Neutron Pickup from ^{46}Ca

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The reaction ^{46}Ca$(d,t)^{45}$Ca was investigated at 10-MeV bombarding energy with 11-keV energy resolution. States in ^{45}Ca at 0 keV ($\frac{7}{2}^-$), 176 keV ($\frac{3}{2}$), and 1886 keV ($\frac{3}{2}^+$) were excited with spectroscopic factors of 6.5±1.5, <0.25, and 1.3±0.4, respectively. The application of the distorted-wave Born approximation to the (d,t) reaction is discussed in some detail. The present data are compared with results from neutron transfer reactions on ^{46}Ca and neighboring nuclei.

I. INTRODUCTION

The present paper reports on results from the reaction ^{46}Ca$(d,t)^{45}$Ca at 10-MeV bombarding energy. This work is a continuation of the previously published investigation of the ^{43}Ca$(d,t)^{42}$Ca reaction.[1]

Only three states were identified in ^{45}Ca, namely, the ground state, the 176-keV and the 1886-keV states. The nature of these states is of some interest, since there is considerable doubt as to where the hole states of ^{45}Ca are situated[2] and as to the amount of the configuration admixture in the predominantly neutron $(1f_{7/2})^6$ ^{46}Ca ground state.

In the ^{43}Ca(d,t) work it was shown that the relative cross sections as well as the angular-distribution shapes could be well reproduced in a distorted-wave (DW) calculation. The present data are used for a further investigation of the DW procedures.

The recent work of Bassel[3] has made it possible to derive absolute spectroscopic factors from (d,t) data; a comparison of all the available (d,t) and (d,p) data[1,4,5] on ^{43,46}Ca is undertaken, and it is shown that consistent results obtain for the $1f_{7/2}$ spectroscopic strengths.

II. EXPERIMENTAL PROCEDURES AND RESULTS

The ^{46}Ca(d,t) data were derived from the same experiment that gave information on the ^{46}Ca(d,d') reaction.[6] The major experimental problem was to obtain reliable, selective scanning of triton tracks in a background of protons, deuterons, and α particles. Because of the low ground-state (g.s.) Q value, the tritons appear at momenta lower than those of elastically scattered deuterons. The position of the ^{46}Ca(d,t) g.s. triton group was calculated for a number of angles near 35° and a particle group was recognized at each of the calculated positions. One scanner was especially trained to recognize the tracks of these groups from all other tracks. This scanner could then selectively pick out the triton group corresponding to the ground state at all other angles and the scanning was reproduceable. The triton spectrum at 35° reaction angle was then scanned (see Fig. 1) and the kinematical behavior of all groups was checked, resulting in the mass assignments shown on the figure. The ^{13}C$(d,t)^{12}$C g.s. group was recognized at 50°. Besides the ground-state group, only two other groups showed mass 45, (d,t) behavior. The angular distributions are shown in Fig. 2 and the Q values and excitation energies are shown in Table I in comparison with other data on ^{45}Ca. The maximum cross sections of any transition missed in the present experiment can be set to <10 μb/sr.

III. DISTORTED-WAVE ANALYSIS

A. Optical-Model Potentials

The deuteron potential X of Ref. 6 was used in the present investigation. This potential gives a reasonable account of the ^{46}Ca(d,d) data at 10-MeV bombarding energy and it gave good results in the analysis of the ^{46}Ca(d,d') and ^{46}Ca(d,p) reactions.[5,6] Calculations were also made for the $l=3$ transition of ^{46}Ca(d,t) using the $Z2$ average potential of Bassel *et al.*[7] obtained by analysis of ^{40}Ca(d,d) scattering from 7–12 MeV; the only result was a reduction in the predicted peak cross section of about 10%. The potentials are defined in Table II.

The triton potential is very uncertain. The current prejudice for triton or ^{3}He potentials of depth about 3 times that for a single nucleon precludes the ^{40}Ca$+t$ potential used in the analysis of the ^{43}Ca(d,t) reaction. Somewhat arbitrarily it was decided to use the same potential that Bassel used[3] for ^{48}Ca$+^3$He in his study

[1] J. H. Bjerregaard, H. R. Blieden, O. Hansen, G. Sidenius, and G. R. Satchler, Phys. Rev. **136**, B1348 (1964).
[2] H. Morinaga and G. Wolzak, Phys. Letters **11**, 148 (1964); O. Ames, G. Garrett, and P. Vajk, Bull. Am. Phys. Soc. **11**, 393 (1966).
[3] R. H. Bassel, Phys. Rev. **149**, 791 (1966).
[4] J. H. Bjerregaard and O. Hansen, Phys. Rev. **155**, 1229 (1967).
[5] J. H. Bjerregaard, O. Hansen, and G. Sidenius, Phys. Rev. **138**, B1097 (1965).
[6] T. A. Belote, J. H. Bjerregaard, O. Hansen, and G. R. Satchler, Phys. Rev. **138**, B1067 (1965).
[7] R. H. Bassel, R. M. Drisko, G. R. Satchler, L. L. Lee, Jr., J. P. Schiffer, and B. Zeidman, Phys. Rev. **136**, B960 (1964).

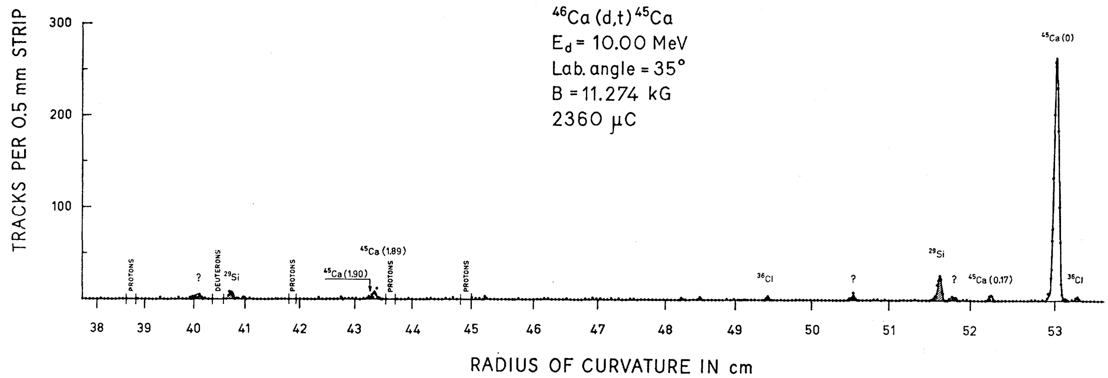

FIG. 1. Triton spectrum from the reaction ^{46}Ca(d,t). Those triton groups which were identified in at least three reaction angles are marked with chemical symbol and mass, pertaining to the final nucleus.

of the normalization of (d,t) and $(d,{}^3\mathrm{He})$ reactions (see Table II, potential $T45$). This has a real depth $V=139$ MeV, whereas the present tendency seems to be toward use of the next deeper family of potentials (pot.) (i.e., those with $V\approx 180$ MeV if a similar radius is used). However, a comparison of the present choice ($V=139$, pot. $T45$) with the potential used in the ^{43}Ca(d,t) analysis ($V=107$, pot. $T42$), for the $l=3$ transition, showed that the deeper potential predicted a peak cross section about 13% smaller (and hence a spectroscopic factor larger by 13%), without changing the angular distribution (Table III). It is expected that use of one of the next deeper sets of potentials would lead to a similar or less change. Of course, one also expects some difference between triton and ^{3}He potentials, although this is believed to be small. Present indications are that triton potentials have slightly weaker surface absorption, which would increase the predicted (d,t) cross sections. These uncertainties, however, are within the over-all uncertainty of the analysis.

A spin-orbit term was tried in the d and t potentials for the ground-state transition. A strength of $V'_{\mathrm{so}}(d)=6$ MeV was used combined with $V'_{\mathrm{so}}(t)=0$ or 8 MeV. The effect on the predicted (d,t) cross sections was negligibly small.

Additional uncertainty attends the choice of parameters for the Saxon well which binds the picked-up neutron. In the ^{46}Ca(d,p) and ^{43}Ca(d,t) analysis, $r_0=1.25$, $a=0.65$ were used, and for consistency the same values were adopted here. It was assumed that the neutron was bound by its separation energy and the well depth was adjusted accordingly. A previous detailed study[8] of the ^{40}Ca(d,p) reaction had used $r_0=1.20$, $a=0.65$. Use of these parameters in the present case reduced the cross section for the $l=3$ transitions by about 20%. Spin-orbit coupling for the bound neutron was included only for the ground-state $1f_{7/2}$ pickup; a coupling of 25 times the Thomas value was used. Its inclusion increased the cross section about 20%; the effect on the $f_{5/2}$ or $d_{3/2}$ would be a reduction of comparable magnitude.

More severe problems attend the definition of the bound-state wave function[9] (more generally, form factor) for the $f_{5/2}$ and $p_{3/2}$ pickup transitions. The calcu-

TABLE I. ^{45}Ca results.

^{44}Ca(d,p)[a] E_x[b] (keV)	l_n	$(2j+1)S$	j^π	^{46}Ca(d,t), present exp. E_x[c] (keV)	l_n[d]	S	Q[c,e] (keV)
0	3	3.4	$(\frac{7}{2})^-$	0	3	6.5 ± 1.5	-4144
176		(nonstripping)	$(\frac{5}{2}^-)$	168	(3)	<0.25	-4312
1433	1	0.5	$(\frac{3}{2},\frac{1}{2})^-$				
1558		(nonstripping)	...				
1584		(nonstripping)	...				
1886	(2)	(0.15, $1d$)	$\frac{3}{2}^+$	1886	(2)	1.3 ± 0.4	-6030
1904	1	2.6	$(\frac{3}{2})^-$		(1)	<0.04	
1973		(nonstripping)	...				

[a] ^{44}Ca(d,p) from J. Rapaport, W. E. Dorenbusch, and T. A. Belote, Phys. Rev. **156**, 1225 (1967). j^π assignments are from the above reference and from Ames *et al.* (Ref. 2).
[b] The estimated uncertainty is 6 keV.
[c] The estimated uncertainty is 10 keV.
[d] An l_n value in parentheses means that l_n was not determined in this experiment but was assumed for the DW estimates of S.
[e] The mass Q value is -4144 ± 10 keV. Good agreement was also found for the ^{13}C(d,t) and ^{30}Si(d,t) g.s. Q values [see e.g., J. Mattauch, W. Thiele, and A. Wapstra, Nucl. Phys. **67**, 32 (1965)].

[8] L. L. Lee, Jr., J. P. Schiffer, B. Zeidman, G. R. Satchler, R. M. Drisko, and R. H. Bassel, Phys. Rev. **136**, B971 (1964).
[9] N. Austern, Phys. Rev. **136**, B1743 (1964); W. T. Pinkston and G. R. Satchler, Nucl. Phys. **72**, 641 (1965); E. Rost, Phys. Letters **21**, 87 (1966).

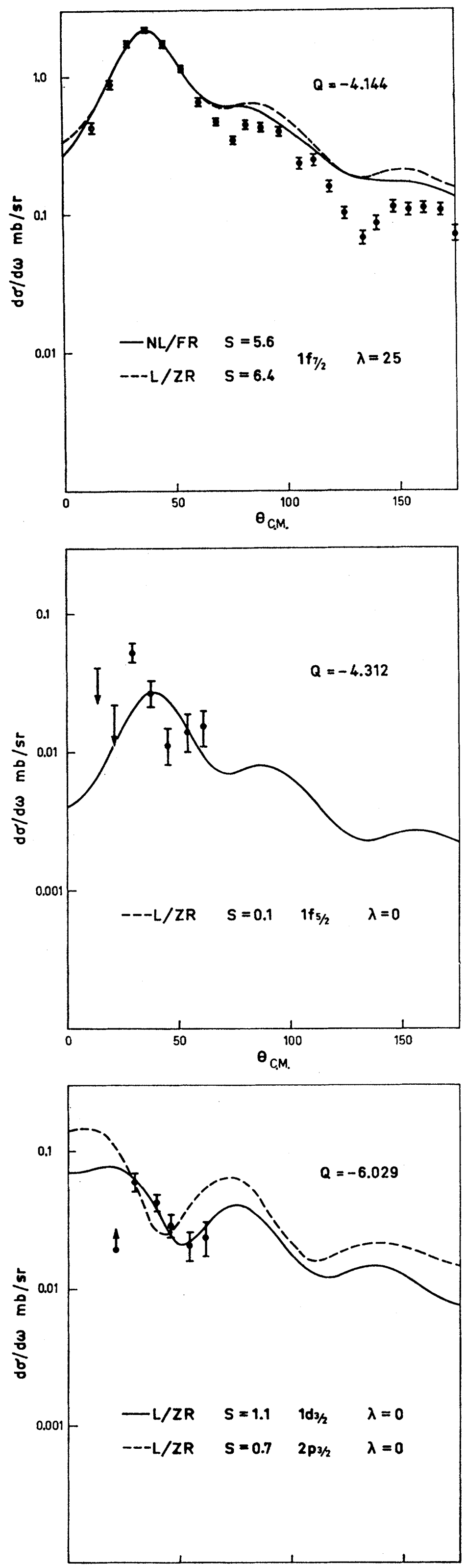

FIG. 2. ^{46}Ca(d,t) angular distributions (E = 10 MeV). The curves are DW calculations as explained in the text.

lations were made assuming single-particle motion in a well of depth V_n with binding B = separation energy. The zero-order $1f_{5/2}$ single-particle level should be some 5 MeV higher in energy (less tightly bound) and the effect of this perturbation on the radial function for the small component present in the 176-MeV level is not known. A sample calculation was made for the $p_{3/2}$ transition, using an effective binding energy 8.4 MeV instead of the separation energy 12.3 MeV. This is roughly the binding one would expect for $2p_{3/2}$ in the same well which binds the $1f_{7/2}$ with 10.4 MeV. This increased the predicted cross section by a factor of 2.0 without changing the angular distribution noticeably. Such a function, of course, has the wrong tail, and this is particularly serious in the present reaction which is especially sensitive to the magnitude of the tail of the wave function. It seems likely that a more realistic form factor for this transition would not produce such a large change.

B. Nonlocality (NL) and Finite-Range (FR) Corrections

These were only studied for the ground-state $l=3$ transition, and were computed in the local-energy approximation.[10] Finite-range effects should definitely be included, and the Gaussian approximation with range 1.69 fm was used.[3] The corrections for nonlocality of the optical potentials are less certain, although there is no doubt some such effect is there. Ascribing all the energy dependence of the phenomenological potentials to nonlocality, we used ranges of 0.54 fm for the deuteron, 0.3 fm for the triton.

Finite-range corrections alone increased the peak cross section by about 15%, and made slight changes in the angular distribution. Surprisingly, the addition of nonlocality corrections has essentially no effect; the peak cross section was reduced by only 2% and no change in shape ensued.

Nonlocality corrections were not applied to the neutron bound-state wave function. There are questions of self-consistency in the choice of equivalent local and nonlocal potentials for bound states which do not arise for scattering. At present it seems better to avoid introducing the correction in an *ad hoc* fashion. There are also indications that more satisfactory results are obtained without it (e.g., Ref. 3).

In Table III we have summarized the discussions of this and the preceding subsections.

C. Spectroscopic Factors

The DW calculations are compared with experiment in Fig. 2. Figure 2 (upper part) shows the results for

[10] F. G. Perey and D. S. Saxon, Phys. Letters **10**, 107 (1964), P. J. Buttle and L. J. Goldfarb, Proc. Phys. Soc. (London) **A83**, 701 (1964).

TABLE II. Optical-model potentials.[a]

Particle	Potential	V (MeV)	r_0 (fm)	a (fm)	W_D (MeV)	W (MeV)	r_0' (fm)	a' (fm)	r_{0c} (fm)	Spin-orbit coupling
d	46X	115	1.0	0.804	13.0	0	1.419	0.660	1.30	b
d	Zav	112	1.0	0.90	18.0	0	1.55	0.47	1.30	b
d	43X	115.5	1.0	0.90	17.5	0	1.55	0.47	1.30	b
t	T45	139	1.08	0.80	0	12.3	1.743	0.721	1.40	b
t	T42	107	1.07	0.854	0	12.0	1.81	0.592	1.40	b
n	N1	c	1.25	0.65	0	0	0	0	0	d
n	N2	c	1.20	0.65	0	0	0	0	0	d

[a] The potential was of the form
$V(r) = -V(1+\exp x)^{-1} + i[-W+4W_D(d/dx')](1+\exp x')^{-1} + V_{\text{Coul}}$; $x=(r-r_0A^{1/3})/a$, $x'=(r-r_0'A^{1/3})/a'$.
[b] A spin-orbit term $V_{so}=2(\hbar/m_\pi c)^2 V_{so}'(1/r)(d/dr)(1+\exp x)^{-1}\mathbf{l}\cdot\mathbf{s}$ could be added as described in the text.
[c] Adjusted to give a binding of 6.26 MeV $-Q_{(d,t)}$ unless otherwise stated. With no spin-orbit coupling the depths were $1f_{7/2}$, 57 MeV; $1f_{5/2}$, 57.2 MeV; $1d_{3/2}$, 43.1 MeV; and $2p_{3/2}$, 63.2 MeV.
[d] In the $1f_{7/2}$ case a spin-orbit potential $V_{so}=V\lambda(\hbar/M_p c)^2\frac{1}{2}(1/r)(d/dr)(1+\exp x)^{-1}\mathbf{l}\cdot\mathbf{s}$ was added with $\lambda=25$ or 0, as described in the text.

the ^{46}Ca$(d,t)^{45}$Ca g.s. transition assuming $1f_{7/2}$ transfer. The 46X, T45, and N1 potentials were used, the latter with a spin-orbit strength of 25 times the Thomas value. The normalization suggested by Bassel[3] is adopted, i.e.,

$$(d\sigma/d\omega)_{d,t} = 3.33 S\sigma_{\text{DW}}\,, \tag{1}$$

where S is the usual spectroscopic factor. The spectroscopic factor deduced by fitting DW prediction and experiment at the maximum cross sections varies from 6.4 (L/ZR) to 5.5 (L/FR). If the bound-state potential N2 is used, $S=7.9$ (L/ZR), and this will be reduced proportionately with the finite range corrections. Thus, a spectroscopic strength of 6.5±1.5 is a reasonable estimate, considering the uncertainties of the analysis and of the measured cross sections. The agreement between theoretical and measured angular distributions is reasonably good.

The data for the transition to the 176-keV$(\frac{5}{2}^-)$ state are poor. It is not clear that the transition proceeds via pickup. It has been shown[11] that in (d,p) this state is excited by a nonstripping transition, i.e., the component of $(f_{7/2})^4$ $(f_{5/2})$ is small. The present data can thus be used only to obtain an upper limit of the amount of $1f_{5/2}$ admixture in the ^{46}Ca ground state. Considering the various uncertainties, it appears that $S<0.25$.

The state excited here with $E_x=1886$ keV appears to be the $\frac{3}{2}^+$ state rather than the $\frac{3}{2}^-$ state. The (d,t) angular distribution gives considerably better agreement with $l=2$ than with $l=1$ (see Fig. 2) and the excitation energy also supports this identification. A spectroscopic factor of $S=1.1$ was assumed for the theoretical curve shown in Fig. 2.

This is appreciably less than the $S=4$ expected if this transition exhausted the $1d_{3/2}$ pickup strength. We note the well depth required to bind a $1d$ neutron at this energy ($B=12.28$) is only $V_n=43.1$ MeV. Even allowing for spin-orbit coupling (which might change V_n to ≈ 47 MeV), this is considerably less than the $V_n=52.9$ MeV (with spin-orbit) which binds the $1f_{7/2}$ neutron. This perhaps implies that the centroid of the $d_{3/2}$ strength is to be found several MeV higher in excitation. (Including neutron spin-orbit coupling would raise S to about 1.3; a more realistic form factor might be closer to the wave function for a more tightly bound $d_{3/2}$ neutron, which also would tend to increase the S value needed.)

The transition to the 1904-keV $\frac{3}{2}^-$ state was not observed and $\sigma_{\text{exp}}(20°)<0.01$ mb/sr. Use of a binding energy equal to the separation energy implies $S<0.04$. If a more realistic form factor were closer to the wave

TABLE III. DW cross sections for the $1f_{7/2}$ g.s. transition.[a]

Elastic potentials: Deuteron Name	Deuteron V_{so}'	Triton Name	Triton V_{so}'	Neutron bound state: Name	λ	Binding (MeV)	Conditions of range and locality		$\sigma_{\max}$
46X	0	T45	0	N1	25	10.4	ZR	L	1.0
Zav	0	T45	0	N1	25	10.4	ZR	L	0.9
46X	0	T42	0	N1	25	10.4	ZR	L	1.15
46X	0	T45	0	N2	25	10.4	ZR	L	0.8
46X	0	T45	0	N1	0	10.4	ZR	L	0.84
46X	0	T45	0	N1	25	10.4	FR	L	1.15
46X	0	T45	0	N1	25	10.4	FR	NL	1.13
46X	6	T45	0	N1	25	10.4	ZR	L	1.0
46X	6	T45	8	N1	25	10.4	ZR	L	1.0
43X	0	T45	0	N1	25	10.4	ZR	L	≈1.1

[a] Summary of the discussions of Sec. III. All $\sigma_{\max}$ are quoted relative to the first value. V_{so}' and λ refer to the potentials given in the footnotes of Table II. The names of the potentials refer to Table II. ZR = zero range, FR = finite range (see text), L = local, NL = nonlocal (see text).

[11] T. A. Belote, W. E. Dorenbusch, O. Hansen, and J. Rapaport, Nucl. Phys. 73, 321 (1965).

function for a less tightly bound neutron, the cross section predicted would increase and the upper limit on the value of S would be further reduced. No other $l=1$ strength was identified.

IV. DISCUSSION

A. Consistency of the DW Analysis

The ^{43}Ca(d,t) experiments of Ref. 1 were performed in a manner very similar to that employed in the present experiment. It therefore seems worthwhile to compare the results of these two experiments in some detail. It is known from (d,p) experiments[4,5,12] on ^{43}Ca and ^{46}Ca that the amounts of core excitation in the ground states of these nuclei are small. The (d,t) data demonstrate that the amounts of $2p$ and of $1f_{5/2}$ admixtures likewise are small. The total $1f_{7/2}$ pickup strengths in ^{43}Ca and ^{46}Ca should therefore be close to 3 and 6, respectively. The observed $1f_{7/2}(d,p)$ strengths are consistent with these pick-up strengths. The ^{43}Ca(d,t) data of Ref. 1 were therefore reanalyzed using the $T45$ triton parameters and bound-state wave functions with a spin-orbit coupling ($N1$ of Table II). The results are quoted in Table IV. The changes relative to the results quoted in Ref. 1 are less than 10% for any one transition and the strength sum is close to 3.0. The ^{46}Ca(d,t) $1f_{7/2}$ strength was also close to 6 (see Table I). Thus, the stripping and pickup experiments on ^{43}Ca and ^{46}Ca yield consistent $1f_{7/2}$ spectroscopic information.

B. Nuclear Structure Information

The present data set limits on the number of $2p$ and $1f_{5/2}$ neutrons in the ^{46}Ca ground state of 0.04 and 0.25 particles, respectively. Previous neutron-pickup data[13] on ^{48}Ca and ^{44}Ca have indicated $2p$ admixtures in these nuclei of ≈ 0.02 and 0.06 particles, respectively.

The amounts of admixtures from the higher-lying shell are important for the understanding of the structure of the low-lying Ca levels. Several calculations have been published,[14] which include $2p$ mixing, but since wave functions are not given in these publications, it is not possible to test the models against the present data.

[12] T. A. Belote, H. Y. Chen, O. Hansen, and J. Rapaport, Phys. Rev. **142**, 624 (1966).
[13] T. W. Conlon, B. F. Bayman, and E. Kashy, Phys. Rev. **144**, 941 (1966).
[14] T. Engeland and E. Osnes, Phys. Letters **20**, 424 (1966); B. J. Raz and M. Soga, Phys. Rev. Letters **15**, 924 (1965); P. Federman and I. Talmi, Phys. Letters **22**, 469 (1966).

TABLE IV. Analysis of the ^{43}Ca(d,t) reactions.[a]

Ex (MeV)	0	1.52	2.42	2.75	3.19	$\sum S$
J^π	0^+	2^+	2^+	4^+	6^+	
S^b	0.65	0.2	0.25	0.75	1.1	3.0
S^c	0.6	0.2	0.2	0.7	1.0	2.7

[a] Data from Ref. 1; the DW calculations were zero range and local. The $T45$ and $N1(\lambda=25)$ potentials of Table II were employed.
[b] Deuteron potential $46X$ (see Table II).
[c] Deuteron potential $43X$ (see Table II).

If good isospin is assumed, the ^{45}Ca $\frac{3}{2}^+$ hole states with $T=\frac{5}{2}$ will have a total strength of $4[1-1/(2T_A+1)]$, i.e., 3.43, (T_A is the target isospin). The ^{45}Ca $\frac{3}{2}^+$ strength found here was estimated to be ≈ 1.5, i.e., only 50% of the available strength. The Ca(p,d) data,[13] data[15] from Ti(p,d) and data[16] from (^{3}He,α) reactions on Cr and Ca isotopes show a similar trend: only 25–50% of the total shell model strength is observed to the lowest $\frac{3}{2}^+$ state. Only one $l=2$ transition was observed in all the (p,d) cases.

The reason for the nonfulfillment of the $d_{3/2}$ sum rule is not clear. It has been suggested that it is connected with the distorted-wave Born approximation (DWBA) method used for extracting the spectroscopic information.[15] However, the fact that the $\frac{3}{2}^+$ strength deduced is similar in (p,d) (d,t) and (^{3}He,α) reactions (albeit not measured on the same target) indicates that the effect might be connected with the nuclear structure, rather than due to a fault of the analysis. If the $\frac{3}{2}^+$ states for example are largely associated with deformed states,[17] a core overlap of less than unity should be taken into account. The remaining strength will then be distributed over more states, i.e., part of the $\frac{3}{2}^+$ strength will be found at higher excitation.

ACKNOWLEDGMENT

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[15] E. Kashy and T. W. Conlon, Phys. Rev. **135**, B389 (1964); R. Sherr, B. F. Bayman, E. Rost, M. E. Rickey, and G. G. Root, *ibid.* **139**, B1272 (1965).
[16] R. Stock [private communications (1966)].
[17] P. Federman, Phys. Letters **20**, 174 (1966).