tensor and two-body spin-orbit as well as central components, can provide a satisfactory description of the observed level energies. A less restricted parametrization of the interaction is therefore considered, with phenomenological corrections in relative s states. A least-squares fit to observed level energies of the isotopes of Ni^{58} to Ni^{62} then yields an interaction which provides an acceptable description of the level structure of the Ni isotopes from Ni^{58} to Ni^{65}. This best-fit interaction is shown to indicate repulsive interactions between identical-nucleon shells and to conserve seniority to a useful degree of approximation. The interaction matrix elements are compared with those of an approximate reaction matrix calculated by Kuo from Hamada and Johnston's free-nucleon interaction; a fair degree of agreement is found when allowance is made for core excitations wherein a single nucleon is promoted from a core orbit to a valence or empty orbit.

Core excitation has a large influence on the effective interaction; it is therefore to be expected that a treatment of moments and transition rates within the configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$ will demand the introduction of effective operators, explicitly modified by the influence of neglected configurations. Analysis of the model and its predictions for transition rates confirms this expectation. With unmodified operators, the model is quite unable to account for the large deviations of the observed magnetic moments from the Schmidt values and the observation that the stripping strength into a given orbit is spread over several states of the residual nucleus. In both cases, the goodness of the seniority quantum number hampers agreement with experiment. Core excitation likewise exerts a large influence on quadrupole moments and on $E2$ transition rates and branching ratios. Here, however, good agreement with experiment can be attained by introducing an effective neutron charge of between $1.5e$ and $2e$.

PHYSICAL REVIEW VOLUME 160, NUMBER 4 20 AUGUST 1967

Shell-Model Calculations for $N=30$ Nuclei*

J. B. McGrory
Oak Ridge National Laboratory, Oak Ridge, Tennessee

(Received 27 February 1967)

The results of shell-model calculations on $N=30$ nuclei with $20\leq Z\leq 27$ are reported. An inert ^{48}Ca core is assumed, protons are restricted to the $1f_{7/2}$ shell, and the two active neutrons can occupy the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ orbits. A Hamiltonian is used which leads to a good fit to selected experimental data in this same mass region. For all nuclei treated here except ^{57}Co, the agreement of the calculated spectra with experimental spectra for states below an excitation energy of about 2.5 MeV is satisfactory. The shell-model wave functions are used to calculate spectroscopic factors for one- and two-nucleon transfer reactions which involve ^{54}Cr, ^{55}Mn, and ^{56}Fe. The qualitative results for the one-nucleon transfer reactions are in satisfactory agreement with the factors extracted from experimental data. The predicted relative strengths for strong transitions in these reactions are not in good agreement with the extracted numbers. The quantitative agreement between the predicted strengths and the strengths extracted from experiment for the $^{54}Fe(t,p)^{56}Fe$ reaction is good.

I. INTRODUCTION

THE degree of success which one can reasonably expect in a shell-model calculation of the structure of a given nucleus depends, in large measure, on the degree to which the assumed closed core is really closed. Some of the difficulties found in shell-model calculations in which ^{16}O and ^{40}Ca are assumed to be closed cores come about because these nuclei are not really good closed cores.[1] There is evidence that ^{48}Ca forms a good closed core. Experimental data on the $^{48}Ca(d,p)^{49}Ca$ (Ref. 2) and $^{48}Ca(p,d)^{47}Ca$ (Ref. 3) reactions indicate that there is a negligible amount of core excitation in ^{48}Ca. Furthermore, shell-model calculations[4,5] of a number of properties of $N=28$ nuclei with $20<Z<28$ have shown that a good account of these properties is given when a ^{48}Ca core is assumed, and protons are restricted to the $\pi f_{7/2}$ shell.[6] A closed-shell ^{48}Ca core is also assumed by Vervier[7] in treating $N=29$ nuclei. He restricts protons to the $\pi f_{7/2}$ shell, and allows the extra core neutrons to occupy the $\nu p_{3/2}$, $\nu p_{1/2}$, and $\nu f_{5/2}$ shells. He found good agreement with experiment for a large number of energy levels, spectroscopic factors and some β decay and electromagnetic-transition rates.

* Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.

[1] G. E. Brown and A. M. Green, Nucl. Phys. **85**, 87 (1966).

[2] E. Kashy, A. Sperduto, H. A. Enge, and W. W. Buechner, Phys. Rev. **135**, B765 (1964).

[3] T. W. Conlon, B. F. Bayman, and E. Kashy, Phys. Rev. **144**, 940 (1966).

[4] S. Goldstein and I. Talmi, Phys. Rev. **105**, 995 (1957).

[5] I. Talmi, Phys. Rev. **126**, 1096 (1962).

[6] Principal quantum numbers will generally be omitted here. The active orbits included are the $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$. π and ν will denote proton and neutron, respectively.

[7] J. Vervier, Nucl. Phys. **78**, 497 (1966).

In this paper, the results of shell-model calculations for $N=30$ nuclei are presented. A ^{48}Ca core is assumed, and configuration mixing for the two extra core neutrons is allowed. The active neutron orbits which are included are the $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ shells. A calculation of a few $N=30$ nuclei[7] in which no neutron configuration mixing is included has been reported. The results suggest that effects of configuration mixing may be very important.

In Sec. II, the underlying assumptions of this calculation are discussed. In Sec. III, the results of energy-level calculations are presented, and they are compared with the experimentally observed spectra. Some preliminary results of the energy-level calculations have been reported previously.[8] For most of the nuclei treated, the calculated spectra compare quite well with experiment. In Sec. IV, the theoretical wave functions are further tested by comparing predicted spectroscopic factors with factors extracted from experiment for nucleon-transfer reactions involving ^{54}Cr, ^{55}Mn, and ^{56}Fe. The gross features of the extracted data are reproduced by the calculation. The quantitative results, in particular the relative strengths of strong transitions to low-lying states in single-neutron-transfer reactions, are not good. In Sec. V there is a summary and discussion of the results.

II. METHOD OF CALCULATION

As indicated above, a ^{48}Ca closed-shell core is assumed here. Thus, the $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, and $2s_{1/2}$ single-particle orbits are completely filled as is the $\nu f_{7/2}$ shell. The protons are restricted to the $f_{7/2}$ shell. However, there is experimental evidence that these assumptions are too restrictive. In some $N=28$ nuclei there are well-known excited states which could arise only from excitations of nucleons from the ^{48}Ca core. One example is the spectrum of ^{48}Ca itself,[9] shown in Fig. 1. Here the first-excited $J=0^+$ state is at 4.3 MeV. In ^{52}Cr, there is a $J=0^+$ state at 2.65 MeV,[10] and in ^{54}Fe, there are extra states starting at 3.16 MeV.[11] One hopes that these excited states are at a high enough energy that they do not mix strongly with the low-lying states. The final restriction is that the neutrons may occupy only the $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ single-particle orbits. These states are strongly populated in the ^{48}Ca$(d,p)^{40}$Ca reaction.[2] States are populated in this reaction by $l=2$ and $l=4$ transitions. These even-parity states are at about the same energy as the $\nu f_{5/2}$ single-particle state, which is included in the calculation. Only configurations in which at least two particles are

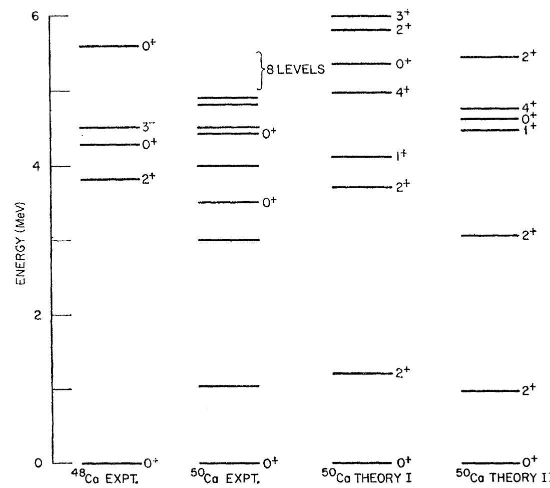

FIG. 1. Comparison of experimental and theoretical excitation spectra for ^{50}Ca.

excited to the even-parity single-particle states can affect the nuclear levels which are discussed here. As the $\pi f_{7/2}$ orbit fills, the $\nu f_{5/2}$ level is lowered rapidly, while the even-parity levels are probably not significantly shifted. Thus, the importance of the $l=2$ and $l=4$ states decreases as Z increases from 20–27.

Under these assumptions, the wave functions of states in a nucleus with Z protons and 2 neutrons outside the core are written as linear combinations of basic states of the form

$$\psi^J=\{(\pi f_{7/2}{}^Z)_{J_\pi}(\nu j_1\times\nu j_2)_{J_\nu}\}^J\,,$$

where j_1 and j_2 are the orbits occupied by the neutrons. The Hamiltonian in this space then has the form

$$\mathcal{H}=Z\epsilon(\pi_{7/2})+\epsilon(\nu j_1)+\epsilon(\nu j_2)+\mathcal{H}_{\pi-\pi}+\mathcal{H}_{\nu-\nu}\times\mathcal{H}_{\pi-\nu}.$$

Here $\epsilon(\pi f_{7/2})$ is the single-particle energy of a proton in the $f_{7/2}$ shell, and $\epsilon(\nu j_i)$ is the single-particle energy of a neutron in the orbit j_i. We will discuss only excitation energies, and therefore set $\epsilon(\pi f_{7/2})=0$ and $\epsilon(\nu p_{3/2})=0$. The neutron single-particle excitations are taken from the ^{49}Ca experimental spectrum.[2]

$$\epsilon(\nu p_{1/2})=2.07\text{ MeV}\quad\text{and}\quad\epsilon(\nu f_{5/2})=3.96\text{ MeV}.$$

The two-body matrix elements and single-particle energies are kept constant for all the $N=30$ nuclei.

$\mathcal{H}_{\pi-\pi}$ is taken from spectroscopic data. In the space which we assume, the states of ^{50}Ti and ^{54}Fe are $(\pi f_{7/2}{}^2)^J$ and $(\pi f_{7/2}{}^{-2})^J$, respectively. If the assumptions of an inert ^{48}Ca core and the constancy of the two-body matrix elements for all $N=30$ nuclei were true, then each of these nuclei would have only four states, with $J=0^+$, 2^+, 4^+ and 6^+, and the relative spacings of these two nuclei would be identical. It has been pointed out above that there are extra states in ^{54}Fe, as there are in ^{50}Ti (Ref. 10). However, the spectra of the first $J=0^+$, 2^+, 4^+ and 6^+ states of the two nuclei

[8] J. B. McGrory, Phys. Letters **21**, 64 (1966).
[9] S. Hinds, J. H. Bjerregaard, O. Hansen, and O. Nathan, Phys. Letters **21**, 328 (1966).
[10] *Nuclear Data Sheets*, compiled by K. Way *et al.* (National Academy of Sciences—National Research Council, Washington 25, D. C.)
[11] M. F. Thomas, A. R. Poletti, and M. A. Grace, Nucl. Phys. **78**, 561 (1966).

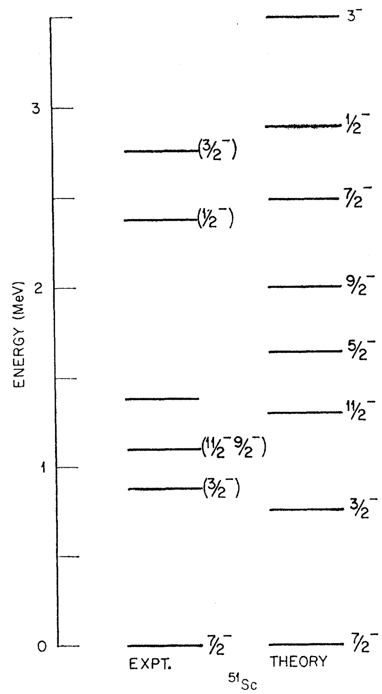

FIG. 2. Comparison of experimental and theoretical excitation spectra for ^{51}Sc.

are very similar: for ^{50}Ti (Ref. 12), $0^+(0)$, $2^+(1.57$ MeV), $4^+(2.70$ MeV), $6^+(3.22$ MeV); for ^{54}Fe (Ref. 11), $0^+(0)$, $2^+(1.41$ MeV), $4^+(2.55$ MeV), $6^+(2.97$ MeV). In the calculations which are reported here, $\mathcal{H}_{\pi-\pi}$ has been taken from ^{54}Fe. A number of the calculations have been repeated in which $\mathcal{H}_{\pi-\pi}$ is taken from ^{50}Ti, and the results are found to be insensitive to the differences in the two interactions.

$\mathcal{H}_{\nu-\nu}$ is taken from a shell-model calculation of the nickel isotopes by Cohen, Lawson, Macfarlane, Pandya, and Soga.[13] They assumed all neutron and proton orbits through the $f_{7/2}$ shell are filled, (i.e., a ^{56}Ni core) and that the heavier nickel isotopes can be described in terms of neutrons distributed across the $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ orbits. They searched for that $\mathcal{H}_{\nu-\nu}$ which minimizes the difference between the experimentally observed spectra and the calculated spectra. The resulting predicted spectra agrees with experiment to within about 200 keV. Since we use this $\mathcal{H}_{\nu-\nu}$, we can expect agreement only to within about 200 keV.

For $\mathcal{H}_{\pi-\nu}$, we use the interaction used by Vervier[7] in studying $N=29$ nuclei. He assumed $\mathcal{H}_{\pi-\nu}$ to be a delta function, and chose the force parameters as follows. In the space which is included here, ^{49}Ca and ^{57}Ni consist of one neutron outside the closed ^{48}Ca and ^{56}Ni cores, respectively. The low-lying spectra of these nuclei should represent the energies of the single-particle orbits outside these respective cores. From these spectra,[2] the following single-particle energies can be deduced:

$$^{49}\text{Ca}:\ \epsilon(\nu p_{3/2})=0,\quad \epsilon(\nu p_{1/2})=2.07\text{ MeV},\quad \epsilon(\nu f_{5/2})=3.96\text{ MeV}.$$

$$^{57}\text{Ni}:\ \epsilon(\nu p_{3/2})=0,\quad \epsilon(\nu f_{5/2})=0.80\text{ MeV},\quad \epsilon(\nu p_{1/2})=1.06\text{ MeV}.$$

There is a distinct difference between the two sets of single-particle energies, because the neutrons are interacting with different cores. Since the core is taken to be ^{48}Ca, the levels in ^{49}Ca are used for single-particle energies. It is assumed that the ^{57}Ni spectrum results from the single-particle energies of ^{49}Ca plus the interaction of the single neutron with the protons which are added in going from ^{48}Ca to ^{56}Ni. The interaction parameters were chosen to reproduce the spectrum of ^{57}Ni when one uses the ^{49}Ca single-particle energy levels.

The shell-model nuclear wave functions are found by diagonalizing $\mathcal{H}$ in the model space. The matrices in the calculations reported here vary in size up to 70×70. The calculation and diagonalization of the matrices was carried out with the use of the Oak Ridge–Rochester shell-model computer programs.[14]

III. RESULTS OF ENERGY-LEVEL CALCULATIONS

In Figs. 1–8, the energy-level spectra predicted by the calculation discussed in the previous section are com-

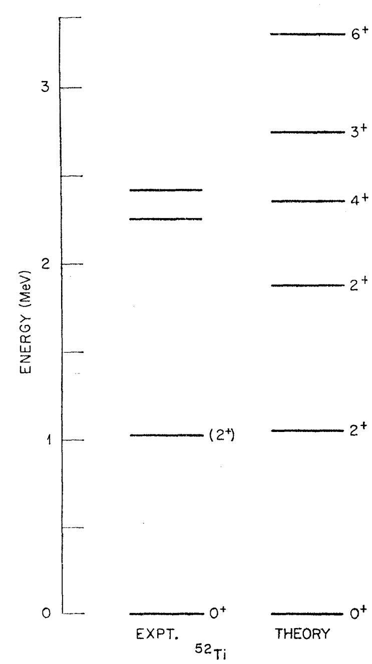

FIG. 3. Comparison of experimental and theoretical excitation spectra for ^{52}Ti.

[12] G. Chilosi *et al.*, Nuovo Cimento **27**, 86 (1963).

[13] S. Cohen, R. D. Lawson, M. H. Macfarlane, S. Pandya, and M. Soga, (to be published).

[14] J. B. French, E. C. Halbert, J. B. McGrory, and S. S. M. Wong (to be published).

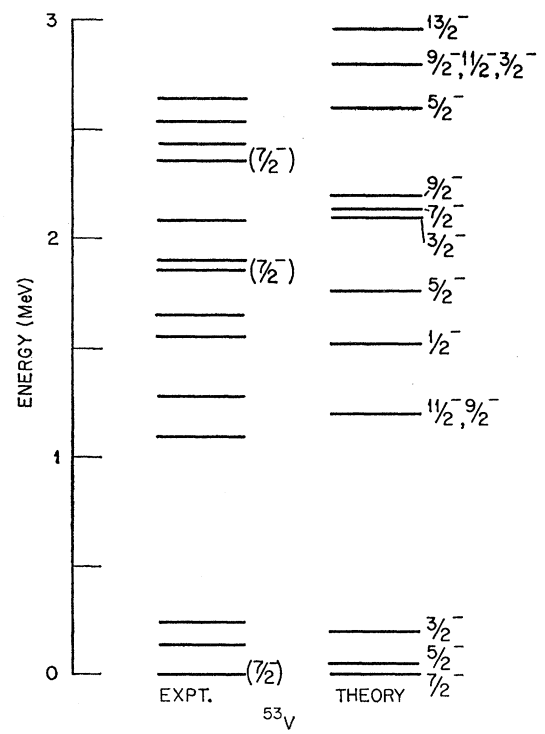

FIG. 4. Comparison of experimental and theoretical excitation spectra for ^{53}V.

pared with experimental spectra. The agreement for levels which lie in the energy region from 0–2 MeV is generally satisfactory, with the striking exception of ^{57}Co.

One general feature that appears in most of the calculated spectra is that the predicted levels lie slightly higher than the corresponding experimental levels. A possible explanation for this uniform deviation is that the excited states in this calculation may be more strongly affected than the ground state by the core-excited configurations which are omitted here, since the ground state is farther away in energy from the core-excited configurations.

We discuss here the results of the energy-level calculations in order of increasing Z.

A. ^{50}Ca

The predicted and experimental spectra for ^{50}Ca are compared in Fig. 1, where the prediction of the present calculation is called Theory I. In this same figure the experimental spectra of ^{48}Ca is also included. The experimental levels were observed in (t,p) reactions on ^{46}Ca and ^{48}Ca, respectively, by Hinds, Bjerregaard, Hansen, and Nathan.[15] The only firm spin assignments are for $J=0^+$ states. The energy of the first excited state in ^{50}Ca is well reproduced here. The agreement for the higher lying states is not so impressive. A similar calculation of ^{50}Ca has been reported by Vervier.[16] He assumes a configuration space identical to the one used here, but he uses a different $\mathcal{H}_{\nu-\nu}$. His predicted spectra is labeled Theory II in Fig. 1. He predicts a second $J=2^+$ state in excellent agreement with the level observed at 2.99 MeV, and a second $J=0^+$ state at 4.56 MeV compared to an experimental level at 4.47 MeV. On the basis of this agreement, it is suggested that there probably is not much mixing between ^{50}Ca shell-model states within the allowed configurations and excited states which come from core-excited configurations related to those seen in the excitation spectrum of ^{48}Ca. The results of our calculation could be interpreted to suggest that this is not quite true. The second $J=2^+$ shell-model state predicted by our model is virtually degenerate with the $J=2^+$ core-excited state at 3.83 MeV in ^{48}Ca. It is conceivable that these two levels do lie fairly close in energy, and that as a result of the mixing of the two states one would obtain two $J=2^+$ levels in good agreement with the levels in ^{50}Ca at 2.99 and 3.99 MeV. The situation is similar for the $J=0^+$ states. There are two excited $J=0^+$ states in the experimental spectrum of ^{50}Ca in Fig. 1, and only one excited $J=0^+$ state predicted in the same energy region. There are two $J=0^+$ states in ^{48}Ca. It is again possible that the mixing of the shell model and core-excited $J=0^+$ states can lead to good agreement with experiment. Indeed, it seems almost necessary to have significant mixing of these states if one attempts to

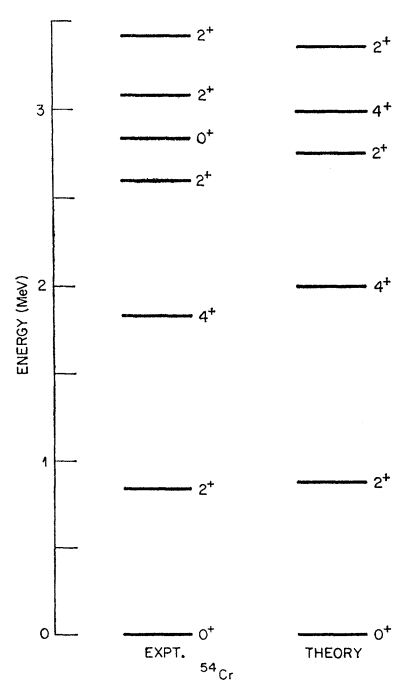

FIG. 5. Comparison of experimental and theoretical excitation spectra for ^{54}Cr.

[15] S. Hinds *et al.*, Phys. Letters **21**, 328 (1966).

[16] J. Vervier, Phys. Letters **22**, 82 (1966).

relate the first $J=0^+$ state in ^{50}Ca with the first $J=0^+$ state in ^{48}Ca, since the state in ^{50}Ca lies almost 1 MeV below the state in ^{48}Ca. The wave functions found by Vervier for the $J=0^+$ states give a reasonable qualitative explanation of the relative strengths of the transitions leading to these states in the ^{48}Ca$(t,p)^{50}$Ca reaction. The ground-state transition is strongly favored compared to the transitions to the excited $J=0^+$ states. The $J=0^+$ wave functions resulting from our calculation are very similar to Vervier's and would lead to a similar reasonable explanation of this data. The uncertainties of shell-model calculations such as these are too great to draw any firm conclusions as to the extent of mixing of core-excited and shell-model states in ^{50}Ca. A detailed analysis of the ^{48}Ca$(t,p)^{50}$Ca experiment might well provide more information on this question.

B. ^{51}Sc

In Fig. 2, the theoretically predicted and experimental spectra of ^{51}Sc are compared. The experimental spectrum is taken from the results of a study of the ^{48}Ca$(\alpha,p)^{51}$Sc reaction by Ginaven, Kossler, and Bernstein.[17] They obtained both excitation energies and angular distributions for transitions to the states shown in the figure. Details of the experiment and its analysis are reported elsewhere,[18] and so we give here only a summary of the analysis. The spin assignments shown in Fig. 2 were made both directly from a distorted-wave analysis of the (α,p) data, and from a qualitative comparison of these angular distributions with the proton distributions observed in the ^{40}Ca$(\alpha,p)^{43}$Sc reaction for transitions to states with known spins. The level ordering and energies of the first three predicted states are in satisfactory agreement with experiment. The observed transitions to the first three states are strong. The angular distribution for the transition to the third state, at 1.1 MeV, does not distinguish between a $J=\frac{9}{2}^-$ or a $J=11/2^-$ spin assignment, but the predicted transition strength for a transition to a $J=\frac{9}{2}^-$ state is weaker, by an order of magnitude, than the strength for a transition to a $J=11/2^-$ state, so that an assignment of $J=11/2^-$ is favored for this state. The predicted strengths for transitions to the $J=\frac{5}{2}^-$, $J=\frac{7}{2}^-$, and $J=\frac{9}{2}^-$ are all very small compared to the strengths to the first three states. Experimentally the transition to the state at 1.4 MeV is very weak. Thus, the experimental data are not inconsistent with the existence of these three predicted states, since it is quite reasonable that they might not be observed in the reaction. The agreement between theory and experiment can therefore be considered quite satisfactory for levels up to about 2 MeV in excitation.

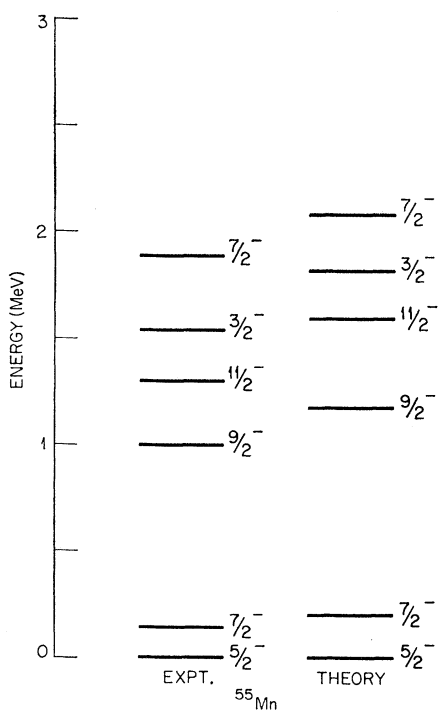

FIG. 6. Comparison of experimental and theoretical excitation spectra for ^{55}Mn.

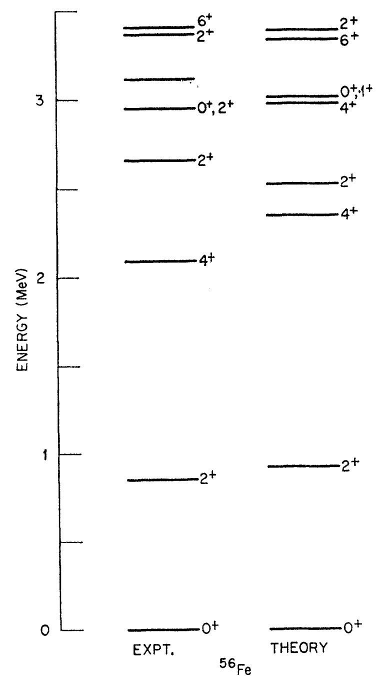

FIG. 7. Comparison of experimental and theoretical excitation spectra for ^{56}Fe.

C. ^{52}Ti

The experimental and theoretical spectra of ^{52}Ti are compared in Fig. 3. The ^{52}Ti spectra is taken from an

[17] R. O. Ginaven, W. J. Kossler, and A. M. Bernstein, Bull. Am. Phys. Soc. 12, 110 (1967).
[18] R. O. Ginaven, A. M. Bernstein, R. M. Drisko, and J. B. McGrory (to be published).

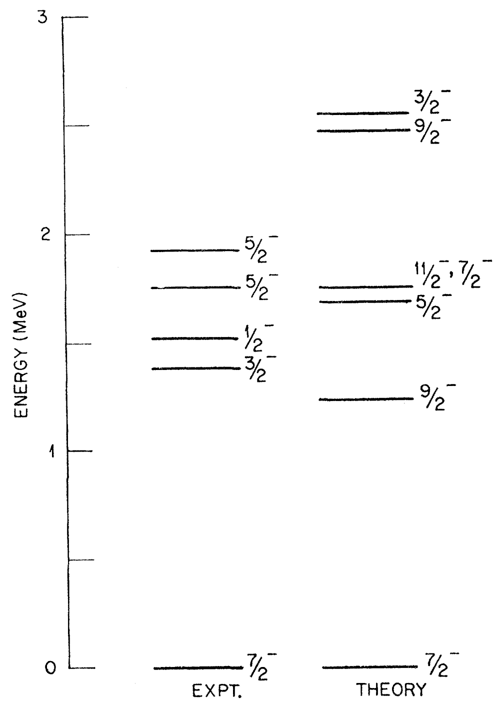

FIG. 8. Comparison of experimental and theoretical excitation spectra for ^{57}Co.

investigation of the ^{50}Ti(t,p)^{52}Ti reaction by Williams, Knight, and Leland.[19] They have reported the excitation energies of only three states, and suggest two spin assignments. The excitation energy of the first excited states is well reproduced by the calculation. The fact that the second-predicted $J=2^+$ state at 1.9 MeV lies below any possible experimental analog is somewhat surprising in view of the general trend discussed above. It is quite possible, in the case of ^{52}Ti, that a state corresponding to the $J=2^+$ state at 1.9 MeV was missed experimentally. Only 7% of the predicted wave function in this state can be excited in the (t,p) reaction, so that the cross section for the reaction may be too small to have permitted the observation of this state.

D. ^{53}V

The theoretical and experimental spectra of ^{53}V are compared in Fig. 4. The experimental data is taken from the work of Hinds and Middleton on the ^{51}V(t,p)^{53}V reaction.[20] There are a number of experimental levels observed in ^{53}V, but very few spin assignments. Only those states populated with $l=0$ transfers from the $J=\frac{7}{2}^-$ target ground state have been unambiguously assigned. The multiple spin assignments on some of the predicted levels indicates degeneracy of states with the indicated spin, and not an uncertainty in the spin of the theoretical level. The gross features of the experimental spectra are nicely reproduced by the theory, i.e., a low-lying triplet of states, a gap of about 1 MeV, and then a region comparatively densely populated with states. The most obvious shortcoming of the predicted spectra is that one of the $J=\frac{7}{2}^-$ states seen experimentally is not accounted for by the theory. The third experimental $J=\frac{7}{2}^-$ level occurs at a high enough energy that it is reasonable to expect that it is affected by proton core excitation. This point will be amplified below.

E. ^{54}Cr

The experimental spectra of ^{54}Cr, shown in Fig. 5, is one proposed by White[21] from the results of a study of the ^{53}Cr(n,γ)^{54}Cr reaction. The agreement between theory and experiment for levels up to about 2 MeV is again good. At around 3 MeV we fail to predict the $J=0^+$ and $J=2^+$ levels that are observed. Levels with these spins are observed at roughly the same energy in ^{52}Cr and ^{54}Fe (Ref. 10). In the space assumed here, the levels of these latter nuclei would be in the $(\pi f_{7/2}{}^4)$ and $(\pi f_{7/2}{}^6)$ configuration. As indicated above, the second $J=0^+$ and $J=2^+$ states in ^{54}Fe must be made up of configurations which are not included here. The spectra of ^{52}Cr has been calculated,[22] and, again, these levels around 3 MeV apparently cannot be explained in terms of $\pi f_{7/2}$ configurations. Presumably, they arise as a result of protons being excited to the $\pi p_{3/2}$ shell. It is reasonable to assume that these excitations will be important in ^{54}Cr and ^{56}Fe at about the same level of excitation. If we assume this explanation for the missing levels, then all levels in ^{54}Cr up to an energy of about 3 MeV could be accounted for.

F. ^{55}Mn

The ^{55}Mn spectrum, shown in Fig. 5, has been determined from fast-neutron inelastic-scattering experiments.[23] The predicted spectrum for this nucleus is in striking agreement with experiment. More will be said about the structure of these states in the discussion of nucleon transfer reactions below.

G. ^{56}Fe

There has been a large amount of data acquired in recent years on the energy levels of ^{56}Fe. The energy spectrum shown in Fig. 2 is a composite of these results.[24–26] Our calculation seems to account for most of

[19] D. C. Williams, J. D. Knight, and W. T. Leland, Phys. Letters **22**, 162 (1966).
[20] S. Hinds and R. Middleton (unpublished).
[21] D. H. White, Phys. Rev. **125**, 777 (1963).
[22] J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. **134**, B515 (1964).
[23] N. Nath, M. A. Rothman, D. M. Van Patter, and C. E. Mandeville, Nucl. Phys. **13**, 74 (1959).
[24] M. H. Shapiro, P. F. Hinrichsen, R. Middleton, and R. K. Mohindra, Phys. Letters **19**, 573 (1965).
[25] J. Kremenek and W. W. Daehnick, Bull. Am. Phys. Soc. **11**, 99 (1966).
[26] B. L. Cohen and R. Middleton, Phys. Rev. **146**, 748 (1966).

the observed experimental levels up to an energy of about 3 MeV. There is an observed $J=2^+$ level at 2.96 MeV for which there is no good theoretical counterpart. Again, this state is presumably dominated by a core-excited configuration. However, here, in contrast to ^{54}Cr, we do predict a $J=0^+$ state in the vicinity of 3 MeV. Since the core-excited $J=0^+$ state is certainly present, this shell-model state must be considered as an extra state. There are a number of spin assignments suggested for the level at 3.12 MeV. It is at least a doublet. One of the states is a high-spin state. It is populated by β decay from the ground state of ^{56}Co, which is a $J=4^+$ state. The β-decay rate indicates the transition could be allowed or first-forbidden. As will be discussed below, the state is populated in the ^{54}Fe$(t,p)^{56}$Fe reaction with an angular distribution which suggests $J=4$. This state γ decays almost exclusively to the first $J=4^+$ state. Another state in the multiplet must be a low-spin state which γ decays about equally to the ground state and first $J=2^+$ state. The state is populated in the ^{56}Fe$(d,t)^{56}$Fe reaction. These facts suggest a state with spin $J=1^+$ or 2^+. There are two predicted states in this energy region with $J=1^+$ and $J=4^+$, respectively. These assignments are consistent with these data.

H. ^{57}Co

The experimental energy levels of ^{57}Co (Ref. 10) are shown in Fig. 8. The disagreement between theory and experiment is as striking here as is the agreement in the previous nuclei. Except for the fact that the ground-state spin is correctly predicted, and one of the $J=\frac{5}{2}^-$ states seems to be accounted for, there is essentially no similarity between the experimental and theoretical spectrum. One of the main discrepancies is the observed existence of several low-spin states, in particular the $J=\frac{1}{2}^-$ state at 1.51 MeV. In the picture of ^{57}Co assumed here, there is one hole in the $\pi f_{7/2}$ shell. Thus there is only one proton state, with spin $J=\frac{7}{2}^-$, so that in order to form a $J=\frac{1}{2}^-$ state in ^{57}Co one must couple an $f_{5/2}$ neutron and a $p_{3/2}$ neutron to either $J_\nu=3$ or $J_\nu=4$. Such states are much too high in energy in this calculation to account for the level at 1.51 MeV. The spectra of ^{59}Co has been calculated with this same model, and the same problem appears. In ^{59}Co there are three $J=\frac{1}{2}^-$ states observed below 1.5 MeV,[10] but we predict none at anywhere near this energy. If one were to expand the space to include excitations to the $\pi p_{3/2}$ shell, there would be several possible states with $J=\frac{1}{2}^-$. Thus it appears that at least by the time one reaches the cobalt isotopes in this mass region, proton excitations from the $f_{7/2}$ shell become significant in relatively low-lying states.

This raises the question as to why the importance of these excitations becomes evident so abruptly. The proton-proton interaction is taken directly from experiment. Those levels which are assumed to be pure $f_{7/2}{}^2$ configurations certainly have admixtures of other configurations. By using an "effective" interaction, we implicitly include in the calculation some of the effects of these other configurations, and thus partially compensate for the fact that these configurations are omitted from the calculation explicitly. In ^{57}Co, the only proton configuration is the one-hole state in the $\pi f_{7/2}$ shell. In this case, the only effect the proton-proton interaction has on the calculation is on the absolute value of the energy levels. It has no effect at all on the relative excitation energies. Therefore, in this case, it is true that the effects of configurations not included in the calculation are no longer included implicitly in the calculation of the level spacings. It is possible that the significance of proton excitations increases with increasing Z in these nuclei, and that the importance of these excitations is masked by the use of the effective proton interaction in this model. It is unlikely, however, that this is the complete explanation for this striking disparity.

As we stated above, this calculation is, for some of the nuclei treated here, an extension of a previous calculation[7] in which neutrons were restricted to the $\nu p_{3/2}$ orbit. In those cases where a comparison can be made, i.e., ^{54}Cr, ^{55}Mn, and ^{56}Fe, the inclusion of configuration mixing leads to distinct improvement in the quality of agreement between theory and experiment. The improvement in ^{54}Cr and ^{56}Fe is mostly in the relative spacings. For ^{55}Mn, the calculation without neutron-configuration mixing inverts the ground and first excited state with respect to the experimental ordering, and it also inverts the lowest-$\frac{3}{2}^-$ and $\frac{9}{2}^-$ states. The configuration-mixing calculation predicts the correct ordering of spins in both cases. While it is true that one cannot place a great deal of significance on the ordering of levels within 200 keV of each other (and this is the case for the ground and first excited state in ^{55}Mn), it is also true that in each case in these three nuclei where there was an apparent disparity in the predicted sequence of spins, the disparity is corrected by the inclusion of configuration mixing of the neutrons.

IV. CALCULATION OF SOME PERTINENT SPECTROSCOPIC FACTORS

It is well known by now that the fact that a shell-model calculation gives good agreement with observed energy spectra does not necessarily mean that the assumed model can be directly interpreted as an adequate description of the nuclei of interest. In a sense, by use of effective interactions one may use the wrong Hamiltonian with the wrong wave functions to obtain correct energy levels. A partial test of the adequacy of the wave functions is the calculation of spectroscopic factors for nucleon-transfer reactions, and the comparison of these numbers with experimentally extracted spectroscopic factors. In this section we present the results of such calculations involving isotopes of Cr, Mn and Fe.

A. ^{53}Cr(d,p)^{54}Cr and ^{54}Cr(p,d)^{53}Cr

We discuss first the results of experiments on the Cr isotopes. There is experimental data on the ^{53}Cr(d,p)^{54}Cr (Refs. 27, 28) and ^{54}Cr(p,d)^{53}Cr (Ref. 29) reactions. In all the calculations which we discuss here, the wave functions which we use for $N=29$ nuclei are generated with the same Hamiltonian which we have used for the $N=30$ nuclei. The results for the $N=29$ nuclei are essentially identical to those of Vervier.[7] In Table I, the theoretical and experimental spectroscopic factors for the ^{53}Cr(d,p)-^{54}Cr reaction are compared. This reaction involves the ground state of ^{53}Cr, and all the states of ^{54}Cr. Since it has already been shown that the ground state of ^{53}Cr is rather well represented in this model, this reaction should provide a fair test of the ^{54}Cr wave functions. The predicted strength of the ground-state transition is too large by a factor of more than 2. The accuracy of the experimental absolute transition strength is not expected to be as good as for relative strengths to different states. Indeed, the reported absolute strength of the ground-state transition for the inverse reaction ^{54}Cr(p,d)^{53}Cr, as shown in Table III, is 0.83, which differs by 30% from the 0.62 for the (d,p) reaction; whereas the numbers should be the same. Nevertheless, the predicted absolute strength is probably too large. The ratio of the strengths of the transitions to the ground state and to the first $J=2^+$ state at 0.83 MeV is fairly well reproduced. The predicted spectroscopic factor for an $l=3$ transition to the first $J=4^+$ state is relatively large. There have been at least two experiments on the ^{53}Cr(d,p)^{54}Cr reaction, and in neither experiment was this state excited. Elwyn and Shull[28] indicate that the cross section for $l=3$ transfers is considerably weaker than for $l=1$ transfers, and that they might have missed this state. The comparison for states in the energy region from about 3 MeV and up is not good. This is not too surprising since, as we discussed above, it is in just this region that configurations not included in this calculation become significant. In Table II the predicted and experimental spectroscopic factors for the ^{54}Cr(p,d)^{53}Cr experiment are compared. This should be a good test of the ^{54}Cr ground-state wave function. Qualitatively, the over-all agreement is reasonable to the extent that the strongly observed transitions are predicted to be strong, and the weak transitions are predicted to be weak. Along this line, it is interesting to note that the $\frac{3}{2}^-$ state at 2.32 MeV is fairly strongly excited in the ^{52}Cr(d,p)^{53}Cr reaction, but very weakly excited in the ^{54}Cr(p,d)^{53}Cr reaction. This behavior is nicely reproduced qualitatively. The predicted strength for the (d,p) reaction is $S=0.12$, and for the (p,d) reaction $S=0.002$, as compared to the observed values $S=0.5$ and $S\leq 0.02$, respectively.

TABLE I. Experimental and predicted spectroscopic factor S for the ^{53}Cr(d,p)^{54}Cr reaction.

Experiment				Calculation			
Energy (MeV)	Spin	l	$(2J+1)S$	Energy (MeV)	Spin	l	$(2J+1)S$
0	0^+	1	0.62	0	0	1	1.45
0.83	2^+	1	1.31	0.89	2	1	3.91
1.84	4^+			2.00	4	3	1.51
2.63	2^+	1	1.80	2.814	2	1	1.15
2.84	0^+	(1)	0.45				
3.08	2^+	1	1.55				
3.16	4^+	1		3.05	4	3	0.27
3.44	2^+	1	1.0	3.39	2	1	3.25
				3.68	0	1	0.33

TABLE II. Experimental and predicted spectroscopic factors S for the ^{54}Cr(p,d)^{53}Cr reaction.

Experiment				Theory			
Energy (MeV)	Spin	l	S	Energy (MeV)	Spin	l	S
0	$\frac{3}{2}^-$	1	0.83	0	$\frac{3}{2}^-$	1	1.45
0.56	$\frac{1}{2}^-$	1	0.31	0.66	$\frac{1}{2}^-$	1	0.30
1.00	$\frac{5}{2}^-$	3	0.51	1.19	$\frac{5}{2}^-$	3	0.22
1.28	$\frac{7}{2}^-$	3	0.70				
1.53	$\frac{7}{2}^-$	3	3.2	1.45	$\frac{7}{2}^-$		0
1.9		(3)	<0.08	1.96	$\frac{5}{2}^-$	3	0.01
2.32	$\frac{3}{2}^-$	(1)	<0.02	2.28	$\frac{3}{2}^-$	1	0.002
2.65	$\frac{5}{2}^-$	(3)	<0.08	2.67	$\frac{5}{2}^-$	3	0.005
2.71	$\frac{1}{2}^-$	(1)	<0.01	2.96	$\frac{1}{2}^-$	1	0
				3.06	$\frac{3}{2}^-$	1	0.005

Quantitatively, the results of the (p,d) calculations are not good. The relative transition strengths to the first three states are in poor agreement with experiment.

One noteworthy fact appears here in the experimental data. That is the appearance of two rather strong $l=3$ transitions to $\frac{7}{2}^-$ states at 1.28 and 1.53 MeV in ^{53}Cr. Since the target in the (p,d) reaction is in a $J=0^+$ ground state, these transitions must proceed by $\nu f_{7/2}$ pickup. This says that the neutron core-excited states play a significant role in states at less than 1.5 MeV, whereas the ^{48}Ca spectrum suggests they are not important below 4.0 MeV.

B. ^{55}Mn(p,d)^{54}Mn

In Table III, the theoretical spectroscopic factors for neutron pickup in reactions leading from ^{55}Mn to ^{54}Mn are compared with experimental numbers determined from the ^{55}Mn(p,d)^{54}Mn reaction.[30] The experimental energy levels for ^{54}Mn obtained from other experiments[31,32] are summarized in the first column of this table. The experimental resolution is rather poor, and

[27] V. P. Bochen *et al.*, Nucl. Phys. **51**, 161 (1964).
[28] A. J. Elwyn and F. B. Shull, Phys. Rev. **111**, 925 (1958).
[29] C. A. Whitten, Ph.D. thesis, Princeton University, 1966 (unpublished).
[30] J. C. Legg and E. Rost, Phys. Rev. **134**, B752 (1964).
[31] B. J. Bjerregaard, P. F. Dahl, O. Hansen, and G. Sidenius, Nucl. Phys. **51**, 641 (1964).
[32] A. Sperduto and W. W. Buechner, Phys. Rev. **134**, B142 (1964).

TABLE III. Experimental and predicted spectroscopic factors S for the ^{55}Mn$(p,d)^{54}$Mn reaction.

Experiment					Theory			
Energy (MeV)	Energy (this reaction) (MeV)	Spin	S $l=1$	S $l=3$	Energy (MeV)	Spin	S $l=1$	S $l=3$
0	"0"		0.83		0.0	3^+	0.74	0.02
0.06					0.07	2^+	0.54	0.02
0.16					0.24	4^+	0.24	0.001
0.37	"0.36"		0.27		0.48	3^+	0.22	0.03
0.41					0.55	5^+	0	
0.84					1.03	4^+	0.02	0.04
1.01					1.37	3^+	0.01	0.02
1.07	"1.12"		0.12	1.2	1.38	1^+		
1.14					1.45	5^+		0.01
1.38					1.53	2^+	0.01	
1.46	"1.46"		0.13	0.67				
1.51								

furthermore the absolute spectroscopic factors are uncertain by a factor of about 2. Taking these facts into consideration, the predictions for transitions to states below 1 MeV can be considered satisfactory. If we assume that the observed ground-state transition is the sum of the strengths to the first two states, and that the transition to the "0.36-MeV level" is the sum of the strengths to the next three levels, then the ratio of the two experimental strengths, ~ 3, is well reproduced by the ratio of the theoretical strengths, 2.7. Above 1.0 MeV, the agreement is not too good. We predict no $l=1$ transitions with any appreciable strength, while two peaks are seen with significant strength. Likewise, the experimental transition to the "levels" at 1.12 and 1.46 MeV have strong $l=3$ components, while we predict no such strength. There are two possible explanations for these discrepancies. The first is simply that our model Hamiltonian does not mix in enough $\nu f_{5/2}$ strength in the low-lying states. The second is the possible importance of neutron core excitations. Such excitations are significant in ^{53}Cr at a similar energy level, so it would be expected that they would be significant here.

C. ^{56}Fe$(p,d)^{55}$Fe and ^{54}Fe$(t,p)^{56}$Fe

In Table IV are compared the results of theory and experiment for the ^{56}Fe$(p,d)^{55}$Fe reaction.[30] The results are similar to those already discussed for Cr and Mn. The strong experimental transitions are strong theoretically, and no strong theoretical $l=1$ transitions are predicted which are not seen experimentally. Again the quantitative agreement is not good, and again the major discrepancy is the existence of strong $l=3$ transitions to $J=\frac{7}{2}^-$ states. There are two $J=\frac{7}{2}^-$ states at 1.32 and 1.42 MeV, and the sum of the spectroscopic factors to these two states is 6.1. These two states are presumably admixtures of that $J=\frac{7}{2}^-$ state predicted at 1.42 MeV, and a neutron core-excited state at about the same energy. The model used here would predict no transition strength to either one of these states.

A distorted-wave analysis of the ^{54}Fe$(t,p)^{56}$Fe experiment[26] has been carried out by Drisko.[33] The experimentally observed cross sections are compared with those predicted by the wave functions determined in the present calculation in Table V. The experimental and theoretical cross section for the transition to the $J=2^+$ state at 0.85 MeV have been set equal to each other as a means of normalizing the data. There is very good agreement for the relative strengths of transitions to states in ^{56}Fe up to an energy of 3.12 MeV, with the exception of the transitions to the states at 2.95 MeV. The striking features of the experimental data which are well reproduced by the calculation are the very large ground-state transition, and the very weak transition to the $J=4^+$ state at 2.084. The predicted strength for the transition to the $J=2^+$ state at 2.66 MeV is somewhat strong. It is again possible that the configurations

TABLE IV. Experimental and predicted spectroscopic factors S for the ^{56}Fe$(p,d)^{55}$Fe reaction.

Experiment				Theory			
Energy (MeV)	Spin	l	S	Energy (MeV)	Spin	l	S
0	$\frac{3}{2}^-$	1	1.0	0.0	$\frac{3}{2}^-$	1	1.46
0.41		1	0.42	0.56	$\frac{1}{2}^-$	1	0.32
0.93	$\frac{5}{2}^-$	3	0.87	1.25	$\frac{5}{2}^-$	3	0.19
1.32		(3)					
1.42	$\frac{7}{2}^-$		6.1	1.43	$\frac{7}{2}^-$		0
1.93		1		1.81	$\frac{3}{2}^-$	1	
2.06		1		1.87	$\frac{1}{2}^-$	1	0.01
2.15		3		1.84	$\frac{5}{2}^-$	3	0.01

TABLE V. Experimental and theoretical cross sections σ for the ^{54}Fe$(t,p)^{56}$Fe.

Experiment			Theory		
Energy (MeV)	J^π	σ	(MeV)	J^π	σ
0	0^+	11 400	0	0^+	10 800
0.85	2^+	2400	0.92	2^+	2400
2.08	4^+	33	2.34	4^+	22
2.66	2^+	1120	2.52	2^+	1700
2.95	0^+	656	3.02	0^+	160
2.95	2^+	890			
3.12	(4^+)	362	2.96	4^+	200

[33] R. M. Drisko (to be published).

in which protons and neutrons are excited from the $f_{7/2}$ shell are significant in this state, as well as the states at 2.95 MeV. There are at least two other possible contributing factors to the relatively poor results for the predictions for transitions to the state at 2.95 MeV. The first is that the states are experimentally unresolved, which complicates the analysis. Secondly, in the calculation of the strength for the $J=0^+$ transition, there are large cancellations between various components of the wave function. When this is the case, the calculated numbers are not only sensitive to the details of the radial-wave-function tails, but also to small components in the wave functions, so that the numbers are not too reliable. This is quite different from the case of the transition to the $J=4^+$ state at 2.08 MeV, which is so small. In that case, the cross section is reduced because the $J=4^+$ wave function contains a large component in which the protons are not coupled to $J=0$. These latter components cannot be populated directly in the (t,p) reaction on ^{54}Fe.

V. SUMMARY AND DISCUSSION

Shell-model calculations of the energy levels and wave functions of $N=30$ nuclei with $20\leq Z\leq 27$, in which an inert ^{48}Ca core is assumed, have been carried out. The residual interaction is determined by fits to selected observed spectra. With the exception of ^{57}Co, the calculated energy-level structure of these $N=30$ nuclei is in satisfactory agreement with observed levels below about 2.5-MeV excitation energy. No really convincing explanation has been offered for the breakdown of the model used here in the case of ^{57} Co. The fact that proton excitations from the $\pi f_{7/2}$ shell may be important in this case is a possible contributing factor to this discrepancy.

The wave functions for ^{54}Cr, ^{55}Mn, and ^{56}Fe produced in this calculation have been used to predict the strengths of nucleon-transfer reactions involving these nuclei. The predicted strengths are not in as good agreement with experiment as the excitation energies. Although the gross features are reproduced, the relative strengths of transitions to several states in any one nucleus are poorly reproduced by the calculation. The existence of strong $l=3$ transitions to relatively low-lying $J=\frac{7}{2}^-$ states in these nuclei is evidence that excitations of the $\nu f_{7/2}$ shell may play a significant role in the low-lying states of these nuclei.

The comparison of spectroscopic factors calculated from theoretical wave functions with those factors extracted from experimental data on nucleon-transfer reactions is meaningful only to the extent that the procedure for extracting the numbers from experiment is accurate. Several papers[34–36] have appeared recently which discuss ways to improve the calculation of form factors in distorted-wave analyses of stripping and pickup reactions. In particular, Prakash[36] has studied the effects on form factors in pickup reactions due to residual interactions between nucleons in the target nucleus. He has analyzed experimental data on the ^{58}Ni$(d,t)^{57}$Ni reaction, and estimates that the inclusion of these effects would reduce the extracted spectroscopic factor for $\nu f_{5/2}$ transitions in this reaction by about 50%. He finds that the extracted factors for $\nu p_{3/2}$ and $\nu p_{1/2}$ transitions in the same reaction are not significantly changed by these effects. The same three neutron orbits which Praskash includes in his treatment of ^{58}Ni are included in the calculation reported here. If changes similar to those found by Prakash in the ^{58}Ni$(d,t)^{57}$Ni reaction were found in the extracted factors for the ^{54}Cr$(p,d)^{53}$Cr and ^{56}Fe$(p,d)^{55}$Fe as a result of a more complete analysis, the calculated spectroscopic factors reported here would be in significantly better agreement with the extracted factors.

The predicted strengths for the two-nucleon-transfer reaction ^{54}Fe$(t,p)^{56}$Fe are in better agreement with experiment than those found for the one-nucleon-transfer reactions. They are good enough to suggest that the wave functions generated in this calculation can be very useful in the analysis of (t,p) reaction data involving the other $N=30$ nuclei treated here.

There are other obvious tests of the wave functions produced in this calculation. One test would be the calculation of $M1$ and $E2$ electromagnetic-transition rates. Vervier carried out such calculations for the $N=29$ nuclei and some of the $N=30$ nuclei. He used effective charges and nucleon moments chosen to fit the observed transition rates in nuclei with simple configurations. These effective charges and moments vary with the model basis. Since the importance of core excitations in these nuclei seems to be established, we plan to attempt shell-model calculations which explicitly include these core excitations first, rather than try to incorporate these effects into the present calculations through effective charges and moments. Such calculations are in progress on the isotopes of iron, cobalt and nickel. The size of the calculation precludes the possibility, at present, of studying any of the lighter $N=30$ nuclei in the expanded space.

ACKNOWLEDGMENTS

The author wishes to thank Dr. J. B. Ball, Dr. E. C. Halbert, and Dr. R. M. Drisko for many helpful discussions in the course of this work. He is grateful to G. H. Holloway for his assistance, and to Dr. R. M. Drisko, Dr. R. D. Lawson, and Dr. R. Middleton for permission to use their unpublished results.

[34] R. Huby and J. L. Hutton, Phys. Letters **19**, 660 (1966).
[35] W. T. Pinkston and G. R. Satchler, Nucl. Phys. **72**, 641 (1965).
[36] A. Prakash, Ph.D. thesis, University of Pittsburgh, 1966 (unpublished).