Shell Model of the Nickel Isotopes*

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The isotopes of nickel are described by a shell model within the identical-nucleon configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$. A least-squares fit to observed level energies yields an effective interaction which satisfactorily reproduces the level structure of the Ni isotopes from Ni^{58} to Ni^{65}. This best-fit interaction is shown to indicate repulsive interactions between identical-nucleon shells and to conserve seniority to a useful degree of approximation. The interaction matrix elements are in fair agreement with those of an approximate reaction matrix computed by Kuo from Hamada and Johnston's free-nucleon interaction, this agreement being obtained only when core polarization is taken into account. Moments and transitions also show the strong influence of core excitation. Observed quadrupole moments and $E2$ transition rates are adequately reproduced with an effective neutron charge of between $1.5e$ and $2e$; in particular, the observed inhibition of the crossover ground-state decay of the second $2^+(2_2^+)$ states of Ni^{60} and Ni^{62} is reproduced with 2_2^+ wave functions for which a two-phonon vibrational description is clearly incorrect. It is shown that the original model cannot account for the large deviations of observed magnetic moments from the Schmidt values, nor for the observed spreading of stripping strength into a given orbit over several states of the residual nucleus. To account for these, it is necessary to introduce effective transition operators, strongly modified by the influence of neglected configurations.

1. INTRODUCTION

THIS paper concerns the shell model for the isotopes of Ni. The choice of configurations is discussed in Sec. 2 and an effective interaction within the chosen configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$ is determined in Sec. 3 by a least-squares fit to observed level energies. The resulting shell-model interaction is similar to that obtained by Kuo[1] from the Hamada-Johnston potential between free nucleons; in Sec. 5 it is shown to conserve seniority to a useful degree of approximation. The energies, moments, and transition rates predicted by the model are compared with experiments in Secs. 4, 6, and 7. In Sec. 8 we relate the present work to published shell-model studies of the Ni isotopes, and we summarize our main conclusions in Sec. 9.

2. CHOICE OF CONFIGURATIONS; SINGLE-PARTICLE ENERGIES

To choose configurations for the shell-model description of the Ni isotopes, consider Fig. 1, which shows the single-particle levels pertinent around mass-number 56. It is clear that the nearest approximate closed shell occurs at $_{28}Ni_{28}^{56}$, whose nucleon numbers $N=Z=28$ correspond to the filling of the $1f_{7/2}$ shell for both neutrons and protons. Then if excitation of the $1f_{7/2}$ shell in the Ni^{56} core can be ignored, an identical-nucleon description of the Ni isotopes emerges; properties of low-lying states arise from the motion of neutrons around a doubly closed-shell core.

This identical-nucleon description cannot, of course, be literally correct. It is incompatible with the very fact that strong $E2$ transitions are observed between low-lying Ni energy levels. Furthermore, there is clear evidence[2] from stripping and pickup experiments that even the lowest levels in the isotopes of Ni involve sizable amounts of core excitation. Since, however, explicit inclusion of core-excited configurations would involve shell-model calculations of ridiculous immensity, we are forced to treat the Ni^{56} core as inert and to hope that the influence of core excitation on level energies can be absorbed by the effective residual interaction between valence nucleons.

Our choice of configurations now becomes $(2p_{3/2}, 1f_{5/2},2p_{1/2},1g_{9/2},\cdots)^n$ where all n active nucleons are neutrons and where we have yet to decide how many single-particle orbits to include. In this connection, consider the level scheme of Ni^{57}. Figure 2 summarizes the Ni^{57} excitation energies determined[3–7] in several experimental studies of the reaction $Ni^{58}(p,d)$ and $Ni^{58}(d,t)$; the energies obtained in the various experiments agree to within 50 keV. Pickup transitions with the angular distribution characteristic of orbital-angular-momentum transfer $l=1$ are observed to the Ni^{57} ground state and to the 1.08-MeV excited state. Levels at 0.78 and 2.6 MeV are excited by $l=3$ pickup. Small differences between the shapes of the two $l=1$ angular distributions and between the shapes of the

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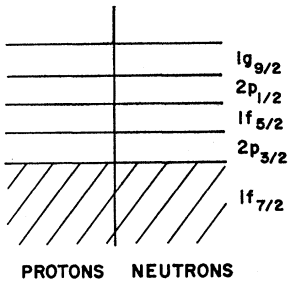

FIG. 1. Single-particle energy levels pertinent around mass 56. The cross-hatching indicates that all orbits up to and including $1f_{7/2}$ are filled with both neutrons and protons. The diagram symbolizes a doubly closed shell with $N=Z=28$.

two $l=3$ angular distributions suggest that the corresponding pairs of states in Ni^{57} have different spins. Since no Ni^{57} states other than the four under consideration have been found up to an excitation energy of 3 MeV, we interpret the ground state and the 0.78- and 1.08-MeV states of Ni^{57}, respectively, as $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ single-particle states. The level at 2.6 MeV is so strongly excited in the pickup reactions that it may reasonably be identified as a $1f_{7/2}$ single-hole state. Although the position of the $1g_{9/2}$ level in Ni^{57} is not known, a reasonable guess can be made from the single-particle level systematics[8] of this mass region. The $1g_{9/2}$ single-particle state is probably at least 3 MeV above the $2p_{3/2}$ level—well above the $1f_{7/2}$ single-hole state. Thus, for levels up to 2 or 3 MeV in the higher Ni isotopes ($A\leqslant 62$), the $1g_{9/2}$ orbit probably contributes less than core-excited configurations. Inclusion of the $1g_{9/2}$ orbit without excitation of the $1f_{7/2}$ core is therefore pointless.

Our choice of configurations is now complete. We shall calculate the properties of low-lying levels of the Ni isotopes within the identical-nucleon configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$. The pertinent single-particle energies, taken directly from the results of the pickup experiments mentioned above, are

$$\begin{aligned}&\epsilon(p_{3/2})=0,\\&\epsilon(f_{5/2})=0.78\text{ MeV},\\&\epsilon(p_{1/2})=1.08\text{ MeV}.\end{aligned}\tag{1}$$

These single-particle energies are of course uncertain to the extent that fragments of single-hole strength may appear at higher excitation energies in the spectrum of Ni^{57} and so may have escaped detection in the pickup experiments. However, explicit calculations have shown that changes of as much as 500 keV in the single-particle energies can be absorbed by the effective interaction between nucleons without significant alteration of the quality of agreement between theory and experiment. We therefore pay no further attention to variations in the single-particle energies; they will remain fixed throughout at the values quoted above.

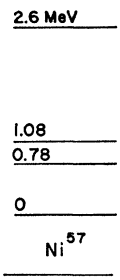

FIG. 2. Observed energy levels below 3 MeV in Ni^{57}. Experimental data are from Refs. 3–7.

3. THE EFFECTIVE TWO-BODY INTERACTION

We turn next to the specification of a residual two-body interaction between the valence neutrons. Our procedure is phenomenological. The effective interaction within the chosen configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$ is parametrized in a fashion to be described and the interaction parameters are determined by a least-squares fit to observed level energies in various isotopes of Ni. The resulting best-fit interaction is then compared with one more directly related to the forces between free nucleons.

A two-body interaction within the configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$ is completely determined by its matrix elements between the antisymmetric two-particle states of the single-particle orbits $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$. There are 30 such two-body matrix elements. Unfortunately, a direct parametrization of the two-body interaction in terms of its two-body matrix elements is not feasible here; the body of available energy-level data is too restricted to allow a sufficient overdetermination of the interaction parameters. We must clearly cut down the number of parameters.

As a first attempt at a more restricted parametrization of the shell-model Hamiltonian, we have considered a potential interaction with central (singlet and triplet), tensor, and two-body spin-orbit parts, namely

$$\begin{aligned}V=V_0f_0(r)\left[\frac{1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4}\right]+V_1f_1(r)\left[\frac{3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4}\right]\\+V_2f_2(r)\left[\frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]\\+V_3f_3(r)[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{l}],\end{aligned}\tag{2}$$

where $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ is the relative coordinate vector of the two nucleons and $\mathbf{l}$ is their relative angular momentum. Each component of the potential is assumed to have a Gaussian radial dependence

$$f_i(r)=\exp(-\beta_ir^2),\quad(i=0,1,2,3)\tag{3}$$

and the single-particle radial wave functions are taken to be of the harmonic-oscillator form

$$\Phi(r_i)\propto\exp(-\tfrac{1}{2}\alpha r_i^2),$$

$i=1,2$. The four range parameters $\lambda_i=(\alpha/2\beta_i)^{1/2}$ $(i=0,1,2,3)$ are fixed once and for all at the values $\lambda_0=\lambda_1=0.7$, $\lambda_2=1.0$, and $\lambda_3=0.5$—values qualitatively

[8] B. L. Cohen, R. H. Fulmer, A. L. McCarthy, and P. Mukherjee, Rev. Mod. Phys. 35, 332 (1963).

consistent with the fact[9] that the tensor range of nucleon-nucleon scattering potentials tends to be longer and the spin-orbit range shorter than the central range. Finally, the four strength parameters V_i are determined by a least-squares fit to observed level energies in the isotopes of Ni; both excitation energies and ground-state binding energies relative to the Ni^{56} core are included in the fitting procedure.

The resulting best-fit interaction potential does not provide a satisfactory description of the Ni spectra. Two systematic shortcomings are particularly evident. Firstly, the calculated 0^+ excited states are much too close to ground; they appear at about 1–1.5 MeV, close to the 2^+ first excited states, instead of at 2 MeV or higher. Secondly, numerous calculated levels intrude in unpopulated regions of the experimental spectrum.

There are several possible ways of improving agreement with experiment without abandoning the restricted four-parameter potential form [Eq. (2)] of the effective interaction within $(2p_{3/2},1f_{5/2},2p_{1/2})^n$. One obvious source of error is the harmonic-oscillator approximation to the single-particle radial wave functions. However, errors of this sort are most serious for weakly bound (or unbound) single-particle states.[10] Since the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ levels are bound by about 10 MeV to the Ni^{56} core, the harmonic-oscillator approximation is unlikely to cause large errors here. Another possibility was that a different set of range parameters λ_i might yield better agreement between theory and experiment. To examine this possibility, we repeated the fitting procedure with various alternative sets of fixed range parameters; agreement with the experimental spectra was not significantly improved. Finally, it can be argued that it is unreasonable to expect our restricted 4-parameter model to fit both binding energies and excitation spectra and that, accordingly, elimination of ground-state binding energies from the fit should yield a more accurate description of the excitation energies. This does not turn out to be the case. Elimination of ground-state binding energies does not significantly change the quality of agreement between the theoretical and experimental excitation energies. The best-fit value of the triplet-central strength V_1 changes from 31.9 to 14.3 MeV, the other three strength parameters change by less than 20%, and the systematic discrepancies mentioned above persist.

Thus, no interaction potential of the form given by Eq. (2), acting within the identical-nucleon configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$, gives a satisfactory description of the Ni spectra. Now it is known[11] that the change in the energy spectra in response to large amounts of core excitation can be simulated by appropriate changes in the matrix elements of the effective two-body interaction. We conclude that the poor agreement between

TABLE I. Calculated and observed level energies in the nuclei Ni^{58} to Ni^{62}. For ground states, the binding energy B of Eq. (6) is tabulated; for all other states, the excitation energy is given. Where no explicit reference is given in the right-hand column to the table, the appropriate references are Ref. 13 for ground-state binding energies, Ref. 14 for even-mass excitation energies, Refs. 7 and 14–16 for odd-mass excitation energies, and Ref. 17 for odd-mass spin assignments. In addition to the fitted levels, the excitation energy of the next calculated state of each spin is given. For even-A nuclei. column two gives the spin of the level and for odd-A nuclei twice the spin is listed.

Nucleus	J or $2J$	Calculated energy (MeV)	Observed energy (MeV)	References
Ni^{58}	0_1	(−2.06)	(−1.93)	
	2_1	1.31	1.45	
	4_1	2.29	2.46	
	0_2	2.54	2.77	18
	2_2	2.88	2.90	19
	0_3	4.23		
	1_1	3.81		
	2_3	3.67		
	3_1	3.49		
	4_2	3.95		
Ni^{59}	3_1	(−0.81)	(−0.67)	
	5_1	0.35	0.34	
	1_1	0.37	0.47	
	3_2	0.84	0.89	
	1_2	1.16	1.32	
	1_3	3.06		
	3_3	1.78		
	5_2	1.45		
	7_1	1.40		
	9_1	1.00		
	11_1	2.14		
Ni^{60}	0_1	(−1.88)	(−1.80)	
	2_1	1.42	1.33	
	2_2	2.17	2.16	
	0_2	2.32	2.29	
	4_1	2.20	2.50	
	3_1	2.75	2.62	
	0_3	3.27		
	1_1	3.45		
	2_3	2.58		
	3_2	3.37		
	4_2	2.80		
	5_1	3.64		
	6_1	3.17		
Ni^{61}	3_1	(0.62)	(0.64)	
	5_1	0.12	0.07	
	1_1	0.02	0.28	
	5_2	0.93	0.91	
	1_2	1.02		
	3_2	1.03		
	5_3	1.67		
	7_1	0.92		
	9_1	1.00		
	11_1	1.48		
	13_1	2.32		
Ni^{62}	0_1	(0.45)	(0.31)	
	2_1	1.53	1.17	
	0_2	2.01	2.05	
	2_2	2.25	2.30	
	4_1	2.20	2.34	
	0_3	2.61		
	1_1	3.57		
	2_3	2.70		
	3_1	2.84		
	4_2	2.76		
	5_1	3.23		
	6_1	3.30		
	7_1	5.70		

[9] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962), Chap. V.

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TABLE II. Calculated and observed level energies in the nuclei Ni^{63} to Ni^{66}. A negative excitation energy means that the theoretical calculation predicted that state to be the ground state. For even-A nuclei, column two gives the spin of the level and for odd-A nuclei twice the spin is listed. For further information, see the caption of Table I.

Nucleus	J or $2J$	Calculated energy (MeV)	Observed energy (MeV)	References
Ni^{63}	1_1	(3.95)	(3.73)	
	5_1	0.08	0.09	
	3_1	0.28	0.16	
	3_2	1.00	0.53	
	1_2	1.25	1.01	
	1_3	2.34		
	3_3	1.29		
	5_2	1.00		
	7_1	1.17		
	9_1	1.24		
	11_1	1.74		
	13_1	2.44		
Ni^{64}	0_1	(4.90)	(4.33)	
	2_1	1.56	1.34	
	2_2	2.37	2.28	22
	4_1	2.25	2.62	21
	0_2	2.15	2.89	22
	0_3	3.55		
	1_1	3.84		
	2_3	2.60		
	3_1	3.07		
	4_2	2.72		
	5_1	3.36		
	6_1	3.71		
Ni^{65}	5_1	(9.50)	(8.46)	
	1_1	−0.02	0.06	
	3_1	0.55	0.32	
	3_2	0.76	0.70	
	1_2	1.40		
	3_3	1.45		
	5_2	1.05		
	7_1	1.21		
	9_1	1.28		
	11_1	2.12		
Ni^{66}	0_1	(11.51)	(9.76)	23
	0_2	2.26		
	1_1	4.14		
	2_1	1.50		
	2_2	2.68		
	3_1	3.36		
	4_1	2.35		

theory and experiment is probably due to the inflexibility of the assumed form of interaction potential. Since the main shortcomings of a Gaussian interaction potential stem from its behavior in relative s states,[12] we introduce as additional parameters the four diagonal radial matrix elements

$$I_{n0}=\int_0^\infty \mathfrak{R}_{n0}^2(r)V(r)r^2dr \quad (n=1-4) \tag{4}$$

of the central interaction in relative s states. There are then two distinct contributions to each interaction matrix element. The first is simply the appropriate matrix element of the potential operator in Eq. (2); the second is the corresponding matrix element of an additional central interaction which is operative only when the nucleons are in relative s states and has the value I_{n0} when the relative s-state wave function of the nucleus is diagonal in n, the number of radial nodes.

The effective two-body interaction now depends on eight parameters—the four interaction strengths V_i and the four radial integrals I_{n0}. These parameters are determined by a least-squares fit to 24 observed level energies in Ni^{58}, Ni^{59}, Ni^{60}, Ni^{61}, and Ni^{62}. The energy levels of the heavier isotopes of Ni are also calculated and compared with experiment; they are not included in the fitting procedure because of the possibility, discussed in Sec. 4, that $g_{9/2}$ admixtures in low-lying states may increase significantly beyond Ni^{62}. Thus, the function

$$\chi^2=\sum_{k=1}^{24}[E_k^{\rm exp}-E_k(V_i,I_{n0})]^2 \tag{5}$$

is minimized numerically as a function of V_i and I_{n0}. The energy E_k is an excitation energy if the state k is excited, but is a binding energy relative to the Ni^{56} core—the quantity B defined by Eq. (6) of Sec. 4—if the state k is a ground state. The necessary experimental level energies are given in Tables I and II, which include results from several groups of workers.[13–23]

The resulting best-fit interaction provides a satisfactory description of the low-lying spectra of the Ni isotopes; the rms deviation between the measured and calculated values of the 24 fitted level energies is about 150 keV. A detailed comparison between theory and experiment is postponed to Sec. 4. The next point to be discussed is the nature of the best-fit interaction. Let us note in this connection that the parameters V_i and I_{n0} have no clear physical significance. We therefore discuss the effective interaction solely in terms of its two-body matrix elements and do not quote values for the strengths V_i and radial integrals I_{n0}.

The matrix elements of the best-fit interaction are given in column C of Table III. For comparison, Table III also gives the matrix elements of effective inter-

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TABLE III. Shell-model interaction matrix elements (in MeV) for the isotopes of Ni. Orbits are labeled by twice their j values ($2p_{\frac{3}{2}} \rightarrow 3$, $1f_{\frac{5}{2}} \rightarrow 5$, $2p_{\frac{1}{2}} \rightarrow 1$). Column A lists the reaction matrix elements, neglecting core polarization, calculated approximately from the Hamada-Johnston potential. Column B gives the same matrix elements when perturbative corrections for core excitations which result from a single nucleon being promoted from a core orbit to a valence (empty) orbit are included. C is the phenomenological best-fit interaction determined by the least-squares procedure of Sec. 3.

State					Matrix elements $\langle j_1 j_2 J \mid V \mid j_3 j_4 J \rangle$		
$2j_1$	$2j_2$	$2j_3$	$2j_4$	J	A	B	C
5	5	5	5	0	−0.27	−1.25	−1.73
				2	−0.32	−0.05	0.28
				4	−0.06	0.46	0.17
5	5	5	3	2	−0.01	0.14	0.54
				4	0.02	0.35	0.48
5	5	5	1	2	−0.05	−0.51	−0.64
5	5	3	3	0	−0.56	−1.26	−0.99
				2	−0.07	−0.24	−0.25
5	5	3	1	2	−0.19	−0.46	−0.22
5	5	1	1	0	−0.20	−0.95	−0.65
5	3	5	3	1	−0.22	0.40	1.06
				2	0.00	0.30	0.57
				3	−0.19	0.37	0.65
				4	−0.37	−0.07	−0.39
5	3	5	1	2	0.18	0.20	0.21
				3	−0.02	0.12	0.02
5	3	3	3	2	0.03	0.14	0.15
5	3	3	1	1	−0.09	−0.04	0.06
				2	0.13	0.17	0.10
5	1	5	1	2	−0.26	−0.17	−0.28
				3	−0.11	0.64	0.86
5	1	3	3	2	−0.09	−0.06	−0.21
5	1	3	1	2	−0.21	−0.33	−0.23
3	3	3	3	0	−0.97	−0.88	−1.04
				2	−0.42	−0.25	0.37
3	3	3	1	2	−0.49	−0.27	−1.10
3	3	1	1	0	−1.24	−0.96	−1.03
3	1	3	1	1	−0.13	0.06	0.71
				2	−0.77	−0.36	−0.41
1	1	1	1	0	−0.09	−0.24	−0.31

actions deduced by Kuo[1] from the nucleon-nucleon potential of Hamada and Johnston.[24] The idea here[1] is that for the effective interaction one uses an approximate reaction matrix corrected where necessary for polarization of the Ni^{56} core. The matrix elements in column A of Table III are those of the reaction matrix calculated approximately from the Hamada-Johnston potential without consideration of core polarization. The matrix elements in column B include perturbative corrections for core excitations wherein a single nucleon is promoted from a core orbit to a valence or empty orbit.

It is clear from Table III that although most of the uncorrected reaction-matrix elements A have the same sign as the corresponding phenomenological matrix elements C, they tend to be considerably smaller. The corrections for core polarization are in the right direction and of roughly the right size; in fact the corrected matrix elements B are in fair agreement with the phenomenological matrix elements C.

Errors in the interaction matrix elements influence the energy spectra in strongly correlated fashion. It is therefore impossible to assess the degree of agreement between two effective interactions by scanning tables of matrix elements; the resulting energy spectra must be compared. When this comparison is made,[25] the energy spectra calculated with interactions B and C agree closely only in the case of Ni^{58}. When there are more than two valence neutrons, the relatively small differences between the interaction matrix elements B and C conspire to produce large discrepancies in the calculated energy spectra[25]; furthermore, these discrepancies increase with the number of valence particles. This behavior probably stems from the fact that the direct use of a reaction matrix as an effective shell-model interaction is strictly justified[26] only for systems with two nucleons outside an inert (or, with perturbative corrections, nearly inert) closed-shell core. We cannot expect detailed agreement for the heavier Ni isotopes without proper consideration of the effects of the additional valence nucleons on the effective interaction and on the core.

Thus, although our effective two-body interaction within the configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$ cannot be said to have been derived rigorously from the force between free nucleons, it is clear that a rough quantitative relationship has been established.

4. COMPARISON OF MEASURED AND CALCULATED LEVEL ENERGIES

The fitting procedure described in Sec. 3 yields a shell-model Hamiltonian specified by the single-particle energies of Eq. (1) and by the interaction matrix elements given in column C of Table III. We now compare the eigenvalues of this model Hamiltonian with the observed level energies of Ni isotopes.

Consider first the ground-state energies and define the quantity

$$B(n) = -\{\text{B.E.}(\text{Ni}^{56+n}) - \text{B.E.}(\text{Ni}^{56}) - n[\text{B.E.}(\text{Ni}^{57}) - \text{B.E.}(\text{Ni}^{56})]\}, \quad (6)$$

where B.E.(Ni^A) is the (positive) ground-state binding energy[13] of the mass-A Ni isotope and n is the number of neutrons outside the Ni^{56} core. $B(n)$ is in fact the amount contributed to the ground-state binding energy of Ni^{56+n} by the single-particle energy splittings and by the two-body interactions between valence neutrons. Now, according to our model of the Ni isotopes, the quantity [B.E.(Ni^{57})−B.E.(Ni^{56})] is the binding energy of a $p_{3/2}$ neutron outside the Ni^{56} core; its measured value[6,7] is 10.26 MeV. Thus, the ground-state binding energy $B(n)$ (in MeV) becomes

$$B(n) = -\{\text{B.E.}(\text{Ni}^{56+n}) - \text{B.E.}(\text{Ni}^{56}) - 10.26n\}. \quad (7)$$

[24] T. Hamada and I. D. Johnston, Nucl. Phys. 34, 382 (1962).

[25] R. D. Lawson, M. H. Macfarlane, and T. T. S. Kuo, Phys. Letters 22, 168 (1966).

[26] R. J. Eden, in *Nuclear Reactions*, edited by P. M. Endt and and M. Demeur (North-Holland Publishing Company, Amsterdam, 1959), Vol. I, Chap. I.

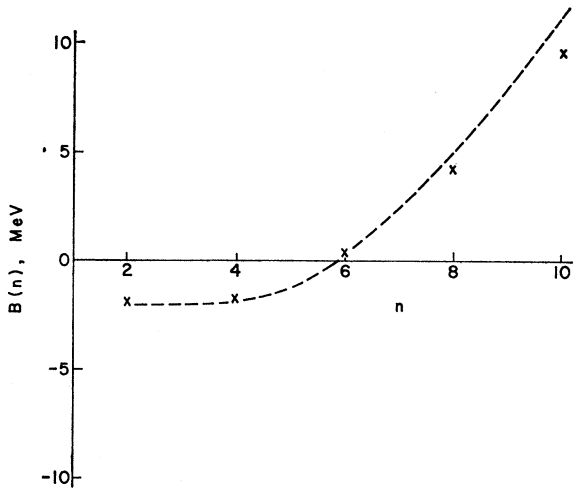

FIG. 3. Ground-state binding energies of the even-mass Ni isotopes, with $B(n)$ as defined by Eq. (6). The crosses represent the measured binding energies from Refs. 6, 7, and 16; the dashed curve gives the calculated values. The odd-mass binding energies follow a parallel curve displaced upward by 1–2 MeV. The quality of agreement between theory and experiment is much the same as for the even-mass binding energies.

The calculated value, the model counterpart of this ground-state binding energy, is the n-particle ground-state eigenvalue of the model Hamiltonian.

Calculated values of $B(n)$ are compared with experiment in Fig. 3. The ground-state binding energies for $n=2$ to $n=6$ are included in the least-squares fit that determines the effective interaction and are fitted with great accuracy; the error is in each case less than 15 keV. The ground-state energies of the heavier isotopes of Ni ($n=7$–10) are not included in the least-squares fit but are nevertheless predicted with satisfactory accuracy by the model; the errors involved amount to a few hundred keV.

It is not our intention here to emphasize these binding-energy results as indicative of a successful physical description of the Ni isotopes. Rather we wish to stress an important qualitative feature of the binding-energy curve in Fig. 3. Our fit to level energies in Ni^{58} to Ni^{62} has led to an effective interaction whose influence on ground-state binding energies turns repulsive for $n>6$. Part of this repulsion arises from the fact that the zero of energy has been taken to be the -10.26 MeV by which the $2p_{3/2}$ level is bound. With this as reference, it follows that the energy of both the $1f_{5/2}$ and $2p_{1/2}$ single-particle levels is positive, the values being 0.78 and 1.08 MeV, respectively. However, even when this single-particle effect is subtracted out, the residual nucleon-nucleon interaction must still be repulsive. This repulsion manifests itself by leading to positive values for the quantities

$$W(j_1j_2)=\frac{2(1+\delta_{j_1j_2})}{\sum_J [J]}\sum_J [J]\langle j_1j_2J|V|j_1j_2J\rangle, \quad (8)$$

TABLE IV. Comparison of the interaction parameters determined in the present study and in Ref. 28. The quantities listed are the pairing matrix elements $\varepsilon_0(j,k)$ and shell-shell interaction energies $W(j,k)$ defined by Eqs. (9) and (8) in the text. The errors quoted are obtained from the appropriate diagonal elements of the error matrices in the least-squares fit of Ref. 28 and the present study; they give only a crude idea of the uncertainties involved since the errors in the different interaction parameters are correlated.

Orbits		$\varepsilon_0(j,k)$ in MeV			$W(j,k)$ in MeV		
j	k	Auerbach[a]	Best fit	Error	Auerbach[a]	Best fit	Error
$\frac{5}{2}$	$\frac{5}{2}$	−1.74	−1.73	0.5	−0.24	0.32	0.7
$\frac{5}{2}$	$\frac{3}{2}$	−1.12	−0.99	0.2	0.58	0.59	0.2
$\frac{5}{2}$	$\frac{1}{2}$	−0.56	−0.65	0.4	0.56	0.77	0.2
$\frac{3}{2}$	$\frac{3}{2}$	−0.92	−1.04	0.7	0.20	0.54	1.2
$\frac{3}{2}$	$\frac{1}{2}$	−0.97	−1.03	0.6	0.18	0.02	0.5
$\frac{1}{2}$	$\frac{1}{2}$	−0.89	−0.31	1.0	...	...	...

[a] Reference 28.

where $[J]\equiv(2J+1)$, the sums over J run from $|j_1-j_2|$ to j_1+j_2, and $\langle j_1j_2J|V|j_1j_2J\rangle$ is the diagonal matrix element of the two-body interaction in the antisymmetric two-particle state $|j_1j_2J\rangle$. Since the interaction energy of n_1 particles in orbit j_1 with n_2 particles in orbit j_2 is proportional to $n_1n_2W(j_1j_2)$, the ground-state energy systematics of the Ni isotopes can be seen to indicate repulsive interactions between identical-nucleon shells.[27] Table IV gives the shell-shell interaction energies $W(j_1j_2)$ obtained by Auerbach[28] and those obtained with the best-fit interaction matrix elements of Sec. 3.

The excitation energies also are in good agreement with experiment. In presenting the detailed comparison between theory and experiment, let us again distinguish the lighter Ni isotopes (which are included in the least-squares fitting procedure which determines the effective interaction) from the heavier Ni isotopes (which are not).

Experimental and theoretical level energies in the isotopes Ni^{58} to Ni^{62} are summarized in Table I. The rms deviation between the measured and calculated energies of the 24 levels of known spin is about 150 keV. There are no serious discrepancies. There are 0^+ excited states at about the right excitation energy. No unwanted theoretical energy levels appear in those parts of the level scheme to which the fitting procedure has been applied, although there are still too many predicted states in the energy region just above that covered in the fit. The model predicts a 3^+ state at 2.75 MeV in Ni^{60}; such a level has indeed been identified at 2.62 MeV, very close to the predicted position.

Table II compares the theoretical and experimental values of the level energies of the heavier Ni isotopes, Ni^{63} to Ni^{65}. Although none of these levels are included in the fitting procedure, agreement with experiment is not qualitatively different from that obtained for the

[27] I. Talmi, Rev. Mod. Phys. 34, 704 (1962).
[28] N. Auerbach, Nucl. Phys. 76, 321 (1966).

lighter Ni isotopes. One sizable discrepancy does, however, emerge; the model energy of the first 0^+ excited state of Ni^{64} is 2.15 MeV. This is 740 keV lower than the measured excitation energy of 2.89 MeV.

For the even-A isotopes from Ni^{58} to Ni^{64}, the theoretical excitation spectra tend to be symmetric about the center of the shell (Ni^{62}). Most of the experimental excitation energies favor this symmetry; an exception is that the first 0^+ excited state in Ni^{64} is about 600 keV higher than the corresponding state in Ni^{60}. Thus the fact that the calculated excitation energies of these states differ by less than 150 keV (in the wrong direction) leads to the disagreement between theory and experiment mentioned in the preceding paragraph.

Two systematic shortcomings of the model deserve mention here although the errors involved are not large. (1) The calculated excitation energies of 4^+ first excited states are too low. (2) The first 2^+ excited states in the model all lie close to 1.5 MeV. The corresponding experimental excitation energies start at around 1.5 MeV in Ni^{58}, drop to a minimum excitation energy of 1.17 MeV in Ni^{62}, and then begin an upward trend again at Ni^{64}. On the whole, however, our model provides a good description of the low-lying Ni energy levels.

We conclude this section with a few further remarks about the ground-state binding-energy curve and the implications for configuration mixing of its repulsive upturn after Ni^{62}. It has been mentioned that the influence of the effective two-body interaction on binding energies is repulsive for $n>6$. Indeed, after subtraction of the single-particle binding energies, Eq. (7), the Ni^{62} ground state is about 4 MeV more strongly bound than the ground state of Ni^{64}. Because of this, there may be little difference in energy between the configuration $(2p_{3/2},1f_{5/2},2p_{1/2})^8$ and the excited configuration $(2p_{3/2},1f_{5/2},2p_{1/2})^6(1g_{9/2})^2$, in spite of the fact that the $1g_{9/2}$ single-particle level probably lies 2–3 MeV above the $2p_{3/2}$. Therefore $1g_{9/2}$ configuration mixing in low-lying states may increase sharply after Ni^{62}. It is with this possibility in mind that we have omitted the levels of the heavier Ni isotopes (Ni^{63} to Ni^{66}) in determining a best-fit effective interaction.

The expectation that the admixture of the $1g_{9/2}$ configuration would increase after Ni^{62} does not seem to be borne out by the energy-level schemes; the quality of agreement between theory and experiment is not markedly poorer in Ni^{63}, Ni^{64}, and Ni^{65} than in the lighter isotopes of Ni. A possible explanation here is that the shell-shell interaction energies between the $1g_{9/2}$ orbit and the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ orbits are themselves strongly repulsive.

An underlying assumption of the above discussion of ground-state energies is that the binding energies of neutrons in the single-particle levels $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ outside the Ni^{56} core are the same for all isotopes of Ni. It is clear, however, that the addition of valence nucleons must change the core and consequently the shell-model potential which it provides. Such changes influence ground-state binding energies much more critically than excitation spectra. It is therefore likely that our phenomenological shell-shell interaction constants contain contributions due to effects that the valence nucleons produce on the Ni^{56} core; accordingly, our discussion of ground-state binding energies has only qualitative significance.

5. THE SENIORITY QUANTUM NUMBER

Quasiparticle methods[29] in the identical-nucleon shell model are closely linked to the concept of seniority; indeed, a v-quasiparticle state is simply a special state of seniority v. Quasiparticle methods are therefore likely to be useful only if seniority is a good quantum number in identical-nucleon systems. We shall now show that in our identical-nucleon model of the Ni isotopes, seniority is indeed fairly well conserved.

The seniority of a state is by definition the number of unpaired particles it contains, a pair being two nucleons occupying the same orbit and coupled to total angular momentum zero. Table V gives the percentage seniority decomposition of the model wave functions for low-lying states in Ni^{61} and Ni^{62}. Similar results are obtained for other isotopes of Ni. There are three main qualitative conclusions (which can probably be assumed to be valid, with a few obvious modifications, for other identical-nucleon systems). These are:

(1) All 0^+ ground states of even-A nuclei have

TABLE V. Seniority decomposition of various model states in Ni^{61} and Ni^{62}. The quantities listed are the percentage intensities (100×squared amplitude) of the various seniority components. A given shell-model basis state will have the configuration $(f_{5/2})^{n_1}\times(p_{3/2})^{n_2}(p_{1/2})^{n_3}$. If the seniorities of the three configurations $(f_{5/2})^{n_1}$, $(p_{3/2})^{n_2}$, and $(p_{1/2})^{n_3}$ are v_1, v_2, and v_3, respectively, then the seniority of the basis state is defined to be $V=\sum_{i=1}^{3} v_i$.

Nucleus	State	Percentage in state with seniority v			
		$v=0$	2	4	6
Ni^{62}	0_1	99.7	···	0.3	···
	0_2	87.3	···	12.7	···
	1_1	···	23.7	71.0	5.3
	2_1	···	99.4	0.5	0.1
	2_2	···	89.1	10.7	0.2
	3_1	···	40.6	59.3	0.1
	4_1	···	92.9	7.0	0.1
		$v=1$	3	5	
Ni^{61}	$\frac{1}{2}_1$	95.9	3.9	0.2	
	$\frac{1}{2}_2$	26.6	72.3	1.1	
	$\frac{3}{2}_1$	92.1	7.3	0.6	
	$\frac{3}{2}_2$	29.8	65.6	4.6	
	$\frac{5}{2}_1$	96.3	3.6	0.1	
	$\frac{5}{2}_2$	20.8	74.3	4.9	

[29] M. Baranger, Phys. Rev. **120**, 957 (1960).

seniority $v=0$; 2^+ first excited states have $v=2$. Admixtures of states of higher seniority are less than $\frac{1}{2}\%$.

(2) There is a class of low-lying states whose seniority is not as pure as that of the 0^+ ground states and 2^+ first excited states but for which the lowest possible seniority still predominates; higher seniority admixtures are here of the order of 10%. This class includes the second 0^+ and 2^+ and the first 4^+ states in even-A isotopes, and the lowest states of spin $\frac{3}{2}^-$, $\frac{5}{2}^-$, and $\frac{1}{2}^-$ (the spins of the underlying single-particle orbits) in odd-mass isotopes.

(3) In other states, such as the lowest states of spin 1^+ and 3^+ and the third 2^+ state in even-A isotopes, seniority is very strongly mixed. Any sensible two-quasiparticle description of the states just mentioned is clearly out of the question.

Auerbach[28] has carried out a lowest-seniority study of states of spin 0^+, $\frac{3}{2}^-$, $\frac{5}{2}^-$, and $\frac{1}{2}^-$ in the Ni isotopes within the configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$; in these calculations it is assumed from the outset that 0^+ states have seniority 0 and that states of spin $\frac{3}{2}^-$, $\frac{5}{2}^-$, and $\frac{1}{2}^-$ have seniority 1. Under these circumstances only eleven independent combinations of interaction matrix elements enter the calculation. These are the six pairing energies

$$\mathcal{E}_0(j_1j_2)=\langle j_1j_10|V|j_2j_20\rangle \tag{9}$$

and the five shell-shell interactions $W(j_1j_2)$ defined in Eq. (8). [The shell-shell interactions provide only five additional parameters because $W(\frac{1}{2}\frac{1}{2})=4\mathcal{E}_0(\frac{1}{2}\frac{1}{2})$.] Auerbach determines the constants $\mathcal{E}_0$ and W by a least-squares fit to the energies of levels of appropriate spin in the isotopes Ni^{58} to Ni^{65}. Table IV compares the resulting interaction constants with those obtained from the best-fit interaction matrix elements of Sec. 3. The two sets of parameters agree to within the uncertainties involved in their determination.

We have seen that some of the states that Auerbach assumes to have lowest seniority in fact contain up to 10% admixtures of higher seniority. The fact that Auerbach's interaction parameters agree with ours therefore implies that 10% admixtures of higher seniority have little effect on level energies. This conclusion can be verified[30] by recalculating the energy spectra of the best-fit model Hamiltonian of Secs. 2 and 3 within the lowest-seniority part of the shell-model basis. Only those states are considered whose lowest-seniority component amounts to 85% or more of the wave function. Corresponding energy eigenvalues in the complete and seniority-truncated bases agree to within 120 keV or less. The errors are substantially less than the rms deviation (about 150 keV) between measured and calculated level energies.

We conclude that the energies of the lowest states of identical-nucleon systems can be adequately treated within the lowest-seniority approximation. On the other hand, higher-seniority components must clearly be retained in any discussion of transition rates since there the influence of 10% admixtures can be very great.[30]

6. ELECTROMAGNETIC PROPERTIES OF THE Ni ISOTOPES

Our phenomenological shell model of the Ni isotopes has been shown to give a satisfactory description of level energies. This has been achieved, however, by absorbing large amounts of configuration interaction in the effective shell-model Hamiltonian. Thus to discuss transition rates within the configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$ we must be prepared to deal with effective operators explicitly modified by the influence of neglected configurations. This is particularly obvious in the case of $E2$ transitions. Indeed, in the situation under consideration there can be no $E2$ transitions at all without participation from the core since we are dealing with all-neutron configurations. We therefore introduce an effective $E2$ operator, using the simple expedient of endowing each valence neutron with a nonzero effective charge e_v. The effective $E2$ operator for the neutron configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$ is then

$$Q_q^2=e_v\sum_{i=1}^{n} r_i^2Y_q^2(\hat{r}_i), \tag{10}$$

where r_i is the angular coordinate of the ith nucleon. Matrix elements of this operator are evaluated with harmonic-oscillator radial wave functions $[\propto\exp(-\frac{1}{2}\alpha r^2)]$ whose oscillator parameter $\alpha=M\omega/\hbar$, where M is the nucleon mass, is given[31] by

$$\hbar\omega=41A^{-1/3}\text{ MeV}. \tag{11}$$

Consider first the reduced $E2$ transition rates $B(E2:0_1^+\rightarrow 2_1^+)$ for Coulomb excitation of the 2^+ first excited states of Ni^{58}, Ni^{60}, and Ni^{62}. The effective charges needed to fit the observed[32] transition rates are given in Table VI. Their average value $1.7e$ is similar to what is encountered for other nuclei in this mass region. For example, McCullen and Zamick[33] give an estimate of $1.9e$ for the effective neutron charge in the $1f_{7/2}$ shell. The fact that e_v decreases by 20% from Ni^{58} to Ni^{60} and again from Ni^{69} to Ni^{62} reflects the failure of the model to reproduce the measured ratio of transition rates. The deviations are, however, small and are, furthermore, physically reasonable; they indicate that the $1f_{7/2}$ proton shell becomes more stable with the addition of valence neutrons.

[30] M. H. Macfarlane, in *Lectures in Theoretical Physics* (University of Colorado Press, Boulder, Colorado, 1966); Vol. VIII-C, p. 583.

[31] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **29**, No. 16 (1955).

[32] P. H. Stelson and F. K. McGowan, Nucl. Phys. **32**, 652 (1962).

[33] L. Zamick and J. D. McCullen, Bull. Am. Phys. Soc. **10**, 485 (1965).

TABLE VI. $E2$ transition rates in the even-mass Ni isotopes. The first column identifies the nucleus under consideration, the second gives the values for the transition rate $B(E2: 0_1^+ \rightarrow 2_1^+)$ measured and reported in Ref. 32. The third column lists the values of the effective neutron charge [Eq. (10)] needed to fit the observed $E2$ rates. The fourth and fifth columns compare measured and calculated branching ratios for the E_2 decay of the second 2^+ states of even-mass Ni isotopes, the measured branching ratios being those of Refs. 20 and 34. The sixth column gives the fraction of the sum rule exhausted by the first model 2^+ state. The sum-rule value is $\sum_i B(E2:0^+ \rightarrow 2_i^+)$ within the chosen configurations $(2p_{3/2}, 1f_{5/2}, 2p_{1/2})^n$.

Nucleus	Measured $B(E2: 0_1^+ \rightarrow 2_1^+)$ (10^{-48} cm^4)	Effective charge	$\frac{B(E2: 2_2^+ \rightarrow 0_1^+)}{B(E2: 2_2^+ \rightarrow 2_1^+)}$ Measured	Calculated	$\frac{B(E2: 0_1^+ \rightarrow 2_1^+)}{\sum_i B(E2: 0_1^+ \rightarrow 2_i^+)}$
Ni^{58}	0.072	1.90	...	33.1	0.87
Ni^{60}	0.091	1.65	0.005	0.05	0.89
Ni^{62}	0.083	1.45	<0.005	0.03	0.96

Ratios of $E2$ transition rates within a given nucleus are tests of the model wave functions rather than indicators of the importance of core excitation. Of particular interest are the branching ratios for the decay of the second excited 2^+ states (2_2^+) of the even-mass Ni isotopes. Such states have two possible modes of electromagnetic de-excitation—either directly to the ground state or by a two-stage decay through the 2^+ first excited state (2_1^+). It is a striking experimental[20,34] property of Ni^{60} and Ni^{62} that the crossover transition $2_2^+ \rightarrow 0_1^+$ is strongly inhibited. Table VI shows that the model wave functions successfully reproduce this property of the measured branching ratios.

It has been argued that the inhibition of the crossover transition $2_2^+ \rightarrow 0_1^+$ indicates vibrational characteristics in the even-mass Ni isotopes. According to this interpretation, 2^+ first excited states are 1-phonon states and second 2^+ excited states are two-phonon states. The crossover transition is forbidden because it involves the transfer of two phonons. Analysis of our model wave functions does not suppor this argument. The second 2^+ model states cannot be approximate two-phonon states since, as stated in Sec. 5, they are predominantly seniority 2, whereas two-phonon states have large seniority-4 components. Nevertheless, the model wave functions correctly reproduce the inhibited crossover transitions. They do so because, as shown in Table VI, the first 2^+ states in the model nearly exhaust the sum rule $\sum_i B(E2: 0_1^+ \rightarrow 2_i^+)$ for ground-state $E2$ transitions within the chosen configurations $(2p_{3/2}, 1f_{5/2}, 2p_{1/2})^n$. An equivalent statement is that 2^+ first excited states are well reproduced by the action of the quadrupole operator Q^2 of Eq. (10) on the corresponding ground state, i.e.,

$$\Psi(2_1^+) \propto Q^2 \Psi(0_1^+). \tag{12}$$

Thus, little of the available $E2$ strength is left for the crossover transition $2_2^+ \rightarrow 0_1^+$. We conclude, in agreement with Hsu and French,[35] that the crossover transition is inhibited by a mechanism that has little to do with the detailed structure of the second 2^+ state.

[34] D. M. van Patter, Nucl. Phys. **14**, 42 (1959).
[35] L. S. Hsu and J. B. French, Phys. Letters **19**, 135 (1965).

The vibrational model predicts that the branching ratio

$$\mathfrak{R} = \frac{B(E2: 3_1^+ \rightarrow 2_1^+)}{B(E2: 3_1^+ \rightarrow 2_2^+)} \tag{13}$$

should be zero since the decay $3_1^+ \rightarrow 2_1^+$ is a 2-phonon transition and is accordingly forbidden . This branching ratio has been measured[20] in Ni^{60} and indeed turns out to be very small ($\mathfrak{R} < 0.004$). Our model wave functions also give an inhibited crossover ($\mathfrak{R} = 0.046$) in Ni^{60}, again in good agreement with experiment. However, the shell-model wave functions give $\mathfrak{R} \approx 1$ in Ni^{62} whereas the vibrational model, as has been seen, gives $\mathfrak{R} = 0$ for all even isotopes. A measurement of the branching ratio $\mathfrak{R}$ in Ni^{62} would clearly be of interest.

The quadrupole moment of Ni^{61} has recently been measured[36]; its value is $+0.128 \pm 0.025$ b. With an effective neutron charge of $1.5e$ (the average of the values given in Table VI for Ni^{60} and Ni^{62}), the shell-model wave functions yield a quadrupole moment of $+0.135$ b. Table VII lists the calculated values for the quadrupole moments of the 2^+ first excited states of even-mass Ni isotopes; experimental values have not yet been determined.

Thus the known quadrupole properties (moments and transition rates) of the Ni isotopes are successfully reproduced by the model wave functions with an effective-charge treatment of the $E2$ operator. The

TABLE VII. Calculated and measured quadrupole moments of various states of the Ni isotopes. The measured quadrupole moment of the $\frac{3}{2}^-$ ground state of Ni^{61} is given in Ref. 36; no measurements have yet been reported for the other states (2^+ first excited states) for which calculated quadrupole moments are given in the table. For Ni^{61} and Ni^{64}, an effective charge of $1.5e$ was used. In all other cases, the effective charge was taken from column 3 of Table VI.

Nucleus	J	Q/e (barns) Calculated	Measured
Ni^{58}	2	−0.251	
Ni^{60}	2	−0.029	
Ni^{61}	$\frac{3}{2}$	0.135	0.128 ± 0.025
Ni^{62}	2	0.032	
Ni^{64}	2	0.078	

[36] W. J. Childs and L. S. Goodman (to be published).

TABLE VIII. Calculated and measured magnetic moments of states in Ni^{61}. Experimental values are taken from Refs. 37 and 38.

Ni^{61} state	Magnetic moment (nuclear magnetons) Measured	Schmidt	Calculated
$\frac{3}{2}_1$	−0.75	−1.91	−1.33
$\frac{5}{2}_1$	0.35	1.36	1.30

data indicate an effective neutron charge of between $1.5e$ and $2e$.

Very little is known about the magnetic dipole properties of the Ni isotopes. Only the magnetic moments of the $\frac{3}{2}^-$ ground state and $\frac{5}{2}^-$ first excited state of Ni^{61} have been measured.[37,38] The observed values lie well inside the Schmidt limits, as is shown in Table VIII; deviations from the Schmidt values are in excess of 1 nm. On the other hand, with the assumption that each valence neutron in Ni^{61} has its free-particle anomalous moment of −1.913 nm, our model wave functions yield magnetic moments which differ insufficiently from the Schmidt values to explain the experimental results. This result is in part a consequence of seniority conservation. The essential point here is that the $M1$ operator is a quasispin scalar[39]; its matrix elements between states of definite seniority are therefore independent of particle number and its expectation values in states of seniority 1 necessarily reduce to the Schmidt moments. The deviations of the calculated magnetic moments from the Schmidt values are thus small because the pertinent model wave functions are predominantly seniority 1. (See Table V.)

We have already remarked that an adequate treatment of transition rates within the configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$ can be expected to demand the introduction of effective transition operators,[40,41] explicitly modified by the influence of neglected configurations. This should certainly be true of magnetic moments, which are known[42] to be acutely sensitive to small configuration impurities. Our results clearly confirm these expectations. There are, however, too few experimental data to permit the phenomenological determination of an effective $M1$ operator.

7. SINGLE-NUCLEON SPECTROSCOPIC FACTORS

As a further test of the model wave functions, consider transition rates in (d,p) and (d,t) reactions, in which a neutron is transferred between two states Ψ_0 and Ψ_1. All the stripping and pickup reactions considered here involve even-A and therefore spin-zero targets; in such situations conservation of angular momentum and parity (with the additional assumption that two orbits that differ only in principal quantum number cannot be simultaneously active) compels capture to take place into or from a definite shell-model orbit j. The appropriate transition operator is then the single-nucleon creation operator $a^{j\dagger}$ for orbit j; the transition rate is determined by the spectroscopic factor $\mathcal{S}$ defined by the equation

$$\mathcal{S}(\Psi_0 \rightleftharpoons \Psi_1)=\frac{1}{(2J_1+1)}|\langle\Psi_1\|a^{j\dagger}\|\Psi_0\rangle|^2, \qquad (14)$$

where the symbol $\langle \| \| \rangle$ indicates a reduced matrix element in the sense of the Wigner-Eckart theorem[43] and J_1 is the spin of the state Ψ_1; the forward arrow in Eq. (14) refers to stripping and the backward to pickup. The spectroscopic factor is in fact a reduced transition rate, a quantity of exactly the same sort as the reduced $E2$ rate $B(E2)$.

It is clear that the total strength of all stripping transitions involving orbit j must be proportional to the number of holes in that orbit in the ground state of the target nucleus. In similar fashion, pickup reactions measure the number of particles. These statements find formal expression in the sum rules[44]

$$\begin{aligned} G_j(d,p)&=\sum_f \frac{2J_f+1}{2J_i+1}\mathcal{S}_j(i\rightarrow f)=(2j+1)-\langle N_j\rangle, \\ G_j(d,t)&=\sum_f \mathcal{S}_j(i\rightarrow f)=\langle N_j\rangle. \end{aligned} \qquad (15)$$

The sums embrace all possible final states Ψ_f for stripping or pickup involving orbit j, and $\langle N_j\rangle$ is the ex-

TABLE IX. Fractional occupation numbers $\langle N_j\rangle/(2j+1)$ of the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ orbits in the ground states of the even-mass Ni isotopes. The errors quoted are explained and the appropriate experimental references cited in the text.

Orbit	$n=$	Occupation numbers $\langle N_j\rangle/(2j+1)$ 2	4	6	8
$p_{3/2}$	(d,p)	0.24±0.10	0.45±0.12	0.59±0.01	0.76±0.03
	(d,t)	0.27	0.58	0.66±0.08	0.85±0.05
	Calc.	0.32	0.52	0.69	0.84
$f_{5/2}$	(d,p)	0.17	0.33	0.55	0.74
	(d,t)	0.12	0.18	0.36	0.56
	Calc.	0.09	0.25	0.39	0.56
$p_{1/2}$	(d,p)	0.02±0.10	0.11±0.25	0.16±0.05	0.28±0.07
	(d,t)	0.12	0.30	0.60±0.16	0.60±0.11
	Calc.	0.09	0.25	0.45	0.64

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TABLE X. The (d,p) spectroscopic factors for some low-lying states in the odd-mass Ni isotopes. Data on Ni^{59} and Ni^{61} are taken from Ref. 16, data on Ni^{65} from Ref. 15. In addition to the spectroscopic factors for states of known spin, the table gives the summed strength $\sum_f(2J_f+1)S$ of all $l=1$ transitions to states whose spins may be either $\frac{1}{2}$ or $\frac{3}{2}$. J_f is the spin of the final (d,p) state under consideration. The sixth column gives the sum-rule limit [Eq. (15)] obtained by summing the calculated values of $(2J_f+1)S$ over all final model states with spin J_f.

Residual nucleus	$(l,J)_f$	Experimental excitation energy (MeV)	$(2J_f+1)S$ (d,p) experiment	Calculated	Sum-rule limit
Ni^{59}	$1,\frac{1}{2}$	0.47	1.24	1.70	1.82
		1.32	0.56	0.10	
	$1,\frac{3}{2}$	0.0	2.77	2.36	2.72
		0.89	0.31	0.33	
	1,?	10 states from 2.5 to 5.0 MeV	0.30		
	$3,\frac{5}{2}$	0.34	5.19	5.24	5.46
Ni^{61}	$1,\frac{1}{2}$	0.29	1.21	1.38	1.50
	$1,\frac{3}{2}$	0.0	1.67	1.73	1.92
	1,?	12 states from 1.0 to 5.0 MeV	1.1		
	$3,\frac{5}{2}$	0.07	3.37	4.35	4.58
		0.91	0.23	0.15	
Ni^{65}	$1,\frac{1}{2}$	0.07	1.23	0.70	0.72
	$1,\frac{3}{2}$	0.32	0.17	0.64	0.64
		0.70	0.62	0.00	
	1,?	3 or 4 states from 1.0 to 2.5 MeV	0.25		
	$3,\frac{5}{2}$	0.0	1.49	2.56	2.64

pectation value in the target ground state $|\Psi_t\rangle$ of the number operator for orbit j. That is,

$$\langle N_j\rangle=\langle\Psi_t|N_j|\Psi_t\rangle. \quad (16)$$

Calculated values of the fractional occupation numbers $\langle N_j\rangle/(2j+1)$ are compared with experiment in Table IX. The experimental occupation numbers listed are obtained with the aid of Eqs. (15) from the results of (d,p) [15,16] and (d,t) [7] experiments on the even-mass Ni isotopes. Spectroscopic factors are extracted from the differential cross sections by an analysis based on the distorted-wave Born approximation; absolute values are separately renormalized for each target such that total pickup and stripping strengths on the target nucleus Ni^{56+n} satisfy the relations

$$\sum_j G_j(d,t)=\sum_j[(2j+1)-G_j(d,p)]=n. \quad (17)$$

The sums $\sum_j$ embrace the orbits $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$. The error limits in Table IX stem solely from the fact that numerous $l=1$ transitions are observed to states whose spin may be either $\frac{1}{2}^-$ or $\frac{3}{2}^-$; no account is taken of other uncertainties in the experiments and in their analysis.

The (d,p) and (d,t) occupation numbers are in good accord with each other only for the $p_{3/2}$ orbit. The (d,p) analysis suggests a more rapid rate of filling for the $1f_{5/2}$ orbit and a correspondingly slower rate of filling for the $2p_{1/2}$ orbit. Table IX shows that the model occupation numbers are in good agreement with the values obtained from the pickup experiments. This suggests that unless there is something seriously wrong with the entire analysis, a significant fraction of the total $1f_{5/2}$ strength has gone undetected in the stripping experiments.

The calculated occupation numbers are of course rather insensitive to the interaction parameters. Indeed, once the valence orbits have been selected—and our occupation-number analysis is evidence that they have been chosen correctly—the model cannot avoid predicting that these valence orbits fill gradually with the addition of valence nucleons. Thus agreement with experiment for the ground-state occupation numbers is a modest, though satisfying, achievement of the theory.

On the other hand, a serious disagreement between theory and experiment emerges on consideration of stripping transitions to individual states of the residual odd-mass Ni isotopes. In every case, the shell model concentrates almost the entire sum-rule strength into a single state of the residual nucleus. As is clear from Table X, the observed $l=1$ strength (where l is the orbital angular momentum of the transferred neutron) is much more widely dispersed over different states of the residual nuclei than the predictions of our model would lead us to expect. Although there is no clear evidence of a similar discrepancy for the $f_{5/2}$ transitions, let us recall that analysis of the ground-state occupation numbers suggests that a significant fraction of the total $l=3$ stripping strength may have escaped detection.

A further prediction of the model is that the state having the bulk of the stripping strength is the lowest of its spin in the residual nucleus in question. In the reaction $Ni^{64}(d,p)Ni^{65}$, the $l=1$ transition to the second $\frac{3}{2}^-$ state is found[15] to be more than three times as strong as that to the lowest $\frac{3}{2}^-$ state.

These discrepancies are serious. It is unlikely that they are due in a significant degree to deficiencies in the distorted-wave stripping theory used to extract "experimental" spectroscopic factors. For example, the cross sections for stripping with target excitation[45] seem to be much too small to account for the observed spreading of the stripping strength. We conclude that an adequate treatment of the single-nucleon spectroscopic factors within the configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$ requires the introduction of an effective transition operator radically different from the simple creation operator $a^{j\dagger}$ of Eq. (14). The requisite effective operator must contain large additional terms of the form $a^\dagger a^\dagger a$, with two creation operators and one destruction operator. Another way of putting this argument is to note that agreement with experiment for the single-nucleon spectroscopic factors undoubtedly demands more seniority mixing than is present in our model wave functions. Since it is very difficult to envisage a sufficiently radical modification of the wave functions, we must incorporate the necessary mixing in the effective transition operator.

8. COMPARISON WITH OTHER SHELL-MODEL STUDIES OF Ni

The earliest detailed shell-model study of the Ni isotopes is that of Arvieu, Salusti, and Veneroni,[46] who consider energy-level schemes and $E2$ transition rates with the aid of quasiparticle approximations. The use of these approximations enables them to extend the chosen configurations to $(2p_{3/2},1f_{5/2},2p_{1/2},1g_{9/2})^n$ but restricts them to lowest seniority. As the effective interaction, Arvieu *et al.* use a spin-dependent central potential with a Gaussian radial dependence. Agreement with experiment is markedly poorer than in the present study. A lowest-seniority shell-model calculation within the configurations $(2p_{3/2},1f_{5/2},2p_{1/2},1g_{9/2})^n$ with the central interaction of Arvieu *et al.* suggests that this poorer agreement is due more to the restricted treatment of the interaction than to errors inherent in the quasiparticle approximations.

Hsu and French[35] calculate the energy-level schemes of the Ni isotopes within the configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$. Their effective interaction acts in relative s states only, apart from a monopole term added to fit the ground-state binding energies. Their main concern is with the accuracy of various types of quasiparticle approximation and there is accordingly little overlap with the present study. One interesting point of agreement concerning the purported vibrational character of states in the even-mass Ni isotopes has already been noted in Sec. 6.

Another study which uses a simple form for the residual two-body force is the work of Plastino, Arvieu, and Moszkowski.[47] These authors study the spectra of many single-closed-shell nuclei, including nickel, by use of the surface-delta-function interaction. This force leads to results in qualitative agreement with experiment. For nickel, in particular, when the interaction strength is fitted to the odd-even mass differences, the excitation energy of the first 2^+ state in the even-A isotopes is close to experiment. However, their calculation leads to the same difficulties as we encountered: (1) the first 4^+ state is consistently predicted at too low an excitation energy, and (2) the energy of the first 2^+ state varies little from one nucleus to another.

In two recent papers, Auerbach[23,48] studies the isotopes of Ni within the identical-nucleon configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$. The first paper, discussed in Sec. 5, makes a least-squares fit to determine the eleven lowest-seniority ($v=0$ and $v=1$) interaction parameters. The second paper removes the restriction to lowest seniority and recalculates the spectra of Ni^{58}, Ni^{59}, Ni^{60}, and Ni^{64}, the necessary additional interaction parameters being taken from the s-state interaction of Kallio and Kolltveit.[49]

Our calculations go beyond those of Auerbach in three important respects. (a) The present study gives complete level schemes and wave functions for the isotopes Ni^{61}, Ni^{62}, and Ni^{63}, which Auerbach treats only in lowest-seniority approximation. (b) Treatment of the mid-shell isotopes without the lowest-seniority approximation enables us to make a better assessment of the goodness of the seniority quantum number. The point here is that seniorities up to 5 are possible in Ni^{61} and Ni^{63}, and up to 6 in Ni^{62}. The other odd-mass Ni isotopes have $v\leqslant 3$, the even-mass isotopes $v\leqslant 4$. (c) The present study treats moments and transition rates, whereas Auerbach considers only the energy spectra.

We have, of course, emphasized here the points at which our work differs from that of Auerbach. Where they overlap, the two studies agree quite closely. Indeed the fact that two different paths lead to the same effective interaction (to within the various uncertainties involved) suggests that were it feasible to carry out an unrestricted least-squares determination of the thirty independent interaction matrix elements for the active orbits $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$, then the resulting effective interaction would not differ very much from that given in Sec. 3 of the present study.

9. CONCLUSIONS

The isotopes of nickel are described by a shell model within the identical-nucleon configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$. It is shown that with this choice of configurations no conventional shell-model potential, even with

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tensor and two-body spin-orbit as well as central components, can provide a satisfactory description of the observed level energies. A less restricted parametrization of the interaction is therefore considered, with phenomenological corrections in relative s states. A least-squares fit to observed level energies of the isotopes of Ni^{58} to Ni^{62} then yields an interaction which provides an acceptable description of the level structure of the Ni isotopes from Ni^{58} to Ni^{65}. This best-fit interaction is shown to indicate repulsive interactions between identical-nucleon shells and to conserve seniority to a useful degree of approximation. The interaction matrix elements are compared with those of an approximate reaction matrix calculated by Kuo from Hamada and Johnston's free-nucleon interaction; a fair degree of agreement is found when allowance is made for core excitations wherein a single nucleon is promoted from a core orbit to a valence or empty orbit.

Core excitation has a large influence on the effective interaction; it is therefore to be expected that a treatment of moments and transition rates within the configurations $(2p_{3/2},1f_{5/2},2p_{1/2})^n$ will demand the introduction of effective operators, explicitly modified by the influence of neglected configurations. Analysis of the model and its predictions for transition rates confirms this expectation. With unmodified operators, the model is quite unable to account for the large deviations of the observed magnetic moments from the Schmidt values and the observation that the stripping strength into a given orbit is spread over several states of the residual nucleus. In both cases, the goodness of the seniority quantum number hampers agreement with experiment. Core excitation likewise exerts a large influence on quadrupole moments and on $E2$ transition rates and branching ratios. Here, however, good agreement with experiment can be attained by introducing an effective neutron charge of between $1.5e$ and $2e$.

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Shell-Model Calculations for $N=30$ Nuclei*

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The results of shell-model calculations on $N=30$ nuclei with $20\leq Z\leq 27$ are reported. An inert ^{48}Ca core is assumed, protons are restricted to the $1f_{7/2}$ shell, and the two active neutrons can occupy the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ orbits. A Hamiltonian is used which leads to a good fit to selected experimental data in this same mass region. For all nuclei treated here except ^{57}Co, the agreement of the calculated spectra with experimental spectra for states below an excitation energy of about 2.5 MeV is satisfactory. The shell-model wave functions are used to calculate spectroscopic factors for one- and two-nucleon transfer reactions which involve ^{54}Cr, ^{55}Mn, and ^{56}Fe. The qualitative results for the one-nucleon transfer reactions are in satisfactory agreement with the factors extracted from experimental data. The predicted relative strengths for strong transitions in these reactions are not in good agreement with the extracted numbers. The quantitative agreement between the predicted strengths and the strengths extracted from experiment for the $^{54}Fe(t,p)^{56}Fe$ reaction is good.

I. INTRODUCTION

THE degree of success which one can reasonably expect in a shell-model calculation of the structure of a given nucleus depends, in large measure, on the degree to which the assumed closed core is really closed. Some of the difficulties found in shell-model calculations in which ^{16}O and ^{40}Ca are assumed to be closed cores come about because these nuclei are not really good closed cores.[1] There is evidence that ^{48}Ca forms a good closed core. Experimental data on the $^{48}Ca(d,p)^{49}Ca$ (Ref. 2) and $^{48}Ca(p,d)^{47}Ca$ (Ref. 3) reactions indicate that there is a negligible amount of core excitation in ^{48}Ca. Furthermore, shell-model calculations[4,5] of a number of properties of $N=28$ nuclei with $20<Z<28$ have shown that a good account of these properties is given when a ^{48}Ca core is assumed, and protons are restricted to the $\pi f_{7/2}$ shell.[6] A closed-shell ^{48}Ca core is also assumed by Vervier[7] in treating $N=29$ nuclei. He restricts protons to the $\pi f_{7/2}$ shell, and allows the extra core neutrons to occupy the $\nu p_{3/2}$, $\nu p_{1/2}$, and $\nu f_{5/2}$ shells. He found good agreement with experiment for a large number of energy levels, spectroscopic factors and some β decay and electromagnetic-transition rates.

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