PHYSICAL REVIEW VOLUME 160, NUMBER 4 20 AUGUST 1967

Comparison Procedure and the Neutron-Neutron Scattering Length*

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(Received 6 April 1967)

At present the only means for obtaining information about the neutron-neutron scattering length a_{nn} and effective range r_0 are studies of reactions with three or more particles in the final state. It was felt that the comparative analysis of the various reactions which exhibit a $^1S_0$ nucleon-nucleon interaction in the final state would be a reliable procedure by which to extract a_{nn}. Thus the nucleon-induced breakup reactions of deuterons and trions were analyzed using the Born approximation and S-state wave functions for the deuteron, the trion and the two nucleons having a low relative energy in the final state. We found the comparative analysis of the trion breakup processes unsuccessful in the energy range considered. The values for a_{nn} extracted in analyses of the reaction D$(n,p)2n$ around 14 MeV cannot be reconciled with the theoretical predictions based on exact charge symmetry and using either a hard-core or a velocity-dependent potential. The present analysis indicates, however, that for incident energies larger than 30 MeV the distorting interference effects are removed and that the deuteron breakup processes become more promising for the determination of a_{nn}.

I. INTRODUCTION

IN 1961, Ilakovac *et al.*[1] discovered a pronounced peak in the proton spectra from deuteron breakup induced by 14.4-MeV neutrons. This proton peak is kinematically associated with two neutrons in the final state having low relative energy, and it was assumed that its shape and magnitude are related to the low-energy neutron-neutron scattering parameters. The first attempt to extract the neutron-neutron scattering parameters from this spectrum using the Born approximation and describing the two neutrons in the final state with a $^1S_0$ wave function led to a neutron-neutron scattering length, a_{nn}, of -22 ± 2 F.[2] Investigations of the reaction D$(n,p)2n$ performed[3–7] later in various laboratories essentially confirmed the results of the original measurement. Because of lack of understanding of the three-nucleon system, it was always emphasized that the quoted uncertainties in the extracted neutron-neutron scattering lengths do not include possible theoretical uncertainties.

The interaction between two particles can be studied either by investigating a two-body system or by investigating a many-body, preferably a few-body system, provided that proper conditions are chosen. Since abundant evidence argues[8] against the existence of a bound dineutron system, the only two-body process that can give information about the neutron-neutron interaction is the scattering of neutrons by neutrons. Colliding-neutron-beam experiments are now feasible, and the accuracy of the proposed measurements is such that it could result in a determination of the scattering length and the effective range with uncertainties not larger than 3% and 50–70%, respectively.[9] Until such experiments are performed, few-body systems remain the sole source of information about the neutron-neutron interaction.

The few-body systems most suitable for extracting information about the nucleon-nucleon interaction are nuclear reactions producing three particles in the final state. Examples of multiparticle reactions which can yield information about the neutron-neutron interaction are D$(n,p)2n$, H$^3(n,d)2n$, H$^3(d,$He$^3)2n$, H$^3($H$^3,\alpha)2n$, D$(\pi^-,\gamma)2n$, He$^3(\pi^-,p)2n$, and He$^4(\pi^-,d)2n$. The conditions can be chosen in such a manner that the observable is dominated by the influence of a particular two-body interaction. Indeed, the experimental data on multiparticle reactions reveal pronounced intensity maxima at specific values of the internal energy of two particles in the exit channel and at specific values of the momentum transfer variable.[10] One distinguishes between sequential processes and quasifree processes.

The only multiparticle process which is at present reasonably well understood[11,12] is the reaction D$(\pi^-,\gamma)2n$. In this case, there is only one pair of strongly interacting particles in the final state. It is estimated[12] that the uncertainty inherent in the theoretical analysis with

* Supported in part by the U. S. Office of Naval Research under Contract No. Nonr 233-(44).
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which the neutron-neutron scattering length can be extracted is of the order of ± 1 F for this reaction.

The present status of experimental techniques allows measurements to be performed from which the neutron-neutron scattering length can be extracted with an uncertainty smaller than ± 1 F. Thus, it is desirable to develop a theoretical procedure which has a smaller intrinsic uncertainty. The exact treatment of the three-body problem in the reaction $D(n,p)2n$, based on the work of Faddeev,[13] Amado *et al.*,[14] Lovelace,[15] and others[16] provides at least in principle the possibility of extracting the neutron-neutron scattering parameters. In order to carry out such an ambitious task, one should use the neutron-proton interaction in its full complexity, one should possess an adequate knowledge of the nuclear off-the-energy-shell interaction, and one should have a fair estimate of possible three-body forces. Such a calculation is still impossible to perform at the present time. A somewhat more modest analysis has been performed by Aaron and Amado[17] using a separable nucleon-nucleon potential. The model used by Amado and co-workers[14] gives a good description of the static properties of the triton, the elastic scattering of neutrons by deuterons and the total inelastic neutron-deuteron cross section. Though the predictions of the model *qualitatively* reproduce the experimental proton spectra from the reaction $D(n,p)2n$ at 14.4 MeV at several angles between 4° and 45°, the disagreement with the data is of such a nature as to prevent a determination of the neutron-neutron scattering length. It is likely that this failure is related to the inadequate description of the nucleon-nucleon interaction used in the model.

It was felt that the comparative analysis[18,19] of the various reactions which exhibit a $^1S_0$ nucleon-nucleon interaction in the final state would be a reliable procedure with which to extract the neutron-neutron scattering length. Besides, such an analysis might eventually yield useful information about three-body interactions. Since the comparative analysis might be the only method to learn about the forces between certain unstable particles, it is worthwhile to discuss its capabilities.

This paper represents a comparative analysis of the processes $D(n,p)2n$ [5,6,7] and $D(p,n)2p$,[20,21] and $H^3(n,d)2n$,[22,23] $He^3(n,d)np$,[24] and $He^3(p,d)2p$.[25–27] In Sec. II the significance of the neutron-neutron scattering length is discussed with specific reference to the charge symmetry and charge independence of nuclear forces and the existence of a hard core in the nucleon-nucleon interaction. Section III presents a discussion of multiparticle reactions, in particular, to what extent certain peaks in the cross section can be identified as related to final-state interactions. Section IV presents general qualitative features of deuteron and trion (H^3 and He^3) breakup processes. The analysis of the nucleon-induced trion and deuteron breakup processes are given in Sec. V. Section VI presents a critical evaluation of the comparison procedure and serves as a conclusion of this work.

II. THE NEUTRON-NEUTRON SCATTERING LENGTH, THE HARD-CORE, AND THE CHARGE INDEPENDENCE OF NUCLEAR FORCES

Early proton-proton and neutron-proton scattering data suggested the charge independence[28] of nuclear forces. Nuclear-structure data supported this hypothesis.[29] The comparison between energy levels of mirror nuclei and between charge-symmetric nuclear reactions reveal the equality of neutron-neutron and proton-proton forces, indicating that the nucleon-nucleon interaction is charge-symmetric. At present, overwhelming evidence has been accumulated demonstrating that the nucleon-nucleon interaction is to a large extent also charge-independent. However, there is some evidence which points to a small departure from charge independence and maybe even from charge symmetry.[5,30–32]

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The most sensitive and least ambiguous way to study a small departure from charge independence is to investigate the low-energy nucleon-nucleon scattering parameters. Since the $^1S_0$ state is almost bound, the scattering length is a magnifying glass for the nuclear potential. A change in depth of the potential is related to a change in the $^1S_0$ scattering length through the expression

$$\Delta a/a = A\Delta V/V\,,$$

where A is of the order of 10, its precise value depending on the shape of the potential.

The scattering lengths which are extracted from the experimental data must be corrected for the effects of the electromagnetic interaction between the nucleons. The largest correction is due to the electrostatic interaction between two point charges and it affects only the proton-proton system, amounting to a change from -7.815 ± 0.008 F as observed experimentally[33] to -16.6 to -16.9 F after the Coulomb potential is turned off.[34] The electromagnetic correction should include terms due to the magnetic interaction, the finite charge and magnetic moment distributions of the nucleons, the mass difference between the neutron and the proton, and the vacuum polarization. It is estimated that the net correction amounts to 0.5 F.[34]

The most reliable value for the neutron-neutron scattering length is at present based on the study[35] of the reaction $D(\pi^-,\gamma)2n$, which gives

$$a_{nn} = -16.4\pm1.9\text{ F}.$$

This value has an additional theoretical uncertainty of ±1.0 F.[12] This result should be compared with

$$a_{pp} = -16.6 \quad\text{to}\quad -16.9\text{ F}\,,$$

and [36]

$$a_{np} = -23.678\pm0.028\text{ F}.$$

These values should also be corrected for the effects caused by the electromagnetic interactions, which have already been pointed out, amount to about 0.5 F.

The following conclusions can be drawn:

(1) The values for a_{nn} and a_{pp} are essentially identical. This implies a verification of the charge symmetry of nuclear forces. More precisely, based on the experimental uncertainty in the measurement of a_{nn}, it follows that the charge symmetry of nuclear forces is valid to within 1.6%.[30]

(2) The charge independence of nuclear forces is obviously violated. The difference between a_{np} and a_{pp} of 6.93 ± 0.15 F implies a departure of (4.8 ± 0.1)% from charge independence.[30] The percentages quoted depend on the shape of the potential.

The validity of charge symmetry has been questioned by Okamoto,[32] who claims that the difference in the binding energies of H^3 and He^3 cannot be accounted for in terms of the Coulomb interaction only. According to Okamoto the neutron-neutron interaction is 0.5 to 1.5% stronger than the proton-proton interaction. A recent calculation,[37] however, resulted in a Coulomb energy large enough to account for the mass difference of H^3 and He^3.

It should be emphasized that the comparison between a_{nn} and a_{pp} is based on calculations which employed potentials with a hard core. The electromagnetic corrections to the scattering lengths depend upon the nature of the potential, i.e., whether it contains a hard core, a soft core, or is velocity-dependent. Although the *n-p* and *p-p* scattering parameters do not result in a distinction in favor of any of these potentials, the neutron-neutron scattering length could possibly shed light on this problem. For instance, the value for a_{nn} based on the exact charge symmetry for velocity-dependent potentials is -19.3 F.[38]

Thus, the precise measurement of a_{nn} in itself cannot solve the problem of the hard core and the validity of charge symmetry. It should be supplemented with additional data, like electron-deuteron scattering and studies of the nucleon-nucleon bremsstrahlung, which might be able to distinguish between the various potentials.

The causes for a violation of charge independence of about 4–5% should be investigated by considering the electromagnetic effects which distort the nuclear interaction between two nucleons. The major electromagnetic effect which influences the nuclear interaction is due to the electromagnetic mass splitting of the mesons. This gives a correction of the right sign and the right order of magnitude to explain the difference between the values for a_{np} and a_{pp}. The possible difference between the *n-n* and *p-p* interactions, if any exist, could be caused by isospin mixing and radiative corrections to the meson-nucleon coupling constants.

The precise value for the neutron-neutron $^1S_0$ scattering length plays a crucial role in the understanding of charge independence and charge symmetry of nuclear forces and the detailed form of the nucleon-nucleon interaction in the interior region.

III. FINAL-STATE INTERACTIONS IN MULTIPARTICLE REACTIONS

The cross section for a reaction leading to N particles in the final state depends upon $3N-4$ independent kinematical variables; e.g., a reaction with three particles in the final state depends upon five kinematical vari-

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ables. Measurements in which all independent kinematical variables are determined are often referred to as "complete" experiments, whereas those in which less than $3N-4$ variables are measured often are called "incomplete" experiments. It has been argued[39] that the incomplete experiments could not be subjected to a meaningful analysis. The argumentation is based on a number of phenomena which are displayed clearly in complete but which are not so obviously present in incomplete experiments, e.g., interference effects and enhancements due to knockout processes. For example, in sequential processes studied through complete experiments, interference effects cause a splitting of a peak into two peaks or a change of the width of a peak from its natural value.[40,41] Such phenomena have not been observed in incomplete experiments. The importance of interference effects and their way of manifestation was realized some time ago. Attempts to consider quantitatively the interference between resonant processes, and between a resonant and a nonresonant process, have been undertaken.[6,42,43]

The pioneering work in describing sequential processes was done by Watson[44] and by Migdal.[45] Later a somewhat different model was proposed by Phillips, Griffy, and Biedenharn,[46] according to which the spatial localization can modify the observables in multiparticle reactions. It has been pointed out that some peaks observed in the energy spectra can be due to spatial localization. The existence of such effects and to some extent their actual physical meaning has not been clarified. Only one experiment has been reported[47] that provides evidence in favor of the spatial localization concept. However, this interpretation is open to criticism.[48] A number of experiments have been performed which should demonstrate spatial localization peaks, if they would exist. Some examples are the reactions $\mathrm{Li}^6(n,d)n\mathrm{He}^4$ and $\mathrm{Li}^7(n,t)n\mathrm{He}^4$,[49] where a strong n-α resonance occurs 1 MeV below the maximum deuteron (or triton) energy. The investigation of this pair of reactions as well as the charge-symmetric reactions leading to Li^5 (Ref. 50) does not give any evidence that could be interpreted in favor of the spatial localization concept. If the spatial localization produces enhancements near the maximum energy of a detected particle in incomplete experiments, they should have been observed in the above-mentioned reactions.

Maxima in the cross section $d^3\sigma/d\Omega_3 d\Omega_4 dE_3$ in reactions of the type $1+2\rightarrow 3+4+5$ have been observed which are completely due to phase space. Since the phase-space factor can always be calculated exactly in complete as well as in incomplete experiments, it should not produce any uncertainties.

Correlation spectra have revealed pronounced peaks related to quasifree scattering. A suspicion has arisen that the energy spectra in incomplete experiments are strongly influenced by quasifree processes. Moreover, the pronounced peaks in these spectra which are often interpreted as indicative of a sequential process, could be just a manifestation of a quasifree scattering. For instance, in the reaction $\mathrm{D}(n,p)2n$, the protons emitted in the forward direction with near-maximum energies originate in a knockout process. But consequently, this quasifree process leaves the two neutrons in a state of low relative energy causing a strong final-state interaction. It has been clearly demonstrated[51] that the quasifree process is significantly influenced by the final-state interaction even under circumstances which are most suitable for a quasifree process to occur and where the final-state interaction is not particularly prominent. The question to what extent the peaks observed in incomplete experiments are caused by the final-state interactions can be answered by studying multiparticle reactions for which the particular strong final-state interaction is isospin-forbidden. The reactions $\mathrm{He}^4(d,\alpha)np$,[25] $\mathrm{He}^4(d,n)p\alpha$,[52] $\mathrm{D}(d,n)pd$,[52] and $\mathrm{D}(d,p)nd$ [53] have been studied in incomplete experiments. In no case has the energy spectrum of a detected single particle revealed a peak. This fact gives in our opinion strong support to the interpretation of the peaks in the energy spectra from the reactions $\mathrm{D}(n,p)2n$, $\mathrm{H}^3(n,d)2n$, etc., in terms of a ${}^1S_0$ final-state interaction. Of course, the final-state interaction peaks may be influenced by a knockout process. Later we will give evidence that the reaction mechanism producing a pair of particles with low-relative energy in the final state significantly influences the energy spectrum. It is interesting to note that a quasifree scattering description has been attached to the results of a complete investigation of the reaction $\mathrm{D}(d,pd)n$.[54] Nevertheless, it was possible to explain[53] the proton spectra in the incomplete experiment by assuming only a simultaneous breakup into three particles and four particles at lower proton energies.

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These two descriptions are not in contradiction, since the quasifree process can be one possible mode of producing simultaneous, nonsequential breakup. Therefore, it seems that although the information from incomplete experiments may be restricted, there is no reason to believe that this information should be incorrect.

The discussion given above leads to the following conclusions:

(1) The analysis of incomplete experiments is not *a priori* predestinated to failure.
(2) The prominent peaks observed in the reactions $D(n,p)2n$, $H^3(n,d)2n$, etc., display the influence of a $^1S_0$ nucleon-nucleon final-state interaction. As stated in the Introduction, there exists at present no adequate theory which would enable an analysis of any single multiparticle reactions and yield reliable information about two-nucleon interactions. It has been suggested that many difficulties would be surmounted by the comparison procedure.

The implicit conditions to the comparison procedure are as follows.

(1) The processes which lead to a neutron-neutron, a neutron-proton, or a proton-proton final-state interaction, respectively, should be analyzed using the same model. If an acceptable model can be constructed that gives good agreement with the experimental data using the known *n-p* and *p-p* scattering parameters, then one can have confidence in the *n-n* scattering parameters extracted by means of that model.
(2) The $^1S_0$ nucleon-nucleon final-state interaction should be dominant in the observed spectra. This condition excludes the group of reactions $H^3(H^3,\alpha)2n$, $H^3(He^3,\alpha)np$, and $He^3(He^3,\alpha)2p$, where the nucleon-α particle final-state interaction is as prominent as the nucleon-nucleon final-state interaction.
(3) All members of a group should have equivalent final states. This rules out a comparison between the reactions $H^3(d,He^3)2n$ and $He^3(d,t)2p$, since the *p-t* system has one more resonance than the *n*-He^3 system.
(4) The reaction mechanism producing the particular final state of interest should be the same for all members of the group.

The knowledge of the reaction mechanisms is at present quite limited. In order to distinguish between knockout, pickup, heavy-particle stripping, or other processes, abundant experimental data are required. Since such information is at the moment not available, one is forced to use simple, often naive pictures. One would expect that the emission of a deuteron in the forward direction in the processes $H^3(n,d)2n$, $He^3(n,d)np$, and $He^3(p,d)2p$ is dominated by a pickup mechanism. Also, that the processes $D(n,p)2n$ and $D(p,n)2p$ in the forward direction are dominated by a knockout mechanism. Caution, however, must be applied in including the reactions $D(n,n')np$ and $D(p,p')np$ in a comparative analysis with the former pair, since the latter two reactions can be considered as a slightly inelastic scattering process.[55] The condition of the identity of the reaction mechanism prevents the use of the reactions $He^3(d,t)2p$ and $D(H^3,He^3)2n$ measured at forward angles in the same comparative analysis, because the first reaction is predominantly a pickup and the latter a charge-exchange process.

The comparison procedure can be made considerably more reliable by the inclusion of experimental information regarding the angular and energy dependence of the energy spectra. The angular dependence of the energy spectra $d^2\sigma/dE_3d\Omega_3$ should help in determining the reaction mechanism or at least establish the equivalence of the reaction mechanism for all the reactions under investigation. It is expected that the presence of the third particle will distort the dominant final-state interaction of two nucleons in a $^1S_0$ state. This distortion can be investigated by studying another group of reactions where the nucleon-nucleon pair appears with another particle in the final state and by studying the reactions at various incident energies which effectuates a change in the relative energy of the third particle and the nucleon-nucleon pair.

As a result of the discussion outlined above, we felt that it was reasonable to explore the comparison procedure in the analysis of the reactions $D(n,p)2n$ and $D(p,n)2p$ as one group of processes and in the analysis of the reactions $H^3(n,d)2n$, $He^3(n,d)np$, and $He^3(p,d)2p$ as a second group of processes. All the available data at various energies were analyzed.

A serious shortcoming of the present procedure is the use of the plane-wave Born approximation rather than the distorted-wave Born approximation (DWBA). It is obvious that distortions are present in both entrance and exit channels. The DWBA was applied in the analysis of the angular distribution of the reaction $He^3(p,d)2p$ and an encouraging agreement with the data was obtained.[27] The distortions should be specified through independent measurements of the elastic scattering. In cases where data on the elastic scattering of the particles in the entrance channel or of the particles in the exit channel are not available, one usually employs an extrapolation based on the elastic-scattering data involving nearby nuclei. Such extrapolations do not seem justified in the treatment of few-nucleon systems, and it was felt that the inclusion of distortion effects at the present state confuses the picture through an increase in the number of free parameters.

IV. GENERAL QUALITATIVE FEATURES OF DEUTERON AND TRION BREAKUP PROCESSES

The small-angle nucleon spectra from the reactions $D(n,p)2n$ and $D(p,n)2p$ at incident energies around 14 MeV reveal two strong final-state interactions: a

[55] A. H. Cromer, Phys. Rev. 129, 1680 (1963).

neutron-neutron (or proton-proton) and a neutron-proton final-state interaction. At higher incident energies the neutron-proton final-state interaction peak is kinematically removed and the spectra are dominated by the interaction between the two undetected nucleons in the final state.

The deuteron energy spectra at small angles from the breakup of trions by nucleons exhibit one pronounced peak corresponding to the only strong final-state interaction present: a nucleon-nucleon final-state interaction. The nucleon-deuteron system does not have any strong resonance,[56] which is exhibited by the smooth energy behavior of the nucleon-deuteron phase shifts.[57] The high-energy portions of the deuteron and trion breakup spectra show a striking similarity. Expressed quantitatively, at incident energies around 14 MeV, the full widths at half-maximum of the peaks which correspond to the final-state interaction of the two undetected particles are after unfolding of the experimental energy resolution 1.0±0.2 MeV for all five processes $D(n,p)2n$, $D(p,n)2p$, $H^3(n,d)2n$, $He^3(n,d)np$, and $He^3(p,d)2p$. The differential cross section $d^2\sigma/dE_3d\Omega_3$ for these breakup processes depends strongly upon the angle θ_3 at which the spectrum is measured.[27,42,58–61]

The Watson-Migdal model as well as the Phillips-Griffy-Biedenharn approach have been quite successful in the interpretation of sequential processes which proceed via resonant states, e.g., He^5 g.s., Li^5 g.s., Be^8 g.s., Be^8 2.9 MeV, etc.[49] However, it has been shown that both models are incapable of explaining the aforementioned deuteron and trion breakup spectra measured in incomplete experiments. Furthermore, both models do not reproduce the experimentally found similarities. The 8° triton spectrum resulting from the reaction $He^3(d,t)2p$ has been compared[19] with predictions using the Watson-Migdal formulation for the final-state interaction of the two undetected protons. This comparison led to a value of the proton-proton scattering length in excellent agreement with the one derived from proton-proton scattering data. A study of the $D(He^3,t)2p$ reaction at 0° and 3° and the same center-of-mass (c.m.) energy, however, led[62] to a value of a_{pp} 30% to 50% smaller ($-12\ \mathrm{F}<a_{pp}<-10\ \mathrm{F}$). Of course, the reaction mechanisms for tritons emitted in the forward and backward directions of the $He^3(d,t)2p$ reaction are different. Presumably, the forward mode is dominated by a pickup process, whereas the backward mode is dominated by a charge-exchange process. The larger absolute value of a_{pp} which was required to fit the triton energy spectra at backward angles implies that the experimental peaks are narrower than those predicted by the Watson-Migdal model using the correct value for a_{pp}. The same situation is encountered in the analysis of the deuteron and trion breakup processes.

The spectra of α-particles from the reactions $H^3(t,\alpha)2n$,[63] $He^3(t,\alpha)np$,[64] and $He^3(He^3,\alpha)2p$ [65,66] are remarkedly influenced by the strong α-particle-nucleon interaction. The peaks in the α-particle spectra corresponding to the nucleon-nucleon final-state interaction seem to increase relative to the peaks corresponding to the alpha-particle-nucleon final-state interaction if the incident energy is increased.[66,67] However, at the energies where these processes have been analyzed the cross section for the sequential decay through He^5 or Li^5 is at least twice as large as the cross section for the sequential decay via the nucleon-nucleon $^1S_0$ state. The Watson-Migdal formulation resulted in satisfactory fits to the experimental data,[65] though the fits become progressively poorer in the region of the nucleon-nucleon final-state interaction. Because of the presence of the He^5 and Li^5 resonance, respectively, it is impossible to conclude whether the Watson-Migdal approach would lead to acceptable values for the nucleon-nucleon scattering length.

V. ANALYSIS OF THE NUCLEON-INDUCED DEUTERON AND TRION BREAKUP PROCESSES

A. Method of Analysis

1. *Trion Breakup Processes*

The deuteron spectra from the nucleon-induced trion breakup reactions have been analyzed using the Born approximation. Thus, plane waves have been used to describe the incident nucleon in the initial state and the motion of the deuteron relative to the c.m. of the remaining two nucleons in the final state.

The perturbing nucleon-nucleon interaction is represented by a general spin-dependent potential.[68] There are eight two-body potentials which are denoted as ${}^1V_{nn}{}^+$, ${}^3V_{nn}{}^-$, ${}^3V_{np}{}^+$, ${}^1V_{np}{}^+$, ${}^3V_{np}{}^-$, ${}^1V_{np}{}^-$, ${}^1V_{pp}{}^+$, and ${}^3V_{pp}{}^-$. The superscript to the left specifies whether the two nucleons are in a singlet or triplet spin state, while the superscript to the right specifies whether the

[56] The present status of the excited states in He^3 have recently been summarized by B. Antolković *et al.*, in Ref. 24.
[57] W. T. H. van Oers and K. W. Brockman, Jr., Nucl. Phys. **A92**, 561 (1967).
[58] V. K. Voitovetskii, I. L. Korsunskii, and Yu. F. Pazhin, Nucl. Phys. **69**, 513 (1965).
[59] C. Wong, J. D. Anderson, C. C. Gardner, J. W. McClure, and M. P. Nakada, Phys. Rev. **116**, 164 (1959).
[60] P. Tomaš (private communication).
[61] A. D. Bacher, T. A. Tombrello, and Y. S. Chen, Bull. Am. Phys. Soc. **11**, 896 (1966).
[62] B. J. Moeton, M. P. Fricke, R. O. Ginaven, E. E. Gross, J. J. Malanify, and A. Zucker, Bull. Am. Phys. Soc. **12**, 16 (1967).
[63] N. Jarmie and R. C. Allen, Phys. Rev. **111**, 1121 (1958).
[64] D. B. Smith, N. Jarmie, and A. M. Lockett, Phys. Rev. **129**, 785 (1963).
[65] A. D. Bacher, Ph.D. thesis, California Institute of Technology, 1966 (unpublished).
[66] K. P. Artjomov, V. J. Chuev, V. Z. Goldberg, A. A. Ogloblin, V. P. Rudakov, and J. N. Serikov, Phys. Letters **12**, 53 (1964).
[67] R. J. Slobodrian, D. J. Clark, J. S. C. McKee, and W. F. Tivol, Bull. Am. Phys. Soc. **11**, 896 (1966).
[68] R. S. Christian and J. L. Gammel, Phys. Rev. **91**, 100 (1953).

potential is for a state of even or odd parity. The subscripts specify the two nucleons. We suppose that all these potentials have the same radial dependence, so that ${}^xV_{ij}{}^y(r)={}^xV_{ij}{}^yU(r)$, the various V's becoming pure numbers. $U(r)$ is taken to be the potential for the ground state of the deuteron, so that ${}^3V_{np}{}^+=1$.

The trion wave function in the initial state is assumed to be completely symmetrical in the spatial coordinates of all three nucleons. In the final state we suppose spatially symmetrical (S-state) wave functions for the deuteron and for the relative motion of the remaining two nucleons.

Let

$$I_1=\int \exp[-i\mathbf{k}'\cdot\tfrac{1}{2}(\mathbf{r}_3+\mathbf{r}_4-\mathbf{r}_1-\mathbf{r}_2)]\phi_d{}^*(|\mathbf{r}_4-\mathbf{r}_3|)\phi_{k''}{}^*(|\mathbf{r}_2-\mathbf{r}_1|)\times U(|\mathbf{r}_2-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{3}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4))]\phi_t(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)d\tau,$$

$$I_2=\int \exp[-i\mathbf{k}'\cdot\tfrac{1}{2}(\mathbf{r}_3+\mathbf{r}_4-\mathbf{r}_1-\mathbf{r}_2)]\phi_d{}^*(|\mathbf{r}_4-\mathbf{r}_3|)\phi_{k''}{}^*(|\mathbf{r}_2-\mathbf{r}_1|)\times U(|\mathbf{r}_3-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{3}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4))]\phi_t(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)d\tau,$$

$$I_3=\int \exp[-i\mathbf{k}'\cdot\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_4-\mathbf{r}_2-\mathbf{r}_3)]\phi_d{}^*(|\mathbf{r}_4-\mathbf{r}_1|)\phi_{k''}{}^*(|\mathbf{r}_3-\mathbf{r}_2|)\times U(|\mathbf{r}_3-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{3}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4))]\phi_t(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)d\tau,$$

$$I_4=\int \exp[-i\mathbf{k}'\cdot\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_4-\mathbf{r}_2-\mathbf{r}_3)]\phi_d{}^*(|\mathbf{r}_4-\mathbf{r}_1|)\phi_{k''}{}^*(|\mathbf{r}_3-\mathbf{r}_2|)\times U(|\mathbf{r}_4-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{3}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4))]\phi_t(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)d\tau,$$

$$K_1=\int \exp[-i\mathbf{k}'\cdot\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3-\mathbf{r}_1-\mathbf{r}_4)]\phi_d{}^*(|\mathbf{r}_3-\mathbf{r}_2|)\phi_{k''}{}^*(|\mathbf{r}_4-\mathbf{r}_1|)\times U(|\mathbf{r}_4-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{3}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4))]\phi_t(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)d\tau,$$

$$K_2=\int \exp[-i\mathbf{k}'\cdot\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_3-\mathbf{r}_2-\mathbf{r}_4)]\phi_d{}^*(|\mathbf{r}_3-\mathbf{r}_1|)\phi_{k''}{}^*(|\mathbf{r}_4-\mathbf{r}_2|)\times U(|\mathbf{r}_2-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{3}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4))]\phi_t(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)d\tau,$$

$$K_3=\int \exp[-i\mathbf{k}'\cdot\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3-\mathbf{r}_1-\mathbf{r}_4)]\phi_d{}^*(|\mathbf{r}_3-\mathbf{r}_2|)\phi_{k''}{}^*(|\mathbf{r}_4-\mathbf{r}_1|)\times U(|\mathbf{r}_2-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{3}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4))]\phi_t(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)d\tau,$$

$$K_4=\int \exp[-i\mathbf{k}'\cdot\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_3-\mathbf{r}_2-\mathbf{r}_4)]\phi_d{}^*(|\mathbf{r}_3-\mathbf{r}_1|)\phi_{k''}{}^*(|\mathbf{r}_4-\mathbf{r}_2|)\times U(|\mathbf{r}_3-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{3}(\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4))]\phi_t(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)d\tau,$$

where $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, and $\mathbf{r}_4$ are the coordinates of the four particles; $\mathbf{k}_0$ is the wave number vector of the incident particle, and $\mathbf{k}'$ is the wave number vector of the emitted deuteron. ϕ_t and ϕ_d are the wave functions for the ground state of the trion and the deuteron, respectively. $\phi_{k''}$ represents the wave function for the relative motion of the two remaining nucleons in the final state with wave number vector $\mathbf{k}''$. The I's and K's are functions of $\mathbf{k}'$ and $\mathbf{k}''/|\mathbf{k}''|$. The various transition amplitudes are found by performing the spin summations. For the reactions $He^3(n,d)np$ and $H^3(p,d)np$ we find for $S=1$, triplet n-p continuum states:

$$M(S=1,\text{ triplet } n\text{-}p \text{ states})=\sum_{m=1}^{4}\alpha_m I_m,$$

where

$$\alpha_1={}^3V_{np}{}^+,$$
$$\alpha_2={}^3V_{nn}{}^-+\tfrac{1}{2}\,{}^3V_{np}{}^-+\tfrac{1}{2}\,{}^1V_{np}{}^-,$$
$$\alpha_3=-{}^3V_{nn}{}^--\tfrac{1}{2}\,{}^3V_{np}{}^--\tfrac{1}{2}\,{}^1V_{np}{}^-,$$
$$\alpha_4=-{}^3V_{np}{}^+;$$

for $S=1$, singlet n-p continuum states:

$$M(S=1,\text{ singlet } n\text{-}p \text{ states})=\sum_{m=1}^{4}\beta_m I_m,$$

where

$$\beta_1=-\tfrac{1}{2}\sqrt{2}\,{}^1V_{np}{}^+,$$
$$\beta_2=-\tfrac{1}{2}\sqrt{2}({}^3V_{nn}{}^-+\tfrac{1}{2}\,{}^3V_{np}{}^++{}^3V_{np}{}^--\tfrac{1}{2}\,{}^1V_{np}{}^+),$$
$$\beta_3=\tfrac{1}{2}\sqrt{2}({}^3V_{nn}{}^--\tfrac{1}{2}\,{}^3V_{np}{}^++{}^3V_{np}{}^-+\tfrac{1}{2}\,{}^1V_{np}{}^+),$$
$$\beta_4=\tfrac{1}{2}\sqrt{2}\,{}^3V_{np}{}^+;$$

and for $S=0$, triplet n-p continuum states:

$$M(S=0,\text{ triplet } n\text{-}p \text{ states})=\sum_{m=1}^{4}\gamma_m I_m,$$

where

$$\gamma_1=\tfrac{1}{2}(\sqrt{6})\,{}^3V_{np}{}^+,$$
$$\gamma_2=\tfrac{1}{2}(\sqrt{6})({}^1V_{nn}{}^++\tfrac{1}{2}\,{}^3V_{np}{}^++\tfrac{1}{2}\,{}^1V_{np}{}^+),$$
$$\gamma_3=\tfrac{1}{2}(\sqrt{6})({}^1V_{nn}{}^++\tfrac{1}{2}\,{}^3V_{np}{}^++\tfrac{1}{2}\,{}^1V_{n,p}{}^+)$$
$$\gamma_4=\tfrac{1}{2}(\sqrt{6})\,{}^3V_{np}{}^+.$$

For $H^3(p,d)np$ the n-n potentials must be replaced by p-p potentials. For the reactions $H^3(n,d)2n$ and $He^3(p,d)2p$ we find for $S=1$, singlet n-n or p-p continuum states:

$$M(S=1,\text{ singlet } n\text{-}n \text{ states})=\sum_{n=1}^{4}\epsilon_n K_n,$$

where

$$\epsilon_1=-{}^1V_{nn}^+,$$
$$\epsilon_2=\tfrac{1}{2}(3\,{}^3V_{nn}^-+{}^1V_{nn}^+-{}^3V_{np}^++{}^3V_{np}^-),$$
$$\epsilon_3=-\tfrac{1}{2}(3\,{}^3V_{nn}^--{}^1V_{nn}^++{}^3V_{np}^++{}^3V_{np}^-),$$
$$\epsilon_4={}^3V_{np}^+.$$

For $He^3(p,d)2p$ the n-n potentials must be replaced by p-p potentials.

Some of the above integrals have a simple physical interpretation. The integrals I_1 and K_1 represent the stripping of the trion with the emission of a deuteron and the incident nucleon plus a dissimilar nucleon from the trion remaining to form an unbound nucleon-nucleon pair. This process is peaked in the backward direction for emission of a deuteron. The integrals I_4 and K_4 represent the pickup of a dissimilar nucleon from the trion by the incident nucleon to form a deuteron. The other two nucleons of the trion remain as an unbound nucleon-nucleon pair. This process is peaked in the forward direction. The integrals I_2, K_2 and I_3, K_3 represent fairly complicated rearrangements which cannot be described in terms of a single primary interaction between two nucleons.

In the evaluation of the various integrals we have used for the trion a wave function of Gaussian form[69]

$$\phi_t(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)=(3\gamma^2/\pi^2)^{3/4}\exp[-\tfrac{1}{2}\gamma(r_{23}^2+r_{24}^2+r_{34}^2)],$$

where $\gamma(H^3)=0.15715\text{ F}^{-2}$; $\gamma(He^3)=0.15400\text{ F}^{-2}$; for the deuteron a Hulthén wave function[70]

$$\phi_d(r)=\left[\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}\right]^{1/2}\frac{\exp(-\alpha r)-\exp(-\beta r)}{r},$$

where $\alpha=0.2317\text{ F}^{-1}$ and $\beta=1.202\text{ F}^{-1}$; and for the unbound nucleon-nucleon pair an S-state scattering wave function[71]

$$\phi_{k''}(r)=f\left[\frac{\cos k''r}{r}+\cot\delta_0(k'')\frac{\sin k''r}{r}-\frac{\exp(-\epsilon r)}{r}\right],$$

which for a proton-proton pair becomes

$$\phi_{k''}(r)=f\left[\frac{G_0(k''r)}{r}+\cot\delta_0(k'')\frac{F_0(k''r)}{r}-\frac{\exp(-\epsilon r)}{C_0 r}\right],$$

where

$$f=\frac{\exp[2i\delta_0(k'')]-1}{2ik''}.$$

In the final expressions $|f|^2$ can be calculated from

$$|f|^2=(k''^2+[k''\cot\delta_0(k'')]^2)^{-1}$$

and

$$k''\cot\delta_0(k'')=-1/a+\tfrac{1}{2}r_0k''^2$$

respectively,

$$C_0^2k''\cot\delta_0(k'')=-1/a+\tfrac{1}{2}r_0k''^2-h(\eta)/R.$$

The values for the scattering lengths and effective ranges are given in Table I. In the proton-proton system

$$R=28.821\text{ F},\quad \eta=(2k''R)^{-1},\quad C_0^2=2\pi\eta/[\exp(2\pi\eta)-1],$$

and

$$h(\eta)=\eta^2\sum_{n=1}^{\infty}\frac{1}{n(n^2+\eta^2)}-\ln\eta-0.57722.$$

We have adopted a Yukawa radial form for the nucleon-nucleon potential:

$$U(r)=U_0\frac{\exp(-\lambda r)}{\lambda r},$$

where $U_0=-68.0$ MeV and $\lambda=0.847\text{ F}^{-1}$. The integrals I_1, K_1, and I_4, K_4 are then readily evaluated. However, difficulties are encountered in evaluating the integrals I_2, K_2 and I_3, K_3 since they are not separable in the coordinates. It should be remarked that it is reasonable to assume that their contributions to the transition amplitudes are small compared to the contributions from the integrals I_1, K_1 and I_4, K_4. Rather than evaluate I_2, K_2 and I_3, K_3 numerically, we estimated their contribution by replacing the Yukawa potential by a δ potential and expanding the deuteron ground-state wave function in a sum of three Gaussians:

$$\phi_d(r)=\sum_{i=1}^{3}Z_i\exp[-(\zeta_i r)^2]$$

i	Z_i ($\text{F}^{-3/2}$)	ζ_i^2 (F^{-2})
1	0.01388	0.01691
2	0.05583	0.09018
3	0.11784	0.42836

In calculating the transition matrix elements we have assumed an exchange mixture of the Serber type, i.e., ${}^3V_{np}^+=1$, ${}^1V_{np}^+=0.69$, and the odd-parity potentials are set to be zero. We have also assumed that the n-n, n-p, and p-p potentials are equal in equivalent states.

The differential cross section for the emission of a deuteron is

$$\frac{d^2\sigma}{dE'd\Omega'}=\frac{4}{(4\pi)^4}\frac{2\mu_0}{\hbar^2}\frac{2\mu''}{\hbar^2}\frac{2m_d}{\hbar^2}\frac{k'k''}{k_0}\left\{\frac{1}{4\pi}\int|M|^2d\Omega''\right\},$$

[69] W. Laskar, C. Tate, and P. G. Burke, *Nuclear Forces and the Few-Nucleon Problem*, edited by T. C. Griffith and E. A. Power (Pergamon Press, London, 1960), Vol. II, p. 559.

[70] M. J. Moravcsik, Nucl. Phys. **7**, 113 (1958).

[71] R. M. Frank and J. L. Gammel, Phys. Rev. **93**, 463 (1954).

TABLE I. Values of the nucleon-nucleon scattering lengths and effective ranges used in the analysis.

	a (F)	r_0 (F)		Ref.
n-n	?	2.85		34
n-p	+5.396	1.726	triplet spin state	36
n-p	−23.678	2.51	singlet spin state	36
p-p	−7.778	2.714		33

where μ_0 is the reduced mass of a nucleon and a trion in the initial state, μ'' is the reduced mass of the nucleon-nucleon system in the final state, and m_d is the mass of the deuteron. The elements of solid angle $d\Omega'$ and $d\Omega''$ are defined by

$$d\mathbf{k}'=k'^2dk'd\Omega' \quad \text{and} \quad d\mathbf{k}''=k''^2dk''d\Omega''.$$

The energy of the emitted deuteron in the c.m. system, E', has the value $E'=(\hbar k')^2/(2m_d)$. The integration over the directions of $\mathbf{k}''$ is straightforward since we have assumed S-state wave functions for the nucleon-nucleon pair in the final state. For the reactions $He^3(n,d)np$ and $H^3(p,d)np$ we find

$$|M|^2=\tfrac{3}{4}|M(S=1,\text{ triplet } n\text{-}p \text{ states})|^2 +\tfrac{3}{4}|M(S=1,\text{ singlet } n\text{-}p \text{ states})|^2 +\tfrac{1}{4}|M(S=0,\text{ triplet } n\text{-}p \text{ states})|^2,$$

and for the reactions $H^3(n,d)2n$ and $He^3(p,d)2p$ we find

$$|M|^2=\tfrac{3}{4}|M(S=1,\text{ singlet } n\text{-}n \text{ or } p\text{-}p \text{ states})|^2.$$

In order to investigate the effects of the structure of the trion wave function on the shape of the theoretical spectra we have performed another Born-approximation calculation assuming that the trion consists of a nucleon plus a deuteron and that this deuteron is ejected by a δ-type interaction with the incident nucleon. In these calculations we have used a trion wave function of the form[72]

$$\phi_t(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)=N\frac{\exp[-\gamma(|\mathbf{r}_2-\frac{1}{2}\mathbf{r}_3-\frac{1}{2}\mathbf{r}_4|)]}{|\mathbf{r}_2-\frac{1}{2}\mathbf{r}_3-\frac{1}{2}\mathbf{r}_4|}\phi_d(|\mathbf{r}_4-\mathbf{r}_3|),$$

where

$\gamma(H^3)=0.4474\ F^{-1}$; $\gamma(He^3)=0.4206\ F^{-1}$; $\phi_d(|\mathbf{r}_4-\mathbf{r}_3|)$

represents a Hulthén wave function. We shall further refer to the above-mentioned calculations by Roman numerals I and II, respectively.

2. *Deuteron Breakup Processes*

The proton and neutron spectra from the reactions $D(n,p)2n$ and $D(p,n)2p$, respectively, have also been analyzed using the Born approximation. The *n-p* final-state interaction which is only prominent in the small angle spectra around 14 MeV has been neglected. Starting with a general spin-dependent nucleon-nucleon interaction, we proceeded in a manner similar to that for the trion breakup reactions. We suppose spatially symmetrical (S-state) wave functions for the deuteron in the initial state and for the neutron-neutron or proton-proton system in the final state. Let

$$I_1=\int \exp[-i\mathbf{k}'\cdot(\mathbf{r}_3-\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2))]\phi_{k''}^*(|\mathbf{r}_2-\mathbf{r}_1|)U(|\mathbf{r}_2-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3))]\phi_d(|\mathbf{r}_3-\mathbf{r}_2|)d\tau,$$

$$I_2=\int \exp[-i\mathbf{k}'\cdot(\mathbf{r}_3-\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2))]\phi_{k''}^*(|\mathbf{r}_2-\mathbf{r}_1|)U(|\mathbf{r}_3-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3))]\phi_d(|\mathbf{r}_3-\mathbf{r}_2|)d\tau,$$

$$I_3=\int \exp[-i\mathbf{k}'\cdot(\mathbf{r}_1-\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3))]\phi_{k''}^*(|\mathbf{r}_3-\mathbf{r}_2|)U(|\mathbf{r}_3-\mathbf{r}_1|)\exp[+i\mathbf{k}_0\cdot(\mathbf{r}_1-\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3))]\phi_d(|\mathbf{r}_3-\mathbf{r}_2|)d\tau,$$

where $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$ are the coordinates of the three particles; and where $\mathbf{k}_0$ is the wave number vector of the incident particle, and $\mathbf{k}'$ is the wave number vector of the observed proton or neutron; ϕ_d is the wave function of the deuteron, and ϕ_k'' represents the wave function for the relative motion of the two neutrons or protons in the final state with wave number vector $\mathbf{k}''$. The I's are functions of $\mathbf{k}'$ and $\mathbf{k}''/|\mathbf{k}''|$. We find for the transition amplitudes for the $S=\frac{1}{2}$, singlet *n-n* or *p-p* continuum states

$$M(S=\tfrac{1}{2},\text{ singlet } n\text{-}n \text{ or } p\text{-}p \text{ states})=\sum_{m=1}^{3}\gamma_m I_m,$$

where

$$\gamma_1=\tfrac{1}{2}(\sqrt{6})\,{}^1V_{nn}^{+},$$

$$\gamma_2=\tfrac{1}{8}(\sqrt{6})({}^3V_{np}^{+}+{}^3V_{np}^{-}+{}^1V_{np}^{+}+{}^1V_{np}^{-}),$$

$$\gamma_3=\tfrac{1}{8}(\sqrt{6})({}^3V_{np}^{+}-{}^3V_{np}^{-}+{}^1V_{np}^{+}-{}^1V_{np}^{-}).$$

For $D(p,n)2p$ the *n-n* potential must be replaced by a *p-p* potential.

Again the integrals can be given a simple physical interpretation. Speaking in terms appropriate to the

[72] J. L. Gammel and A. D. MacKellar, Phys. Rev. 133, B1476 (1964).

reaction $D(n,p)2n$, integral I_1 represents the pickup of a neutron by the incident neutron, forming a "dineutron." This process is peaked in the backward direction. Integral I_2 represents the knockout of a proton by the incident neutron, and integral I_3 can be interpreted as a charge-exchange process. The latter two processes are peaked in the forward direction. We have neglected in our calculations a contribution from the first spatial integral, I_1. A δ-type potential was adopted in evaluating I_2 and I_3. With a δ-type potential the integrals I_2 and I_3 are identical and the Born approximation becomes equivalent to the impulse approximation. The wave functions ϕ_d and $\phi_{k''}$ were the same as defined before. The expression for $|M|^2$ is

$$|M|^2=\tfrac{1}{3}|M(S=\tfrac{1}{2}, \text{singlet } n\text{-}n \text{ or } p\text{-}p \text{ states})|^2.$$

All spectra were calculated for the laboratory angles specified using the relation

$$\left(\frac{d^2\sigma}{dE'd\Omega'}\right)_L=\frac{k_L'}{k'}\frac{d^2\sigma}{dE'd\Omega'},$$

where k_L' is the wave number of the observed particle in the laboratory system.

B. Critical Evaluation of the Experimental Data

In order to compare the calculated predictions with the experimental data one must fold in the finite energy and angular resolutions. Except for the (p,n) reactions where time-of-flight techniques have been used to measure the neutron spectra, the energy resolution $\Delta E/E$ for the various experimental data is inversely proportional to the energy E. In other words, ΔE remains constant. It should be emphasized that the proper treatment of the experimental energy resolution is a crucial point in the comparative analysis. The high-energy side of the experimentally observed peak and the theoretical prediction after folding in the true energy resolution have to coincide in all cases except for a proton-proton final-state interaction. Searches were made to find whether slight changes in the energy resolution would improve the fits. The energy resolutions quoted with the experimental data are in general satisfactory. It is estimated that the uncertainty in the energy resolutions is approximately 5–15% of the quoted values. None of the conclusions drawn in this paper would be affected even if the energy resolution would differ by as much as 20% from those values. However, if one wants to determine the neutron-neutron scattering length to ± 0.5 F, then the energy resolution should be known with an uncertainty smaller than 5%.

The angular resolution mainly affects the comparison between the absolute values of the experimental differential cross sections and the calculated cross sections. Since the angular dependence of the calculated spectra is found to be much smaller than demanded by the experimental data, it was decided to calculate the spectra for the mean scattering angle and to neglect the angular resolution.

In the first attempts[2] to explain the shape of these breakup spectra it was necessary to allow a correction for the energy calibration given. A small energy shift was allowed also in the present analysis in order to achieve a better fit to the data in the immediate vicinity of the $^1S_0$ final-state interaction peak. The energy resolutions used in the analysis as well as the energy shifts are listed in Table II.

The over-all accuracy of the experimental data used in the analysis is quite good, except for possibly the $H^3(n,d)2n$ data at 20.8 MeV. None of the conclusions of the analysis is hampered by the experimental uncertainties. If, however, the neutron-neutron scattering length has to be determined to ± 0.5 F, the relative differential cross sections $d^2\sigma/dE_3d\Omega_3$ should be known to $\pm$(1–2)%, and the energy calibration to ± 20 keV.

C. Comparison with the Experimental Data

In Fig. 1(a) the deuteron spectrum from the reaction $H^3(n,d)2n$ [22] at an incident neutron energy of 14.4 MeV is compared with the results of calculations I and II. Calculation II (the dashed curve) produces a peak broader than, while calculation I (solid curves) produces a peak narrower than, the experimental one. These statements remain valid for values of a_{nn} in the range of -17 to -23 F. A change in a_{nn} of 6 F causes a change of about 15% in the theoretical peak width. The dependence on the effective range, r_0, is negligible. The dotted and dashed curve presents the prediction according to the Watson-Migdal formulation.

Similar results are obtained for the deuteron spectrum from the reaction $H^3(n,d)2n$ at 20.8 MeV[23] [see Fig. 1(b)].

TABLE II. Pertinent information regarding the experimental spectra used in the analysis.

Reaction	Incident energy (MeV)	Scattering angle (degrees)	Full width at half-maximum Gaussian resolution function (MeV)	Energy shift (MeV)	Ref.
$H^3(n,d)2n$	14.4	5	0.50	+0.04	22
$H^3(n,d)2n$	20.8	0	0.45	+0.60	23
$He^3(n,d)np$	14.4	5	0.50	+0.06	24
$He^3(p,d)2p$	11.9	6.0	···	···	25
$He^3(p,d)2p$	25.5	11.0	0.15	−0.15	26
$He^3(p,d)2p$	30.2	10.0	$f(E_{Ld})$	···	27
$D(n,p)2n$	13.9	4.5	0.65	+0.22	6
$D(n,p)2n$	14.4	4.8	0.70	+0.02	1
$D(n,p)2n$	22	7.5	1.20	−0.40	7
$D(p,n)2p$	14.1	3	$f(E_{Lp})$	+0.02	20
$D(p,n)2p$	30.1	0	$f(E_{Lp})$	−0.10	21
$D(p,n)2p$	49.4	0	$f(E_{Lp})$	···	21

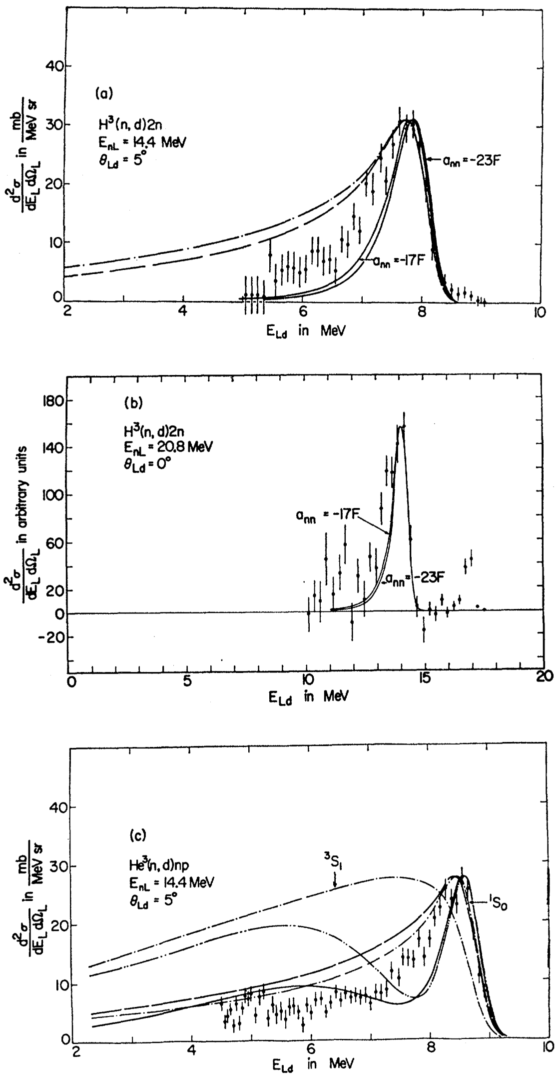

FIG. 1. Analysis of the deuteron spectra from nucleon+trion breakup reactions. (a) Deuteron spectrum from the reaction $H^3(n,d)2n$ at 14.4 MeV and a mean laboratory scattering angle of 5°. (b) Deuteron spectrum from the reaction $H^3(n,d)2n$ at 20.8 MeV and a laboratory scattering angle of 0°. (c) Deuteron spectrum from the reaction $He^3(n,d)np$ at 14.4 MeV and a mean laboratory scattering angle of 5°. The experimental data are given by the dots and error bars. The dotted and dashed curves correspond to the predictions of the spectra according to the Watson-Migdal formulation. The labels to these dotted and dashed curves in figure (c) refer to $^3S_1$ or $^1S_0$ *n-p* final-state interaction. The dashed curves result from calculation II. The solid lines are the results of calculation I, retaining only the contribution of the dominant pickup process. Where two solid curves are shown, the respective values of the scattering lengths used are indicated in the figure. The double dotted and dashed curve of figure (c) includes a contribution of the heavy-particle stripping process. The experimental energy resolution has been folded in to all curves.

A comparison of the calculations for the $H^3(n,d)2n$ and $He^3(n,d)np$ [Fig. 1(c)] spectra[24] suggests that a better fit can be obtained using a $^2S_{1/2}$ trion wave function which has a small component of mixed spatial symmetry.[73] In fact such a wave function has been used in a previous analysis,[22] resulting in good agreement between the experimental and theoretical spectra.

The $He^3(p,d)2p$ spectrum at 11.9 MeV[25] cannot be explained by the dominant pickup contribution to the cross section in calculation I [see Fig. 2(a)]. This difficulty also persists at 25.5 MeV,[26] where both calculations I and II produce enhancements which are much too broad, though calculation II gives a narrower curve than calculation I [see Fig. 2(b)]. The analysis of the 30-MeV data[27] essentially confirms these results

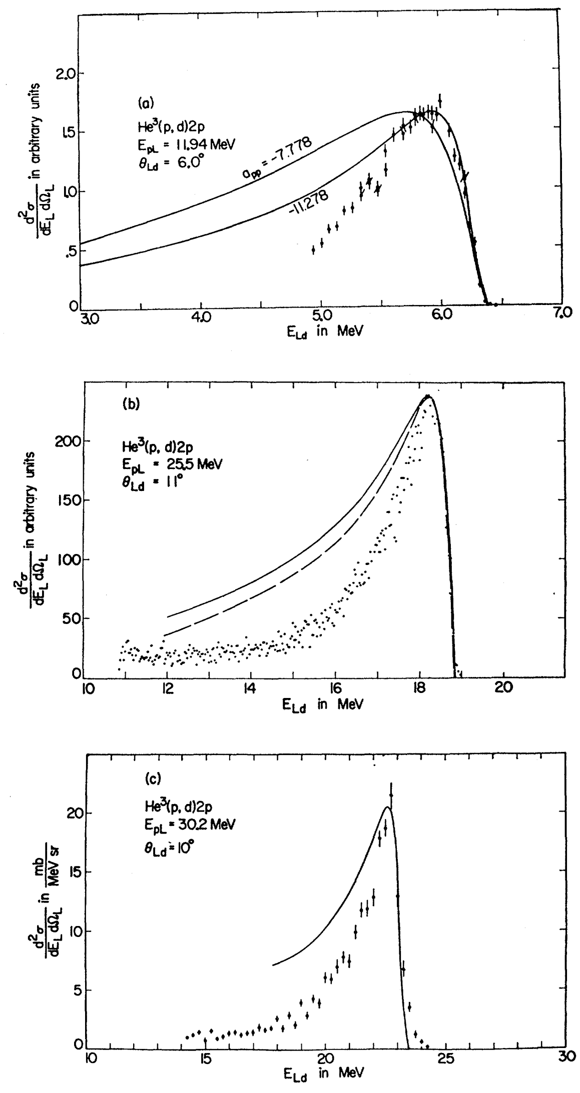

FIG. 2. Analysis of the deuteron spectra from nucleon+trion breakup reactions. (a) Deuteron spectrum from the reaction $He^3(p,d)2p$ at 11.9 MeV and a laboratory scattering angle of 6°. (b) Deuteron spectrum from the reaction $He^3(p,d)2p$ at 25.5 MeV and a laboratory scattering angle of 11°. (c) Deuteron spectrum from the reaction $He^3(p,d)2p$ at 30.2 MeV and a laboratory scattering angle of 10°. The legend for the curves shown is the same as in Fig. 1.

[73] B. F. Gibson and L. I. Schiff, Phys. Rev. **138**, B26 (1964).

[Fig. 2(c)]. The fits to the $He^3(p,d)2p$ spectra do not improve using a trion wave function of the type described above.

Figure 3 shows the calculated spectra for the reaction $He^3(p,d)2p$ at various angles. The present calculation is unable to reproduce the observed angular variation of the spectra.

The analysis of the proton spectrum from the $D(n,p)2n$ reaction at 14.4 MeV[2] reproduces the previous result[5] in favoring $a_{nn}=-21$ F to -22 F. This is to be expected since the present calculation is in essence the same as performed previously.[2] The same conclusion is drawn from the analysis of the 13.9-MeV data[6] [see, Figs. 4(a),(b)]. It is interesting to note that the shapes of both experimental spectra at the low-energy side of the peaks differ qualitatively in the same manner from the calculated predictions. The 22-MeV data[7] favor values for a_{nn} between -15 F and -18 F [Fig. 4(c)]. In view of the very small change in the width of the theoretical peak for a change in the value of a_{nn} from -17 F to -23 F combined with the non-negligible experimental uncertainties, the only conclusion that can be drawn from the analysis of the $D(n,p)2n$ reaction at 22 MeV is that it favors lower values of $|a_{nn}|$ than does the 14-MeV data.

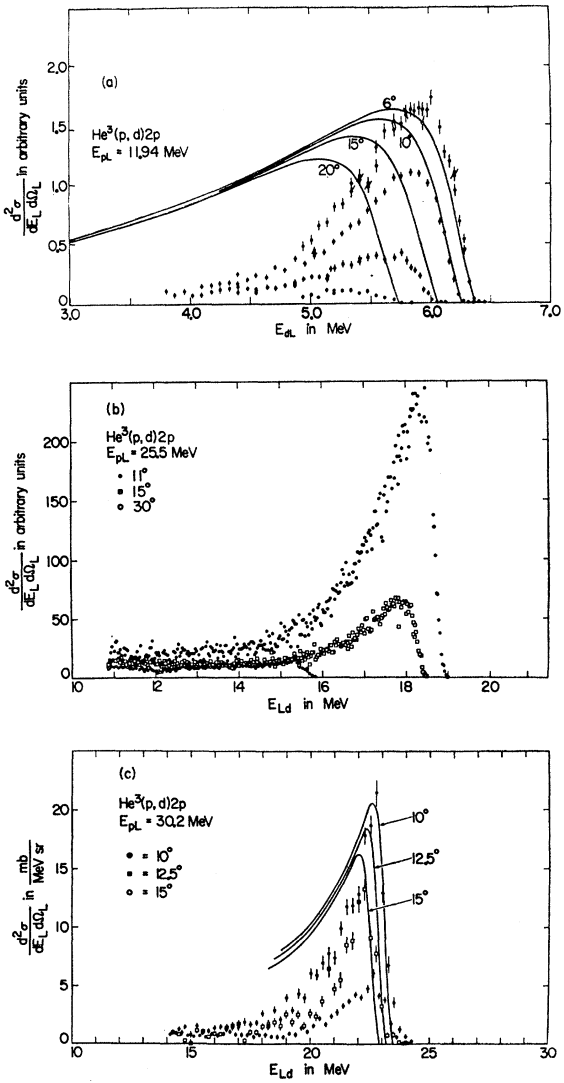

FIG. 3. Comparison between the angular dependence of the experimental spectra and the calculated spectra according to calculation I. As expected, use of the Born approximation prevents a quantitative agreement with the experimental angular distribution.

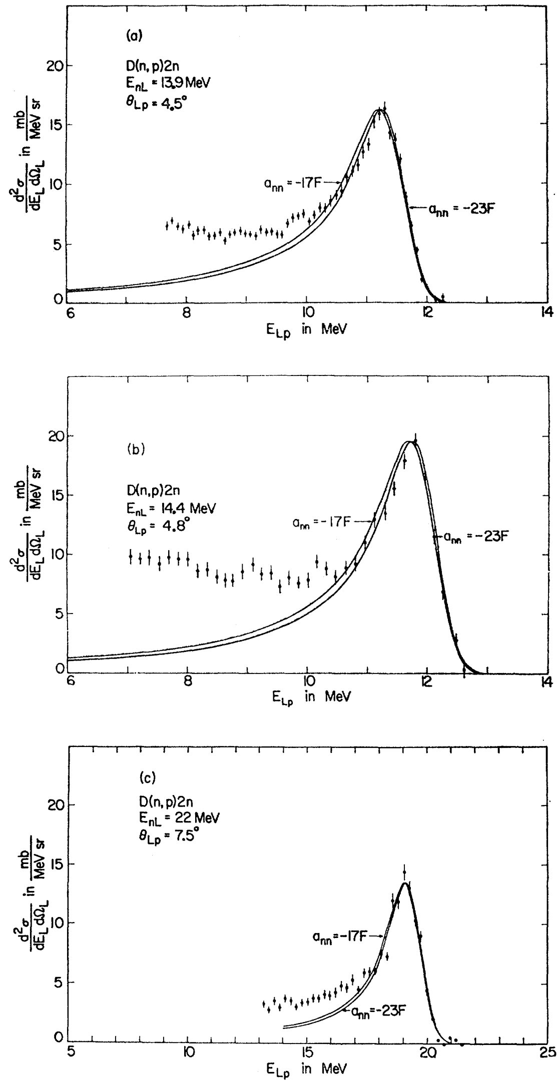

FIG. 4. Analysis of the nucleon spectra from nucleon+deuteron breakup reactions. (a) Proton spectrum from the reaction $D(n,p)2n$ at 13.9 MeV and a mean laboratory scattering angle of 4.5°. (b) Proton spectrum from the reaction $D(n,p)2n$ at 14.4 MeV and a mean laboratory scattering angle of 4.8°. (c) Proton spectrum from the reaction $D(n,p)2n$ at 22 MeV and a mean laboratory scattering angle of 7.5°. The experimental data are given by the dots and error bars. The solid lines give the predictions for the spectra according to the calculations outlined in the text. Where more than one solid curve is shown, the respective values for the scattering lengths used are indicated in the figure. The experimental energy resolution has been folded into all curves.

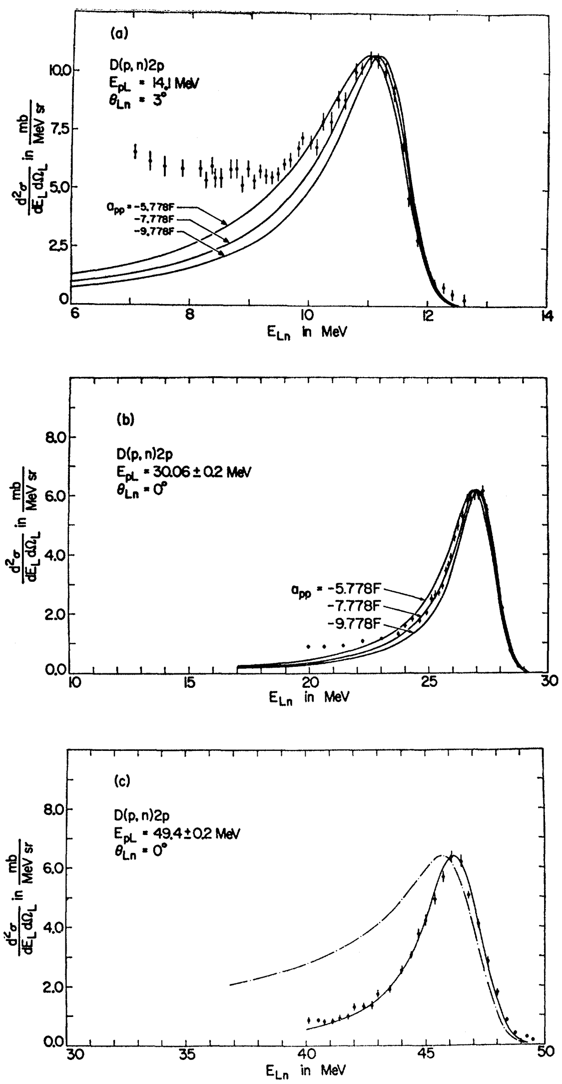

FIG. 5. Analysis of the nuclear spectra from nucleon+deuteron breakup reactions. (a) Neutron spectrum from the reaction $D(p,n)2p$ at 14.1 MeV and a laboratory scattering angle of 3°. (b) Neutron spectrum from the reaction $D(p,n)2p$ at 30.1 MeV and a laboratory scattering angle of 0°. (c) Neutron spectrum from the reaction $D(p,n)2p$ at 49.4 MeV and a laboratory scattering angle of 0°. The dotted and dashed curve of (c) corresponds to the prediction for the spectrum according to the Watson-Migdal formulation. For the rest the legend is the same as for Fig. 4.

The high-energy parts of the neutron spectra from the reaction $D(p,n)2p$ at 14.1 MeV,[20] 30.1 MeV, and 49.4 MeV [21] are compared with the theoretical curves in Fig. 5. The 14.1-MeV data yield a value for a_{pp} of -7.0 F, but at higher energies the experimental spectra are consistent with $a_{pp}=-7.78$ F, which is the value of a_{pp} derived from *p-p* scattering data. This situation can be understood since at the higher energies the *n-p* final-state interaction is kinematically removed and the interference term is therefore reduced. Figure 5(c) also shows the Watson-Migdal prediction for the shape of the spectrum due to a *p-p* final-state interaction. There is no agreement for realistic values of a_{pp}.

VI. CRITICAL EVALUATION OF THE COMPARISON PROCEDURE

The comparison procedure is certainly potentially a far-reaching method, but it must be used cautiously. The inability of the model presented in this paper to give a reasonable explanation of the nucleon-induced trion breakup processes is quite instructive. In particular, if one realizes that these processes are dominated by a *single*, strong final-state interaction, then one is bound to conclude that the comparative analysis of the trion breakup processes is not successful in the energy range considered. Consequently, these processes cannot at present be exploited to extract the neutron-neutron scattering length. The deuteron spectra from these reactions seem to be determined chiefly by the first step of a sequential process.

Recently the processes $H^3(d,He^3)2n$ and $He^3(d,t)2p$ have been explored in a comparative analysis,[19] and it was claimed that a_{nn} is determined to ± 1 F and r_0 to ± 1.6 F. The critical evaluation of the comparison procedure in this paper casts doubt on its application also to the above mentioned reactions. The $H^3(d,He^3)2n$ and $He^3(d,t)2p$ reactions are faced with additional difficulties. Strong resonances have been shown[54,74] to occur in the *p-t* and *n*-He^3 systems. This is even aggravated by the presence of an additional resonance in the *p-t* system (the 20.1-MeV level in He^4). The very accurate measurement[25] of the $He^3(d,t)2p$ spectrum at a lower energy reveals that the Watson-Migdal expression does not satisfactorily explain the high-energy side of that spectrum. The $He^3(d,He^3)np$ spectrum cannot be explained by the Watson-Migdal expression either[25,75] (Fig. 6). The angular distribution of the $He^3(d,t)2p$ re-

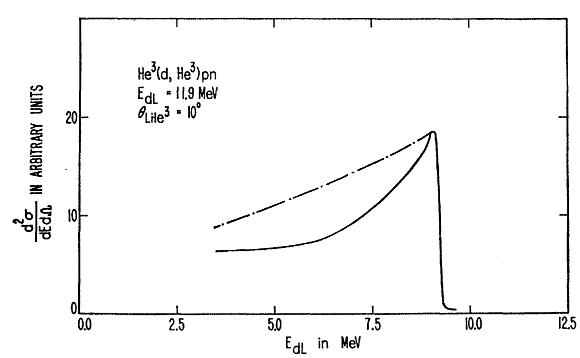

FIG. 6. He^3 spectrum from the reaction $He^3(d,He^3)np$ at 11.9 MeV. The experimental data are represented by the solid line. The dotted and dashed curve corresponds to the prediction for the spectrum according to the Watson-Migdal formulation including both the $^1S_0$ and $^3S_1$ *n-p* final-state interactions.

[74] R. W. Zurmühle, Nucl. Phys. **72**, 225 (1965).

[75] In Ref. 25 the He^3 spectrum is compared with only the $^1S_0$ *n-p* enhancement. The inclusion of a contribution from the $^3S_1$ *n-p* final-state interaction still does not give a satisfactory fit.

action suggests that the charge exchange and stripping processes are modulated in a complex fashion by the p-p final-state interaction.[76–78]

The analysis of the deuteron breakup processes was reasonably successful and encouraging. There is a clear indication that these processes in the energy region between 30 to 100 MeV and even better between 100–200 MeV [79] are the most promising candidates to extract precise information regarding the neutron-neutron interaction.

In this paper the comparison procedure has been applied exclusively to the analysis of incomplete experiments. The comparison procedure should be applied also to complete experiments. At present, the available data are still quite scarce. Only the reaction $D(p,2p)n$ has been studied extensively[80] and even there much more remains to be done. A few attempts have been made to analyze the data obtained in complete experiments.[81,82] The agreement between the data and the calculated spectra based on either the Watson-Migdal model[44,45] or the Phillips, Griffy, and Biedenharn model,[46] or the Frank and Gammel approach[71] is only qualitative. In none of these complete experiments has the neutron-neutron interaction been investigated. There has been reported[83] only one study of the process $D(n,2n)p$. The data obtained so far are not good enough to encourage any analysis.

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PHYSICAL REVIEW VOLUME 160, NUMBER 4 20 AUGUST 1967

Nuclear g Factors of the First Excited 2^+ States in Samarium-152 and -154 and Gadolinium-156, -158, and -160*

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(Received 3 April 1967)

The nuclear g factors of the Coulomb-excited 2^+ rotational states in samarium-152 and -154 and gadolinium-156, -158, and -160 have been measured using the pulsed-beam technique. The precession of the excited nuclei in an external magnetic field was detected by observing the angular distribution of the de-excitation γ radiation as a function of time. The measured g factors and mean lives τ were $g=0.277\pm0.028$ and $\tau=2.12\pm0.07$ nsec for the 122-keV state in ^{152}Sm, $g=0.288\pm0.029$ and $\tau=4.37\pm0.07$ nsec for the 82-keV state in ^{154}Sm, $g=0.296\pm0.018$ and $\tau=3.29\pm0.08$ nsec for the 89-keV state in ^{156}Gd, $g=0.315\pm0.025$ and $\tau=3.69\pm0.08$ nsec for the 79-keV state in ^{158}Gd, and $g=0.303\pm0.026$ and $\tau=3.92\pm0.08$ nsec for the 75-keV state in ^{160}Gd. For ^{152}Sm the precession was measured as a function of target temperature in order to understand better the internal field at the nucleus. It was found that the internal field did not have the theoretically expected temperature dependence above room temperature. The samarium results were obtained by using samarium metal targets, which were noticeably perturbed by electric quadrupole interactions. The gadolinium experiments utilized liquid-metal targets, and no perturbations were observed.

I. INTRODUCTION

THE determination of the nuclear g factors of rotational states is of interest because it gives information about the nature of the collective motion of the nucleus. In particular the g factor relates the relative contributions of the neutron and proton collective flow.[1,2]

In these experiments the 2^+ states at 122 keV in ^{152}Sm, 82 keV in ^{154}Sm, 89 keV in ^{156}Gd, 79 keV in ^{158}Gd, and 75 keV in ^{160}Gd were Coulomb excited by the nanosecond-pulsed beam from the Case Institute of Technology Van de Graaff accelerator. The precession of the excited nuclei in an external magnetic field was detected by observing the de-excitation γ radiation as a function of time. This technique has been described in earlier papers.[3–5] This differential γ-ray angular dis-

* This work was supported in part by the U. S. Atomic Energy Commission.
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