Theory of Partially Conserved Axial-Vector Current and Mesonic Exchange Effects in Nuclear Beta Decay

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By assuming that the axial-vector current is partially conserved, a relationship is obtained between the phenomenological meson-exchange operator in beta decay and the two-body pion-production operator. An analysis of the experimental results for the process $p+p\rightarrow d+\pi^+$ in terms of this operator is made, and the strength of the meson-exchange operator is then deduced from the relationship. It is found that the strength does not agree with that necessary to account for the ft value of the beta-decay process $H^3\rightarrow He^3+e^-+\bar{\nu}$. Possible reasons for the disagreement are discussed.

1. INTRODUCTION

It is common knowledge[1] that the phenomenon of nuclear β decay is well described by an interaction Hamiltonian density having the form

$$\mathcal{H}_\beta(x)=(G_\beta/\sqrt{2})[J_\mu(x)j_\mu(x)+\tilde{J}_\mu(x)\tilde{j}_\mu(x)],\quad(1)$$

where G_β is the β-decay coupling constant ($G_\beta\approx1.4\times10^{-10}$ erg F^3), and the four-currents J_μ and j_μ refer to the hadrons and leptons, respectively.[2] Explicitly, $j_\mu=-i\bar{\psi}_e\gamma_\mu(1+\gamma_5)\psi_\nu$, where ψ_e and ψ_ν are the usual field operators for the electron and the neutrino. The hadron current J_μ can be further decomposed into polar-vector (V) and axial-vector (A) parts, thus

$$J_\mu=J_\mu^{(V)}+J_\mu^{(A)}.\quad(2)$$

There is now strong experimental evidence (e.g., Ref. 3) that $J_\mu^{(V)}$ is conserved (CVC theory) so that, neglecting electromagnetic effects,

$$\partial_\mu J_\mu^{(V)}(x)=0.\quad(3)$$

In addition, during the last few years it has become fashionable to assume that $J_\mu^{(A)}$ is partially conserved (PCAC theory). This theory was originally proposed by Nambu, Gell-Mann, and others,[4] and for the purposes of this paper we take it to be defined by the following continuity equation:

$$\partial_\mu J_\mu^{(A)}=a\phi_\pi,\quad(4)$$

where ϕ_π is the renormalized field operator which creates the π^+, and a is a constant given by[5]

$$a=\frac{\sqrt{2}M\mu^2G_A}{G_\beta G_{\pi NN}K_{\pi NN}(0)}.\quad(5)$$

Here M is the nucleon mass, μ is the pion mass, G_A is the β-decay axial-vector coupling constant ($\approx-1.2G_\beta$), $G_{\pi NN}$ is the rationalized, renormalized pion-nucleon coupling constant [$G_{\pi NN}/4\pi\approx14.6$], and $K_{\pi NN}(0)$ is the invariant pion-nucleon vertex function normalized so that $K_{\pi NN}(-\mu^2)=1$.

This theory has been successful in giving a simple account of the Goldberger-Treiman[6] relation and, in conjunction with current commutation relations, has enabled satisfactory calculations to be carried out of the renormalization of the β-decay axial-vector coupling constant.[7] Kim and Primakoff[8] have also used the theory in the context of β decay in complex nuclei treating the different nuclei as "elementary" particles. In this way they were able to obtain a "nuclear" Goldberger-Treiman relation and to relate axial-vector nuclear matrix elements to experimentally measurable quantities in pion-nuclear processes. Unfortunately, few, if any, of the necessary experimental data are at present available so no real test of the theory could be made.

The object of the present paper is to investigate the implications of PCAC theory for the two-body meson-exchange corrections[9] which seem to be present in beta-decay processes.[10] In particular, a relationship between the β-decay meson-exchange operator and a two-body effective Hamiltonian operator for pion production is established. In Sec. 2 the contribution of exchange effects to axial-vector beta-decay matrix elements is discussed, and a phenomenological two-body beta-decay interaction is introduced. In Sec. 3 this operator is related via PCAC theory to a phenomenological two-body pion production operator. An analysis of the process $p+p\rightarrow d+\pi^+$ is given in Sec. 4 in terms of an effective Hamiltonian consisting of one- and two-body terms. In Sec. 5 the explicit form for the exchange

[1] T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **15**, 381 (1965); R. J. Blin-Stoyle and S. C. Nair, Advan. Phys. **15**, 493 (1966).
[2] A metric is used in which $x=(x_1,x_2,x_3,x_4=x,y,z,ict)$ and in order to ensure that H_β is a relativistic scalar it has been necessary to introduce the notation $\tilde{J}_\mu=[J_1^\dagger,J_2^\dagger,J_3^\dagger,-J_4^\dagger]$ where the dagger implies "Hermitian conjugate."
[3] R. J. Blin-Stoyle, Nucl. Phys. **57**, 232 (1964).
[4] Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento **17**, 757 (1960); M. Gell-Mann and M. Lévy, *ibid.* **16**, 705 (1960); J. Bernstein, M. Gell-Mann, and W. Thirring, *ibid.* **16**, 560 (1960).
[5] S. L. Adler, Phys. Rev. **137**, B1022 (1965).
[6] M. L. Goldberger and S. B. Treiman, Phys. Rev. **109**, 193 (1958).
[7] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965).
[8] C. W. Kim and H. Primakoff, Phys. Rev. **139**, B1447 (1965); C. W. Kim and H. Primakoff, *ibid.* **147**, 1034 (1966).
[9] J. S. Bell and R. J. Blin-Stoyle, Nucl. Phys. **6**, 87 (1957); R. J. Blin-Stoyle, V. Gupta, and H. Primakoff, *ibid.* **11**, 44 (1959); R. J. Blin-Stoyle, V. Gupta, and J. S. Thompson, *ibid.* **14**, 685 (1959/60).
[10] R. J. Blin-Stoyle, Phys. Rev. Letters **13**, 55 (1964); R. J. Blin-Stoyle and S. Papageorgiou, Nucl. Phys. **64**, 1 (1965).

operator is obtained and finally in Sec. 6 the results are applied to the β-decay process $H^3 \to He^3 + e^- + \bar{\nu}$. It is found that the currently accepted sign and magnitude of β-decay exchange effects are inconsistent with the predictions of PCAC theory. Possible reasons for this inconsistency are suggested and discussed.

2. EXCHANGE EFFECTS IN NUCLEAR BETA DECAY

In the case of allowed axial-vector matrix elements, the relevant β-decay operator can be written[9,10]

$$\mathbf{H}_\beta = \mathbf{H}_\beta^{(1)} + \mathbf{H}_\beta^{(2)}, \tag{6}$$

where

$$\mathbf{H}_\beta^{(1)} = G_A \sum \boldsymbol{\sigma}_i \tau_i^{\pm}, \tag{7}$$

and

$$\begin{aligned}\mathbf{H}_\beta^{(2)} = G_A \sum_{i<j} \{ & [g_{\mathrm{I}}(r)\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j + g_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\cdot \hat{r}\,\hat{r}](\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)^{\pm} \\ & + [h_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j) + h_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\cdot \hat{r}\,\hat{r}](\tau_i^{\pm} - \tau_j^{\pm}) \\ & + [j_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) + j_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\cdot \hat{r}\,\hat{r}] \\ & \times (\tau_i^{\pm} + \tau_j^{\pm})\}.\end{aligned} \tag{8}$$

Here $\mathbf{H}_\beta^{(1)}$ is the usual one-body β-decay operator and $\mathbf{H}_\beta^{(2)}$ is the most general, static form for the operator representing exchange effects in β decay.[9] The notation is standard and for the present the g, h, and j are taken to be arbitrary functions of the internucleon separation r $(=r_{ij})$.

The firmest evidence for the presence of exchange effects in β decay comes from a comparison of the $n \to p$ and $H^3 \to He^3$ ft values.[10] Assuming that the ground state of H^3 and He^3 is basically a fully space-symmetric ${}^2S_{1/2}$ state with a 6% admixture of ${}^4D_{1/2}$ state,[11] the implication is that the axial-vector matrix element is enhanced by about 10% by exchange contributions. In calculating this contribution using the operator $\mathbf{H}_\beta^{(2)}$ given in (8), it is sufficient to assume that the three-body state is 100% ${}^2S_{1/2}$. In this case, the exchange contribution to the axial-vector matrix element $M_A^{(2)}$ is

$$\begin{aligned}M_A^{(2)} = 4\sqrt{3}G_A\langle \phi(1,2,3) | & g_{\mathrm{I}}(r_{12}) + \tfrac{1}{3}g_{\mathrm{II}}(r_{12}) \\ & + h_{\mathrm{I}}(r_{12}) + \tfrac{1}{3}h_{\mathrm{II}}(r_{12}) | \phi(1,2,3)\rangle,\end{aligned} \tag{9}$$

where $\phi(1,2,3)$ is the fully space-symmetric radial function for H^3 and He^3.

We now take

$$\phi(1,2,3) = N \exp[-\tfrac{1}{2}k(r_{12}+r_{13}+r_{23})] \tag{10}$$

and for simplicity assume that the combination of radial functions in (10) can be represented by a Yukawa function, i.e.,

$$g_{\mathrm{I}}(r) + \tfrac{1}{3}g_{\mathrm{II}}(r) + h_{\mathrm{I}}(r) + \tfrac{1}{3}h_{\mathrm{II}}(r) = \Gamma \frac{\exp(-\alpha r)}{\alpha r}. \tag{11}$$

Here, experience in nuclear-physics calculations using phenomenological potentials suggests that this latter approximation is not a restrictive one. In the above expressions $k = 0.78$ F^{-1}, N is a normalizing constant, and Γ and α are phenomenological parameters. It is then straightforward to obtain corresponding value of Γ and α consistent with $M_A^{(2)} \approx +\frac{1}{10}M_A^{(1)}$ (see Ref. 10). Here $M_A^{(1)}$ is the usual matrix element of $\mathbf{H}_\beta^{(1)}$ for the ${}^2S_{1/2}$ state and has the value $G_A\sqrt{3}$. Assuming, for example, that $\alpha =$ (pion Compton wavelength)$^{-1} \approx 0.7$ F^{-1}, requires $\Gamma \approx 0.1$.

3. RELATION BETWEEN $\mathbf{H}_\beta$ AND THE PION-PRODUCTION OPERATOR

We now return to PCAC theory. On the basis of this theory Adler,[5] for example, has shown that

$$\langle \beta | \partial_\mu J_\mu(0) | \alpha \rangle = \frac{a(2\pi)^{3/2}}{q^2+\mu^2} T(\pi^+ + \alpha \to \beta), \tag{12}$$

where α and β are nuclear states (say), $q = p_\beta - p_\alpha$ and $T(\pi^+ + \alpha \to \beta)$ is the transition amplitude for the process $\pi^+ + \alpha \to \beta$ or, conversely, by time-reversal invariance, $\beta \to \pi^+ + \alpha$. Our purpose now is to relate $T(\pi^+ + \alpha \to \beta)$ to an "effective" Hamiltonian H_π which we take to be responsible for the pion-production process. H_π is assumed to have the following form:

$$H_\pi = \int S_\mu(\mathbf{r}) \partial_\mu \phi_\pi(\mathbf{r}) d^3r, \tag{13}$$

where $S_\mu(\mathbf{r})$ is a four-vector function of the coordinates (momenta, spins, and isospins) of the nucleons involved in the pion production and ϕ_π is the pion-field operator. The detailed form of $S_\mu(\mathbf{r})$ will be discussed shortly.

Given (12) and (13) and the usual relationship between the transition amplitude and the matrix element of a perturbing Hamiltonian, it follows that

$$\begin{aligned}\delta(\mathbf{p}_\beta - \mathbf{p}_\alpha - \mathbf{q}) & \langle \beta | \partial_\mu J_\mu^{(A)}(\mathbf{r}) | \alpha \rangle \\ & = \delta(\mathbf{p}_\beta - \mathbf{p}_\alpha - \mathbf{q}) \frac{a(2\pi)^{3/2}}{q^2+\mu^2} T(\pi^+ + \alpha \to \beta) e^{-i\mathbf{q}\cdot\mathbf{r}} \\ & = -\frac{a(2\pi)^{3/2}(2\omega_q)^{1/2}}{q^2+\mu^2} \langle \beta | H_\pi | \alpha\pi^+ \rangle e^{-i\mathbf{q}\cdot\mathbf{r}} \\ & = -\frac{a(2\pi)^{3/2}(2\omega_q)^{1/2}}{q^2+\mu^2} \langle \beta | \int S_\mu(\mathbf{r}') \partial_\mu \phi_\pi(\mathbf{r}') d^3r' | \alpha\pi^+ \rangle e^{-i\mathbf{q}\cdot\mathbf{r}}.\end{aligned} \tag{14}$$

Now in allowed β decay $|\alpha\rangle$ and $|\beta\rangle$ are nuclear states having the same parity so that selection rules prohibit any contribution from J_4 and similarly from S_4 apart from small retardation effects. These terms can therefore be dropped from (14). In addition the momentum transfer is small and so Eq. (14) can be evaluated in the limit

[11] J. M. Blatt and L. M. Delves, Phys. Rev. Letters 12, 544 (1964).

$q \to 0$. Carrying out a partial integration on the right-hand side and using the usual pion annihilation property of ϕ_π, it is then straightforward to show that

$$\langle\beta|\nabla\cdot\mathbf{J}^{(A)}(\mathbf{r})|\alpha\rangle\delta(\mathbf{p}_\beta-\mathbf{p}_\alpha) = (a/\mu^2)\langle\beta|\nabla\cdot\mathbf{S}(\mathbf{r})|\alpha\rangle\delta(\mathbf{p}_\beta-\mathbf{p}_\alpha). \quad (15)$$

Here it is to be noted that we might at an earlier stage have identified $J_\mu(\mathbf{r})$ essentially with $(a/\mu^2)S_\mu(\mathbf{r})$, since in a gradient coupling theory the identification of $J_\mu^{(A)}$ with the pion source term (apart from a constant) leads at once to the PCAC Eq. (4) (see, e.g., Gell-Mann and Lévy[4]). However, the procedure adopted above is rather more general and does not imply the assumption of a *fundamental* gradient coupling theory but only an *effective* Hamiltonian involving gradient coupling. Multiplying both sides of (15) by $\mathbf{u}\cdot\mathbf{r}$, where $\mathbf{u}$ is a constant vector, and integrating over d^3r gives

$$\langle\beta|\int\mathbf{J}^{(A)}(\mathbf{r})\cdot\mathbf{u}d^3r|\alpha\rangle\delta(\mathbf{p}_\beta-\mathbf{p}_\alpha) = (a/\mu^2)\langle\beta|\int\mathbf{S}(\mathbf{r})\cdot\mathbf{u}d^3r|\alpha\rangle\delta(\mathbf{p}_\beta-\mathbf{p}_\alpha). \quad (16)$$

If $\mathbf{u}$ is taken to be the space-independent part of the lepton $(V-A)$ covariant, it is then clear that in an allowed Gamow-Teller transition the effective β-decay operator can be taken to be

$$\mathbf{H}_\beta=\frac{a}{\mu^2}G_\beta\int\mathbf{S}(\mathbf{r})d^3r. \quad (17)$$

The next problem, therefore, is to consider the form of $\mathbf{S}(\mathbf{r})$ as evidenced by the pion-production process. To this end we consider the process $p+p\to d+\pi^+$.

4. ANALYSIS OF THE PION-PRODUCTION PROCESS $p+p\to d+\pi^+$

The part of the effective Hamiltonian for pion production [Eq. (13)] in which we are interested has the form

$$H_\pi=\int\mathbf{S}(\mathbf{r})\cdot\nabla\phi_\pi(\mathbf{r})d^3r \quad (18)$$

and is therefore responsible for p-wave pion production. In the process $p+p\to d+\pi^+$, p-wave production follows from initial $^1S_0$ and $^1D_2$ states. An analysis of the various experimental data (angular distribution, energy dependence, polarization phenomena) near threshold due to Woodruff gives for the corresponding transition amplitudes

$$a(^1S_0)=\{(0.60\pm0.20)\times\exp[i(2.6_{-0.2}^{+0.4})]\}\eta^{3/2}\,\mathrm{mb}^{1/2}, \quad (19)$$
$$a(^1D_2)=(1.93\pm0.10)\eta^{3/2}\,\mathrm{mb}^{1/2},$$

where η is the center-of-mass pion momentum and the phase is taken relative to that of $a(^1D_2)$.

The notation used here is due to Mandl and Regge[12] and is such that the cross section for the process is given by

$$4\pi\frac{d\sigma}{d\Omega}=\tfrac{1}{4}[\{|a(^1S_0)|^2+\tfrac{1}{2}|a(^1D_2)|^2+\sqrt{2}\,\mathrm{Re}[a(^1S_0)^*a(^1D_2)]+|a(^3P_1)|^2\} + \{\tfrac{1}{2}|a(^1D_2)|^2-\sqrt{2}\,\mathrm{Re}[a(^1S_0)^*a(^1D_2)]\}3\cos^2\theta]. \quad (20)$$

Various theoretical treatments of the above process have been given in the past,[13,14] the most sophisticated of which allow for rescattering of the pion. The intention in this paper is to interpret the experimental data in terms of an effective Hamiltonian consisting of the usual one-body terms and of a phenomenological two-body term. To this end, $\mathbf{S}(\mathbf{r})$ is taken to have the form

$$\mathbf{S}(\mathbf{r})=\sum_i\sqrt{2}\frac{G_{\pi NN}}{2M}K_{\pi NN}(0)\tau_i^+\boldsymbol{\sigma}_i\delta(\mathbf{r}-\mathbf{r}_i)+\sum_{i<j}\{[S_{ij}^\sigma T_{ij}^\tau+T_{ij}^\sigma S_{ij}^\tau][\alpha_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)+\alpha_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\,\hat{r}](\tau_i^+-\tau_j^+)$$
$$+[T_{ij}^\sigma T_{ij}^\tau+S_{ij}^\sigma S_{ij}^\tau][\beta_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)+\beta_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\,\hat{r}](\tau_i^+-\tau_j^+)$$
$$+[\gamma_{\mathrm{I}}(r)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)+\gamma_{\mathrm{II}}(r)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\hat{r}\,\hat{r}](\tau_i^++\tau_j^+)\}\tfrac{1}{2}[\delta(\mathbf{r}-\mathbf{r}_i)+\delta(\mathbf{r}-\mathbf{r}_j)], \quad (21)$$

where S_{ij}^σ and T_{ij}^σ are the singlet and triplet projection operators in ordinary spin space and S_{ij}^τ and T_{ij}^τ are similar operators in isobaric spin space. The $\alpha(r)$, $\beta(r)$, and $\gamma(r)$ are arbitrary functions of the internucleon separation $r\ (=|\mathbf{r}_i-\mathbf{r}_j|)$. Two-body terms antisymmetrical in the δ functions have not been included since they do not contribute at pion threshold.

The spirit of this approach, therefore, is to use an effective Hamiltonian including the usual one-particle pion-nucleon interaction and to account for all multi-

[12] F. Mandl and T. Regge, Phys. Rev. **99**, 1478 (1955).
[13] A. E. Woodruff, Phys. Rev. **117**, 1113 (1960).
[14] See Ref. 13 for references to other theoretical work on the process $p+p\to d+\pi^+$.

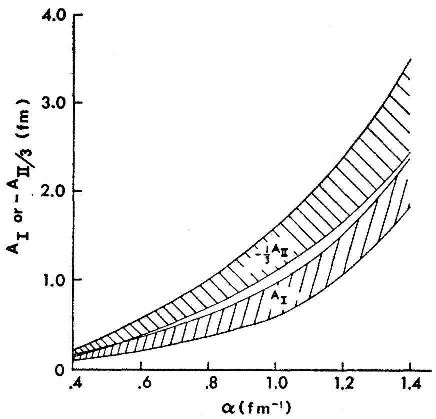

FIG. 1. Bands showing the allowed values of A_{I} and $-\frac{1}{3}A_{\text{II}}$ as a function of the range parameter α.

nucleon effects in terms of a two-body operator. The use of a two-body term is similar to the method used by others[15] in interpreting the pion capture process which, however, is mainly from the pion S state. The two-nucleon term is completely phenomenological; however, it is to be expected that the radial function will have a range of the order of the pion Compton wavelength.

Now in calculating transition amplitudes using the Hamiltonian (18) with $\mathbf{S}(\mathbf{r})$ given as in (21) the results will be obtained in the form

$$\begin{aligned} a(^1S_0) &= (b(^1S_0)+c(^1S_0))e^{i\tau_0}\eta^{3/2}\,\text{mb}^{1/2}, \\ a(^1D_2) &= (b(^1D_2)+c(^1D_2))\eta^{3/2}\,\text{mb}^{1/2}, \end{aligned} \tag{22}$$

where the amplitudes b and c derive from the one-body and two-body terms in $\mathbf{S}(\mathbf{r})$, respectively. τ_0 is the phase of $a(^1S_0)$ relative to $a(^1D_2)$ and depends on the phase factors occurring in the initial diproton wave function.

As far as $b(^1S_0)$ and $b(^1D_2)$ are concerned, part of Woodruff's[13] calculations can be taken over at once.[16] Using the Gammel-Thaler potential[17] to determine the diproton function and the Gartenhaus[18] deuteron wave function, he obtains

$$\begin{aligned} b(^1S_0) &= 0.24, \\ b(^1D_2) &= 0.86, \\ \tau_0 &= 2.65. \end{aligned} \tag{23}$$

Inserting these values in (22) and comparing with the experimental results given in (19) then implies that

$$\begin{aligned} c(^1S_0) &= 0.36\pm0.20, \\ c(^1D_2) &= 1.08\pm0.10. \end{aligned} \tag{24}$$

The problem, therefore, is to choose values for the parameters appearing in the two-body term in (21) so that the above values (24) for c are obtained. Since the transition in the process $p+p\rightarrow d+\pi^+$ with which we are concerned is from a spin-singlet, isospin-triplet to a spin-triplet, isospin-singlet, it is clear that the only set of two-body terms in (21) which can contribute is

$$\sum_{i<j} T_{ij}{}^{\sigma}S_{ij}{}^{\tau}[\alpha_{\text{I}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)+\alpha_{\text{II}}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\,\hat{r}] \times(\tau_i{}^+-\tau_j{}^+)\tfrac{1}{2}[\delta(\mathbf{r}-\mathbf{r}_i)+\delta(\mathbf{r}-\mathbf{r}_j)]. \tag{25}$$

In order to calculate the nuclear-matrix element of (25), the functions $\alpha_{\text{I}}(r)$ and $\alpha_{\text{II}}(r)$ are taken to have the Yukawa forms

$$\alpha_{\text{I}}(r)=A_{\text{I}}\frac{e^{-\alpha r}}{\alpha r}, \quad \alpha_{\text{II}}(r)=A_{\text{II}}\frac{e^{-\alpha r}}{\alpha r}, \tag{26}$$

where α is a range parameter and A_{I} and A_{II} are constants. We take the deuteron wave function to be

$$\Psi_d=\left(\frac{\mu}{4\pi}\right)^{1/2}\frac{1}{r}[u(r)\chi_t+w(r)\chi_D]\eta_s, \tag{27}$$

where $\chi_t(\chi_s)$ is a triplet (singlet) spin state and $\chi_D=(1/2\sqrt{2})S_{12}\chi_s$, S_{12} being the usual tensor operator. η_s is a singlet isospin function. The S- and D-state radial functions $u(r)$ and $w(r)$ are taken from the work of Kottler and Kowalski[19] which uses the Yale internucleon potential.[20]

The initial diproton wave function is taken to have the form

$$\Psi_{pp}=\left(\frac{2}{V}\right)^{1/2}\{R_0(kr)-\tfrac{5}{2}[3(\hat{k}\cdot\hat{r})^2-1]R_2(kr)\}e^{-i\mathbf{K}\cdot\mathbf{R}}\chi_s\eta_t, \tag{28}$$

where $\mathbf{k}$ and $\mathbf{K}$ are the relative and the total momenta of the two protons and the radial functions are approximated by $R_0(kr)=0$ for $r<r_c$, $R_0(kr)=[\sin k(r-r_c)]/kr$ for $r>r_c$ with $r_c=0.5$ F and $R_2(kr)=j_2(kr)$. The expression for $R_0(kr)$ takes account of the hard core in the

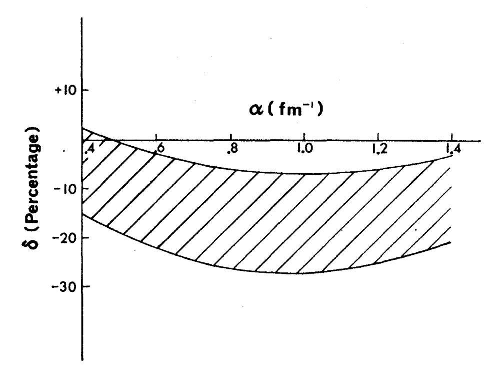

FIG. 2. Magnitude of the β-decay exchange effect (expressed as a percentage) as a function of the range parameter α.

[15] S. G. Eckstein, Phys. Rev. **129**, 413 (1963); P. P. Divakaran, *ibid.* **139**, B387 (1965).

[16] The actual terms contributing are, in Woodruff's notation, A_1, A_2, B_1, and B_2, where A_1 and B_1 are direct p-wave production terms and A_2 and B_2 stem from the Galilean invariant s-wave interaction coupled with retardation effects.

[17] L. J. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957).

[18] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).

[19] H. Kottler and K. L. Kowalski, Nucl. Phys. **53**, 334 (1964).

[20] K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).

internucleon potential; it does not have the correct asymptotic form for large r, but this is unimportant since we are only concerned with the behavior of R_0 for small r. Finally, the pion is taken to be represented by a plane wave function.

Using the above functions the calculation of the amplitudes $c(^1S_0)$ and $c(^1D_2)$ is straightforward and yields

$$\begin{aligned} c(^1S_0)&=\frac{C}{\alpha}\left[(A_{\rm I}+\tfrac{1}{3}A_{\rm II})\int u(r)R_0(kr)j_0\left(\frac{qr}{2}\right)e^{-\alpha r}dr\right.\\ &\qquad\left.-\tfrac{1}{3}\sqrt{2}A_{\rm II}\int w(r)R_0(kr)j_0\left(\frac{qr}{2}\right)e^{-\alpha r}dr\right],\\ c(^1D_2)&=\frac{C}{\alpha}\left[(A_{\rm I}+\tfrac{2}{3}A_{\rm II})\int w(r)R_2(kr)j_0\left(\frac{qr}{2}\right)e^{-\alpha r}dr\right.\\ &\qquad\left.-\tfrac{1}{3}\sqrt{2}A_{\rm II}\int u(r)R_2(kr)j_0\left(\frac{qr}{2}\right)e^{-\alpha r}dr\right].\end{aligned} \tag{29}$$

Here q is the pion momentum, and the constant C, near threshold, is given by

$$C=8\mu^2(M/\mu)^{1/4}. \tag{30}$$

Since we are dealing with a threshold phenomenon ($q\approx 0$), it is permissible to replace $j_0(qr/2)$ by 1 in the above integrals. After evaluating the integrals, values of $A_{\rm I}$, $A_{\rm II}$, and α can be chosen which lead to the values of $c(^1S_0)$ and $c(^1D_2)$ quoted in (24). Allowed values of $A_{\rm I}$ and $-\frac{1}{3}A_{\rm II}$ are plotted as bands in Fig. 1 for 0.4 $\text{F}^{-1}<\mu<1.4\ \text{F}^{-1}$. For example, for $\alpha=m_\pi=0.7\ \text{F}^{-1}$, $A_{\rm I}=(0.38\pm0.10)\ \text{F}^{-1}$, $A_{\rm II}=-(1.95\pm0.38)\ \text{F}^{-1}$.

5. EXPLICIT FORM FOR THE BETA-DECAY EXCHANGE OPERATOR

We now use the results of Sec. 3 [Eq. (17)] to obtain an explicit form for the effective beta-decay operator. Substituting (21) into (17) and carrying out the integration gives

$$\begin{aligned}\mathbf{H}_\beta=\frac{aG_\beta}{\mu^2}\Big\{&\sum_i\sqrt{2}\frac{G_{\pi NN}}{2M}K_{\pi NN}(0)\tau_i^+\boldsymbol{\sigma}_i+\sum_{i<j}\{[S_{ij}^\sigma T_{ij}^\tau+T_{ij}^\sigma S_{ij}^\tau][\alpha_{\rm I}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)+\alpha_{\rm II}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\,\hat{r}](\tau_i^+-\tau_j^+)\\ &+[T_{ij}^\sigma T_{ij}^\tau+S_{ij}^\sigma S_{ij}^\tau][\beta_{\rm I}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)+\beta_{\rm II}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\,\hat{r}](\tau_i^+-\tau_j^+)\\ &+[\gamma_{\rm I}(r)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)+\gamma_{\rm II}(r)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\hat{r}\,\hat{r}](\tau_i^++\tau_j^+)\}\Big\}.\end{aligned} \tag{31}$$

Using the value for a quoted in (5), it is clear that the one-body term in (31) has the form

$$\mathbf{H}_\beta^{(1)}=G_A\sum\tau_i^+\boldsymbol{\sigma}_i,$$

which is the usual one-body Gamow-Teller beta-decay operator [Eq. (7)]. This result, of course, is built into PCAC theory and the value of a is chosen in order to achieve it. Turning now to the two-body terms, by using relations such as

$$\begin{aligned}2T_{ij}^\sigma(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)&=(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)+i(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j),\\ 2S_{ij}^\sigma(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)&=(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)-i(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j),\end{aligned} \tag{32}$$

it is straightforward to show that they have the same form as given in (8), namely,

$$\begin{aligned}\mathbf{H}_\beta^{(2)}=G_A\sum_{i<j}\{&[g_{\rm I}(r)\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j+g_{\rm II}(r)(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j)\cdot\hat{r}\,\hat{r}](\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^+\\ &+[h_{\rm I}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)+h_{\rm II}(r)(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot\hat{r}\,\hat{r}](\tau_i^+-\tau_j^+)\\ &+[j_{\rm I}(r)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)+j_{\rm II}(r)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\hat{r}\,\hat{r}](\tau_i^++\tau_j^+)\}.\end{aligned}$$

The g, h, and j are given by

$$\begin{aligned}g_{\rm I}(r)&=\lambda[\alpha_{\rm I}(r)-\beta_{\rm I}(r)], & g_{\rm II}(r)&=\lambda[\alpha_{\rm II}(r)-\beta_{\rm II}(r)],\\ h_{\rm I}(r)&=\lambda[\alpha_{\rm I}(r)+\beta_{\rm I}(r)], & h_{\rm II}(r)&=\lambda[\alpha_{\rm II}(r)+\beta_{\rm II}(r)],\\ j_{\rm I}(r)&=2\lambda\gamma_{\rm I}(r), & j_{\rm II}(r)&=2\lambda\gamma_{\rm II}(r),\end{aligned} \tag{33}$$

where the constant λ has the value

$$\lambda=\frac{M}{\sqrt{2}G_{\pi NN}K_{\pi NN}^{(0)}}. \tag{34}$$

We thus have explicit expressions for the radial functions occurring in the Gamow-Teller beta-decay exchange operator in terms of the functions occurring in the two-body pion-production operator. These relationships are now exploited for the case of triton beta decay.

6. APPLICATION TO THE BETA-DECAY PROCESS $H^3\rightarrow He^3+e^-+\bar{\nu}$

In the beta decay of H^3 we have seen in Sec. 2 that in order to account for the ft value it seems necessary to appeal to exchange effects. It was shown that using a phenomenological exchange operator the corresponding beta-decay matrix element depended on the combination $g_{\rm I}(r)+\frac{1}{3}g_{\rm II}(r)+h_{\rm I}(r)+\frac{1}{3}h_{\rm II}(r)$. But from (33)

$$\begin{aligned}g_{\rm I}(r)+\tfrac{1}{3}g_{\rm II}(r)+h_{\rm I}(r)+\tfrac{1}{3}h_{\rm II}(r)&\\ &=2\lambda[\alpha_{\rm I}(r)+\tfrac{1}{3}\alpha_{\rm II}(r)]\\ &=2\lambda(A_{\rm I}+\tfrac{1}{3}A_{\rm II})(e^{-\alpha r}/\alpha r),\end{aligned} \tag{35}$$

using the Yukawa forms for $\alpha_{\rm I}(r)$ and $\alpha_{\rm II}(r)$ given in (26). On comparing with (11), we have the relationship

$$\Gamma=2\lambda(A_{\rm I}+\tfrac{1}{3}A_{\rm II})=\frac{2M(A_{\rm I}+\frac{1}{3}A_{\rm II})}{\sqrt{2}G_{\pi NN}K_{\pi NN}^{(0)}} \tag{36}$$

relating Γ to the parameters $A_{\rm I}$ and $A_{\rm II}$.

Taking the values of $A_{\rm I}$ and $A_{\rm II}$ necessary to account for the pion-production process, Γ can then be calculated as a function of the range parameter α. Correspondingly, the enhancement δ of the axial-vector matrix element due to exchange effects can be deduced. In Fig. 2, δ (as a percentage) is plotted against α.

It can be seen that far from giving an *enhancement* of the order $+10\%$ there is a *reduction* in the matrix element which may be small but which could be quite large. Within the range of values considered for α, only for a very long-range two-body term ($\alpha \lesssim 0.5$ F^{-1}) does the possibility arise that δ might be slightly positive.

We now consider possible causes of this discrepancy. Taking the theory set out in this paper, it is clear that a number of approximations have been made. Most serious of these, perhaps, is the use of a plane wave for the initial diproton state. This is probably justifiable, however, since the $^1S_0$ and $^1D_2$ phase shifts are small ($\approx 10°$) at the energy in which we are interested.

As an overall check on the calculation, a comparison can be made with Woodruff's work.[13] From his paper it is possible to deduce separately the contribution to the pion-production amplitude from the four transitions $^1S_0 \rightarrow {}^3S_1$, $^1S_0 \rightarrow {}^3D_1$, $^1D_2 \rightarrow {}^3S_1$ and $^1D_2 \rightarrow {}^3D_1$ where the right-hand states refer to the S- and D-wave components of the deuteron wave function. In particular, inspection of the $^1S_0 \rightarrow {}^3S_1$ amplitude shows that the contribution from rescattering, etc., which corresponds to our two-body term, has magnitude[21] -0.33. But from Eq. (29) the contribution of the two-body term to the $^1S_0 \rightarrow {}^3S_1$ amplitude is proportional to $(A_{\rm I}+\frac{1}{3}A_{\rm I})$ which, in turn, from Eq. (36), is proportional to Γ. Thus a negative sign for Γ, and hence for δ, is again implied.

Of course, this last conclusion and, indeed, the values taken for $b(^1S_0)$ and $b(^1D_2)$ in (23) depend on the model used by Woodruff[13] and so some uncertainty must be assumed here. In addition, the experimental values for $a(^1S_0)$ and $a(^1D_2)$ given in (19) are based on not particularly accurate experimental work carried out over 10 years ago, so leading to further uncertainty. This being so, it is perhaps unwise to regard the discrepancy as a severe and significant one. Even so, it is interesting to reconsider the experimental data on which the arguments for a positive exchange contribution given in Sec. 2 rest. The two crucial measurements are those currently accepted for the ft values of the neutron and H^3, namely,

$$(ft)_n = 1180 \pm 35 \text{ sec (Ref. 22)},$$

$$(ft)_{\rm H^3} = 1137 \pm 20 \text{ sec (Ref. 23)}.$$

Both experiments were carried out in 1959 and have not been repeated since. The neutron lifetime experiment is, of course, an extremely difficult one and doubts have been expressed about the accuracy claimed.[24] It seems likely that the uncertainties in the experiment are such that the actual ft value may be significantly *smaller* than the above value.

In addition, using the above ft value including electromagnetic radiative corrections[25] together with the value $G_A/G_V = -1.250 \pm 0.044$ obtained by Conforto[26] from an analysis of neutron β-decay angular correlation and polarization data leads to $G_V = (1.337 \pm 0.058) \times 10^{-10}$ erg F^3. This is to be compared with $G_V = (1.4052 \pm 0.0049) \times 10^{-10}$ erg F^3 obtained from data on $0^+ \rightarrow 0^+$ transitions (e.g., see Ref. 1). The values are hardly in agreement and again suggest that $(ft)_n$ should be several percent smaller than the value quoted above.

In the case of the H^3 decay, the uncertainty lies in the end-point energy E_0. Here it should be noted that the ft value is extremely sensitive to the value taken for E_0. This point has been discussed in some detail by Bahcall[27] who notes that two conflicting values for E_0 have been obtained:

$$E_0 = 18.61 \pm 0.1 \text{ keV (Ref. 23)},$$

$$E_0 = 17.95 \pm 0.1 \text{ keV (Ref. 28)}.$$

Here the experimental uncertainties work in such a direction that E_0 tends to be underestimated.[24] This being so, the value quoted above for $(ft)_{\rm H^3}$ should be regarded as a lower limit.

Thus, it could be that experiment is wrong to such an extent that there is no need for positive exchange effects at all. Indeed, negative exchange effects as indicated by the calculations of this paper may be necessary.[29] Clearly it is extremely important that the ft values of the neutron and H^3 be carefully remeasured. In addition, more detailed measurements and analysis of the pion-production process $p+p \rightarrow d+\pi^+$ would also help to clarify things. But the over-all situation must be regarded as considerably uncertain. However, we do feel that the approach adopted in this paper coupled with more accurate experimental results could lead to further information about the validity of PCAC theory.

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[21] In the notation of Ref. 13 the two-particle contribution is given by $A_5+A_6+A_7+A_8+A_{10}$.

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[29] Here, however, it should be noted that some recent work by W.-K. Cheng [Ph.D. thesis, University of Pennsylvania (unpublished)] using a quite different approach suggests that PCAC theory *might* lead to a positive exchange effect.