PHYSICAL REVIEW VOLUME 160, NUMBER 4 20 AUGUST 1967

Finite-Range Effects in the Distorted-Wave Impulse Approximation*

R. M. HAYBRON
Oak Ridge National Laboratory, Oak Ridge, Tennessee
(Received 6 March 1967)

The usual (zero-range) distorted-wave, impulse approximation applied to the inelastic scattering of high-energy protons by nuclei ignores the averaging of the two-nucleon t matrix over the range of momentum transfers introduced by the distorted waves. The effects of including this averaging (the finite-range calculation) are investigated here at 50, 100, and 150 MeV for quadrupole transitions in ^{12}C and ^{40}Ca using a much simplified form of the two-nucleon t matrix. It is found that the finite-range effects are not negligible even at the highest energy; in particular, the 150-MeV cross section for ^{12}C is reduced by a factor of two in the forward direction, while the peak cross section for ^{40}Ca is reduced by 20% at this energy. An attempt is also made to fit the data on the 2^+ level of ^{12}C at 45.5 MeV using the finite-range calculation, and it is found that the strength of the transition predicted by the impulse approximation (using the simplified t matrix) is too small by a factor of more than three. Exchange effects were ignored in the finite-range calculations.

I. INTRODUCTION

THE distorted-wave impulse approximation (DWIA)[1] applied to the inelastic scattering of high-energy protons from nuclei has proved to be a powerful tool in the determination of the structure of the nuclear levels involved in the transition.[2,3] At energies of a few hundred MeV, proton wavelengths are small enough to give a rather sharp picture of the overlap of the final and initial nuclear states, and this knowledge can be of considerable value in testing predictions of structure calculations. Although the calculations to be done in the DWIA are lengthy, they can be performed quite easily with the use of high-speed computers and available computer codes.[4]

The significant feature of the DWIA is the fact that the force assumed to be producing the inelastic scattering is just the *free*, two-nucleon interaction *acting once*. One then obtains a transition matrix element describing the process which is expressed in terms of initial and final distorted waves (which are determined from the elastic scattering), the free, two-nucleon transition matrix (which is determined from the study of nucleon-nucleon scattering), and the nuclear transition density (essentially the overlap of the initial and final nuclear states), the latter being the quantity to be determined. In effect then, the only unknown quantity is that which is to be determined, namely, the transition density.

This brief description indicates that, according to this picture, one should be able to somehow invert the high-energy proton scattering data and obtain in an unambiguous way the nuclear transition density. This is not the case, however. In the first place, these data are as yet relatively difficult to collect and are not extensive or very precise. Also, the computational difficulties involved in "solving for" the transition density appear to be virtually insurmountable. In the third place, and most important, the use of the DWIA involves a series of approximations which are difficult to test, so that the calculations which have been done thus far must be regarded as testing the DWIA as well as the transition densities used. It is the purpose of this note to look at one of the simplest corrections to the DWIA as it is currently applied.

In the form in which the DWIA is almost always used, one evaluates the two-nucleon, t matrix (which is a function of bombarding energy and momentum transfer) at the incident energy of the projectile, E_0, and at the net momentum transfer to the nucleus $\mathbf{q}=\mathbf{k}_0-\mathbf{k}_f$, where $\mathbf{k}_0$ is the initial projectile momentum (in units of $\hbar$) and $\mathbf{k}_f$ is the final projectile momentum. If the motion of the projectile were adequately described by plane waves, this would be exactly correct. However, the use of distorted waves, which is necessary to account for the influence of the nucleus upon the propagation of the projectile, introduces a spread of initial and final momenta. Since the inelastic scattering takes place within the nucleus, the projectile momenta which pertain just before and just after the inelastic collision are not $\mathbf{k}_0$ and $\mathbf{k}_f$, but instead a *range* of initial momenta $\mathbf{k}_a'$ and a *range* of final momenta $\mathbf{k}_b'$, the distribution of components depending on the location in the nuclear volume. Then in order to properly apply the DWIA, one should somehow average the two-nucleon, t matrix over the spread of bombarding energy and momentum transfer produced by distortion effects. In fact, the

* Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.
[1] A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N.Y.) **8**, 551 (1959).
[2] For instance, see C. A. Levinson, and M. K. Banerjee, Ann. Phys **3**, 67 (1958); G. P. McCauley and G. E. Brown, Proc. Phys. Soc. (London) **71**, 893 (1958); E. J. Squires, Nucl. Phys. **6**, 504 (1958); D. J. Hooton and G. R. Allcock, Proc. Phys. Soc. (London) **73**, 881 (1959); A. B. Clegg and G. R. Satchler, Nucl. Phys. **27**, 431 (1961); E. A. Sanderson, *ibid.* **26**, 420 (1916); *ibid.* **35**, 557 (1962); D. F. Jackson, *ibid.* **35**, 194 (1962); L. R. B. Elton and D. F. Jackson, *ibid.* **35**, 209 (1962); D. F. Hooton and N. W. Ashcroft, Proc. Phys. Soc. (London) **81**, 193 (1963); D. J. Rowe, G. L. Salmon, A. B. Clegg, and D. Newton, Nucl. Phys. **54**, 193 (1964); T. Erikson, *ibid.* **55**, 497 (1964), and references cited therein.
[3] R. M. Haybron and H. McManus, Phys. Rev. **140**, B638 (1965).
[4] R. M. Haybron and H. McManus, Phys. Rev. **136**, B1370 (1964).

energy dependence of the t matrix is not too strong so that it is probably adequate to use the matrix elements corresponding to the initial projectile energy. On the other hand, the dependence of the t matrix upon momentum transfer is strong, so that ignoring the "average" over $\mathbf{q}'$ $(=\mathbf{k}_a'-\mathbf{k}_b')$ could lead to serious errors in the cross section, particularly at forward angles where refraction effects produce momentum differences large compared to the asymptotic momentum transfer. (Quantities such as momentum or momentum transfer which are characteristic of the nucleon-nucleus system at large separation will be referred to as asymptotic; quantities referring to the system in interaction will be called local.)

The effects of the momentum transfer averaging will be explored here to determine the importance of refraction in computing the inelastic cross section. Since the real two-nucleon, t matrix is a rather complicated quantity, the investigation will employ a much simplified form, namely, a Yukawa in the momentum transfer, with a range chosen so as to reproduce the q dependence of the central part of the actual t matrix as closely as possible. Calculations will be performed for quadrupole transitions in ^{12}C and ^{40}Ca as characteristic of the type of transition and range of atomic weight for which the DWIA has been applied. These calculations will be done at 50, 100, and 150 MeV. The two higher energies are in the range where the DWIA is expected to apply; the 50 MeV calculations are included for reasons to be discussed below.

It would be of great benefit if one could use the DWIA at much lower-bombarding energies, for instance at 50 MeV. In this energy region many more nuclear levels can be resolved than at the higher energies and because of the larger cross sections, data are much more detailed and precise. However, it has been assumed that the impulse approximation would not be good at such low energies. If one restricts his attention to single scattering, the two-body operator which produces the inelastic scattering can be written as $\tau=v+v(1/E-H_0-H_B)\tau$, where v is the two-nucleon potential, H_B is the Hamiltonian for the bound (target) nucleon, and H_0 is the Hamiltonian for the projectile. The impulse approximation consists in replacing H_B by the kinetic energy of the target particle, in which case this is just the equation for free, two-nucleon scattering. This step is expected to be good at large projectile energies where the binding potential term in H_B will contribute a negligible amount to the propagator. On the other hand, at low-projectile energies it is expected that one must use the propagator as it is, that is, the scattering cannot be properly described in terms of the free, two-nucleon operator. However, this statement depends on the region of the nucleus where the inelastic event takes place. If, due to absorption into other reaction channels, the inelastic scattering is restricted to occur at the nuclear surface, then binding might still not be too important and the impulse approximation might hold where at first glance it would be expected to fail.

Attempts are currently being made to describe moderate energy ($\sim$50 MeV) inelastic proton-scattering data using simple, effective two-nucleon interactions.[5,6] If such an interaction can be found it would be of great utility in the interpretation of the many nuclear levels which can be excited at these energies. Although we cannot hope too seriously to apply the impulse approximation at these low energies, it may be that at some intermediate energy we can find an overlap which will help to delineate the effective interaction mentioned above.

For the reasons discussed above, calculations at 50 MeV have been included here. Clearly at these energies where the incident energy of the projectile is comparable to the depth of the optical potential, one would expect large refraction effects and hence the momentum transfer averaging will be important. In addition, calculations using the approximate t matrix (which represents the central part of the real t matrix fairly accurately) for the excitation of the 4.43-MeV level of ^{12}C have been included and compared to experiment in order to ascertain by what amount the DWIA fails in predicting the strength of this transition. This procedure ignores the spin-dependent part of the interaction. However, the spin dependence of the effective interaction proposed at low energies (Ref. 5) seems quite weak, so that this is probably not a serious omission for these preliminary calculations.

A few remarks are in order regarding nomenclature in the following. In the usual form of the DWIA, the interaction takes the form $t(E_0,\mathbf{q})\times\delta(\mathbf{r}-\mathbf{r}_j)$ where $\mathbf{r}$ is the projectile coordinate and $\mathbf{r}_j$ the coordinate of the jth target nucleon. This form has come to be called the *zero-range* form. In fact it would correspond to a true zero-range interaction only if $t(E_0,\mathbf{q})=1$, but even so we shall use the simple, if inaccurate, title just mentioned. In the calculations where refraction is taken into account by averaging over momentum transfers, it will be seen that in effect the δ function in the form above is replaced by a function of finite range; hence, this type of calculation will be called *finite-range*.

We have neglected exchange scattering produced by antisymmetrizing the projectile-target wave function in the finite-range calculations. Since these exchange terms involve matrix elements where the coordinates of the final nucleon differ from those of the initial nucleon, they are quite complicated to evaluate and since we are mainly interested here in the relation of distortion effects to the form of the interaction we have elected to ignore them. This point is discussed briefly in Sec. IV.

[5] W. S. Gray, R. A. Kenefick, J. J. Kruashaar, and G. R. Satchler, Phys. Rev. **142**, 735 (1966); M. B. Johnson, L. W. Owen, and G. R. Satchler, *ibid.* **142**, 748 (1966).

[6] N. K. Glendenning and M. Veneroni, Phys. Rev. **144**, 839 (1965).

II. THEORY

A. The DWIA Matrix Element

The transition amplitude for the inelastic scattering of a proton from an initial momentum state (in the proton-nucleus center-of-mass system) $\mathbf{k}_a$ to a final state $\mathbf{k}_b$ is given in the DWIA by

$$T_{f0}=\int \chi^{(-)*}(\mathbf{k}_b,\mathbf{r}')\{\langle\psi_f(\varrho')|\sum_{j=1} t(\mathbf{r}',\varrho_j':\mathbf{r},\varrho_j)|\psi_0(\varrho)\rangle\} \times\chi^{(+)}(\mathbf{k}_a,\mathbf{r})d\mathbf{r}d\mathbf{r}', \quad (1)$$

where $\psi_0(\varrho)\equiv\psi_0(\varrho_1,\varrho_2,\cdots\varrho_j\cdots\varrho_A)$ is the initial nuclear wave function and ψ_f is the final nuclear wave function. The functions $\chi^{(+)}$ and $\chi^{(-)}$ are distorted waves in the initial and final channels (we are ignoring spin-orbit coupling for simplicity so that $\chi^{(+)}$ and $\chi^{(-)}$ do not have spin indices). These functions can also be assumed to contain projectile spin functions, although we shall be mostly interested in the case where t is a spin scalar. The quantity in curly brackets which is obtained from the average of the t operator over the nuclear wave functions will be called, for convenience, the "effective potential" $\mathcal{V}(\mathbf{r}',\mathbf{r})$, and it is this quantity which is of central interest. In writing (1) we have made the multiple scattering and impulse approximations so that "t" is the free, two-nucleon transition operator. It should perhaps be emphasized that t contains the action of the two-nucleon potential v to all orders in a given nucleon-nucleon collision.

At this point we should emphasize that the ranges of validity of both the multiple-scattering approximation and the impulse approximation are of primary importance to what we shall do although they are not investigated here.

B. The Quantity $\mathcal{V}(r',r)$

We now wish to calculate the effective interaction in Eq. (1). Since the two-body t matrix is measured for nucleons in states of sharp linear momentum, it is necessary to write $\mathcal{V}(\bar{r}',\bar{r})$ in terms of the momentum space components of the quantities involved. Then one has

$$\mathcal{V}(\bar{r}',\bar{r})=\frac{1}{(2\pi)^6}\int\psi_f^*(\mathbf{l}_4)e^{i\mathbf{l}_3\cdot\mathbf{r}'}\langle\mathbf{l}_4,\mathbf{l}_3|t|\mathbf{l}_2,\mathbf{l}_1\rangle \times e^{-i\mathbf{l}_1\cdot\mathbf{r}}\psi_0(\mathbf{l}_2)d\mathbf{l}_1d\mathbf{l}_2d\mathbf{l}_3d\mathbf{l}_4. \quad (2)$$

In Eq. (2) we have written just one of the A terms in Eq. (1), for instance the one for which $j=1$. Then

$$\psi_0(\mathbf{l}_2)=\frac{1}{(2\pi)^{3/2}}\int e^{-i\mathbf{l}_2\cdot\rho_1}\psi_0(\varrho_1,\varrho_2\cdots\varrho_A)d\varrho_1,$$

etc. Now if t were a known operator in momentum space it would be a straightforward, though probably complicated task, to evaluate (2). However, we do not know this operator, but only certain matrix elements of it from free, two-nucleon scattering, so we must restrict the momentum integrals occurring in (2), or alternatively ignore the unknown parts of the t matrix in the performance of these integrals.

We assume that the t-matrix elements at most depend on three quantities:

$$|\mathbf{l}_2-\mathbf{l}_1|^2\propto \text{ initial energy of relative motion},$$
$$|\mathbf{l}_4-\mathbf{l}_3|^2\propto \text{ final energy of relative motion},$$

and the momentum transfer defined to be

$$\mathbf{q}'=\mathbf{l}_1-\mathbf{l}_3.$$

With conservation of linear momentum, the t-matrix element in (2) can then be written

$$\langle\mathbf{l}_4,\mathbf{l}_3|t|\mathbf{l}_2,\mathbf{l}_1\rangle=\delta(\mathbf{l}_1+\mathbf{l}_2-\mathbf{l}_3-\mathbf{l}_4) \times t(|\mathbf{l}_4-\mathbf{l}_3|^2,\ |\mathbf{l}_2-\mathbf{l}_1|^2,\ \mathbf{q}'). \quad (3)$$

In the free, two-nucleon scattering (ignoring p-$p'\gamma$ experiments) we cannot determine the t matrix for the range of variables in (3) required by the integrals in (2). For free scattering, one measures the subset of matrix elements

$$\langle|t|\rangle=\delta(\mathbf{l}_1+\mathbf{l}_2-\mathbf{l}_3-\mathbf{l}_4)t(|\mathbf{l}_2-\mathbf{l}_1|^2,\ |\mathbf{l}_2-\mathbf{l}_1|^2,\ \mathbf{q}'),$$

with

$$0\leq\mathbf{q}'\leq2|\mathbf{l}_2-\mathbf{l}_1|,$$

that is, one obtains the elastic-scattering matrix elements in a restricted range of momentum transfer. The two-nucleon matrix elements required to evaluate the nucleon-nucleon interaction, however, corresponds to inelastic scattering, and, of course, are not determined by the free, two-body scattering. In order to circumvent this difficulty, it is conventional to ignore the dependence of the t matrix on the final state energy. Although difficult to justify without appealing to a model for the two-nucleon interaction, this approximation is intuitively plausible for high-bombarding energies and low-energy excitations and we shall use it throughout.

The t-matrix element in (2) can now be written in the simplified form

$$\langle\mathbf{l}_4,\mathbf{l}_3|t|\mathbf{l}_2,\mathbf{l}_1\rangle=\delta(\mathbf{l}_1+\mathbf{l}_2-\mathbf{l}_3-\mathbf{l}_4)t(|\mathbf{l}_2-\mathbf{l}_1|^2,\ \mathbf{q}'). \quad (4)$$

If we now substitute (4) into (2), change variables and perform the integral over the δ function, the expression for the effective interaction becomes

$$\mathcal{V}(\mathbf{r}',\mathbf{r})=\frac{1}{(2\pi)^3}\int\psi_f^*(\mathbf{l}_2+\mathbf{q}')t(|\mathbf{l}_2-\mathbf{l}_1|^2,\ \mathbf{q}')\psi_0(\mathbf{l}_2) \times e^{i\mathbf{l}_1\cdot(\mathbf{r}-\mathbf{r}')}e^{-i\mathbf{q}'\cdot\mathbf{r}'}d\mathbf{l}_1d\mathbf{l}_2d\mathbf{q}'. \quad (5)$$

C. Plane-Wave Limit

The effective interaction as expressed in Eq. (5) is not in a tractable form for calculations. In particular it requires a knowledge of the two-body, t matrix over a continuous range of energies and this information is not available without interpolation between the energies at which experiments have been performed.

The expression in (5) has been obtained by assuming the limit of high-energy scattering. In this limit the distorted waves approach the plane waves which greatly simplifies the effective form of (5). Substituting Eq. (5) for the curly brackets in (1) and replacing the distorted waves by plane waves, one obtains

$$T_{f0}=\frac{1}{(2\pi)^3}\int e^{-i\mathbf{k}_b\cdot\mathbf{r}'}\left\{\frac{1}{(2\pi)^3}\times\int\psi_f^*(\mathbf{l}_2+\mathbf{q}')t(|\mathbf{l}_2-\mathbf{l}_1)^2,\mathbf{q}')\psi_0(\mathbf{l}_2)e^{i\mathbf{l}_1\cdot(\mathbf{r}-\mathbf{r}')}\times e^{-i\mathbf{q}'\cdot\mathbf{r}'}d\mathbf{l}_1d\mathbf{l}_2d\mathbf{q}'\right\}e^{i\mathbf{k}_a\cdot\mathbf{r}}d\mathbf{r}d\mathbf{r}',\quad(6)$$

where we have ignored the projectile spin. If the $\mathbf{r}$ integral in (6) is performed, one obtains a δ function, $\delta(\mathbf{l}_1+\mathbf{k}_a)$. Then the integral over $\mathbf{l}_1$ yields the expression

$$T_{f0}=\frac{1}{(2\pi)^3}\int e^{-i\mathbf{k}_b\cdot\mathbf{r}'}\times\left\{\int\psi_f^*(\mathbf{l}_2+\mathbf{q}')t(|\mathbf{l}_2+\mathbf{k}_a|^2,\mathbf{q}')\psi_0(\mathbf{l}_2)\times e^{i\mathbf{k}_a\cdot\mathbf{r}'}e^{-i\mathbf{q}'\cdot\mathbf{r}'}d\mathbf{l}_2d\mathbf{q}'\right\}d\mathbf{r}'.\quad(7)$$

Equation (7) has the appearance of the result one would obtain with a local-effective interaction

$$\mathcal{V}(\mathbf{r}',\mathbf{r}')=\int\psi_f^*(\mathbf{l}_2+\mathbf{q}')t(|\mathbf{l}_2+\mathbf{k}_a|^2,\mathbf{q}')\psi_0(\mathbf{l}_2)\times e^{-i\mathbf{q}'\cdot\mathbf{r}'}d\mathbf{l}_2d\mathbf{q}',\quad(8)$$

so that replacing the initial distorted wave by a plane wave removes the nonlocality. Now if the $\mathbf{r}'$ integral in (7) is performed, it yields the delta function $\delta(\mathbf{k}_a-\mathbf{k}_b-\mathbf{q}')$. If we define the asymptotic momentum transfer $\mathbf{q}$ by

$$\mathbf{q}=\mathbf{k}_a-\mathbf{k}_b,\quad(9)$$

the integral over $\mathbf{q}'$ in (7) then yields

$$T_{f0}=\int\psi_f^*(\mathbf{l}_2+\mathbf{q})t(|\mathbf{l}_2+\mathbf{k}_a|^2,\mathbf{q})\psi_0(\mathbf{l}_2)d\mathbf{l}_2.\quad(10)$$

For large $\mathbf{k}_a$ and for a t matrix which is slowly varying with energy, one might reasonably expect that the variation of $t(|\mathbf{l}_2+\mathbf{k}_a|^2,\mathbf{q})$ over the important range of $\mathbf{l}_2$ as determined by the nuclear wave functions can be ignored, in which case Eq. (10) reduces to

$$T_{f0}=t(\mathbf{k}_a^2,\mathbf{q})\int\psi_f^*(\mathbf{l}_2+\mathbf{q})\psi_0(\mathbf{l}_2)d\mathbf{l}_2,\quad(11)$$

which can immediately be written as

$$T_{f0}=t(\mathbf{k}_a^2,\mathbf{q})\int e^{i\mathbf{q}\cdot\mathbf{r}}\psi_f^*(\mathbf{r})\psi_0(\mathbf{r})d\mathbf{r}.\quad(12)$$

It can be noted in (12) that the integral is related to the Born approximation expression for the form factor determined in inelastic electron scattering.[7] The important thing to note from the standpoint of performing DWIA calculations is that in (12) we have the transition matrix element expressed in terms of the two-nucleon, t matrix evaluated at the incident projectile's energy and the asymptotic momentum transfer.

D. Simplified Effective Interaction

The simple expression in Eq. (12) was obtained in the limit where the distorted waves approached plane waves. We therefore expect that, to lowest order, the DWIA matrix element will have a similar form as regards the role of the two-nucleon, t matrix.

The results of the plane-wave calculation suggests that we ignore the dependence of the t matrix in (5) on the integral over $\mathbf{l}_1$ and evaluate it at $\mathbf{l}_1=\mathbf{k}_2$. The criterion for the validity of this approximation is related to the requirement that if the incident distorted wave contains a spread of momenta $\Delta\mathbf{k}_a$ around $\mathbf{k}_a$, the two-body, t matrix must be essentially constant over the corresponding spread in energy. Clearly this approximation also depends on the range of $\mathbf{l}_2$ which is important in the integral and if it is to hold, we should also have $\mathbf{l}_2\ll\mathbf{k}_a$ for the important values of $\mathbf{l}_2$, in which case we can ignore the average of the t matrix over $\mathbf{l}_2$. However, if we assume that the controlling factor is a sharp peaking of the incident distorted wave in momentum space at $\mathbf{l}_1\approx\mathbf{k}_a$, the effective interaction in (5) becomes

$$\mathcal{V}(\mathbf{r}',\mathbf{r})=\delta(\mathbf{r}-\mathbf{r}')\int\psi_f^*(\mathbf{l}_2+\mathbf{q}')t(|\mathbf{l}_2-\mathbf{k}_a|^2,\mathbf{q}')\times\psi_0(\mathbf{l}_2)e^{-i\mathbf{q}'\cdot\mathbf{r}'}d\mathbf{l}_2d\mathbf{q}'.\quad(13)$$

If we make the additional approximation that $\mathbf{l}_2\ll\mathbf{k}_a$ in (13) the interaction reduces to

$$\mathcal{V}(\mathbf{r}',\mathbf{r})=\delta(\mathbf{r}-\mathbf{r}')\int\psi_f^*(\mathbf{l}_2+\mathbf{q}')t(\mathbf{k}_a^2,\mathbf{q}')\psi_0(\mathbf{l}_2)\times e^{-i\mathbf{q}'\cdot\mathbf{r}'}d\mathbf{l}_2d\mathbf{q}',\quad(14)$$

which can be transformed as in going from (11) to (12)

[7] See, for example, V. Gillet and M. A. Melkanoff, Phys. Rev. 133, B1190 (1964).

to yield

$$\mathcal{V}(\mathbf{r}',\mathbf{r})=\delta(\mathbf{r}-\mathbf{r}')\int\left\{\int e^{i\mathbf{q}'\cdot\boldsymbol{\rho}}\psi_f^*(\boldsymbol{\rho})\psi_0(\boldsymbol{\rho})d\boldsymbol{\rho}\right\}\times t(\mathbf{k}_a^2,\mathbf{q}')e^{-i\mathbf{q}'\cdot\mathbf{r}'}d\mathbf{q}'. \quad (15)$$

Finally, we remember that in the plane-wave calculation we obtained the result that the local momentum transfer $\mathbf{q}'$ was equal to $\mathbf{q}$, the asymptotic momentum transfer. If we follow this and set $\mathbf{q}'=\mathbf{q}$ in the t matrix in (15), the effective interaction reduces to

$$\mathcal{V}(\mathbf{r}',\mathbf{r})=\delta(\mathbf{r}-\mathbf{r}')(2\pi)^3t(\mathbf{k}_a^2,\mathbf{q})\times\int\psi_f^*(\boldsymbol{\rho})\delta(\boldsymbol{\rho}-\mathbf{r})\psi_0(\boldsymbol{\rho})d\boldsymbol{\rho}. \quad (16)$$

In (16) we have not performed the integral over the δ function since this is the form in which the DWIA is usually written.

As the previous statement implied, most DWIA calculations to date have been done using the expression in (16), wherein one ignores the average over local bombarding energy and local momentum transfer implied by the expression in Eq. (5).[8] Since the energy dependence of the two-nucleon, t matrix is fairly weak, we shall continue to ignore it. However the dependence of t on momentum transfer is not weak and we propose to investigate the importance of the average over local momentum transfer by using Eq. (15) and comparing the results to those obtained from (16). In order to conveniently distinguish between (15) and (16), we shall call the latter expression the "zero-range approximation" due to the presence of the δ function in the target-projectile coordinates and the former expression the "finite-range approximation." Strictly speaking, the expression in (16) corresponds to a zero-range interaction only if $t(\mathbf{k}_a^2,\mathbf{q})$ is independent of $\mathbf{q}$, but we shall ignore this in the interests of simplicity.

One would expect especially important differences between the zero-range and finite-range interactions at small scattering angles where the asymptotic momentum transfer is small. In this region of scattering angle, refraction effects in the distorted waves introduce finite local momentum transfers even when the local momentum transfer approaches zero. It is for just this reason that the inelastic cross section for 0^+ to 0^+ transitions such as the excitation of the 7.7-MeV level of ^{12}C shows a strong forward peak whereas in the plane-wave approximation this cross section would be zero. That is, the plane-wave cross section is proportional to the square of $\langle\psi_{\text{excited}}|j_0(qr)|\psi_{\text{ground}}\rangle$. For $q\rightarrow 0$, this matrix element goes to zero due to the orthogonality of the wave functions. The fact that the distorted wave result peaks in the forward direction even at 150 MeV indicates that finite momentum transfers have been introduced by refraction effects. Since the two-nucleon, t matrix is, in general, a rather sharply decreasing function of $\mathbf{q}$ for small $\mathbf{q}$, we should expect that finite-range corrections could substantially reduce the cross section at forward angles, at just this effect has been found by Kawai and Terasawa (Ref. 7) for the excitation of the 15.1 MeV spin-flip level of ^{12}C by 156-MeV protons.

We should not expect large corrections due to finite-range effects for high-energy scattering (except possibly for forward angles as just discussed) since we anticipate that the approximations which have been made to obtain the zero-range form for the effective interaction are well satisfied for bombarding energies above 100 MeV. As the bombarding energy is lowered, we would expect that, in general, the finite-range effects will increase in importance so that the zero-range interaction no longer gives an adequate representation of the process, and this is one of the things we are looking for. Of course, at lower energies with relatively larger refraction effects, it is a distinct possibility that one might be forced to use the more general form in Eq. (5) where the average over the local bombarding energy is included producing a nonlocal effective interaction. However, the two-nucleon, t matrix seems to vary rather slowly with energy down to perhaps 40 MeV so that the nonlocality of the interaction may well be ignorable to the first approximation. More seriously, it may be expected that at some lower energy the impulse approximation as well as the single-scattering approximation will begin to fail. If this is the case, then we hope to detect such a failure as a systematic and increasing discrepancy between the results obtained using the finite-range DWIA and experiment at progressively lower-bombarding energies. That the simple picture presented here to describe the inelastic scattering will fail at sufficiently low energies is not in doubt. The complexity of the problem, however, makes it extremely difficult to estimate just where it will fail, and it seems that direct comparison with experiment may be the only means to answer the question. It is unfortunate that there is virtually no proton data as yet in the energy range between 60 and 150 MeV to compare to, but it is expected that this situation will be materially altered in the near future.

E. Nonphysical t-Matrix Elements

The effective interaction for the finite-range DWIA is from Eq. (15)

$$\mathcal{V}(\mathbf{r})=\int e^{-i\mathbf{q}'\cdot\mathbf{r}}t(\mathbf{k}_a^2,\mathbf{q}')\left\{\int e^{i\mathbf{q}'\cdot\boldsymbol{\rho}}\psi_f^*(\boldsymbol{\rho})\psi_0(\boldsymbol{\rho})d\mathbf{p}\right\}d\mathbf{q}', \quad (17)$$

where for simplicity the delta function in (15) has been dropped and has been written as a local interaction.

We would first of all like to investigate the range of

[8] See, however, M. Kawai, T. Terasawa, and K. Izumo, Nucl. Phys. 59, 289 (1964).

$\mathbf{q}'$ required in the integral in (17), since as was pointed out after Eq. (3) the range of $\mathbf{q}'$ permitted in the free, two-nucleon scattering experiment is $0 \leq \mathbf{q}' \leq 2\mathbf{k}_a$. Suppose we consider the scattering of a nucleon of lab momentum k_a from a free nucleon at rest. The center-of-mass momentum of the projectile is K_0, where

$$K_0 = k_a/2 \tag{18}$$

and the momentum transfer Q is given by

$$Q = 2K_0 \sin(\theta/2), \tag{19}$$

where θ is the center-of-mass scattering angle. Now if a projectile nucleon of the same laboratory momentum scatterers from a nucleus of mass M it has a nucleon-nucleus, center-of-mass momentum k_0 given by

$$k_0 = \frac{M}{M+1} k_a \tag{20}$$

(we are assuming that $M_p = M_n = 1$) and a momentum transfer q' given by

$$q' = 2k_0 \sin(\theta/2), \tag{21}$$

where θ is the scattering angle. We use in (16) or (17) elements of the t matrix corresponding to $q' = Q$ which from (19) and (21) gives

$$\sin\left(\frac{\theta}{2}\right) = \frac{k_0}{K_0} \sin(\theta/2) \tag{22}$$

which is

$$\sin\left(\frac{\theta}{2}\right) = \frac{2M}{M+1} \sin(\theta/2) \tag{23}$$

from (18) and (20). Equation (23) shows that the nucleon-nucleus scattering requires elements of t for nonphysical (off-the-energy shell) scattering angles. Since the maximum physical value of θ is 180° in the nucleon-nucleon system, the nucleon-nucleus scattering requires nonphysical momentum transfers for

$$\theta > 2 \sin^{-1}\left(\frac{M+1}{2M}\right). \tag{24}$$

For ^{12}C, this angle is $\simeq 65°$. It should be pointed out that this restriction applies both to the zero-range interaction in (16) and the finite-range interaction in (17), that is for q' above the kinematic limit the expression in (16) is not defined and the integrand in (17) is not defined.

A rather simple computation can illustrate the importance of the momentum cutoff. If the zero-range expression (16) is substituted into (1), the plane-wave limit gives

$$T_{f0} = t(\mathbf{k}_a^2, \mathbf{q}) \int e^{-i\mathbf{q}\cdot\mathbf{r}} \psi_f^*(\mathbf{r}) \psi_0(\mathbf{r}) d\mathbf{r}. \tag{25}$$

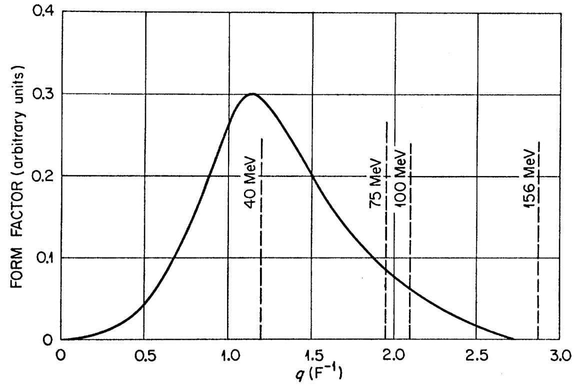

FIG. 1. The nuclear form factor corresponding to the excitation of the first 2^+ level of ^{12}C with this level represented as a $p_{1/2}$ particle, $p_{3/2}$ hole is compared to the physical cutoff of the two-nucleon, t matrix at various energies.

Now suppose we consider excitation of the first 2^+ level of ^{12}C where the ground state is a closed $p_{3/2}$ shell and the excited state is a $(p_{1/2})(p_{3/2})^{-1}$ particle-hole state. Then a simple computation yields

$$T_{f0} \propto t(k_a^2, q) q^2 e^{-q^2/8\alpha} \tag{26}$$

if the oscillator wave functions have the form

$$\mathfrak{U}_{1p} = N_{1p} r e^{-\alpha r^2}. \tag{27}$$

All the spectroscopic factors, etc. have been ignored in (26) as inessential to the point to be made. The cross section computed from (26) will be given by

$$\frac{d\sigma}{d\Omega} \alpha |t(k_a^2, q)|^2 q^4 e^{-q^2/4\alpha}. \tag{28}$$

The factor in (28) due to the nuclear part of the matrix element peaks at $q = \sqrt{(8\alpha)}$, independent of energy. This factor is shown in Fig. 1 compared to the physical cutoffs of t at several energies. Clearly at 156 MeV and at higher energies the cutoff will make little difference. In the zero-range calculation the cross sections become unobservably small for angles approaching the cutoff and in the finite-range calculation, although the integral in (17) presumably goes over all values of q, the peaking in the distorted waves around the asymptotic momentum transfer will, in fact, make large q values ineffective. At lower-bombarding energies, however, the cutoff momentum occurs at values of the cross section which are observable, and hence one must have a procedure to extend the t-matrix to nonphysical q values if this picture is to be applied. Alternately, one could think of inverting the problem and determining these "nonphysical" t-matrix elements from the low-energy experiments, but in view of the approximations underlying the finite-range DWIA, this does not seem to be a very likely procedure. The desired matrix elements could be generated using one of the existing two-nucleon potentials,[9] and this is presumably the way the calculation will ultimately be performed. At the moment, however, we are interested in the importance of refraction effects rather than in detailed fits to experiment so that we will use the simplest possible procedure to define the integrand in Eq. (17), namely guessing a not unreasonable form for the large-q matrix elements of t.

F. The t Matrix

The two-nucleon, t-matrix may be written in terms of the spin, isospin operators of the two nucleons as[10]

$$t=A+B(\boldsymbol{\sigma}\cdot\hat{n})(\boldsymbol{\sigma}_t\cdot\hat{n})+C(\boldsymbol{\sigma}+\boldsymbol{\sigma}_t)\cdot\hat{n} +E(\boldsymbol{\sigma}\cdot\hat{q})(\boldsymbol{\sigma}_t\cdot\hat{q})+F(\boldsymbol{\sigma}\cdot\hat{p})(\boldsymbol{\sigma}_t\cdot\hat{p})\,, \quad (29)$$

where the unit vectors $\hat{p}$, $\hat{q}$ and $\hat{n}$ form an orthogonal coordinate system defined by

$$\hat{q}=(\mathbf{k}_f-\mathbf{k}_0)/q\,,\quad \hat{n}=\mathbf{k}_0\times\mathbf{k}_f/|\mathbf{k}_0\times\mathbf{k}_f|\,,\quad \hat{p}=\hat{q}\times\hat{n}\,. \quad (30)$$

The operator $\boldsymbol{\sigma}$ refers to the projectile while $\boldsymbol{\sigma}_t$ refers to the target. $\mathbf{k}_0$ and $\mathbf{k}_f$ are the initial and final projectile momenta. The coefficients A, B, C, E, and F are in turn operators in the isospin of the nucleons. For instance

$$A=\tfrac{1}{4}(3A_1+A_0)+\tfrac{1}{4}(A_1-A_0)\boldsymbol{\tau}\cdot\boldsymbol{\tau}_t\,, \quad (31)$$

etc., where A_1 is the coefficient for the triplet isospin state and A_0 is the coefficient for the singlet state. A_0, A_1, B_0, B_1, etc., are all functions of bombarding energy k_0^2 and momentum transfer q. In (31) $\boldsymbol{\tau}$ is the projectile isospin, while $\boldsymbol{\tau}_t$ is the target isospin.

The form of the t matrix in Eq. (29) presents an immediate complication in the performance of the integral in Eq. (17). This is due to the fact that not only do the coefficients in the expansion of the t matrix depend upon the magnitude of the momentum transfer, but the unit vectors also depend upon the directions of the local momenta before and after the scattering as can be seen in the Eqs. (30). This is an essential complication. In order to know the directions of the local momenta before and after the scattering, one must decompose the distorted waves into their momentum space components and such a procedure is prohibitively difficult.

The situation can be mitigated to some extent by noting that for the low-lying levels of a variety of nuclei wherein transitions are predominantly collective in nature, the "spin-flip" terms in (29) turn out to be relatively unimportant, so that the part of the t matrix which is active in the transition is just (see Ref. 3)

$$t=A+C(\boldsymbol{\sigma}\cdot\hat{n})\,. \quad (32)$$

This simplifies the problem but does not totally alleviate it since even in (32) one must account for the "wobbling" of the scattering normal $\hat{n}$.

In view of the complications attendant to using either (29) or (32) in the finite-range calculations, we shall replace the "correct" form for the t matrix by a simple form depending only on the magnitude of the momentum transfer. This will vastly simplify the calculations, and will still allow us to ascertain the importance of refraction corrections.

III. USE OF AN APPROXIMATE t MATRIX

A. Yukawa t Matrix

The form of the two-nucleon, t matrix in Eq. (2) is the quantity we wish to insert in Eq. (17) to perform the finite-range DWIA calculations. However, the qualitative aspects of the contrast between the zero-range and finite-range forms for the effective interaction can be studied much more simply by employing a simple analytic form for the t matrix as a function of momentum transfer, postponing at this time the complications attending the use of the real, two-nucleon, t matrix.

The zero-range and finite-range results for a t matrix having the Yukawa form

$$t(q)=t_0(\beta^2+q^2)^{-1} \quad (33)$$

will be studied in this section with β chosen to agree as nearly as is possible with the small q dependence of the central part of the real t matrix. These studies will be performed at 50, 100, and 150 MeV in an attempt to determine the importance of finite-range effects as a function of energy, for transitions of multipolarity $l=2$ in ^{12}C and ^{40}Ca. Quadrupole excitations have been selected for this study because of their common appearance in inelastic scattering at all energies.

We still need the nuclear wave functions to evaluate (17), or more accurately we need

$$f(\mathbf{q}')=\int e^{i\mathbf{q}'\cdot\boldsymbol{\rho}}\psi_f^*(\boldsymbol{\rho})\psi_0(\boldsymbol{\rho})d\boldsymbol{\rho}\,. \quad (34)$$

$f(\mathbf{q}')$ is essentially the form factor measured in inelastic-electron scattering and a search code has been developed to obtain a best fit form factor for the high-energy electron data.[11] In this procedure, the form factor is parametrized in the form

$$f(q)=\sum_i C_i q^i e^{-\gamma/2\,q^2} \quad (35)$$

and a χ^2 search is performed on the data to obtain the coefficients C_i and γ in (35). Such a form factor is

[9] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).
[10] See Ref. 1.
[11] R. M. Haybron, M. B. Johnson, and R. J. Metzger, Phys. Rev. **156**, 1136 (1967).

TABLE I. The optical parameters used in the calculations performed here are displayed. All strengths are in MeV and lengths in fermis.

Nucl.	E_0	V	W	r_0	r_c	a	V_s	W_s	r_0'	a'	W_0
^{12}C	150	22.1	15.9	0.902	1.33	0.452	4.31	−1.07	1.186	0.556	0.0
^{40}Ca	150	22	16.7	1.012	1.32	0.548	4.26	−2.14	1.364	0.542	0.0
^{12}C	100	22.6	11.1	1.0	1.33	0.5	2.88	−1.53	1.34	0.5	0.0
^{40}Ca	100	29.5	12.2	1.086	1.32	0.659	6.28	−1.07	1.4	0.6	0.0
^{12}C	50	37.6	5.2	1.18	1.33	0.7	7.5	0.0	1.4	0.7	0.0
^{40}Ca	50	37.1	7.6	1.16	1.32	0.77	8.3	0.0	1.44	0.66	0.0

available for the 4.43-MeV, 2^+ level of ^{12}C and it was used here. The expansion coefficients are $C_2=0.23$, $C_4=0.0104$, $C_6=-0.0052$ and $\gamma=1.434$. There is no particular benefit accrued to using this relatively accurate form factor for the computations in this section, but it is, in practice, no more difficult to use than the simplest form one could choose. For the study on ^{40}Ca, the form in (35) was used with $C_2=1$ and $\gamma=2.28$. This, of course, does not correspond to any real level of ^{40}Ca, although γ was chosen to match the electron-scattering radius.

The distorted waves required to calculate the transition matrix element in Eq. (1) were generated in the usual way as solutions of the optical potential

$$U(r)=-V(1+e^{x})^{-1}-i\left(W-4W_0\frac{d}{dx'}\right)(1+e^{x'})^{-1}+(\hbar/M_\pi c)^2(V_s+iW_s)\sigma\cdot l(1/r)\frac{d}{dr}(1+e^{x})^{-1}+V_c(r)\,, \quad (36)$$

where

$$x=(r-R_0)/a\,,\quad R_0=r_0A^{1/3}\,,$$
$$x'=(r-R_0')/a'\,,\quad R_0'=r_0'A^{1/3}\,,$$

and $V_c(r)$ is the Coulomb potential produced by a uniform sphere with a charge of magnitude Z and radius r_c. The coefficients in (36) are conventionally chosen to give a best fit to the elastic-scattering data. The parameters used in this calculation are displayed in Table I.

In order to choose sensible parameters for the Yukawa interaction (35), this form squared was compared to $|A(q)|^2=[\mathrm{Re}A(q)]^2+[\mathrm{Im}A(q)]^2$ at 40, 90, and 156 MeV where $A(q)$ was expressed in terms of the Gammel-Thaler phase shifts, and t_0 and β were chosen to match the small q behavior of $|A(q)|^2$ as closely as possible.

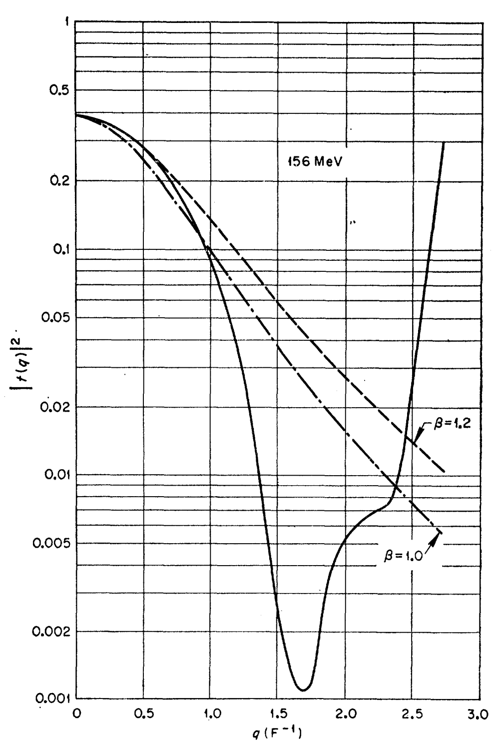

FIG. 2. The solid line is $|A(q)|^2$ at 156 MeV while the dashed lines show the results for a Yukawa t matrix with the indicated range parameters.

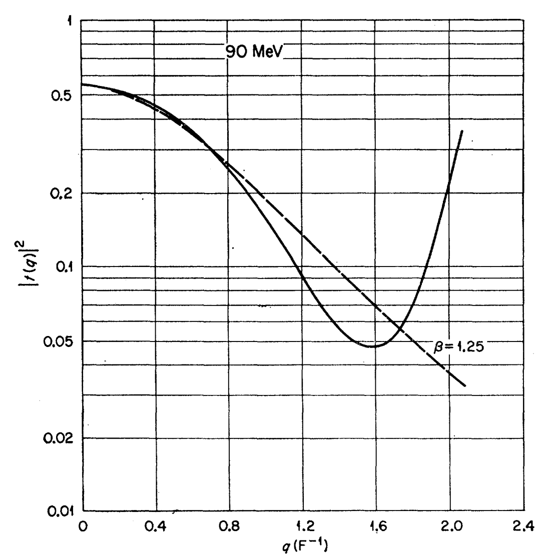

FIG. 3. The solid line is $|A(q)|^2$ at 90 MeV and the dashed line is the result for a Yukawa interaction with range parameter $\beta=1.25$ F^{-1}.

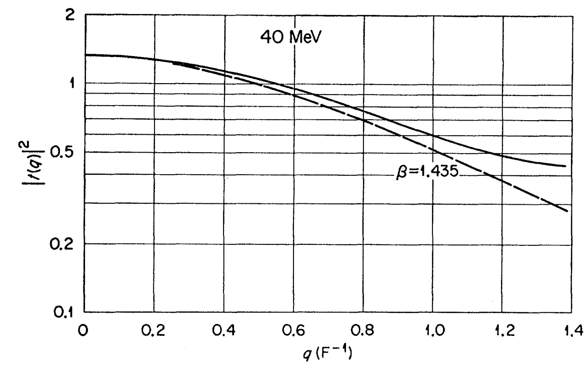

FIG. 4. This shows $|A(q)|^2$ at 40 MeV compared to a Yukawa with $\beta=1.435$ F^{-1}. $A(q)$ in each case was obtained from the Gammel-Thaler phase shifts as tabulated in Ref. 1.

The results of this comparison are displayed in Figs. 2–4 along with the values of t_0 and β used. It can be seen that the Yukawa form is quite good at 40 MeV, but not too representative at the higher energies as one would expect. However, inspection of Fig. 1 indicates that the peak inelastic cross section for the quadrupole level of ^{12}C is at $q\sim 1$ F^{-1} and Figs. 3 and 4 indicate that the simple Yukawa is representing the central part of the real interaction relatively well over the region of primary importance. One refinement which could have been included would have been to fit the real and imaginary parts of $A(q)$ separately with Yukawas and use the resulting complex form of (35) for these studies, but since we are primarily interested in refraction effects rather than choosing a very accurate representation of the t matrix, this was not done.

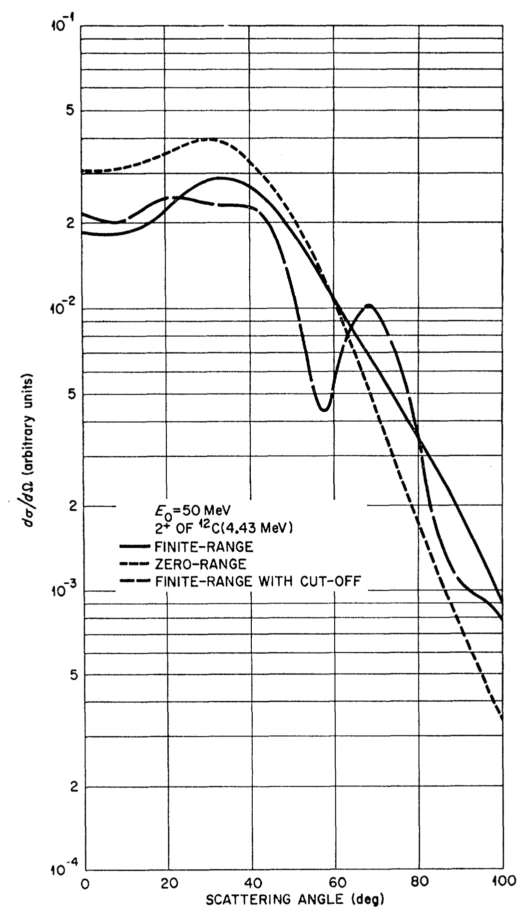

FIG. 5. The zero-range, finite-range and finite-range with cutoff results are shown for the 0^+ to 2^+ excitation of ^{12}C at 50 MeV.

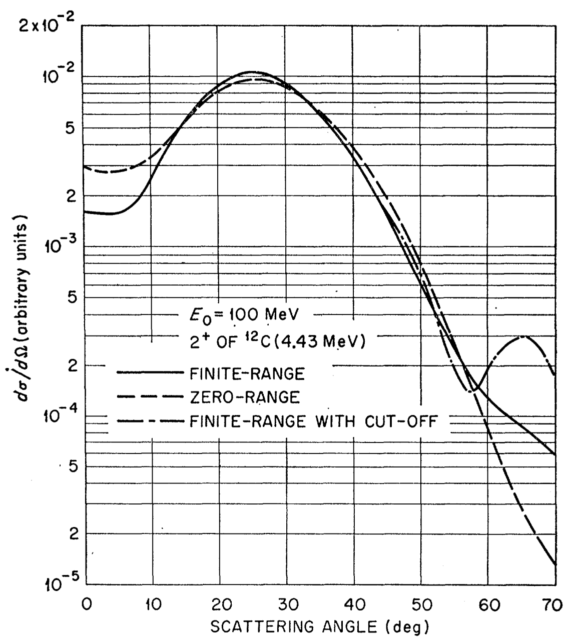

FIG. 6. The excitation of the 2^+ level of ^{12}C for the three approximations is shown for 100 MeV.

B. Discussion of Results for the Yukawa t Matrix

The results obtained for the studies described in the preceding section are displayed in Figs. 5–10. In each case the finite-range results are given by a solid line and the corresponding zero-range results are shown with a dashed line. Figure 7 shows the effects of varying the range β in the effective interaction at 150 MeV as well as the results for the "best fit" to the real t matrix with $\beta=1.2$.

One can summarize the results by noting that the effects of finite range at all the energies is to reduce the cross section rather substantially at small scattering angles, and to enhance it at large scattering angles. The small-angle behavior is simply explained: It results from the well-known phenomenon that at zero-scattering angle (transfer of zero asymptotic momentum neglecting the momentum change due to the inelastic excitation) one has a finite *local* momentum transfer produced by refraction. An incoming particle enters the region of the nuclear potential and is deflected. Subsequently it collides with a target nucleon (producing the inelastic transition) and is scattered with a finite

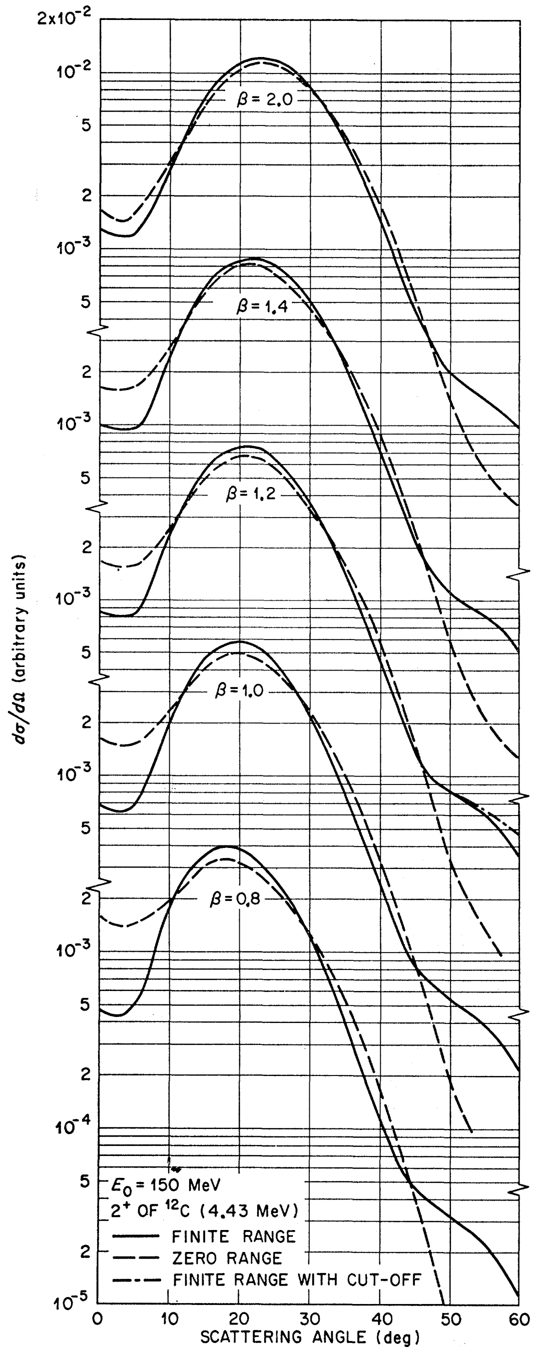

FIG. 7. The zero-range and finite-range results are shown for the 2^+ level of ^{12}C at 150 MeV for several values of the range parameter. The cutoff result is also shown for $\beta=1.2$ F^{-1}.

momentum transfer. Leaving the region of the nuclear potential, the projectile is deflected once more to emerge at zero-scattering angle. Thus scattering in the forward direction is characterized by an average over finite local-momentum transfers and since the effective t matrix is monotonically decreasing function of q, the net result of including refraction effects is to reduce

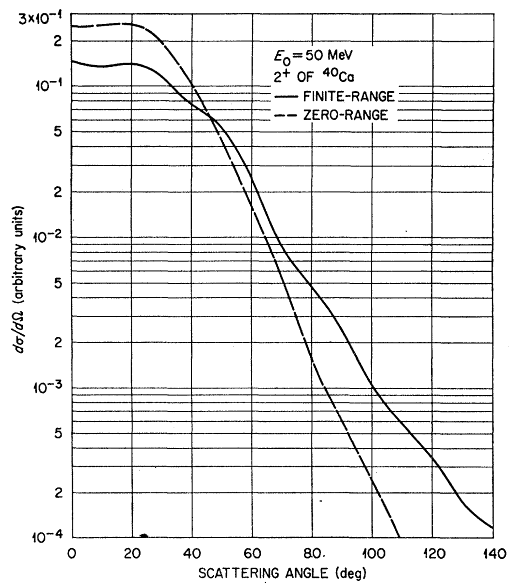

FIG. 8. The zero-range and finite-range results are shown for a 2^+ excitation of ^{40}Ca at 50 MeV.

the small angle-scattering cross section as compared to the result obtained where refraction effects are ignored. Similar ray-tracing arguments could also be applied to the large-angle scattering with the opposite result that the average momentum transfer is now smaller than the asymptotic momentum transfer.

These computations have established that the average momentum transfer at a given scattering angle is, in general, not equal to the asymptotic momentum transfer as is assumed in the zero-range theory of the DWIA. One could, using the results obtained in this section, deduce the average momentum transfer as a function of scattering angle at the three energies where the computations have been performed for the two levels looked at. This average momentum transfer q_{av} is defined by

$$\left[\frac{t_0}{\beta^2+q_{\text{av}}^2}\right]^2\left(\frac{d\sigma}{d\Omega}\right)_{\text{zero range}}=\left(\frac{d\sigma}{d\Omega}\right)_{\text{finite range}}, \quad (37)$$

where the zero-range cross section is that obtained by setting $\beta=\infty$ in Eq. (33). However this procedure is not of any particular utility since the angular dependence of the average momentum transfer would undoubtedly be a function of the optical potential used as well as the level being looked at.

The inclusion of finite-range effects reduces the inelastic cross section in the forward direction by a factor of about two and this result was previously found by Terasawa and Kawai in their study of the excitation of the 15.1 MeV spin-flip level of ^{12}C at 150 MeV which is an $l=0$ transition and peaks in the forward direction. This reduction of the forward cross section relative to the zero-range result seems to be present in the levels

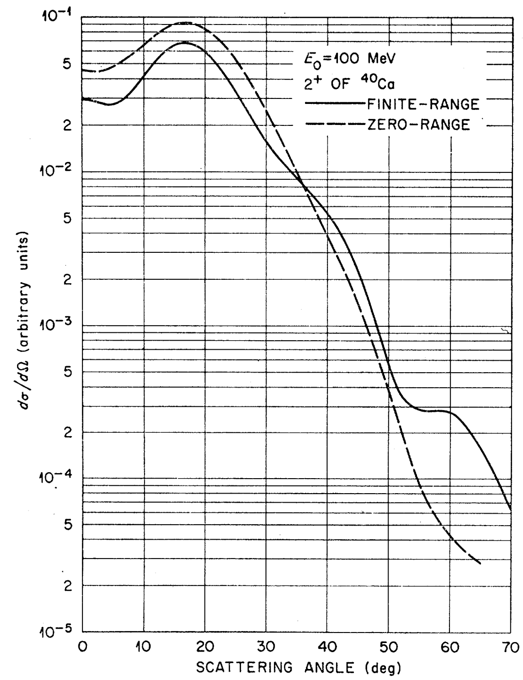

FIG. 9. The zero-range and finite-range results are shown for a 2^+ excitation of ^{40}Ca at 100 MeV.

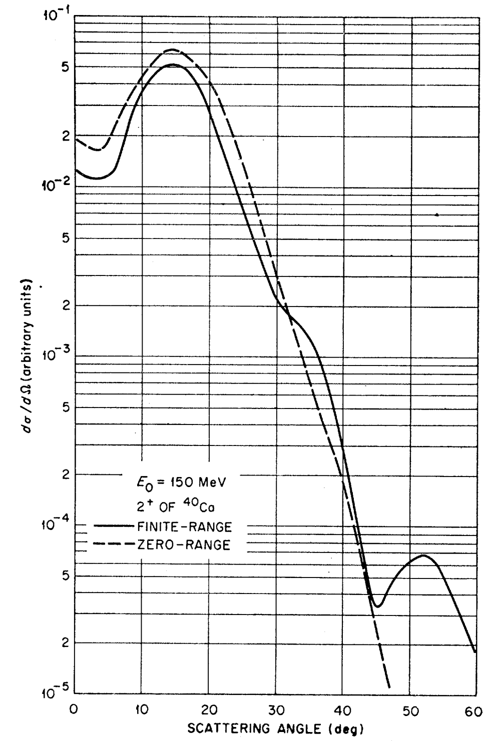

FIG. 10. The zero-range and finite-range results are shown for a 2^+ excitation of ^{40}Ca at 150 MeV.

previously looked at in the DWIA using the zero range. The surprising thing is that the reduction in forward cross section is about the same for both levels looked at here at all three energies. One might well expect that the effects of refraction would become more pronounced at lower energies. That this is not the case is probably due to the fact that the range of the effective interaction ($1/\beta$) is decreasing as one goes to lower energy as can be seen in Figs. 2–4.

The finite-range corrections at 100 and 150 MeV do not shift the location of the peak cross section appreciably for either level and in the case of ^{12}C do not substantially alter its magnitude so that except for forward directions (the cross sections at large angles where substantial corrections are required are unobservably small) the zero-range DWIA should work quite well for ^{12}C (and presumably other very light nuclei) at or above 100 MeV. However, in the case of ^{40}Ca the peak cross section is reduced by nearly 20% at 150 MeV and more than that at 100 MeV. If this reduction of 20% is applied to the results obtained for the 3.7-MeV, 3^- level of ^{40}Ca in Ref. 3, the agreement with the data is substantially improved, since there the theoretical prediction is about 20% too high compared to experiment. This indicates that finite-range effects may have to be taken into account even at high energies for levels in nuclei as heavy as, or heavier than, ^{40}Ca.

The finite-range corrections obviously must be made at energies as low as 50 MeV as expected. The fact that the corrections are not more severe is encouraging. We have not, of course, demonstrated that the DWIA can, in fact, be used at low energy. This will be looked at in the next section where we will be concerned with the strength of the 4.43-MeV cross section as predicted by the finite-range DWIA as compared to experiment.

The discussion of Sec. IIE dealt with the fact that at sufficiently large momentum transfers the t matrix is no longer defined so that the integral in Eq. (17) is indeterminate without some assumption regarding the behavior of $t(q)$ for q larger than the physical cutoff. In order to determine the importance of the large q part of the t matrix, calculations were performed at the three energies for ^{12}C (the calculations were done only for $\beta=1.2$ at 150 MeV) with $t(q)=0$ for $q>q_{\rm phys}$. These results are shown in Figs. 5–7 by the curves with long and short dashes. At 150 MeV the effects of the cutoff can be seen to be negligible, with the same nearly true at 100 MeV. However, at 50 MeV the cutoff introduces a large amount of "noise" into the calculation so that some procedure of extending the t matrix to nonphysical q values is clearly necessary. (The zero-range curves would simply stop at the scattering angle corresponding to the maximum physical q transfer.) This does not necessarily mean that one could determine the particular form of the nonphysical part of $t(q)$ at 50 MeV from comparison with the data, but only that a sharp cutoff is unacceptable.

C. 0^+-0^+ Transition

The fact that the difference between the zero-range and finite-range calculations at 150 MeV is a strong

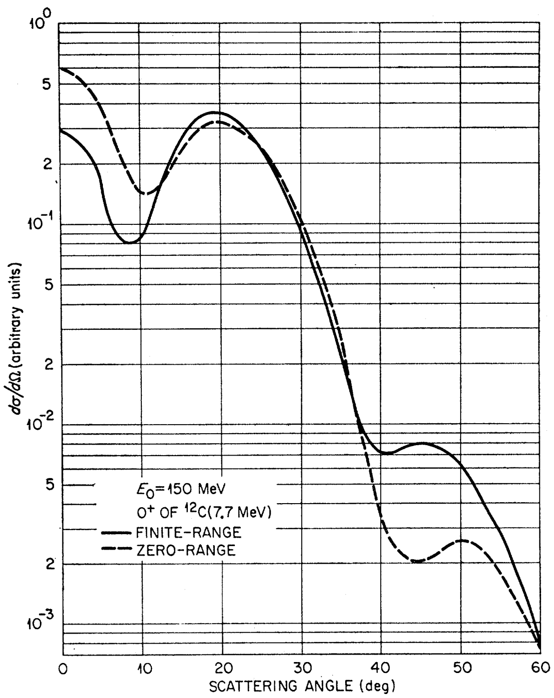

FIG. 11. The zero-range and finite-range results are shown for a 0^+–0^+ transition in ^{12}C at 150 MeV.

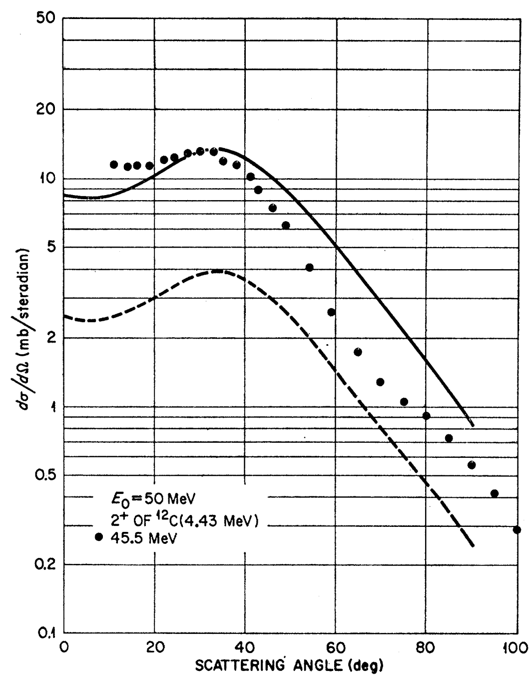

FIG. 12. The cross section for the 4.43-MeV, 2^+ level of ^{12}C is shown compared to data taken at 45.5 MeV (Ref. 13). The dashed curve was obtained for a Yukawa potential with a range parameter $\beta = 1.435$ F^{-1}, and is absolutely normalized. The solid curve was obtained with an enhancement of a factor of 3.4 in the cross section or a factor of 1.8 in the strength of the approximate t matrix.

function of angle in the forward direction indicates that the study of 0^+ to 0^+ transitions will be of particular interest. It has previously been found that the excitation of the 7.7-MeV, 0^+ level of ^{12}C is predicted to have an angular distribution which peaks at 0°, has a minimum at about 10°, and peaks again in the neighborhood of 20° (Ref. 4), and this general shape has been found by experiment.[12] With such a shape, the effects of finite-range ought to be particularly striking. The zero-range and finite-range cross sections were therefore computed using a transition density corresponding to a $1p$-$2p$ excitation and are displayed in Fig. 11. As expected, the finite-range calculation is very different quantitatively, and would most certainly be important in an analysis of data on such levels.

D. Comparison to Data at 45 MeV

From the comparison of DWIA calculations to the experimental data at 156 MeV, it is known that the impulse approximation gives a good estimate of the strength of the transitions studied as well as a reasonably good description of the angular distributions. It is of interest, therefore, to make a comparison of the results obtained in the previous section at 50 MeV to experiment to determine how well the impulse approximation can be expected to do at these low energies. Since the neglected spin-dependent terms in the t matrix are expected to be weak in this energy range, a reasonably good estimate of the situation should be provided using the simple Yukawa form.

If the finite-range cross section is absolutely normalized, the result given in Fig. 12 by the dashed curve is obtained. Compared to the 45.5-MeV data,[13] it can be seen that some enhancement is necessary. The solid curve in Fig. 11 corresponds to an enhancement of 3.4 in the cross section of ~1.8 in the strength of the t matrix.

The angular distribution as predicted bears a qualitative resemblance to the data. This is really not surprising since it has been previously shown by Satchler (Ref. 5) that a Yukawa interaction with a range $\beta \sim 1$ yields angular distributions in fair agreement with experiment at low energies for excitations in ^{90}Zr. It might be noted that use of a smaller value of β ($\beta = 1.435$ in Fig. 12) would improve the shape of the computed curve relative to the data.

It was previously remarked that a better procedure to choose the effective t matrix in (33) would have been to let $t(q)$ be complex with strengths and ranges chosen to agree with the real and imaginary parts of $A(q)$ separately. This would have yielded a complex form factor and perhaps a somewhat different estimate of the enhancement factor required to fit the data. However, if one examines the form of Re$A(q)$ and Im$A(q)$

[12] D. Hasselgren, P. U. Renberg, O. Sundberg, and G. Tibell, Phys. Letters 9, 166 (1964).

[13] I. Slaus (unpublished).

at 40 MeV, it can be seen that the ranges required to fit them separately with Yukawa potentials are not very different. In that case, the procedure we have used here is quite adequate. The use of a complex interaction is required only if the real and imaginary parts of the t matrix have substantially different behavior as a function of q.

IV. DISCUSSION

Corrections to the DWIA due to the averaging of the t matrix over the range of momentum transfers introduced by distortion effects have been studied for quadrupole transitions in ^{12}C and ^{40}Ca from 50–150 MeV. These studies have indicated that the so-called finite-range effects are important over this whole energy range, being essential at 50 MeV.

At 150 MeV where there is little doubt that the DWIA is valid, one finds that the finite-range corrections reduce the cross sections by a factor of two in the forward direction for ^{12}C which is important for $l=0$ transitions and reduce the peak cross section for an $l=2$ transition in ^{40}Ca by 20%. Thus far, quantitative fits to experimental data even at these high energies one should probably allow for corrections to the zero-range DWIA.

We have attempted to apply the impulse approximation (in a simplified form) to the analysis of 50-MeV proton scattering and find that for the 4.43-MeV level of ^{12}C, one must increase the strength of the "real" two-nucleon, t matrix by a factor of about 1.8. In addition the predicted shape of the cross section does not match the data. This seems to indicate a failure of the impulse approximation at such a low energy which, in fact, should be expected. We might emphasize here that the actual two-nucleon amplitude was not used for these studies, but rather a much simplified form which only approximately reproduced the behavior of the central part of the real amplitude. Use of the full, complex t matrix might change some of the results obtained here quantitatively, but it is not likely that the qualitative conclusions made here would be altered.

It might be observed that the finite-range effects which have proved to be important for forward angles in the excitation of the 4.43-MeV level of ^{12}C could be responsible for the conspicuous discrepancy between the measured and computed inelastic polarization[14] for this level at 156 MeV in the forward direction. We are not yet able to do the finite-range calculations where the noncentral parts of the t matrix are included. As indicated in the text, such calculations are much more involved than what we have done here. However, we can conjecture that due to the "wobbling" of the local scattering normal n', one would only see an average projection on the asymptotic normal n, resulting in a reduced polarization, and such a reduction is necessary to align theory and experiment.

It is important to note that we have not included the effects of exchange scattering, that is the process wherein the coordinates of the final nucleon are different from those of the incident nucleon due to the antisymmetrization of the over-all projectile-target wave function. In the zero-range calculation these terms vanish (or more properly are taken into account automatically by the use of an antisymmetrized two-nucleon t matrix). In the finite-range case such terms are present and have been included by several authors who have demonstrated that they are not negligible.[15] However, a study of the reaction $^{28}Si(n,p)$ indicated that a Wigner interaction produced a smaller cross section than did a Majorana force.[16] Since for this reaction the nonexchange force is analogous to the exchange term in inelastic scattering, these results tend to support the assumption that exchange contributions may be neglected to a first approximation.[17] It should be pointed out that since we have neglected exchange, the estimate of the transition strength at 45 MeV made in Sec. IIID is probably too weak; that is, inclusion of exchange would undoubtedly increase the magnitude of the cross section predicted by the unenhanced t matrix. A discussion of exchange contributions in the DWIA will be presented in a forthcoming paper.

ACKNOWLEDGMENTS

The author is grateful to G. R. Satchler, H. McManus, and R. M. Drisko for helpful discussions.

[14] References 3 and 11. Also, see B. Tatischeff, B. Geoffrion, J. LeGuyader, N. Marty, C. Roland, and A. Willis, Phys. Letters **16**, 282 (1965).

[15] K. L. Lim and I. E. McCarthy, Nucl. Phys. **88**, 433 (1966); K. A. Amos, K. L. Lim, V. A. Madsen, and I. E. McCarthy, International Conference on Nuclear Physics, Gatlinburg, Tennessee, 1966 (unpublished).

[16] A. Agodi and G. Schiffer, Nucl. Phys. **50**, 337 (1964); see also T. Une, S. Yamaji, and H. Yoshida, Progr. Theoret. Phys. (Kyoto) **35**, 1010 (1966).

[17] G. R. Satchler, Nucl. Phys. **77**, 481 (1966).