Fine Structure in Nuclear Resonances*

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A K-matrix formulation is given for the shell-model approach to a unified reaction theory. This K matrix differs from the R matrix of Wigner and Eisenbud in using distorted waves of a diffuse potential. The penetrabilities and channel phase shifts are therefore those for a diffuse potential rather than those for a square well with hard-sphere boundary conditions at some radius. This K matrix is used to discuss the fine structure produced by a doorway-hallway system, and a result obtained earlier by Ferrell and MacDonald is rederived. A resonance expansion is then found which explicitly exhibits the widths and resonance energies in a multilevel formula. The distribution of the widths is found, a sum rule is derived, and the average cross section is derived. A general resonance formula is then derived for the case of any number of hallway and doorway states. This is used to generalize the previously obtained doorway-hallway results to the case of hallways coupled to the continuum. In certain cases, a characteristic asymmetry is shown to result.

I. INTRODUCTION

THE success of the shell model in predicting the characteristic features of the ground state and low-lying excited states of nuclei suggests extending the shell model to include continuum states. This extension[1] results in a unification of the shell model with nuclear reaction theory which abandons the "black-box" description of reaction amplitudes in terms of resonance parameters. The reaction dynamics are described from the point of view of the independent-particle picture plus residual interactions.

One of the key consequences of this new picture is that narrow resonances can be described as arising from the discrete states of an independent-particle Hamiltonian in which all nucleons are in bound single-particle states. The total energy for such a state can be positive with respect to the threshold for nucleon emission, but this energy is parceled out among two or more excited nucleons so that none have sufficient energy to escape. These states will be imbedded in a continuum of states which have at least one particle in the continuum but with a lower excitation energy for the other nucleons. The addition of a residual two-particle interaction to the independent-particle Hamiltonian couples the discrete and continuum states. In other words, the particle interaction provides a means for one of the nucleons in the discrete state to exchange energy with another and to acquire enough energy to escape. Conversely, these intermediate discrete states can be reached when an incident nucleon undergoes one or more interactions with the target nucleus through a two-nucleon force.

This picture leads to the concept of a doorway state as a discrete state which can be reached by a single collision with a target nucleon. Other discrete states can be reached by subsequent collisions. This leads to the concept of a hierarchy of states among the discrete states with the doorway states having a unique importance *in certain situations* as shown in Fig. 1.

In the case of only one open channel, for example, a doorway state will determine the angular distribution and polarization of the resonant elastic scattering. The scattering cross section, however, may display a complex resonance structure which contrasts strongly with the energy independence of the angular distribution. This structure is associated with the secondary excitation of more complex states which do not couple directly to the incident channel. In an earlier paper[2] the cross section was derived for a simple model of one doorway state coupled to a number of "hallway" states. The hallway states were defined as being the first hierarchy of states beyond the doorway state. The secondary excitation of the hallway states was shown to introduce fluctuations in the cross section, and an exact expression was derived for the resonant phase shift in a form somewhat different from the resonance expansion familiar from older reaction theories.

A more general discussion of fine structure generated by a doorway-hallway system can be given[3] using the unified reaction theory of one of the authors. The earlier results are then obtained in a more general context which provides explicit expressions for all interaction matrix elements and leads to a simple calculation of the average cross section. However, examination of the results for the special case of a doorway-hallway system reveals that a T-matrix formulation of the shell-model approach to reaction theory is not well adapted to describe fine structure associated with such systems. This is particularly true for the case of "overlapping" levels. The difficulties arise from the diagonalization of a non-Hermitian Hamiltonian whose complex eigenvalues are the resonance energies *and* the level widths.[4]

In this paper we shall relate the S matrix to a K

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[1] W. M. MacDonald, Nucl. Phys. **54**, 393 (1964); **56**, 636 (1966).
[2] R. A. Ferrell and W. MacDonald, Phys. Rev. Letters **16**, 187 (1966).
[3] L. S. Rodberg, *Symposium on Fine Structure, 1966* (University of Kentucky Press, Lexington, Kentucky, to be published).
[4] This point has been explored in detail by C. Mahaux and H. Weidenmuller [Nucl. Phys. **A91**, 241 (1967)].

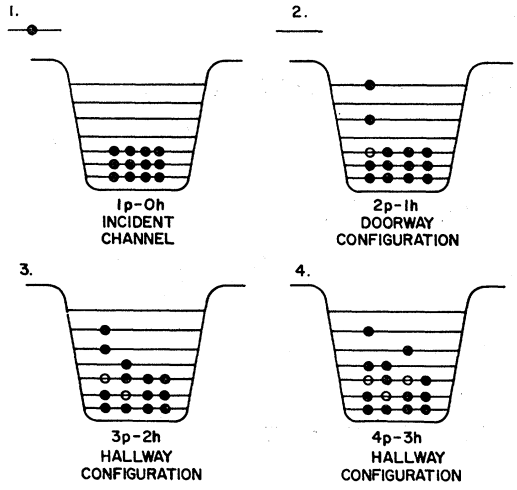

FIG. 1. The generation of the hierarchy of states through the process of multiple scattering in a shell-model description of nuclear reactions.

matrix which is explicitly given in terms of shell-model wave functions. This K-matrix representation is shown to be especially suitable for discussing fine structure and to lead to simple expressions for the reaction amplitude and cross section which are valid for overlapping levels. The K matrix will also be expressed in terms of resonance energies and level widths in a form suitable for determining these from experimental data. The distribution of microscopic widths will be derived for a doorway-hallway system, and an obvious sum rule will be proved. The average cross section will be related to the strength function for the widths.

II. DERIVATION OF THE SHELL-MODEL K MATRIX

We shall derive the K matrix which results from the shell-model approach to reaction theory following the general lines of a development due to Bloch.[5] The T matrix can of course be derived in the same way. The entire development uses antisymmetrized wave functions and the resulting expressions for the S matrix include both direct and exchange terms. The objections raised by Lane and Robson[6] to a unified reaction theory of this form[1] will be seen to have been answered.

The total Hamiltonian for the A-particle system is broken up into two parts, an independent-particle Hamiltonian H_0, and a residual interaction V.

$$H_0=-\sum_i \frac{\hbar^2}{2m}\Delta_i+\sum_i U(i). \tag{2.1}$$

Here $U(i)$ is a finite central potential. The eigenstates of H_0 are antisymmetrized products of the bound and continuum single-particle states for the finite potential U. We restrict ourselves to a limited subspace of these eigenfunctions. The only states to be considered are the states in which all A particles are bound, or states in which only one particle is in the continuum and the other $A-1$ particles are in bound states. The most general wave function that can be obtained from this limited subspace of states is just a linear combination of these states.

$$|\psi\rangle=\sum_\alpha|\alpha\rangle\langle\alpha|\psi\rangle+\sum_{\beta jl}\int d\epsilon|\beta;\, jl\epsilon\rangle\langle\beta;\, jl\epsilon|\psi\rangle. \tag{2.2}$$

Written in configuration space these are

$$\begin{aligned}\langle \mathbf{r}_1,\cdots\mathbf{r}_A|\alpha\rangle&=\mathcal{A}\sum_{i=1}^{A}u_{\alpha_i}(\mathbf{r}_i),\\ \langle \mathbf{r}_1,\cdots\mathbf{r}_A|\beta;\, jl\epsilon\rangle&=\mathcal{A}\prod_{i=1}^{A-1}u_{\alpha_i}(\mathbf{r}_i)u_{jl\epsilon}(\mathbf{r}_A),\end{aligned} \tag{2.3}$$

where antisymmetrized products are indicated. The continuum functions are normalized to a delta function in energy and have the asymptotic forms:

neutron:

$$u_{jl\epsilon}(\mathbf{r})\sim\left(\frac{2M}{\pi\hbar^2k}\right)^{1/2}\frac{1}{r}\sin(kr+\delta_{jl}-\tfrac{1}{2}l\pi)\times\sum_{\sigma' m'}\langle lm'\tfrac{1}{2}\sigma'|\,jm\rangle Y_{lm'}(\Omega)\chi_{\sigma'};$$

proton:

$$u_{jl\epsilon}(\mathbf{r})\sim\left(\frac{2M}{\pi\hbar^2k}\right)^{1/2}\frac{1}{r}\sin(kr+\delta_{jl}-\tfrac{1}{2}l\pi-\eta\ln 2kr)\times\sum_{\sigma' m'}\langle lm'\tfrac{1}{2}\sigma'|\,jm\rangle Y_{lm}(\Omega)\chi_{\sigma'}, \tag{2.4}$$

where δ_{jl} includes the pure Coulomb phase shift and $\eta=MZe^2/\hbar^2k$. The orthonormality condition is

$$\langle jl\epsilon|\,j'l'\epsilon'\rangle=\delta_{jj'}\delta_{ll'}\delta(\epsilon-\epsilon'). \tag{2.5}$$

The solution of the Schrödinger equation

$$(H_0+V)|\psi\rangle=E|\psi\rangle \tag{2.6}$$

is found by inserting the expansion given in Eq. (2.2) and obtaining a set of coupled equations for the expansion coefficients. One set of coupling matrix elements requiring special attention are

$$\langle\beta;\, jl\epsilon|\,V|\beta';\, j'l'\epsilon'\rangle=\delta(\epsilon-\epsilon')\delta_{jj'}\delta_{ll'}\langle\beta|\,V_r|\beta'\rangle+\langle\beta;\, jl\epsilon|\,V_a|\beta';\, j'l'\epsilon'\rangle. \tag{2.7}$$

The first term arises from the interaction of the $(A-1)$ target nucleons without any involvement of the

[5] C. Bloch, Lectures of the Varenna Summer School, 1965 (unpublished).
[6] A. M. Lane and D. Robson, Phys. Rev. **151**, 774 (1966).

continuum nucleon. The Schrödinger equation in this basis yields the coupled linear equations,

$$(E_\alpha{}^A-E)\langle\alpha|\psi\rangle+\sum_{\alpha'}\langle\alpha|V|\alpha'\rangle\langle\alpha'|\psi\rangle$$
$$+\sum_{\beta jl}\int d\epsilon\,\langle\alpha|V|\beta;\,jl\epsilon\rangle\langle\beta;\,jl\epsilon|\psi\rangle=0,$$
$$(E_\alpha{}^{A-1}+\epsilon-E)\langle\beta;\,jl\epsilon|\psi\rangle+\sum_{\beta'}\langle\beta|V_r|\beta'\rangle\langle\beta';\,jl\epsilon|\psi\rangle$$
$$+\sum_{\alpha'}\langle\beta;\,jl\epsilon|V|\alpha'\rangle\langle\alpha'|\psi\rangle$$
$$+\sum_{\beta'j'l'}\int d\epsilon\,\langle\beta;\,jl\epsilon|V_d|\beta';\,j'l'\epsilon'\rangle$$
$$\times\langle\beta';\,j'l'\epsilon'|\psi\rangle=0. \quad (2.8)$$

By choosing a superposition of target states that diagonalizes the residual interaction, we can remove the coupling matrix elements of V_r:

$$|\bar\beta;\,jl\epsilon\rangle=\sum_\beta|\beta;\,jl\epsilon\rangle\langle\beta|\bar\beta\rangle.$$

The transformation matrix satisfies the equation

$$E_\beta{}^{A-1}\langle\beta|\bar\beta\rangle+\sum_{\beta'}\langle\beta|V_r|\beta'\rangle\langle\beta'|\bar\beta\rangle=\bar E_\beta{}^{A-1}\langle\beta|\bar\beta\rangle, \quad (2.9)$$

which is the Schrödinger equation for the target states on the (limited) basis of discrete states.

We can also simplify the equations by diagonalizing the residual interaction on the set of discrete states for the A-nucleon system

$$|\bar\alpha\rangle=\sum_\alpha|\alpha\rangle\langle\alpha|\bar\alpha\rangle,$$
$$E_\alpha{}^A\langle\alpha|\bar\alpha\rangle+\sum_{\alpha'}\langle\alpha|V|\alpha'\rangle\langle\alpha'|\bar\alpha\rangle=\bar E_\alpha{}^A\langle\alpha|\bar\alpha\rangle. \quad (2.10)$$

This equation is that for the usual shell-model diagonalization on the discrete states of a finite potential.

On this new basis Eq. (2.8) becomes

$$(\bar E_\alpha{}^A-E)\langle\bar\alpha|\psi\rangle+\sum_{c'}\int d\epsilon_c'\,\langle\bar\alpha|V|c',\epsilon_c'\rangle\langle c',\epsilon_c'|\psi\rangle=0,$$
$$(\epsilon_c-E)\langle c,\epsilon_c|\psi\rangle+\sum_{c'}\int d\epsilon_c'\langle c,\epsilon_c|V_d|c',\epsilon_c'\rangle\langle c',\epsilon_c'|\psi\rangle$$
$$+\sum_{\bar\alpha'}\langle c,\epsilon_c|V|\bar\alpha'\rangle\langle\bar\alpha'|\psi\rangle=0, \quad (2.11)$$

where we have defined the channel functions and eigenvalues

$$|c,\epsilon_c\rangle=|\bar\beta;\,jl\epsilon_c\rangle,\quad \epsilon_c=\bar E_\beta{}^{A-1}+\epsilon. \quad (2.12)$$

The two equations are simply reduced to the single equation

$$(\epsilon_c-E)\langle c,\epsilon_c|\psi\rangle+\sum_{c'}\int d\epsilon_c'\,f(c,\epsilon_c|c',\epsilon_c')$$
$$\times\langle c',\epsilon_c'|\psi\rangle=0, \quad (2.13)$$

with

$$f(c,\epsilon_c'|c',\epsilon_c')\equiv\langle c,\epsilon|V_d|c',\epsilon_c'\rangle$$
$$+\sum_\alpha\frac{\langle c,\epsilon_c|V|\bar\alpha\rangle\langle\bar\alpha|V|c',\epsilon_c'\rangle}{E-\bar E_\alpha}. \quad (2.14)$$

The solution of Eq. (2.13) is

$$\langle c,\epsilon_c|\psi\rangle=A_c\delta(\epsilon-\epsilon_c)+\frac{1}{E-\epsilon_c}\sum_{c'}K_{cc'}(\epsilon_c)A_{c'}, \quad (2.15)$$

where

$$K_{cc'}(\epsilon_c)=f(c,\epsilon_c|c',\epsilon_c)$$
$$+\sum_{c''}\int d\epsilon_c''\frac{f(c,\epsilon_c|c'',\epsilon_c'')}{E-\epsilon_c''}K_{c''c'}(\epsilon_c''). \quad (2.16)$$

Define the surface function φ_c

$$\varphi_c\equiv\langle\mathbf{r}_1,\cdots,\mathbf{r}_{A-1}|\bar\beta\rangle\sum_{m',\sigma'}\langle lm'\tfrac12\sigma'|jm\rangle$$
$$\times Y_{lm'}(\Omega)\chi_{\sigma'}(A), \quad (2.17)$$

and let F_c and G_c be the regular and irregular solutions of the radial Schrödinger equation with the following asymptotic forms:

neutrons:

$$F_c\sim\left(\frac{2M}{\pi\hbar^2k}\right)^{1/2}\sin(kr+\delta_c-\tfrac12l\pi)/r,$$
$$G_c\sim\left(\frac{2M}{\pi\hbar^2k}\right)^{1/2}\cos(kr+\delta_c-\tfrac12l\pi)/r; \quad (2.18)$$

protons:

$$F_c\sim\left(\frac{2M}{\pi\hbar^2k}\right)^{1/2}\sin(kr+\delta_c-\tfrac12l\pi-\eta\ln2kr)/r,$$
$$G_c\sim\left(\frac{2M}{\pi\hbar^2k}\right)^{1/2}\cos(kr+\delta_c-\tfrac12l\pi-\eta\ln2kr)/r. \quad (2.19)$$

From the expansion of $|\psi\rangle$

$$|\psi\rangle=\sum_{\bar\alpha}|\bar\alpha\rangle\langle\bar\alpha|\psi\rangle+\sum_c\int d\epsilon_c|\,c,\epsilon_c\rangle\langle c,\epsilon_c|\psi\rangle \quad (2.20)$$

and from Eq. (2.15) we deduce the asymptotic form

$$\langle r_A|\psi\rangle\underset{r_A\to\infty}{\sim}\sum_c\varphi_c\{A_cF_c-\pi\sum_{c'}K_{cc'}A_{c'}G_c\}. \quad (2.21)$$

We then write this in terms of ingoing and outgoing spherical waves of the radial free Schrödinger equation which are related to F_c and G_c:

$$\begin{aligned}\mathcal{O}_c&=e^{-i\delta_c}(G_c+iF_c),\\ \mathcal{I}_c&=e^{-i\delta_c}(G_c-iF_c),\end{aligned}\tag{2.22}$$

$$\langle r_A|\psi\rangle\sim-\frac{1}{2i}\sum_c \varphi_c\{(A_c+i\pi\sum_{c'}K_{cc'}A_{c'})e^{-i\delta_c}\mathcal{I}_c - (A_c-i\pi\sum_{c'}K_{cc'}A_{c'})e^{i\delta_c}\mathcal{O}_c\}. \tag{2.23}$$

The S matrix is defined by

$$\mathcal{O}_c=\sum_{c'}S_{cc'}\mathcal{I}_{c'}. \tag{2.24}$$

From Eq. (2.23) it then follows that the S matrix is obtained from the K matrix by the equation

$$S=\omega(1-i\pi K)(1+i\pi K)^{-1}\omega, \tag{2.25}$$

where

$$\omega_{cc'}=e^{i\delta_c}\delta_{cc'}. \tag{2.26}$$

The S-matrix elements are quite explicitly obtained by performing the matrix product indictated. In the next section we derive an expression for the matrix K which is more useful than Eq. (2.16).

III. SEPARATION OF K INTO RESONANT AND NONRESONANT AMPLITUDES

The K matrix given by Eq. (2.16) contains resonances which arise from the discrete states of H_0 for A nucleons. We shall separate the K matrix into resonant and nonresonant parts.

We first recognize that the expansion given in Eq. (2.20) uses eigenstates of the Hamiltonian

$$\bar{H}_0=H_0+V_r+P_dVP_d, \tag{3.1}$$

where P_d projects on the discrete states for A nucleons of H_0. We shall denote by V_c the part of the interaction V which couples the continuum states to each other and to the discrete states. We also define the off-energy-shell operator $\mathbf{K}$ by

$$K=\mathbf{K}\delta(E-\bar{H}_0). \tag{3.2}$$

We can now show from Eq. (2.16) that this operator satisfies the equation

$$\mathbf{K}=V_c+V_c(E-\bar{H}_0)^{-1}\mathbf{K}. \tag{3.3}$$

The same reduction used in earlier papers by one of the authors[1] yields a decomposition of $|\mathbf{K}\rangle$ into a resonant and a nonresonant part. The nonresonant part V_e is given by

$$V_e=V_c+V_c(E-\bar{H}_0)^{-1}P_cV_e. \tag{3.4}$$

The operator P_c projects on the continuum states of $\bar{H}_0$. The full equation for $|\mathbf{K}\rangle$ is

$$\mathbf{K}=V_e+V_eP_d(E-\bar{H}_0-P_dV_eP_d)^{-1}P_dV_e.$$

The V_e is seen here to play the role of an effective interaction which differs from V_c by the inclusion of the effect of virtual transitions to continuum states.

In the resonance expansion of the matrix elements of K which we shall give in the next section, the resonance energies are found from a diagonalization of the Hamiltonian

$$H_s=\bar{H}_0+P_dV_eP_d \tag{3.5}$$

on the discrete states of H_0. This is a shell-model calculation using a restricted basis formed by the configurations for which all single-particle orbitals are bound in a finite potential. The K-matrix elements needed to calculate the S matrix of Eq. (2.25) are

$$K_{cc'}(\epsilon_c)=\langle c,\epsilon_c|V_e|c',\epsilon_c\rangle +\langle c,\epsilon_c|V_eP_d(E-H_s)^{-1}P_dV_e|c',\epsilon_c\rangle. \tag{3.6}$$

This K matrix contains only matrix elements of the effective interaction of Eq. (3.4). The most difficult chore in finding the K matrix is the evaluation of the inverse of the finite matrix $(E-H_s)$. This can be done directly, or by first diagonalizing $(E-H_s)$ with the eigenvectors of H_s. Both procedures will be used in discussing fine structure arising from a doorway-hallway system of discrete states of H_s.

IV. FINE STRUCTURE FOR ONE DOORWAY-HALLWAY SYSTEM

We restrict ourselves in this paper to the case of a single open channel. This approximation does appear to be satisfied for the fine structure observed in Ar^{40}-$(p,p)Ar^{40}$, which will be discussed in detail in another paper. But the principal reason for considering only one open channel is merely to avoid introducing complications which obscure the main physical ideas. There is no difficulty in extending the discussion to inelastic-scattering and charge-exchange reactions.

We are considering the situation in which the discrete states can be divided into two kinds of states, the "doorway" states and the "hallway" states. The doorway states are characterized by a strong coupling to the continuum, and the hallway states by a very weak or zero coupling to the continuum. This circumstance can arise for a two-body nuclear force, which can only couple to the continuum states which differ by no more than a single particle-hole excitation of the target. The hallway states will then differ from the continuum by the excitation of two particles and two holes.

The existence of a doorway-hallway system does not, however, follow *necessarily* from the two-body character of the nuclear force. A large amount of core excitation of the target or a strong mixing of doorway states will destroy this *a priori* basis for the doorway-hallway fine

structure. However, a doorway-hallway system may originate in other ways.

Without pursuing the question of the structure of the doorway and hallway states, we simply postulate the existence of a single doorway and N hallway states. We shall then diagonalize the submatrix of H_s referring to the hallway states in order to remove the coupling of these states among themselves:

$$\langle h_i|H_s|h_j\rangle=\epsilon_i\delta_{ij}. \quad i=1,\cdots,N.$$

This subdiagonalization does not alter the character of the hallway states and therefore we also have

$$\langle h_i|V_e|c,E\rangle=0, \quad i=1,\cdots,N.$$

The doorway state will be coupled both to the continuum and to the hallways:

$$\begin{aligned}&\langle D|H_s|D\rangle=\epsilon_D,\\&\langle h_i|V_e|D\rangle=M_{Di}, \quad i=1,\cdots,N,\\&\langle D|V_e|c,E\rangle=(2\pi)^{-1/2}\Gamma_D{}^{1/2}(E).\end{aligned} \tag{4.1}$$

With these matrix elements the shell-model Hamiltonian matrix has a simple form:

$$\begin{bmatrix}\epsilon_D & M_{D1}{}^* & M_{D2}{}^* & \cdots & M_{DN}{}^*\\ M_{D1} & \epsilon_1 & 0 & & \\ M_{D2} & 0 & \epsilon_2 & & \\ \vdots & & & & \\ M_{DN} & & & & \end{bmatrix}. \tag{4.2}$$

The K-matrix element must be computed from Eq. (3.6). The first term is a nonresonant direct-interaction term which we shall neglect. The resonant part given by the second term is easily evaluated:

$$\begin{aligned}&\pi K_{cc}{}^R(E)=-\frac{\Gamma_D(E)/2}{E-\epsilon_D-\Sigma_H},\\&\Sigma_H\equiv\sum_i|M_{Di}|^2/(E-\epsilon_i).\end{aligned} \tag{4.3}$$

The S matrix follows from Eq. (2.25):

$$S_{cc}=e^{2i\delta_c}\frac{E-\epsilon_D-\Sigma_H-\frac{1}{2}i\Gamma_D(E)}{E-\epsilon_D-\Sigma_H+\frac{1}{2}i\Gamma_D(E)}. \tag{4.4}$$

From this equation a resonant phase shift can be defined

$$\tan\delta_R=-\frac{\Gamma_D(E)/2}{E-\epsilon_D-\Sigma_H}. \tag{4.5}$$

This is just the expression obtained in an earlier paper.[2] At each hallway energy the resonant amplitude has a zero. At first sight it seems strange that the minima and not the maxima of the resonant amplitude should occur at the hallway energies. On closer inspection of the matrix of Eq. (4.2), however, one sees that the hallway states do not actually fall at the energies ϵ_i but are shifted by their interaction with the doorway. The maxima actually occur exactly where they should be expected—at the shifted hallway-state energies.

This can be seen in a different representation of the resonant amplitude which exhibits the resonance energies and the widths associated with each resonance. This representation expresses the fact that each hallway state is actually coupled *indirectly* to the continuum through the doorway state. Each hallway state therefore acquires a width which is borrowed from the doorway. The system has the classical analog of a number of oscillators coupled to a single central one which is being driven. The driving of one produces oscillations in the others, and the full range of normal modes of the system is excited.

A representation of the K matrix which exhibits the resonance energies and their widths can be found simply by diagonalizing the matrix of H_s. The states $|\psi_\lambda\rangle$ which do this will be a superposition of the doorway and hallway states,

$$|\psi_\lambda\rangle=|D\rangle\langle D|\psi_\lambda\rangle+\sum_i|h_i\rangle\langle h_i|\psi_\lambda\rangle, \quad \lambda=1,\cdots,N+1 \tag{4.6}$$

satisfying the equations

$$\langle\psi_\mu|H_s|\psi_\lambda\rangle=E_\mu\delta_{\lambda\mu}. \tag{4.7}$$

The eigenvalues E_μ are zeros of the determinant of this set of equations:

$$E_\mu-\epsilon_D=\Sigma_H(E_\mu). \tag{4.8}$$

These eigenvalues are immediately seen from Eq. (4.5) to be the resonance energies.

A graphical solution for the resonance energies is illustrated in Fig. 2, where the eigenvalues are obtained as the intersection of the straight line $(E-\epsilon_D)$ with the function $\Sigma_H(E)$.

The K matrix assumes a simple form in terms of the eigenstates $|\psi_\lambda\rangle$:

$$\pi K_{cc}{}^R(E)=\tfrac{1}{2}\Gamma_D(E)\sum_{\lambda=1}^{N+1}|\langle D|\psi_\lambda\rangle|^2/(E-E_\lambda). \tag{4.9}$$

This leads to the S matrix

$$S_{cc}=e^{2i\delta_c}\frac{1-\frac{1}{2}i\sum_\lambda\Gamma_\lambda(E)/(E-E_\lambda)}{1+\frac{1}{2}i\sum_\lambda\Gamma_\lambda(E)/(E-E_\lambda)}. \tag{4.10}$$

The widths associated with the resonances E_λ are given by the expression

$$\Gamma_\lambda(E)=\Gamma_D(E)|\langle D|\psi_\lambda\rangle|^2. \tag{4.11}$$

We should note that Eq. (4.10) for S is a multilevel expression, valid for both the case of isolated resonances and for the case of strongly overlapping levels. The latter situation is likely to be the more common one for fine structure in medium to heavy nuclei. In case of

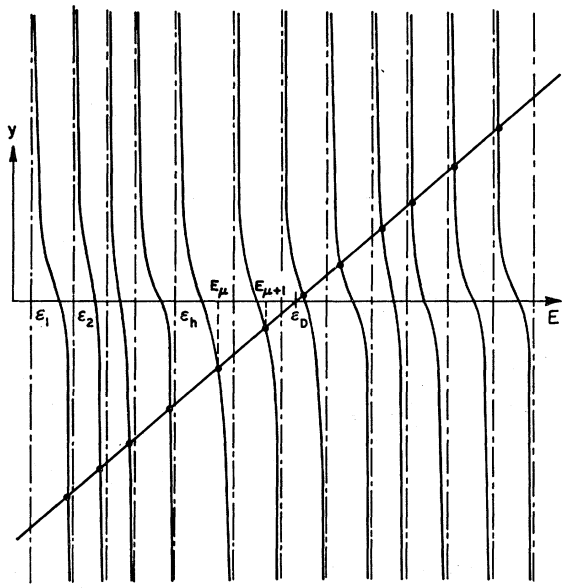

FIG. 2. The graphic solution for the energy eigenvalues of the effective Hamiltonian H_s in the restricted space of bound states. The ordinate represents the function $E-\epsilon_D$ and Σ_H. The intersections between the two curves give the new energy eigenvalues.

"overlapping levels," by which we mean that $\Gamma_\lambda(E)$ is larger than the level spacing, the *apparent* resonance widths are not given by Eq. (4.11). In the case of a single open channel the resonance amplitude must go to zero between every pair of "unperturbed" hallway energies ϵ_i. (If more than one channel is open, this need not be true.) Therefore the widths will appear to be less than the resonance spacing. Nevertheless, a detailed fit of high-resolution data using the multilevel formula will give the widths of Eq. (4.11).

A sum rule can be given for the widths defined by Eq. (4.11). It follows from the unitarity of the transformation given by Eq. (4.6) and from the orthonormality of the discrete states:

$$\sum_\lambda |\langle D|\psi_\lambda\rangle|^2=1. \tag{4.12}$$

If the energy dependence of the width $\Gamma_D(E)$ across the region of the fine structure can be neglected, this sum rule can be read as

$$\sum_\lambda \Gamma_\lambda=\Gamma_D(\epsilon_D). \tag{4.13}$$

The usefulness of the K-matrix approach for the discussion of fine structure is most apparent in the existence of this sum rule. Although a sum rule can be given for the widths which appear in a resonance expansion of the T matrix,[3] this sum rule does not have a form which lends itself to the obvious interpretation of Eq. (4.13). It should be emphasized that the widths which appear in a T matrix, or S matrix, resonance expansion are not the same as those which appear in the K matrix.

Although useful, the resonance expansion of K^R given in Eq. (4.9) does not always exhibit explicitly the energy dependence of K^R. This is because the matrix elements of the second term of V_e given in Eq. (3.4) may be energy-dependent. In particular it is important to choose H_0 so as to avoid having very narrow single-particle resonances in the continuum states. The problem has been discussed in detail[7] for the $d_{3/2}$ resonance of O^{16} and a method has been outlined for obtaining a V_e which is nearly energy-independent.

The distribution of the widths Γ_λ as a function of E_λ is to be found from the matrix Eq. (4.7) by using the normalization condition on the $|\psi_\lambda\rangle$:

$$|\langle D|\psi_\lambda\rangle|^2=\left[1+\sum_i \frac{|M_i|^2}{(E_\lambda-\epsilon_i)^2}\right]^{-1}. \tag{4.14}$$

This expression cannot be evaluated analytically for the general case. However, we can obtain a closed expression for the special case of the "picket-fence model." This is a model of hallways with equal spacing D and the same matrix element M to the doorway. In this model we have

$$\Sigma_H(E)=\frac{\pi M^2}{D}\cot\frac{\pi(E-\epsilon_0)}{D},$$

where the hallway energies occur at $nD+\epsilon_0$. Using the equation

$$|\langle D|\psi_\lambda\rangle|^2=\left[1-\frac{d}{dE_\lambda}\Sigma_H(E_\lambda)\right]^{-1},$$

we find for the distribution of widths

$$\frac{\Gamma_\lambda}{D}=\frac{\Gamma_D\Gamma_s/2\pi}{(E_\lambda-\epsilon_D)^2+(\Gamma_s/2)^2+\Gamma_sD/2\pi}. \tag{4.15}$$

The result contained in Eq. (4.15) is quite surprising at first sight. If $\Gamma_s\gg D$, the width of the distribution is not $2M$, as one would expect from perturbation theory, but Γ_s. The point is that *all* the widths decrease as Γ_s increases and the strength function (Γ_λ/D) becomes very broad and the maximum less pronounced:

$$2\pi\Gamma_\lambda/D=\frac{\Gamma_D\Gamma_s}{(E_\lambda-\epsilon_D)^2+(\frac{1}{2}\Gamma_s)^2}. \tag{4.16}$$

The other limit of small spreading width is also given by this equation. If $\Gamma_s\gg D$, then one eigenvalue approaches ϵ_D while for the others $E-\epsilon_D$ remains finite. Thus $\Gamma_\lambda\to\Gamma_D$ at ϵ_D and $\Gamma_\lambda\to 0$ elsewhere.

Although these results have been derived only for a particular model, we shall take it to be generally true if we replace Γ_λ/D by the average value over intervals larger than D but small compared to Γ_s.

[7] L. Garside and W. MacDonald, Phys. Rev. 138, B582 (1965).

The above equations give the distribution of widths, but the *amplitude* of the cross section oscillates between the unitary maximum and zero. This follows from Eq. (4.10), or from the trivial observation that we only have one open channel. Only if the cross section is *averaged*, or *other* channels open, do we see *amplitude* modulation in the cross section. The cross section for one open channel will be proportional to $|1-S|^2$, with other factors which are well known in the literature[8]:

$$\sigma_a \propto |1-S_{cc}|^2 = \left| e^{-2i\delta_c} - 1 - \frac{i\sum_\lambda \Gamma_\lambda/(E-E_\lambda)}{1+\frac{1}{2}i\sum \Gamma_\lambda/(E-E_\lambda)} \right|^2 . \quad (4.17)$$

For the case of isolated levels $\Gamma_\lambda < D$, the level spacing, we have the approximate form

$$|1-S_{cc}|^2 \cong \left| e^{-2i\delta_c} - 1 - i\sum_\lambda \frac{\Gamma_\lambda}{E-E_\lambda+\frac{1}{2}i\Gamma_\lambda} \right|^2 .$$

V. AVERAGE CROSS SECTION FOR ONE DOORWAY-HALLWAY SYSTEM

The average cross section is of considerable interest because of the effect of finite energy resolution in determining the structure which is observed. The average cross section is obtained by averaging $|1-S_{cc}|^2$ over an energy interval I which is greater than the hallway spacing but less than the width of the doorway state. From the unitarity of S for one channel we have

$$\langle |1-S_{cc}|^2 \rangle = 2(1-\mathrm{Re}\langle S_{cc}\rangle). \quad (5.1)$$

Since the poles of S_{cc} lie in the lower half of the complex plane, an average of S_{cc} can be found by evaluating it at an energy $E+iI$[9]:

$$\langle S_{cc}(E)\rangle = S_{cc}(E+iI). \quad (5.2)$$

We then evaluate the average from Eq. (4.4) using the equation

$$\Sigma_H(E+iI) \simeq \Delta - i\Gamma_s/2,$$

$$\Delta = \mathcal{P}\int \frac{dE'}{D_H(E')} \frac{|M(E')|^2}{(E-E')} . \quad (5.3)$$

The Γ_s is the spreading width defined in Eq. (4.16).

The result for the averaged S matrix is

$$\langle S_{cc}\rangle = e^{2i\delta_c} \frac{E-\epsilon_D-\frac{1}{2}i(\Gamma_D-\Gamma_s-2I)}{E-\epsilon_D-\frac{1}{2}i(\Gamma_D+\Gamma_s+2I)}, \quad (5.4)$$

where we have included the shift Δ in ϵ_D. The average cross section is proportional to

$$\langle |1-S_{cc}|^2 \rangle = 4\sin^2\delta_c + \frac{\Gamma_D(\Gamma_s+\Gamma_D+2I)\cos 2\delta_c - 2\Gamma_D(E-\epsilon_D)\sin 2\delta_c}{(E-\epsilon_D)^2+\frac{1}{4}(\Gamma_s+\Gamma_D+2I)^2} . \quad (5.5)$$

The first term gives the potential-scattering cross section, and the second term contains both the resonant amplitude and an interference term. If there were no coupling of the doorway to the hallway states, the cross section would exhibit a resonance of width Γ_D at an energy ϵ_D, together with the usual interference with the potential scattering. The effect of the fine structure is to give a broadening of the resonance amplitude, as can be seen from the resonance denominator. However, the resonance amplitude is no longer Lorentzian even in the absence of potential scattering, for which

$$\langle |1-S_{cc}|^2 \rangle = \frac{\Gamma_D(\Gamma_s+\Gamma_D+2I)}{(E-\epsilon_D)^2+\frac{1}{4}(\Gamma_s+\Gamma_D+2I)^2} . \quad (5.6)$$

The effect of the spreading width is to reduce the maximum cross section at the resonance energy and to increase the width. The effect of Γ_s on the elastic scattering is precisely the same as if another channel had opened. This fact is apparently the origin of the term "downward-going."[10] This expression is somewhat of a misnomer if only one channel is open because the only decays which are possible for the discrete states are those going through the doorway state. The width of the doorway state is actually decreased by the mixing to hallway states since some of the strength is removed and distributed to form other *separated* resonances. The Γ_s is strictly a property of an *average* cross section and not of some physical state. On the other hand, if there are so many channels open that the hallways are coupled to the continuum through many doorways, and even directly, the spreading width for a particular doorway does have a physical meaning. The lifetime of the doorway will be h/Γ_s in the "no-return" approximation of Danos and Greiner.[11] In this the fine structure will also be absent and the cross section for elastic scattering will be the same as the average cross section.

It is important to note that elastic-scattering experiments having an energy resolution less than the fine-structure spacing can not distinguish between the situation in which a resonant cross section has been averaged over fine structure and the situation in which reactions are contributing to the total width. However, the two cases can be distinguished by measuring the cross sections for other energetically possible reactions and checking whether the total width is the sum of the various partial widths.

[8] J. M. Blatt and L. Biedenharn, Rev. Mod. Phys. **24**, 258 (1965).

[9] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

[10] H. Feshbach, in *Proceedings of the International Conference on the Study of Nuclear Structure with Neutrons, Antwerp, 1965* (North-Holland Publishing Company, Amsterdam, 1966).

[11] M. Danos and W. Greiner, Phys. Rev. **138**, B876 (1965).

We conclude this section by indicating the connection between the strength function and the average cross section. The strength function is defined as the average of Γ_λ/D, the microscopic widths divided by the level spacing. The explicit dependence of $\langle\Gamma_\lambda/D\rangle$ on the energy is given for the picket-fence model in Eq. (4.15). A more general, but approximate, expression can be derived by using the equation

$$\Gamma_D/2(E-\epsilon_D-\Sigma_H)=\sum_\lambda \Gamma_\lambda/(E-E_\lambda).$$

We then evaluate both sides at $E+iI$ and equate the imaginary parts of both sides to obtain an equation for the strength function:

$$s\equiv\langle\Gamma_\lambda/D\rangle=\frac{\Gamma_D\Gamma_s/2\pi}{(E-\epsilon_D)^2+(\Gamma_s/2)^2}. \tag{5.7}$$

Equating the real parts of both sides gives

$$K_0=\mathcal{P}\int\frac{dE_\lambda}{D_\lambda}\frac{\Gamma_\lambda}{(E-E_\lambda)}\simeq\frac{(E-\epsilon_D)\Gamma_D/2}{(E-\epsilon_D)^2+(\Gamma_s/2)^2}. \tag{5.8}$$

These last two equations enable us to reconcile the easily found result that

$$\langle|1-S_{cc}|^2\rangle=\frac{2\pi s(1+\pi s/2)}{(1+\pi s/2)^2+K_0{}^2}, \tag{5.9}$$

with Eq. (5.6). For $|E-\epsilon_D|\gg\Gamma_s$, the term in K_0 can be neglected:

$$\langle|1-S_{cc}|^2\rangle=2\pi s(1+\pi s/2)^{-1}. \tag{5.10}$$

This should be compared with

$$1-|\langle S_{cc}\rangle|^2=2\pi s(1+\pi s/2)^{-2}, \tag{5.11}$$

which is used to define the transmission factor:[8]

$$T_c=1-|\langle S_{cc}\rangle|^2=2\pi s(1+\pi s/2)^{-2}. \tag{5.12}$$

The difference between the averages performed in Eqs. (5.10) and (5.11) defines the "fluctuation cross section,"

$$\sigma_{\mathrm{fl}}\propto 2(\pi s)^2(1+\pi s/2)^{-2}. \tag{5.13}$$

The fluctuation cross section also satisfies

$$\sigma_{\mathrm{fl}}\propto\frac{\Gamma_D{}^2}{(E-\epsilon_D)^2+(\Gamma_D+\Gamma_s)^2/4} \tag{5.14}$$

for comparison.

VI. GENERAL RESONANCE STRUCTURE

In this section we shall generalize the preceding case of one doorway coupled to N hallways to the general case of any number of doorway and hallway states coupled in any manner. Two different expressions for the K-matrix elements can usually be given, corresponding to Eqs. (4.4) and (4.10) for the S matrix in the one doorway-hallway case.

The first equation for the K matrix is simply obtained by evaluating Eq. (3.6) explicitly. Let the doorway states be designated as $|D_i\rangle$. Then arrange the matrix elements of H_s into blocks corresponding to the doorway and hallway states:

$$H_s=\begin{bmatrix} D & M_{DH}{}^* \\ M_{DH} & H \end{bmatrix}.$$

The blocks designated by M_{DH} will contain the coupling matrix elements between the doorway and hallway states. Then the K-matrix elements will be

$$K_{cc'}=\langle c|V_e|c'\rangle+\sum_{i,j}\frac{\langle c|V_e|D_i\rangle M_{ji}\langle D_j|V_e|c'\rangle}{\det(E-H_s)}. \tag{6.1}$$

The M_{ij} is the cofactor of $(E-H_s)_{ij}$ in the matrix of $(E-H_s)$. This equation is exact and contains no approximations with regard to the energy dependence of the matrix elements of H_s.

The disadvantage of Eq. (6.1) is that it contains no explicit information on the energies at which resonances occur or on their widths. As in Sec. 4, we can obtain another representation of the K matrix which does exhibit this information. This representation is obtained by diagonalizing H_s on the set of doorway and hallway states. The eigenvectors will be

$$|\psi_\lambda\rangle=\sum_i|D_i\rangle\langle D_i|\psi_\lambda\rangle+\sum_i|h_i\rangle\langle h_i|\psi_\lambda\rangle, \tag{6.2}$$

satisfying

$$\langle\psi_\lambda|H_s|\psi_\mu\rangle=E_\lambda\delta_{\lambda\mu},\quad\langle\psi_\lambda|\psi_\mu\rangle=\delta_{\lambda\mu}. \tag{6.3}$$

The K-matrix elements are then

$$K_{cc'}=\langle c|V_e|c'\rangle+(2\pi)^{-1}\sum_\lambda\frac{\Gamma_{\lambda c}{}^{1/2}\Gamma_{\lambda c'}{}^{1/2}}{E-E_\lambda}, \tag{6.4}$$

where

$$\Gamma_{\lambda c}{}^{1/2}=(2\pi)^{1/2}\sum_j\langle D_j|\psi_\lambda\rangle^*\langle D_j|V_e|c\rangle. \tag{6.5}$$

Since we are working with the K matrix, all matrix elements can be taken as real.

Although Eq. (6.4) appears to exhibit explicitly the energy dependence, this is not exactly true. The matrix elements which appear in H_s not only contain the direct coupling of the discrete states to each other, but the indirect coupling through intermediate continuum states. This coupling is introduced through the second term of Eq. (3.4) for V_e. The energy dependence of such terms has been studied[12] in O^{16} and found to be very

[12] L. Garside and W. MacDonald, in *Proceedings of the International Conference on the Study of Nuclear Structure with Neutrons, Antwerp, 1965* (North-Holland Publishing Company, Amsterdam, 1966); W. Beres, Phys. Rev. Letters **17**, 1180 (1966).

small when virtual single-particle resonances are properly handled.

Within the approximation of energy-independent matrix elements between the discrete states, the resonance expansion for the K matrix exhibits clearly that the calculation of the fine structure can be neatly separated into two stages. The first step is the calculation of the eigenvalues and eigenvectors the shell-model Hamiltonian. This calculation involves the diagonalization of an Hermitian matrix with real eigenvalues, even if the resonance widths are later found to correspond to strongly overlapping levels. The only refinement of the usual shell-model calculation that is indicated is the inclusion of the coupling through continuum states. The second step is the calculation of the level widths by Eq. (6.5). This is the stage at which the distinction between doorway and hallway states is made. Only the doorway amplitudes in the resonance states contribute to the widths.

The resonance expansion for the K matrix in Eq. (6.4) resembles that for the R matrix of Wigner and Eisenbud. There are, however, a number of important differences. First, the widths $\Gamma_{\lambda c}$ contain matrix elements from discrete states to distorted-wave continuum states. Therefore the penetrability factors which enter are those for a diffuse potential, and not those for a square well. Second, this K matrix contains a direct term which is entirely absent in R-matrix theory by a large number of terms corresponding to contributions from "distant levels." Finally, the channel phase shifts appearing in the matrix ω of Eq. (2.26) are those for a diffuse potential rather than for "hard-sphere" scattering.

In the K-matrix resonance expansion the widths satisfy a physically obvious sum rule. This sum rule follows from the orthonormality of the eigenvectors $|\psi_\lambda\rangle$ and, separately, of the doorway and hallway states:

$$\sum_\lambda \Gamma_{\lambda c}=\sum_j \Gamma_{D_j,c}. \tag{6.6}$$

The partial widths on the right side of this equation are those for decay of the doorway states in channel c. It should be noted that unfortunately this obvious sum rule is satisfied only approximately by the widths which appear in the resonance expansion of the S matrix. The reason is that these expansions involve the diagonalization of a non-Hermitian matrix with complex eigenvalues.[1,3] The diagonalization is usually done by diagonalizing H_s and assuming that the level matrix is then also diagonal. This approximation is untenable for doorway-hallway systems.

An interesting application of the general resonance-structure formulas (6.1) is to the case in which we have a single doorway strongly coupled to the continuum and to a set of hallways which are also directly coupled to the continuum but with this coupling much weaker than for the doorway. Because of this difference in strength of the coupling to the continuum, we can still speak of these latter states as hallways. We will assume we have diagonalized H_s on the hallway set, so that couplings only exist between the doorway and hallway states. The next step is to remove the couplings between the doorway and hallway set to give a new set of states (6.2) which satisfy the restricted Schrödinger equation (6.3). Since H_s is diagonal among the hallway states, the transformation coefficients $\langle D|\psi_\lambda\rangle$ and $\langle h_i|\psi_\lambda\rangle$ are easily found to be related by the equation

$$\langle h_i|\psi_\lambda\rangle=\langle h_i|V_e|d\rangle\langle D|\psi_\lambda\rangle/(\epsilon_i-E). \tag{6.7}$$

The value of $\langle D|\psi_\lambda\rangle$ can be found from the normalization condition and is given by Eq. (4.14). The eigenvalues E_λ still satisfy Eq. (4.8). The structure problem represented by Eq. (4.7) is unaffected, in fact, by the introduction of a coupling of the hallways to the continuum, except for a slight change in V_e which results from second-order terms involving continuum states.

The significant change is in the equation for the widths Γ_λ, which are now the result of the coherent contribution of decay amplitudes:

$$\Gamma_{\lambda c}=2\pi\left|\langle D|V_e|c,\epsilon_c\rangle-\sum_{i=}^{N}\frac{\langle D|V_e|h_i\rangle\langle h_i|V_e|c,\epsilon_c\rangle}{E_\lambda-\epsilon_i}\right|^2 \times|\langle D|\psi_\lambda\rangle|^2. \tag{6.8}$$

The sum rule on the widths becomes

$$\sum_\lambda \Gamma_{\lambda c}=\sum_i \Gamma_{h_i,c}+\Gamma_D, \tag{6.9}$$

where the $\Gamma_{h_i,c}$ are the hallway widths in the absence of coupling to a doorway:

$$\Gamma_{h_i,c}=2\pi|\langle h_i|V_e|c,\epsilon_c\rangle|^2. \tag{6.10}$$

Far from the doorway the widths Γ_λ become equal to $\Gamma_{h_i,c}$. The sum rule therefore can be expressed in a form containing only the widths about the analog:

$$\sum(\Gamma_{\lambda c}-\Gamma_{h_i,c})=\Gamma_D. \tag{6.11}$$

This expression is most convenient if the $\Gamma_{h_i,c}$ (or the matrix element) are slowly varying across the doorway resonance energy.

Under certain conditions the distribution in widths given by Eq. (6.8) will exhibit an asymmetry about the doorway state. Robson[13] has drawn attention to this feature of the fine-structure pattern associated with analog resonances in certain nuclei. Such an asymmetry and a suppression of the widths at some energy will occur if the magnitude and the relative phase of matrix elements $\langle A|V_e|h_i\rangle$ and $\langle h_i|V_e|c,\epsilon_c\rangle$ remain the same across the region of enhanced fine structure. In this

[13] D. Robson, Phys. Rev. **137**, B535 (1965).

case the "picket-fence" model leads to the expression for the widths

$$\Gamma_\lambda = 2\pi \left| \langle h | V_e | c,\epsilon_c \rangle - \frac{\langle h | V_e | A \rangle \langle A | V_e | c,\epsilon_c \rangle}{E_\lambda - \epsilon_D} \right|^2 \times \frac{(E_\lambda - \epsilon_D)^2}{(E-\epsilon_D)^2 + (\Gamma_s/2)^2 + \Gamma_s D/2\pi}, \quad (6.12)$$

where $|A\rangle$ denotes the analog state.

This equation shows that the width is the coherent contribution of decay amplitudes arising from the *direct* coupling of the hallway to the continuum and of the indirect coupling to the continuum through the analog state. The two amplitudes interfere destructively on one side of the doorway and constructively on the other side if the phase of the matrix element $\langle h | V_e | c,\epsilon_c \rangle$ remains the same relative to the phase of $\langle h | V_e | A \rangle \times \langle A | V_e | c,\epsilon_c \rangle$. In this case the equation for the widths can be written in a form which displays the asymmetry which is illustrated in Fig. 3:

$$\Gamma_\lambda = 2\pi |\langle h | V_e | c,\epsilon_c \rangle|^2 \frac{(E_\lambda - E_s)^2}{(E_\lambda - \epsilon_D)^2 + (\Gamma_s/2)^2 + \Gamma_s D/2\pi},$$

$$E_s = \epsilon_D + \frac{\langle h | V_e | D \rangle \langle D | V_e | c,\epsilon_c \rangle}{\langle h | V_e | c,\epsilon_c \rangle}. \quad (6.13)$$

This formula is very reminiscent of one derived by Robson,[13] but the parameters which appear in it are quite different. This equation indicates an enhancement of the hallway widths which extends over an energy range of width $(\Gamma_s^2 + 2\Gamma_s D/\pi)^{1/2}$, which is therefore determined by the analog hallway coupling. This contrasts with the modulation function derived by Robson, which shows a width Γ_D (or Γ_A, since the analog state is the doorway in the situation being discussed). Furthermore, the energy at which the fine structure is suppressed is not the energy ϵ_D (or E_A), but is the energy at which the decay amplitude *contributed* by the doorway (analog) is equal and opposite to the decay amplitude of the hallway in the absence of the analog.

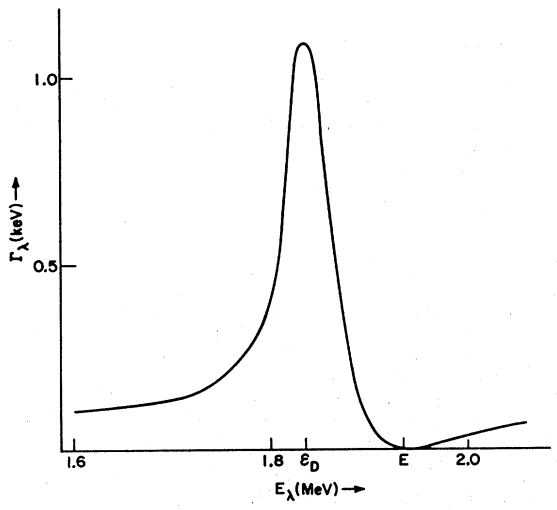

FIG. 3. The distribution of widths as a function of energy for the case in which an asymmetry exists.

The above discussion is intended to be schematic and we have not discussed a number of important points. Questions on the *structure* of the analog state, the effect of isobaric spin selection rules upon various coupling matrix elements, and other aspects of fine structure in analog resonance will be discussed in a separate paper.

VII. SUMMARY

In this paper we have developed a K-matrix approach to nuclear reactions which uses the independent-particle wave functions of a finite potential. All wave functions for the system are properly antisymmetrized, and both direct and exchange amplitudes for single-nucleon reactions are included. The advantage of the K matrix over the corresponding T matrix (also easily derived) is that overlapping levels which are strongly coupled to the same channels are easily described in a multilevel formula. The reason is that the resonance expansion of the K matrix contains real resonance energies and orthonormal intermediate states which are obtained by diagonalizing a Hermitian matrix. Simple sum rules result from the unitarity of the diagonalizing transformation.

This is in marked contrast to the resonance expansion for the S matrix or T matrix which contains complex energies and involves nonunitary transformations.[1,3] The pole expansion of the S matrix is inappropriate for the discussion of doorway-hallway systems, as shown by Weidenmuller and Mahaux.[4]

We have discussed the fine structure associated with a doorway-hallway system using the K matrix. An exact expression for the S matrix has been obtained in the form derived earlier[2] which displays the zeros of the resonant amplitude. However, we have also given the resonance expansion of the K matrix which exhibits the resonance energies and level widths. The level widths have been shown to follow a Lorentzian distribution and to sum to the doorway width. The resonance energies are found by diagonalizing a shell-model Hamiltonian. The average cross section is found and relates to the strength function.

This model of one doorway coupled to the continuum and of N hallways is easily generalized to the case in which the hallways are also coupled to the continuum. A characteristic asymmetry in the pattern of fine structure is shown to result under certain circumstances. The width distribution has a width approximately equal to the spreading width, but with a zero on one side or the other of the doorway. The distribution of widths does not agree with that found by Robson.[13]