Taking matrix elements for the various terms, one has

$$\langle\alpha|[h^{(0)}\,\rho^{(2)}]|\beta\rangle=(\epsilon_\alpha{}^0-\epsilon_\beta{}^0)\langle\alpha|\rho^{(2)}|\beta\rangle\,, \quad \text{(A19)}$$

$$\langle\alpha|[J_x,\rho^{(1)}]|\beta\rangle=\sum_\gamma \{\langle\alpha|J_x|\gamma\rangle\langle\gamma|\rho^{(1)}|\beta\rangle -\langle\alpha|\rho^{(1)}|\gamma\rangle\langle\gamma|J_x|\beta\rangle\}\,, \quad \text{(A20)}$$

$$\begin{aligned}\langle\alpha|[\mathrm{Tr}_2\rho_2{}^{(1)}V_{12},\rho_1{}^{(1)}]|\beta\rangle &=\sum_\gamma \{\langle\gamma|\rho^{(1)}|\beta\rangle\sum_\delta \langle\alpha_1\delta_2|\rho_2{}^{(1)}V_{12}|\gamma_1\delta_2\rangle -\langle\alpha|\rho^{(1)}|\gamma\rangle\sum_\delta \langle\alpha_1\delta_2|\rho_2{}^{(1)}V_{12}|\beta_1\delta_2\rangle\}\\ &=\sum_{\gamma,\delta,\eta} \langle\delta|\rho^{(1)}|\eta\rangle\{\langle\gamma|\rho^{(1)}|\beta\rangle\langle\alpha\eta|V|\gamma\delta\rangle -\langle\alpha|\rho^{(1)}|\gamma\rangle\langle\gamma\eta|V|\beta\delta\rangle\}\,,\end{aligned} \quad \text{(A21)}$$

$$\langle\alpha|[\mathrm{Tr}_2\rho_2{}^{(2)}V_{12},\rho_1{}^{(0)}]|\beta\rangle=(u_\beta-u_\alpha) \times\sum_{\gamma\delta} \langle\alpha\delta|V|\beta\gamma\rangle\langle\gamma|\rho^{(2)}|\delta\rangle. \quad \text{(A22)}$$

Thus, the matrix elements $\rho_{\gamma\delta}{}^{(2)}$ of $\rho^{(2)}$ are subject to the set of linear equations

$$\mathcal{B}_{\alpha\beta}{}^{\gamma\delta}\rho_{\gamma\delta}{}^{(2)}=\mathfrak{D}_{\alpha\beta}\,, \quad \text{(A23)}$$

where

$$\mathcal{B}_{\alpha\beta}{}^{\gamma\delta}=\mathcal{A}_{\alpha\beta}{}^{\gamma\delta} \quad \text{(A24)}$$

and

$$\begin{aligned}\mathfrak{D}_{\alpha\beta}=&\sum_\gamma \{\langle\alpha|J_x|\gamma\rangle\langle\gamma|\rho^{(1)}|\beta\rangle-\langle\alpha|\rho^{(1)}|\gamma\rangle\langle\gamma|J_x|\beta\rangle\}\\ &-\sum_\gamma\sum_{\delta,\eta} \langle\delta|\rho^{(1)}|\eta\rangle\{\langle\gamma|\rho^{(1)}|\beta\rangle\langle\alpha\eta|V|\gamma\delta\rangle -\langle\alpha|\rho^{(1)}|\gamma\rangle\langle\gamma\eta|V|\beta\delta\rangle\}\,.\end{aligned} \quad \text{(A25)}$$

The Thouless-Valatin formula is obtained when one restricts oneself to the lowest order in ω and neglects the off-diagonal elements of A:

$$\mathcal{A}_{\alpha\beta}{}^{\gamma\delta}\rightarrow\mathcal{A}_{\alpha\beta}{}^{\gamma\delta}\delta_{\alpha\gamma}\delta_{\beta\delta}\,.$$

The Inglis formula is obtained when one neglects not only the off-diagonal terms, but the two-body matrix elements in the diagonal terms as well.

In the case where the space of allowed single-particle orbits is truncated, the problem of finding $\rho^{(1)}$ and $\rho^{(2)}$ becomes finite. It can be solved with exactitude numerically. The only operation of any complexity is the inversion of the matrix $\mathcal{A}$ which appears both in the $\rho^{(1)}$ and $\rho^{(2)}$ equations.

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Scattering of Alpha Particles by Oxygen. I. Bombarding Energy Range 5.8 to 10.0 MeV*

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Absolute differential cross sections for the elastic scattering of α particles by O^{16} have been measured as a function of bombarding energy in the range 5.8–10.0 MeV. Measurements were made at center-of-mass angles of 90.0°, 109.9°, 114.0°, 125.3°, 131.4°, 140.8°, 149.4°, 154.0°, 158.8°, and 163.8°. Detailed angular distributions have been measured at 6.97, 8.63, and 9.92 MeV (lab). Sixteen resonances have been observed corresponding to energy levels in Ne^{20} at 9.50, 9.99, 10.30, 10.49, 10.55, ~10.7, 10.83, (10.93), 11.03, 11.29, ~11.6, (11.89), 11.99, 12.27, 12.39, and 12.58 MeV (c.m.). Spin and parity assignments have been made for six of these levels, tentative assignments are suggested for seven, and two or more possible assignments are given for two levels. Phase shifts have been extracted from the angular distribution data at 6.97, 8.63, and 9.92 MeV (lab). New rotational bands in Ne^{20} are suggested by the data and previously reported bands have been extended. Information about the levels in Ne^{20} is compared with that obtained in previous studies of other nuclear reactions. The correspondence with the results of an $O^{16}(\alpha,\gamma)Ne^{20}$ investigation is generally good. The set of Ne^{20} levels found in this work is somewhat different from the set determined by the $C^{12}(C^{12},\alpha)Ne^{20}$ reaction experiments, and this difference is discussed.

I. INTRODUCTION

THE elastic scattering of α particles by O^{16} has been studied from 0.94- to 4.0-MeV bombarding energy by Cameron[1] and from 3.7–6.5 MeV by McDermott *et al.*[2] In these experiments the data were analyzed to yield the spins, parities, and other level parameters of a number of states in the compound nucleus Ne^{20} between 5.48 and 9.93 MeV (c.m.). Prior to both of these experiments, Ferguson and Walker[3] had elastically scattered α's from O^{16} using α particles from RaC′, and observed resonances at bombarding energies of 5.5 and 6.5 MeV.

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[1] J. R. Cameron, Phys. Rev. 90, 839 (1953).

[2] L. C. McDermott, K. W. Jones, H. Smotrich, and R. E. Benenson, Phys. Rev. 118, 175 (1960).

[3] A. J. Ferguson and L. R. Walker, Phys. Rev. 58, 666 (1940).

Below 10-MeV bombarding energy, the collisions between α particles and O^{16} are completely or predominantly elastic scattering events. Experimental results and analysis in the beam-energy range 5.8–10.0 MeV are presented in this paper. In Paper II[4] (the following paper), data are reported for alpha-particle energies of 10–19 MeV where at least two channels are open, and rather different analyses are required in the interpretation of the data.

Excitation curves have been measured at ten angles and the data have been analyzed to yield information about the spins and parities of 15 of the 16 resonances observed. Detailed angular distributions have been measured at 6.97, 8.63, and 9.92 MeV (lab) and sets of phase shifts have been extracted.

In Secs. II and III, the experimental procedure is discussed and the data are presented. Section IV contains an outline of the analysis and its application to the data. The results are interpreted in Sec. V and compared with previously available information about levels in Ne^{20}. Certain of the resonances observed in the present and the following paper[4] extend the rotational-band systematics in Ne^{20} which was initially proposed by Litherland *et al.*[5]

II. EXPERIMENTAL PROCEDURE

Negative helium ions were injected[6] into the Florida State University tandem Van de Graaff accelerator to produce α-particle beams (5–12 nA) of continuously variable energy. Oxygen targets were prepared by vacuum deposition of thin layers of lithium metal onto thin carbon backings with subsequent oxidation of the lithium to form LiOH. To estimate the unresolved yield due to contaminants such as nitrogen and fluorine, back-angle pulse-height distributions were compared with those for thin quartz (SiO_2) targets. No impurities with mass number approximately 16 were detected.

Targets 25–35 keV thick to 10-MeV α particles were mounted in a scattering chamber equipped with p-type silicon-junction detectors, three of which were movable from without the chamber via rotating vacuum seals. For the excitation-curve measurements, as many as eight detectors were operated simultaneously. The pulses from each detector were amplified and fed into a 64-channel analyzer (one quadrant of a TMC 256-channel analyzer).

The experimental arrangement for the detailed angular-distribution measurements was the same except that the fixed counters were removed from the chamber to allow rotation of the movable detectors. Each detector had a resolution on the order of 150 keV, and the collimating slits of each subtended an angle of 2°. A typical pulse-height spectrum is shown in Fig. 1.

[4] M. K. Mehta, W. E. Hunt, and R. H. Davis, following paper, Phys. Rev. **159**, 791 (1967).

[5] A. E. Litherland, J. A. Kuehner, H. E. Gove, M. A. Clark, and E. Almqvist, Phys. Rev. Letters **7**, 98 (1961).

[6] E. B. Carter and R. H. Davis, Rev. Sci. Instr. 34, 93 (1963).

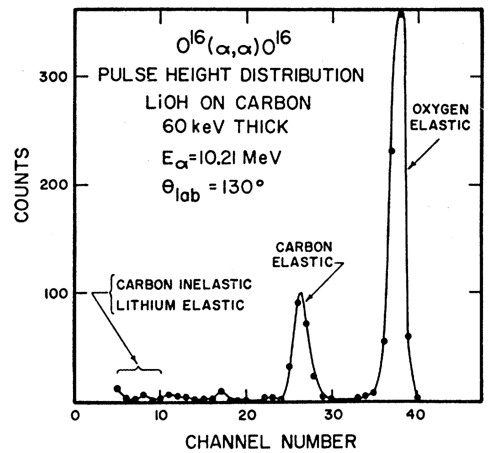

FIG. 1. A typical pulse-height spectrum from a LiOH target.

The transmitted beam was collected in a well-insulated Faraday cup and charged a precision-decade capacitor to a voltage of 3 V or less. An electron suppressor was employed at the entrance to the Faraday cup to prevent escape of secondary electrons from the cup. Positive target biasing was used to prevent secondary electrons from the target from entering the detector and thereby affecting the resolution.

The calibration of the 90° analyzing magnet was determined by a measurement of the well-known $C^{13}(p,n)N^{13}$ threshold with an assumed threshold energy of 3.237 MeV.[7] Additional checks on the calibration and linearity of the magnet were provided by a series of (p,n) threshold measurements[8,9] and by threshold measurements of the $Li^6(\alpha,n)B^9$ and $Li^7(\alpha,n)B^{10}$ reactions.[10] The $Li^7(\alpha,n)B^{10}$ threshold at 4.37 MeV (lab) was determined with both singly and doubly charged helium beams. The maximum calibration error was not greater than ± 20 keV at any energy in the range of this experiment, and probably not more than ± 10 keV.

III. DATA

Excitation curves were measured at 10 angles such that the first 8 Legendre polynomials vanish in the center-of-mass system. There were two angles corresponding to the vanishing of the fourth- and sixth-order polynomials, respectively. Partial waves with l values greater than 8 were not considered below 10-MeV bombarding energy since the largest l value expected from the maximum-impact parameter was $l=6$, and that from penetrability calculations was $l=8$. The size

[7] R. O. Bondelid and C. A. Kennedy, Phys. Rev. **115**, 1601 (1959).

[8] J. W. Nelson, H. S. Adams, R. H. Davis, J. D. Fox, N. P. Heydenburg, H. S. Plendl, R. K. Sheline, and G. M. Temmer, Bull. Am. Phys. Soc. **5**, 424 (1960).

[9] J. W. Nelson, H. S. Plendl, and R. H. Davis, Phys. Rev. **125**, 2005 (1962).

[10] M. K. Mehta, W. E. Hunt, H. S. Plendl, and R. H. Davis, Nucl. Phys. **48**, 90 (1963).

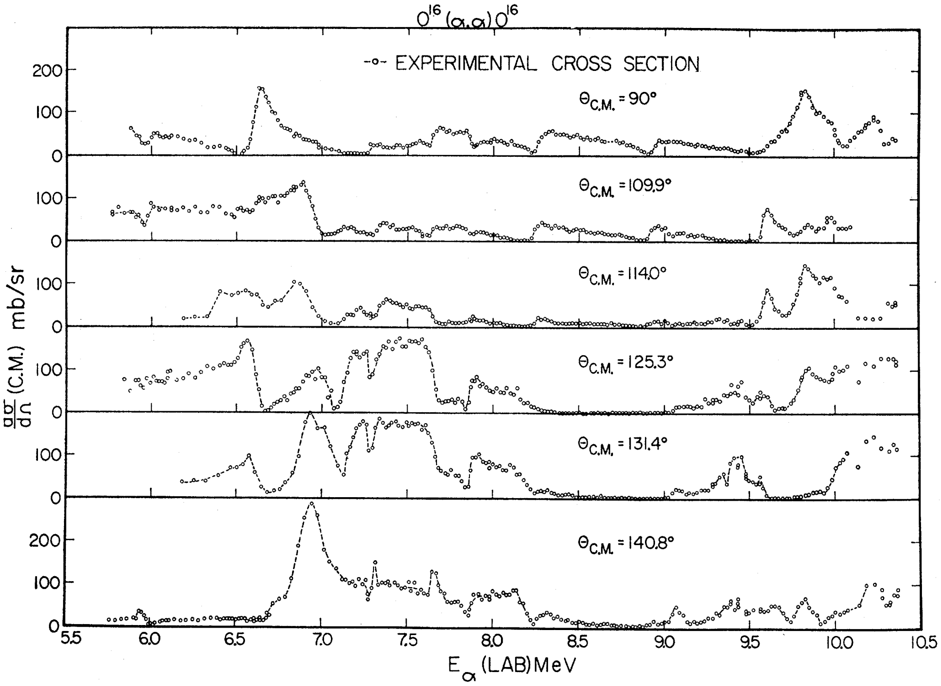

FIG. 2. $O^{16}(\alpha,\alpha)O^{16}$ differential cross sections versus bombarding energy. The center-of-mass observation angles are 90.0°, 109.9°, 114.0°, 125.3°, 131.4°, and 140.8°.

of the energy steps was governed by the detail in the excitation curves and by the target thickness. Targets were 25–35 keV thick to 10-MeV α particles and energy steps were normally 25 keV.

Detailed angular distributions were measured in steps of 3° (lab) at bombarding energies of 6.97, 8.63, and 9.92 MeV. The maximum backward laboratory angle at which the detector could be set without interrupting the beam was 175°, and the minimum forward angle was limited by the inability of the detectors to resolve the elastic groups of oxygen and carbon, approximately 45° at 6.97-MeV and 35° at 9.92-MeV bombarding energy.

Because of uncertainties in determination of the exact oxygen content of the LiOH targets, the absolute cross sections were assigned by normalizing the data to

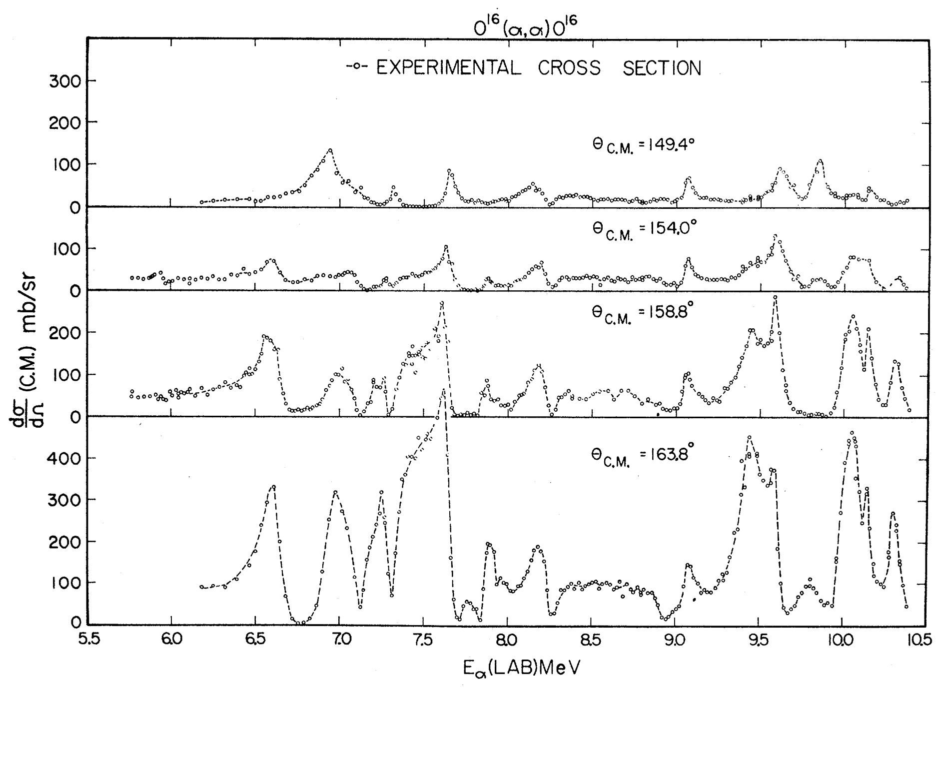

FIG. 3. $O^{16}(\alpha,\alpha)O^{16}$ differential cross sections versus bombarding energy. The center-of-mass observation angles are 149.4°, 154.0°, 158.8°, and 163.8°.

cross sections measured with SiO_2 targets whose thicknesses were known to within 6%. The SiO_2 targets were prepared from blown quartz, and their thicknesses were determined by weighing. Exclusive use of quartz targets was precluded by the frequent overlap of silicon inelastic and oxygen elastic groups. The estimated error in the cross sections is ±12%, except for the region of excitation below 6.3 MeV, where statistical errors of ±10% resulted from very poor beam currents (the order of 1 nA). Statistical errors also increased the cross-section errors near the minimum of the angular distribution where the cross section is less than 10 mb/sr.

The excitation-curve data is displayed in Figs. 2 and 3. The region below 6.5 MeV (lab) overlaps the work by McDermott *et al.*[2] An overlap with the work above 10 MeV (lab) of Mehta *et al.*[4] is also shown. The experimental data for the detailed angular distributions is presented in Figs. 4–6, accompanied by theoretical fits obtained from a phase-shift extraction.

IV. ANALYSIS OF DATA

Anomalies in the excitation curves have been analyzed to yield the approximate resonance energies, tentative spins and parities, and in some cases, observed widths. The angular-distribution data have been fitted with the conventional phase-shift expression, and the extracted sets of phase shifts have been interpreted with a dispersion-theory analysis where possible.

The differential cross section for the elastic scattering of spinless particles by zero-spin nuclei is expressed by[2]

$$\frac{d\sigma}{d\Omega}=k^{-2}\Big|-(\eta/2)\csc^2(\theta/2)\exp[i\eta\ln\csc^2(\theta/2)]+\sum_l(2l+1)P_l(\theta)\exp[i(\alpha_l+\delta_l)]\sin\delta_l\Big|^2,$$

where

$$k=\mu v/\hbar,$$
$$\eta=ZZ'e^2/\hbar v,$$

θ is the center-of-mass scattering angle, μ is the reduced mass, v is the relative velocity, δ_l is the lth partial-wave phase shift, and α_l is the Coulomb phase shift. The quantity α_l is given by the expression

$$\exp(i\alpha_l)=\prod_{m=1}^{l}((m+i\eta)/(m-i\eta)),$$
$$\exp(i\alpha_0)=1.$$

It can be seen from the expression for the cross section that the effects due to the lth partial wave are not present at those angles corresponding to the zeros of the lth Legendre polynomial. Consequently, there is no contribution to the cross section at those angles resulting from the formation of a state in the compound nucleus whose angular momentum is l. Since the total

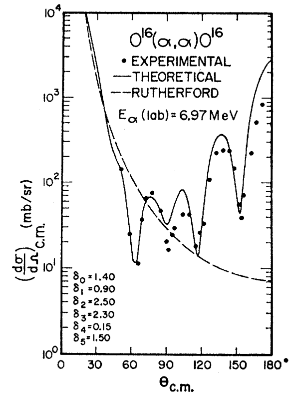

FIG. 4. The angular distribution of the differential cross section at E_α(lab) = 6.97 MeV and a phase-shift analysis.

spin and parity of any state formed in Ne^{20} by α-particle bombardment of O^{16} is $J=l$ and $\pi=(-1)^l$, respectively, visual inspection of the excitation curves at a sufficient

TABLE I. Summary of Ne^{20}-level parameters determined from the data of the present experiment. Tentative J^π assignments are shown in parentheses.

E_R (lab) (MeV)	E_x (c.m.) (MeV)	J^π	Γ (MeV) estimated
5.95 ±0.03	9.50	2^+	0.05[a]
6.58 ±0.03	9.99	4^+	0.15
6.97 ±0.03	10.30	5^-	0.15
7.20 ±0.03	10.49	(3^-)	
7.27 ±0.03	10.55	(0^+)	0.05[a]
∼7.5 ±0.03	∼10.7	4^+	1.00
7.63 ±0.03	10.83	(0^+)	0.05[a]
(7.75)±0.03	(10.93)		0.10
7.87 ±0.03	11.03	4^+	
8.20 ±0.03	11.29	(2^+)	0.15
∼8.6 ±0.03	∼11.6	(2^+)	0.60
(8.95)±0.03	(11.89)	$(2^+,6^+)$	
9.07 ±0.03	11.99	(1^-)	
9.42 ±0.03	12.27	(4^+)	
9.58 ±0.03	12.39	$(0^+,1^-,3^-)$	0.1
9.82 ±0.03	12.58	6^+	0.13

[a] Upper limit for widths.

number of angles (10 in this case) is a means for determining l and J^π for a given level. Determination of the resonance energies, and where possible the observed widths, has also been done by inspection.

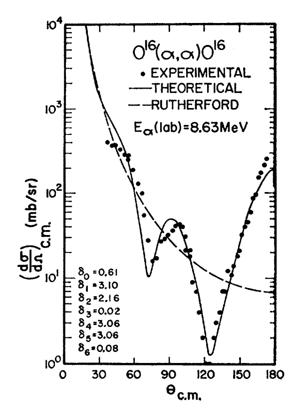

FIG. 5. The angular distribution of the differential cross section at E_α(lab) = 8.63 MeV and a phase-shift analysis.

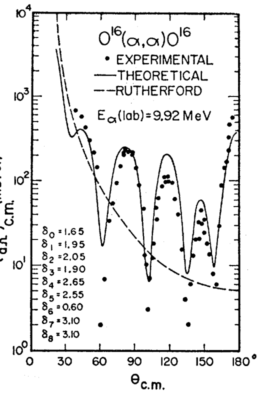

FIG. 6. The angular distribution of the differential cross section at E_α(lab) = 9.92 MeV and a phase-shift analysis.

Of the 16 resonances observed here between 5.8- and 10.0-MeV bombarding energy, J^π assignments have been made for six levels, tentative assignments have been made for seven levels, and two of the levels have more than one possible assignment. These results have been tabulated and are shown in Table I.

The following discussion of the observed levels deals first with the narrow levels (widths less than 200 keV) and second with the broad levels.

A. Narrow Resonances

The level appearing at an α-particle bombarding energy of 5.95 MeV is the same level previously reported by McDermott *et al.*[2] at 6.03 MeV (lab). The level reported by Pearson and Spear[11] at 5.94 MeV in a study of the $O^{16}(\alpha,\gamma)Ne^{20}$ reaction is believed to be the same level. McDermott's assignment of 2^+ for the level has been confirmed in the present experiment.

The strong resonance appearing at 6.58 MeV is present at all angles except 109.9° and 149.4°, both of which correspond to the vanishing of the $l=4$ partial wave. Consequently an assignment of 4^+ has been made for this level. It should be mentioned that the energy dependence of the differential cross section does not support a 4^+ assignment for an isolated resonance. The resonance is not isolated, and insufficient information is available about the α-particle interaction to analyze the energy dependence in detail.

Examination of the excitation-curve data reveals another strong resonance at approximately 6.97 MeV (lab). The behavior of the cross section at the 10 observation angles suggests that the resonance is characterized by either $l=1$ or $l=5$. A detailed (3° steps) angular distribution was measured at this energy, and the analysis establishes the spin and parity 5^-. Further discussion is deferred to the section on angular-distribution analysis.

The resonances at 7.20 and 7.27 MeV (lab) are not well resolved. A careful examination of the data indicates that fluctuations in the cross sections due to the 7.20-MeV resonances are absent at 90°, implying an odd l value for that state. At 140.8°, corresponding to the vanishing of $P_3(\theta)$, the shape of the 7.27-MeV resonance appears as a "dip-rise," while the effect of the 7.20-MeV resonance apparently is absent. Consequently, the level associated with the 7.20-MeV resonance has been tentatively assigned a spin and parity of 3^-. Since effects of the resonance at 7.27 MeV appear at all angles, a 0^+ assignment is tentatively given.

Inspection of the excitation curves reveals a resonance at 7.63 MeV, the effects of which are present at all 10 angles. It has been given a tentative assignment of 0^+. It is obviously superimposed on a very broad resonance, which will be discussed in the section on broad levels.

There is considerable uncertainty about the anomaly occurring at 7.75 MeV. It appears as a small, well-resolved resonance at 163.8°, but does not appear clearly at many of the other angles, and it is listed as a possible resonance with no assignment suggested.

The resonance at 7.87 MeV (lab) is clearly present at all angles except 109.9° and 149.4°, at both of which the P_4 term vanishes. Consequently, this level has been assigned a spin and parity of 4^+.

The resonance at 8.20 MeV has been tentatively identified as 2^+. Its effects are present at all angles except 125.3°, the null angle for $P_2(\theta)$. The variation of the cross section with angle provides additional evidence for this assignment.

At some angles a small anomaly appears in the excitation curves near 8.95 MeV. It is not well resolved, but shows up as a dip at 90°, implying an even l-value state. Furthermore, this anomaly is not present at either of the two angles corresponding to zeros of $P_6(\theta)$. The anomaly also seems to vanish at 125.3°, which is a zero for $P_2(\theta)$. Consequently, an assignment of 2^+ or 6^+ is tentatively given to this weak and possibly spurious anomaly.

The resonance at 9.07 MeV does not vanish at any angle except 90° where all odd-l partial waves vanish. Moreover the cross section increases rather smoothly with increasing angles. Therefore, this level has been given a tentative 1^- assignment.

The resonance appearing at 9.42 MeV behaves somewhat strangely. It vanishes at both angles corresponding to the vanishing of the $l=4$ partial wave, but it is also very small at 90°, where the absence of odd-order partial waves is required. Because of the unusual behavior at 90°, possibly due to an accidental cancellation, an assignment of 4^+ is tentatively proposed for this resonance.

The resonance at 9.58 MeV is present at all angles except possibly 90° and 140.8°, the latter being a null angle of $P_3(\theta)$. Since there is considerable ambiguity as to where the resonance is vanishing, an assignment of 0^+, 1^-, or 3^- is suggested.

Inspection reveals one more resonance below 10 MeV located at 9.82 MeV. Effects of this resonance are present at all angles except 131.4° and 158.8°, both of

[11] J. D. Pearson and R. H. Spear, Nucl. Phys. 54, 434 (1964).

which correspond to the vanishing of $P_6(\theta)$. Consequently, this level has been assigned as 6^+.

Above 10 MeV, inelastic-scattering cross sections become significant and can no longer be neglected in the analysis of elastic-scattering data.

B. Broad Resonances

A close examination of the excitation-curve data between 5.8 and 10.0 MeV (lab) reveals the existence of at least two broad resonances. In the vicinity of 7.5 MeV, the fine structure appears to be superimposed on a broad resonance the width of which is of the order of 1 MeV. This broad structure is apparently absent at 109.9° and strikingly absent at 149.4°. Both of these angles correspond to zeros of $P_4(\theta)$. Consequently, this broad resonance has been assigned as 4^+. The maximum single-particle width calculated from the Wigner limit[12] is 1.0 MeV for an $l=4$ resonance at 7.5 MeV (lab). Thus it is possible that this anomaly is a single α-particle state.

The other broad "resonance" is located between 8.3 and 9.0 MeV. In contrast to the broad level at 7.5 MeV, there is very little, if any, superimposed fine structure. There is some question as to whether the term "resonance" is applicable to this region, even though the cross section departs markedly from the Coulomb cross section. In order to understand this region better, a detailed angular distribution was measured at 8.63 MeV, which is very near the center of the region. The analysis of the angular distribution (discussed in the next section) mildly favors an $l=2$ assignment, but no phase-shift value strongly suggests a resonance.

C. Angular Distributions

Several detailed angular distributions were measured to supplement the 10 excitation curves. The angular distribution at 6.97 MeV was measured to clarify the spin and parity assignment for the resonance at that energy. To learn more about the "broad"-resonance region between 8.3 and 9.0 MeV, an angular distribution was measured at 8.63 MeV. The 9.92-MeV angular distribution is measured at an energy where additional channels are open.

The angular distributions are shown in Figs. 4–6, accompanied by fits obtained in a phase-shift analysis. It was relatively easy to find sets of phase shifts (δ_l) that yielded good fits at either the forward angles or the backward angles, but rather difficult to find sets which yielded satisfactory fits at all angles. The best fits are shown in Figs. 4–6.

Trial fits to the angular distributions were based on the analysis of the excitation curves. Once reasonably good fits had been obtained, a minimum of the χ^2 function[13] was sought for a variation of the phase shifts with the aid of an IBM 709 computer.

[12] T. Teichman and E. P. Wigner, Phys. Rev. **87**, 123 (1952).
[13] James N. Snyder, Phys. Rev. **96**, 1333 (1954).

From dispersion theory, the partial-wave phase shifts (δ_l)[14,15] may be defined by

$$\delta_l=\beta_l-\phi_l,$$

where $\phi_l=\tan^{-1}(F_l/G_l)_{r=a}$. The quantities F_l and G_l are the regular and irregular Coulomb wave functions and a is the interaction radius. For the case where no two levels of the same spin and parity are interfering, β_l can be expressed as

$$\beta_l=\tan^{-1}\left(\frac{\gamma_{\lambda l}^2 k/A_l^2}{E_\lambda+\Delta_{\lambda l}-E}\right),$$

and A_l^2 is given by

$$A_l^2=F_l^2+G_l^2.$$

In the above equations, E_λ is the resonant energy, $\gamma_{\lambda l}^2$ is the reduced width, and $\Delta_{\lambda l}$ is the level shift.

The interaction radius used here to determine ϕ_l and subsequently β_l from the partial-wave phase shifts is $r=1.30(16^{1/3}+4^{1/3})\times10^{-13}$ cm $=5.34\times10^{-13}$ cm, the same value used by McDermott *et al.*[2] below 6.5 MeV (lab). The assumed separation energy of the α particle is that used by Pearson and Spear,[11] a value of 4.730 MeV.

From the best set of δ_l extracted at 6.97 MeV, the resonant parts of the phase shifts have been calculated. The value of β_5 is approximately 90° which implies a 5^- assignment for the resonance at 6.97 MeV. A poorly resolved 1^- resonance is also suggested by the value of the β_1 phase shift ($\beta\cong100°$).

The excitation curves in the vicinity of the angular distribution at 8.63 MeV are relatively smooth. The phase shifts have been extracted from the angular distribution, and none of the resonant parts (β_l) is approximately equal to 90°. Either 8.63 MeV is very different from laboratory resonant energy for a single level, or many distant resonances govern the behavior of the excitation curves in this vicinity.

Several prominent anomalies appear in the vicinity of 9.92 MeV. The angular-distribution data at this energy have been fitted with a set of phase shifts, but no interpretation of the phase shifts in terms of resonance parameters has been attempted.

V. DISCUSSION OF RESULTS

The assignments of level parameters described in the analysis section depends on the behavior of the excitation curves measured at the zeros of Legendre polynomials and on a limited number of angular distributions. These are secure where the anomaly in the excitation curve vanishes at the nulls of only one Legendre polynomial, or where the angular-distribution best fit is unique. Precise values for the resonant energies and the level widths were not extracted from the data, because the excitation curves were not fitted.

[14] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).
[15] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

TABLE II. Rotational bands in Ne^{20}. Columns represent rotational bands, and the labels A through I identify the corresponding plots in Fig. 7. An unnatural parity or a tentative assignment is shown below the excitation-energy value. Band members proposed in the $O^{16}(\alpha,\alpha)O^{16}$ work have a superscript, and the remaining band assignments are those of the Chalk River group. Several previously known levels are shown in the new bands. In three cases the band members proposed by the Chalk River group differ from those determined in the $O^{16}(\alpha,\alpha)O^{16}$ work, and both are shown.

J^π	Band A	B	C	D	E	F	G	H	I
	Excitation energy (MeV)								
0^+	0.00			6.722	7.202	~8.7		10.55[a] (0^+)	10.83 (0^+)
1^-			5.80				8.882		
2^+	1.632	4.969 2^-		7.434	7.838	~8.8	9.50		11.29[a] (2^+)
3^-		5.631	7.166				10.49[a] (3^-)		
4^+	4.248	7.02 4^-		9.076	9.99[a]	~10.7[a]	{10.57 11.03[a]}	12.27[a] (4^+)	12.77[b] (4^+)
5^-		8.46	10.30[a]						
6^+	8.79	10.64 6^-		{12.19 12.58[a]}	13.94[b] (6^+)	14.3[b]			
7^-		{13.39 13.69[b]} (7^-)	15.43[b]						
8^+	11.99								

[a] From this experiment. [b] From Mehta *et al.*, Ref. 4.

Estimates of the total width are most accurate for the prominent anomalies. Here the α-particle partial width may be a substantial fraction of the Wigner limit and approximately equal to the total width. Such states are of special interest when they are members of the rotational-energy bands proposed by Litherland *et al.*[5] The moment of inertia may be related to the width of the band member, as suggested by Davis.[16]

The single α-particle strength is usually distributed among several states some of which can be observed in unusual but analytically convenient reactions, such as $C^{12}(C^{12},\alpha)Ne^{20}$. This reaction has been important in recent studies of the rotational bands in Ne^{20}.

A. Rotational Bands in Ne^{20}

Litherland *et al.*[5] arranged a number of low-lying states in Ne^{20} in series that obeyed rotational-band systematics. Following this work, the levels in Ne^{20} have been intensively studied at the Chalk River Tandem Accelerator Laboratory,[17–20] and further evidence for the existence of rotational bands in Ne^{20} has been found. Extensions of these rotational bands based on data reported here and in the paper by Mehta *et al.*[4] are presented in this section. The possible existence of new bands is also discussed.

The columns of Table II are levels in Ne^{20} that are members of the same band. The superscripts label the band members proposed in this work, and the unlabeled entries are band members proposed by the Chalk River group or are previously known[21,22] levels in Ne^{20}. Unnatural parity assignments and tentative assignments are given below the excitation energy where appropriate.

In several instances, the energy of the band member proposed in this work differs from that in the Chalk River scheme, and both entries are shown. A possible explanation is that the α-particle strength is not equally shared by compound-system states in the vicinity of a single-particle state. The scattering experiment and analysis tend to select states with significant α-particle widths. States formed in reactions may have α-particle partial widths which are small compared with the partial widths associated with nearby states seen in the α-particle scattering work. A preliminary report on a model proposed by Robson in which single α-particle states are coupled with compound-system states has been given by Davis.[23]

In Fig. 7 the energy levels of Ne^{20} are plotted as a function of $J(J+1)$. Where two entries appeared in Table II, the value obtained in this work is plotted. Unnatural parity is indicated by a plus or minus sign next to the point. Points represented by dots are band members proposed by the Chalk River group, and the triangles are those proposed in this paper.

[16] R. H. Davis, *Proceedings of the Third Conference on Reactions Between Complex Nuclei, Asilomar, 1963* (University of California Press, Berkeley, 1963), p. 67.
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[21] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959).
[22] T. Lauritsen and F. Ajzenberg-Selove, in *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington, D. C., 1962), Sets 5 and 6.
[23] R. H. Davis, in *Proceedings of the Symposium on Recent Progress in Nuclear Physics with Tandems*, edited by W. Hering (Max Planck Institute for Nuclear Physics, Heidelberg, 1966).

TABLE III. Levels in Ne^{20}. The results of the present work are compared with those previously published. The 10.30-MeV state observed in this work is thought to differ from the states found at about this energy in the $F^{19}(d,n)Ne^{20}$ reaction, and it is given a separate line. Two levels that have been observed in different reactions and that have different J^π assignments (one tentative), are reported at 11.99 MeV. These are shown on the same line although two different levels may be present.

$O^{16}(\alpha,\alpha)O^{16}$ This work E_x (MeV)	J^π	Ref. 2 E_x (MeV)	J^π	$O^{16}(\alpha,\gamma)Ne^{20}$ Ref. 11 E_x (MeV)	J^π	$C^{12}(C^{12},\alpha)Ne^{20}$ $C^{12}(C^{12},\alpha\gamma)Ne^{20}$ Refs. 18–20 E_x (MeV)	J^π	Ref. 28 E_x (MeV)	J^π	$F^{19}(d,n)Ne^{20}$ and $F^{19}(d,n\gamma)Ne^{20}$ Ref. 29 E_x (MeV)	J^π	Ref. 25 E_x (MeV)	J^π	Ref. 27 E_x (MeV)	J^π	Ref. 26 E_x (MeV)	J^π
9.50	2^+	9.55	2^+	9.48	2^+	9.48	2^+			9.50							
										9.60							
						9.80											
9.99	4^+			10.02	(4^+)			9.96		9.92							
										10.00							
				10.270	2^+	10.24	2^+										
10.30	5^-																
										10.30				10.31	(1^+)	10.33	(2^-)
10.49	(3^-)																
10.55	(0^+)																
						10.57	4^+										
								10.61									
						10.64	6^-										
∼10.7	4^+																
10.83	(0^+)																
(10.93)																	
11.03	4^+			11.08	(4^+)	11.0				11.00		11.11				11.03	
												(11.19)					
11.29	(2^+)			11.27	$(1^-,2^+)$			11.30		11.32		11.33					
												(11.42)					
∼11.6	(2^+)			11.56[a]		11.7				11.66		11.69					
(11.89)	$(2^+,6^+)$											11.87					
11.99	(1^-)					11.99	8^+										
				(12.13)		12.19	6^+					12.19					
12.27	(4^+)			12.25[a]								12.27					
12.39[b]				12.39													
												12.51					
12.58	6^+																

[a] The 11.56- and 12.25-MeV states both have J^π assignments of 0^+, 1^-, 2^+, 3^- or 4^+.
[b] The 12.39-MeV state has a J^π assignment of (0^+, 1^-, or 3^-).

Bands A through E were initially suggested by Litherland *et al.*[5] Band A now includes a recently identified 8^+ state at 11.99 MeV.[20] From the shell-model point of view, a continuation of the band will require a change in configuration.[17] Such changes may yield smoothly joined rotational spectra as shown by the rotational-band spectrum of Be^8.[24]

A possible 7^- state at 13.69 MeV observed by Mehta *et al.*[4] has been added to band B. A 7^- state at 13.39 MeV has been proposed by Kuehner and Almquist.[19]

Band C has been extended by the addition of a 5^- state at 10.30 MeV and a 7^- level at 15.43 MeV. The α-particle width of the C-band members is large, and the moment of inertia is consistent with the values of the width.[16]

A 6^+ level at 12.58 MeV is shown in band D of Fig. 7. This and the 12.19 MeV, 6^+ state found by the Chalk River group are listed in Table II.

Two levels have been added to band E, a 4^+ level at 9.99 MeV and a (6^+) level at 13.94 MeV.

All of the members of band F are wide (800 keV) and the resonant energies of each are known only to within a few hundred keV. By comparison, the width of the 3^- level at 9.16 MeV is narrow, and this suggests a rather different configuration. For this reason, the line connecting the members of band F is not drawn through this point.

Series labeled G, H, and I in Table II and on Fig. 7 are possible new bands. A state in Ne^{20} at approximately 10.93 MeV is tentatively indicated, but its spin and

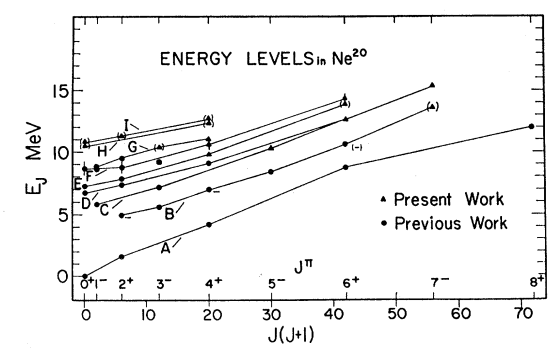

FIG. 7. Rotational bands in Ne^{20}. Excited-state energies are plotted as a function of $J(J+1)$. Except for a few previously known states which are members of new bands, the dots represent band members proposed by the Chalk River group. Unnatural parity assignments are plotted with a minus or plus sign beside the point.

[24] R. H. Davis and C. Mayer-Böricke, *Comptes Rendus du Congres International de Physique Nucléaire* (Centre National de la Recherche Scientifique, Paris, 1964), Vol. II, p. 313.

parity have not been determined. A 2^+ assignment would suggest that this state should be a member of band H. No attempt has been made to fit the several 6^+ levels found[4] between 17 and 20 MeV into a rotational-band scheme.

B. Comparison with Other Results

Previously available information about levels in Ne^{20} in the excitation-energy region of this experiment has been obtained by the study of the $O^{16}(\alpha,\gamma)Ne^{20}$, $C^{12}(C^{12},\alpha)Ne^{20}$, and $F^{19}(d,n)Ne^{20}$ reactions. This information is compared with the results of the $O^{16}(\alpha,\alpha)O^{16}$ experiment in Table III.

A number of the energy levels are identified by excitation energy alone, and a comparison is less certain than is the case for more completely specified levels. A 5^- level at 10.30-MeV excitation observed in this work is shown on a separate line from that for the level found at about the same energy in several of the $F^{19}(d,n)Ne^{20}$ experiments.[25–29] The existence of two different levels is assumed since neither of the assignments[26,27] from the $F^{19}(d,n)Ne^{20}$ agree with that from the $O^{16}(\alpha,\alpha)O^{16}$ experiment, and the latest energy determination places the excitation energy at 10.33 MeV.[26] Two levels that have been observed in different reactions and that have different spin and parity assignments (one tentative) are shown at 11.99 MeV. Two different levels with a small energy separation may be present.

The α-particle capture data[11] bears the closest resemblance to the elastic-scattering data, and both reactions proceed via the same entrance channel. Different channels are obviously involved in the $C^{12}(C^{12},\alpha)Ne^{20}$ and $F^{19}(d,n)Ne^{20}$ reactions. Important spin and parity assignments to levels observed in the $C^{12}(C^{12},\alpha)Ne^{20}$ reaction have been made in the elegant studies at Chalk River.[5–20] These have been discussed in the section on rotational bands (see Sec. V A). A simple correspondence between these results and those of the $O^{16}+\alpha$ experiments does not appear in the comparison (Table III), but a number of the levels found in the $O^{16}(\alpha,\alpha)O^{16}$ work can be arranged in rotational bands. The differences between the band members proposed in this and the $C^{12}(C^{12},\alpha)Ne^{20}$ work may be due to differences in coupling of the reaction channels with compound system states.[23]

Many of the previously reported levels between 9.50- and 12.58-MeV excitation in Ne^{20} were found in investigations of the $F^{19}(d,n)Ne^{20}$ and $F^{19}(d,n\gamma)Ne^{20}$ reactions. It is probable that many of these states have unnatural parity and cannot be observed in the $O^{16}(\alpha,\alpha)O^{16}$ elastic-scattering experiment. An identification of some of these states with those found in this work is shown in Table III.

No levels have been reported in the literature which correspond to the Ne^{20} levels observed at 10.30, 10.49, 10.7, 10.83, 10.93, and 12.58 MeV. While the comparison with other work is satisfactory where a one-to-one correspondence is expected, it demonstrates the need for a better understanding of the dependence of resonance mechanisms on the type of reaction.

VI. SUMMARY

Sixteen resonances have been observed in the range of α-particle bombarding energy 5.8 to 10.0 MeV that correspond to levels in Ne^{20} in the excitation-energy region 9.37 to 12.73 MeV. Six have been given spin and parity assignments, seven tentative assignments have been made, and several possible assignments are suggested for two resonances.

These results are in satisfactory agreement with the previously published results of an $O^{16}(\alpha,\gamma)Ne^{20}$ experiment,[11] but are significantly different from those obtained by the Chalk River group in the analysis of $C^{12}(C^{12},\alpha)Ne^{20}$ data. The distinction is most clearly drawn in the discussion of the rotational-band interpretation of levels in Ne^{20}. Energy levels identified in this work (and in Paper II[4]) have been arranged to extend the bands and suggest new bands. In several instances, these levels differ from the band members proposed in the $C^{12}(C^{12},\alpha)Ne^{20}$ work. This apparent difference in the band members determined in the two types of experiments may arise from a difference in the intermediate steps of the reactions.[23]

The excitation energies of many of the levels found in the studies of $F^{19}(d,n)Ne^{20}$ reaction agree with those found in this work within experimental error. More information about these levels is necessary before a firm correspondence can be established.

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