with the spectrometer set at 5° and the NaI crystal at 125°. The α-particle angular distribution to this level showed a strong $l_n=0$ pattern which indicates a spin and parity of $\frac{1}{2}^+$. Therefore the γ rays will be isotropic and the branching ratios can be measured uniquely at any angle. Although the spectrum was somewhat distorted by γ rays from the first two levels in ^{15}O, it was clear that the 4.353-MeV level in ^{23}Mg decays almost exclusively to the ground state and the strength of all other possible transitions is less than 10%.

VI. SUMMARY OF THE EXPERIMENTAL RESULTS

Magnetic analysis of the α particles from the ^{24}Mg(^{3}He,α)^{23}Mg reaction has revealed all of the previously known levels in ^{23}Mg up to 4.35-MeV excitation. No new level was discovered in this energy range. However, sixteen new levels were found between 4.35- and 6.82-MeV excitation.

Alpha-particle angular distributions have been measured to many of these levels and a distorted-wave Born-approximation (DWBA) analysis made. Angular distributions which are fitted by $l_n=0$, 1, and 2 calculations are quite distinctive and are thought to give reliable assignments. The results as presented in Figs. 2, 4, and 5 are summarized in Fig. 12. The ground and first excited states were found to give $l_n=2$ patterns in agreement with previous work.[5]

Particle-γ angular correlations have been measured for most levels up to 4.353-MeV excitation and spin assignments, mixing parameters, and branching ratios have been determined. These are summarized in Table IV and Fig. 12.

A short report on the present investigation has been given earlier.[16] At that time our results on the angular distributions for the α particles to the ground state and the first two excited states were confirmed by Joyce

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[16] J. Dubois and L. G. Earwaker, Bull. Am. Phys. Soc. **11**, 908 (1966).

et al.[17] Recently, Kozub[18] reported a study of the ^{24}Mg$(p,d)^{23}$Mg reaction using high-energy protons from a sector-focused cyclotron. The deuteron angular distributions for levels in ^{23}Mg at 0.00, 0.45, and 5.32-MeV levels indicate $l_n=2$ pickup, while levels at 2.33 and 4.37 MeV have distributions characteristic of $l_n=0$ pickup, in agreement with our assignments.

Two recent investigations on particle-γ correlations using the ^{24}Mg(^{3}He,α)^{23}Mg reaction have also been reported. Haun *et al.*[19] assign $\frac{7}{2}$ to the 2.05-MeV level and find the angular correlations for the γ rays from the 2.36-MeV level to be isotropic. Lisle *et al.*[20] have obtained results for the 0.45, 2.05, 2.35, and 2.90 levels, which also agree very well with ours except for some minor differences for the 2.90-MeV level.

VII. DISCUSSION

In the present investigation, we have studied the γ decay of ^{23}Mg levels up to 4.35-MeV excitation energy. It is interesting to compare these results with those for the corresponding levels in the mirror nucleus ^{23}Na. The decay schemes for the two nuclei are shown in Fig. 13, the results for ^{23}Na being taken from the compilation by Endt and Van der Leun[6] and from recent papers by Poletti and Start[3] and by Wernbom-Selin and Arnell.[21] The decay of the 4.43-MeV level in ^{23}Na has been obtained from resonance fluorescence by Metzger.[22] The similarity between the decay schemes is very striking except for the third excited states. In two instances we can infer that the ordering of close doublets has been inverted. The close agreement of the branching ratios has stimulated us to examine the comparison in more detail, and in particular to compare the multipole mixing ratios for corresponding γ-ray transitions.

TABLE VI. Comparison between $E2/M1$ mixing ratios for corresponding γ-ray transitions in the ^{23}Na and ^{23}Mg mirror nuclei. Levels with isotropic angular correlations have not been included.

Transition E_a-E_b ^{23}Na (MeV)	^{23}Mg (MeV)	Spin combination $J_a{}^\pi$	$J_b{}^\pi$		Mixing ratio $\delta(E2/M1)$ ^{23}Na	^{23}Mg
0.44–0	0.45–0	$\frac{5}{2}+$	$\frac{3}{2}+$		$+0.08\pm0.02$	-0.08 ± 0.03
2.08–0.44	2.05–0.45	$\frac{7}{2}+$	$\frac{5}{2}+$		$+0.20\pm0.03$	-0.18 ± 0.03
2.98–0	2.90–0	$\frac{3}{2}+$	$\frac{3}{2}+$		$+0.11\pm0.05$	-0.21 ± 0.02
				or	$+2.70\pm0.40$	$+21\ \pm10$
		$\frac{5}{2}+$	$\frac{3}{2}+$		$+0.54\pm0.11$	$+0.24\pm0.02$
2.98–0.44	2.90–0.45	$\frac{3}{2}+$	$\frac{5}{2}+$		$-0.3\ \pm0.3$	$+0.01\pm0.04$
					$-3.0\ \pm2$	$-4.9\ \pm1.0$

It has been pointed out by Morpurgo[23] that if charge symmetry holds and Coulomb forces are neglected, then $E1$ and all magnetic multipole transitions between corresponding levels in mirror should have about equal strengths but opposite signs. When collective effects enhance the transition strengths for the $E2$ radiations, it is also to be expected[24] that the $E2/M1$ mixing ratios for corresponding mirror transitions will differ in sign. With the help of the parity assignments from the l_n values obtained in the present investigation, it is possible to select those transitions in ^{23}Mg which have an $E2/M1$ mixture. These are collected in Table VI together with the mixing ratios for corresponding transitions in ^{23}Na measured by Poletti and Start.[3] No information on the mixing ratios is available for the $J=\frac{1}{2}$ levels. From Table VI it is evident that the theoretical predictions are well fulfilled for the two lowest excited states. For the 2.90-MeV level in ^{23}Mg and the analog at 2.98 MeV in ^{23}Na, there are two spin possibilities, $\frac{3}{2}+$ and $\frac{5}{2}+$. The rule is only obeyed, however, for the $\frac{3}{2}+$ assignment. The results of many attempts to determine the spin of the 2.98-MeV level[3,6] in ^{23}Na are, in fact, more consistent with $\frac{3}{2}$ than $\frac{5}{2}$.

As was pointed out in the Introduction, it has been well established that nuclei with mass number around 25 possess a stable deformation. Many attempts have been made to fit the levels of ^{23}Na into rotational bands and it is fairly well established that the ground state and the first two excited states form a $K^\pi=\frac{3}{2}+$ rotational band with the last proton being in Nilsson orbit 7. It is possible that the 2.71-MeV level is the $\frac{9}{2}+$ member of the band, and the strong similarity of the decay schemes suggests that the 2.712-MeV level in ^{23}Mg is the analog.

The ^{24}Mg(^{3}He,α)^{23}Mg reaction should populate the first two members of the ground-state rotational band rather strongly since they can be formed by direct pickup of one of the two $d_{3/2}$ neutrons in the ^{24}Mg target nucleus. Using Satchler's theory[25] on the probabilities of populating different members of a rotational band by a direct reaction, we can predict that the first-excited state should be more strongly fed than the ground state and that the $\frac{7}{2}+$ and $\frac{9}{2}+$ members of the band should not be populated at all. These predictions are largely substantiated by the experimental results except that the $\frac{7}{2}+$ and $\frac{9}{2}+$ levels also were populated. This can be explained, however, by postulating double excitation and compound nucleus formation, or perhaps direct pickup of higher-angular-momentum neutrons mixed into the predominantly $l_n=2$ ground state of ^{24}Mg. We have assumed that the 2.712-MeV level is the $\frac{9}{2}+$ member of the band for this comparison.

The strengths of the γ-ray transitions within the members of the ground-state rotational band can be calculated and gives us another possible test of the validity of applying the collective model to ^{23}Mg. If the

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[22] F. R. Metzger, Phys. Rev. **136**, B374 (1964).

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[25] G. R. Satchler, Ann. Phys. (N. Y.) **3**, 275 (1958).

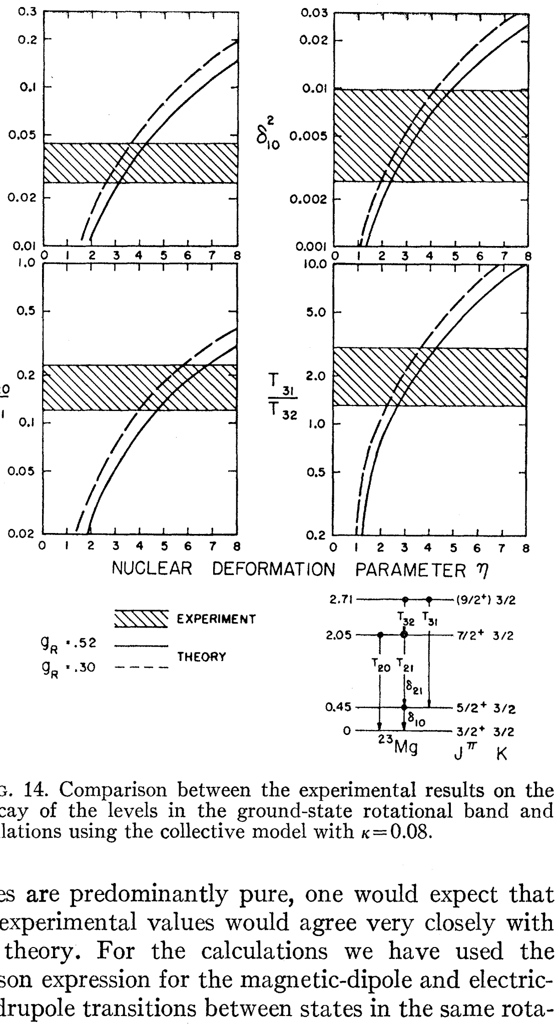

FIG. 14. Comparison between the experimental results on the γ decay of the levels in the ground-state rotational band and calculations using the collective model with $\kappa=0.08$.

states are predominantly pure, one would expect that the experimental values would agree very closely with the theory. For the calculations we have used the Nilsson expression for the magnetic-dipole and electric-quadrupole transitions between states in the same rotational band as given by Howard *et al.*[2] The gyromagnetic ratio of the core, g_R, enters into the calculations. The collective model predicts $g_R \cong Z/A$ but empirical evidence favors $g_R \cong 0.3$ in this mass region.[26] The strength of the spin-orbit coupling term in the nuclear potential is determined by the constant κ, which is assumed to be a slowly varying function of mass number. The calculations were performed using $\kappa=0.08$ and 0.10. In Fig. 14 the theoretical predictions are presented as a function of the nuclear deformation together with the experimental results. Only theoretical curves corresponding to $\kappa=0.08$ and $g_R=0.52$ and 0.30 are shown. When using $\kappa=0.10$, the curves are shifted upwards slightly. From Fig. 14 we conclude that the γ decay of the levels belonging to the ground-state rotational band in ^{23}Mg can be well fitted to the predictions of the collective model, assuming a deformation η between 3 and 4. The only result which shows poor agreement is the branching ratio for the $\frac{7}{2}^+$ level.

Poletti and Start,[3] in their investigation on ^{23}Na, also made predictions for the transition strengths for the two first excited states in ^{23}Mg using $\eta=4$. They predict the minus sign for the mixing ratios in agreement with experiment, but their magnitudes are about a factor 2 too large.

We realize that the theoretical predictions for the strengths of the electromagnetic transitions are strongly dependent on the choice of the parameters κ, g_R, and η. However, by calculating the ratios between two $M1$ or two $E2$ transitions, the dependence on the parameters can be eliminated. Since the absolute strengths of the $M1$ and $E2$ transitions are not yet known, we have to combine the $E2$ and $M1$ ratios to be able to compare:

$$\frac{\delta_{21}^2}{\delta_{10}^2}=\frac{T_{21}(E2)}{T_{21}(M1)}\,\frac{T_{10}(M1)}{T_{10}(E2)}$$

$$=\frac{E_{21}^2}{E_{10}^2}\,\frac{\langle J_2 2\tfrac{3}{2}0|J_1\tfrac{3}{2}\rangle^2\langle J_1 1\tfrac{3}{2}0|J_0\tfrac{3}{2}\rangle^2}{\langle J_2 1\tfrac{3}{2}0|J_1\tfrac{3}{2}\rangle^2\langle J_1 2\tfrac{3}{2}0|J_0\tfrac{3}{2}\rangle^2}.$$

In the present investigation we obtained $\delta_{10}=-0.075\pm0.024$ and $\delta_{21}=-0.182\pm0.025$ and that gives us an experimental ratio of 5.9 in very good agreement with the theoretical predictions of 5.9. However, as can be seen, the experimental mixing ratios are subject to appreciable errors. If we apply the formula to ^{23}Na, where[3] $\delta_{10}=0.08\pm0.02$ and $\delta_{21}=0.20\pm0.03$, we get a theoretical value of 6.5, which compares favorably with the experimental value of 6.3.

The good agreement between experiment and the predictions of the collective model for γ-ray transitions within the ground-state rotational band makes it encouraging to look for additional rotational bands in ^{23}Mg. An inspection of the Nilsson diagram suggests that there should be two $K^\pi=\frac{1}{2}^+$ band heads, fairly close to the ground state of ^{23}Mg, one due to promoting the odd neutron into orbit 9 and the other by creating a neutron hole in orbit 6. Calculations of the total binding energy of ^{23}Mg as a function of deformation suggest that for stable deformations the $\frac{1}{2}^+$ state associated with a neutron in orbit 9 should be 2 or 3 MeV higher in energy than the ground state in which the neutron is in orbit 7 ($K^\pi=\frac{3}{2}^+$). The $\frac{1}{2}^+$ level corresponding to the hole in orbit 6 should be another 2 MeV higher in excitation energy. With the ^{24}Mg(^{3}He,α)^{23}Mg reaction, one should expect to populate the hole state very strongly by a direct pickup process while the particle state, if pure, should not show strong direct-reaction behavior.

In the present experiment, however, we observe two strong $l_n=0$ transitions to $\frac{1}{2}^+$ levels at 2.356 and 4.353 MeV, and it would appear that there is considerable mixing of the two $K=\frac{1}{2}^+$ bands. It is also possible, however, that the ^{24}Mg target nucleus in its ground state has a large component consisting of a neutron pair promoted into orbit 9.

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[26] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter 1, No. 8 (1959).

The level spacings between levels in a $K=\frac{1}{2}$ band are dependent on the decoupling parameter a, which can be calculated from the Nilsson wave functions. A $K^\pi=\frac{1}{2}^+$ band based on orbit 9 should have a decoupling parameter close to 0 for $\eta=4$, so that even if Coriolis mixing of different bands perturbs the level spacings, the 2.904-MeV level is the only reasonable candidate for the $\frac{3}{2}^+$ member of a $K^\pi=\frac{1}{2}^+$ band based on the 2.356-MeV level. Furthermore, the 3.968-MeV level is a possible candidate for the $\frac{5}{2}^+$ member. In agreement with Satchler's theory, it was not populated as strongly as the $\frac{3}{2}^+$ member, and in spite of the comparatively low-energy difference, it has a γ-ray branch to the $\frac{3}{2}^+$ member of the same band. Using the experimental values $\hbar^2/2I=0.17$ MeV (here, I is the moment of inertia) and the decoupling constant $a=-0.26$ obtained from the three lowest members of this band, we should expect the $\frac{7}{2}^+$ member at about 4.80 MeV if Coriolis mixing with other bands does not perturb the level position too much. The only level discovered in that region in the present investigation is at 4.675 MeV. It was weakly populated (Fig. 1) as is expected from Satchler's theory and the α particles to that level showed an angular distribution which might be interpreted as a $l_n=4$ angular-momentum transfer. The corresponding level in ^{23}Na is probably at 4.78 MeV and a $\frac{7}{2}^+$ assignment[6] is in fact favored for that level.

The $K^\pi=\frac{1}{2}^+$ band based on orbit 6 should have a decoupling parameter of about 2, which means that the $\frac{3}{2}^+$ member will move up and the $\frac{5}{2}^+$ member down in energy so that the level order should be $\frac{1}{2}^+$, $\frac{5}{2}^+$, and $\frac{3}{2}^+$. Satchler's theory predicts that the $\frac{1}{2}^+$ and the $\frac{5}{2}^+$ members should be much more strongly populated than the $\frac{3}{2}^+$ member. With the $\frac{1}{2}^+$ level at 4.353 MeV, the strongly populated level at 5.289 MeV with $l_n=2$ could be the $\frac{5}{2}^+$ member of that band.

Levels with odd parity ($l_n=1$) were found at 2.768, 3.792, and 5.992 MeV. From the Nilsson diagram, it is evident that odd-parity levels can be formed by promoting the odd neutron into orbit 14 ($K^\pi=\frac{1}{2}^-$) or by lifting a neutron up from orbit 4 ($K^\pi=\frac{1}{2}^-$). The ^{24}Mg(^{3}He,α)^{23}Mg reaction should preferentially populate the members of the rotational band based on the latter, but in order to explain all three $l_n=1$ levels we have to assume that both $K^\pi=\frac{1}{2}^-$ bands are populated. Calculation of the binding energy suggests that the band based on orbit 14 should be lower in energy than that based on orbit 4. Orbit 14 has, however, a large negative decoupling constant which suggests that the $\frac{3}{2}^-$ member of the $K^\pi=\frac{1}{2}^-$ rotational band should, if unperturbed by band mixing, be the lowest member with a close-lying $\frac{7}{2}^-$ member. Indeed, the possible $l_n=3$ assignment for the 2.712-MeV level may indicate that it is the $\frac{7}{2}^-$ member; the next negative parity state at 3.792 MeV should then be $\frac{1}{2}^-$, which is not the case.

A $K^\pi=\frac{1}{2}^-$ band based on orbit 4, on the other hand, has a decoupling constant equal to 0.63 for $\eta=4$, which accounts fairly well for 2.768 ($\frac{1}{2}^-$)- and 3.792 ($\frac{3}{2}^-$)-MeV levels, leaving the 2.712-MeV level as the $\frac{9}{2}^+$ member of the ground-state rotational band. The $\frac{1}{2}^-$ assignment for the 2.768-MeV level is somewhat strengthened by the fact that the $\frac{3}{2}^-$ assignment requires improbably large $M2/E1$ mixing ratios for the ground-state transition (Table IV), and furthermore, we should expect a measurable γ-ray transition to the 0.451-MeV level. The $\frac{5}{2}^-$ member must be close to the $\frac{3}{2}^-$ member; it could be the level at 3.856. Even though the level was weakly populated, the measured angular distribution of the α particle at 12-MeV bombarding energy could fairly well be fitted to an $l_n=3$ angular-momentum transfer. The proposed analog state in ^{23}Na at 3.92 MeV shows a γ decay which is consistent with that expected from a $\frac{5}{2}^-$ level. From the first three members proposed for the $K^\pi=\frac{1}{2}^-$ band in ^{23}Mg, we obtain $\hbar^2/2I=0.18$ MeV and $a=0.94$. We can therefore expect a close-lying $\frac{7}{2}^-$ and $\frac{9}{2}^-$ doublet at about 6.5 MeV belonging to the same band.

The third $l_n=1$ level at 5.992 MeV could then possibly be the $\frac{1}{2}^-$ or $\frac{3}{2}^-$ member of a rotational band based on orbit 14.

In conclusion, it seems from this discussion that the collective model provides satisfactory explanation of the spins and parities for all known levels in ^{23}Mg up to 5 MeV and also for the dynamic characteristics as they are revealed in the γ decays of the levels for the ground-state rotational band. For the levels in the higher-lying rotational bands, a considerable band mixing must take place which makes a comparison of the dynamic behavior of the levels with theory less straightforward.

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