Conductivity of a Plasma in a Steady Magnetic Field

T. PRADHAN AND B. DASGUPTA
Saha Institute of Nuclear Physics, Calcutta, India

(Received 17 March 1967)

We derive expressions for the complex conductivity tensor of a homogeneous classical plasma in an external uniform magnetic field, in terms of electric field correlations, using the Kubo theory of transport phenomena. The main aim is to bring out explicitly the magnetic field dependence of the conductivity tensor. Exact relations between the conductivity tensor in the presence of the magnetic field and the same tensor in the absence of the magnetic field have been obtained.

I. INTRODUCTION

THIS work is based on the general theory of conductivity developed by Kubo and others.[1] The conductivity tensor can be rigorously expressed as a time integral (correlation function) over the spontaneous fluctuation of the current in the system. Using this formalism Kubo *et al.*[2] have obtained a relation between zero-frequency conductivity and the electric field correlation for a quantum plasma. We propose to obtain exact relationships between conductivity and electric field correlation for all frequencies for a classical plasma. We start with the Kubo formula for conductivity expressed as a velocity correlation which can be written as a certain integral in phase space over the Green's function for the Liouville equation. By making use of this equation we reduce the velocity correlations to electric field correlations. From the resultant expressions we obtain a relation between the complex conductivity tensor in the presence of the magnetic field and the same tensor in the absence of the magnetic field.

II. REDUCTION OF VELOCITY CORRELATION TO ELECTRIC FIELD CORRELATION

We consider a fully ionised homogeneous plasma in local thermodynamic equilibrium. The electrons move against the background of uniformly distributed smeared-out static positive ions. The magnetic field is taken along the Z axis. In the framework of Kubo theory, the conductivity tensor of such a plasma is given by[3]

$$\sigma_{\mu\nu}(z)=e^2\beta\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' v_{r\mu}R(x,v\,|\,x',v';z)v_{s\nu}', \quad (1)$$

where $\mathbf{v}_r=(1/m)[\mathbf{p}_r-(e/c)\mathbf{A}(\mathbf{x}_r)]$ [in which $\mathbf{p}_r$= momentum of the rth electron and $\mathbf{A}(\mathbf{x}_r)=\frac{1}{2}\mathbf{B}\times\mathbf{x}_r$ with $\mathbf{B}$= magnetic field strength]; $(x,v)\equiv(\mathbf{x}_1,\mathbf{v}_1;\mathbf{x}_2,\mathbf{v}_2;\cdots\mathbf{x}_N,\mathbf{v}_N)$; $d\Gamma=(dxdv)$, the element of volume in phase space; $\beta=(kT)^{-1}$; and

$$R(x,v\,|\,x',v';z)=G(x,v\,|\,x',v';z)\times f_N(x',v')$$

[in which $G(x,v\,|\,x',v';z)$ is the Fourier transform of Green's function of the Liouville operator of the system of interacting N-electrons and $f_N(x,v)$ is the equilibrium distribution function]. We shall find it more convenient to work with $\sigma_{++}(z)$, $\sigma_{+-}(z)$, $\sigma_{-+}(z)$, and $\sigma_{--}(z)$ rather than $\sigma_{11}(z)$, $\sigma_{21}(z)$, $\sigma_{12}(z)$ and $\sigma_{22}(z)$. In the expression for these tensor components, $v_\pm=v_1\pm iv_2$ are used in places of v_1, v_2 etc. The two sets are related by

$$\begin{aligned}\sigma_{++}&=\sigma_{11}-\sigma_{22}+i(\sigma_{12}+\sigma_{21}),\\ \sigma_{+-}&=\sigma_{11}+\sigma_{22}-i(\sigma_{12}-\sigma_{21}),\\ \sigma_{-+}&=\sigma_{11}+\sigma_{22}+i(\sigma_{12}-\sigma_{21}),\\ \sigma_{--}&=\sigma_{11}-\sigma_{22}-i(\sigma_{12}+\sigma_{21}).\end{aligned} \quad (2a)$$

Because of the symmetry of the problem $\sigma_{11}=\sigma_{22}$. Therefore

$$\begin{aligned}\sigma_{11}&=\tfrac{1}{4}(\sigma_{+-}+\sigma_{-+}),\\ \sigma_{12}&=(1/2i)[\sigma_{++}-\tfrac{1}{2}(\sigma_{+-}-\sigma_{-+})],\\ \sigma_{21}&=(1/2i)[\sigma_{++}+\tfrac{1}{2}(\sigma_{+-}-\sigma_{-+})].\end{aligned} \quad (2b)$$

Our main aim is to bring out explicitly the magnetic field dependence of the right-hand side of Eq. (1). For this we shall make use of the equations

$$\left\{iz+\sum_{j=1}^{N}\left[\mathbf{v}_j\cdot\frac{\partial}{\partial\mathbf{x}_j}+\frac{e}{m}\sum_{i\neq j=1}^{N}\left(\frac{\partial}{\partial\mathbf{x}_j}\frac{e}{|\mathbf{x}_i-\mathbf{x}_j|}+c^{-1}\mathbf{v}_j\times\mathbf{B}\right)\cdot\frac{\partial}{\partial\mathbf{v}_j}\right]\right\}R(x,v\,|\,x',v';z)=\delta(x-x')\delta(v-v')f_N(x,v), \quad (3a)$$

$$\left\{-iz+\sum_{j=1}^{N}\left[\mathbf{v}_j'\cdot\frac{\partial}{\partial\mathbf{x}_j'}+\frac{e}{m}\sum_{i\neq j=1}^{N}\left(\frac{\partial}{\partial\mathbf{x}_j'}\frac{e}{|\mathbf{x}_i'-\mathbf{x}_j'|}+c^{-1}\mathbf{v}_j'\times\mathbf{B}\right)\cdot\frac{\partial}{\partial\mathbf{v}_j'}\right]\right\}R(x,v\,|\,x',v';z)=-\delta(x-x')\delta(v-v')f_N(x',v'), \quad (3b)$$

[1] R. Kubo, J. Phys. Soc. (Japan) 12, 570 (1957); H. Nakano, Progr. Theoret. Phys. (Kyoto) 15, 77 (1956); 17, 145 (1957); M. S. Green, J. Chem. Phys. 20, 1281 (1952); 22, 398 (1954).
[2] R. Kubo, H. Hasegawa, and N. Hashitsume, J. Phys. Soc. (Japan) 14, 56 (1959).
[3] We use a form of Kubo theory given by S. F. Edwards and J. J. Sanderson [Phil. Mag. 6, 71 (1961)] and by R. Balescu [*Statistical Mechanics of Charged Particles* (Interscience Publishers, Inc., New York, 1963), Chap. 13].

which are obtained from the Liouville equations for the Green's functions and the distribution $f_N(x,v)$. $\delta(x-x')$ stands for the product

$$\delta(\mathbf{x}_1-\mathbf{x}_1')\delta(\mathbf{x}_2-\mathbf{x}_2')\cdots\delta(\mathbf{x}_N-\mathbf{x}_N').$$

If we multiply Eq. (3a) by v_{r1} and then integrate over $d\Gamma$ and similarly multiply Eq. (3b) by v_{s1}' and then integrate over $d\Gamma'$, we obtain

$$\int d\Gamma\left[izv_{r1}-\frac{e}{m}E_1(\mathbf{x}_r)-\Omega v_{r2}\right]\times R(x,v\mid x',v';z)=v_{r1}'f_N(x',v'), \quad (4a)$$

$$\int d\Gamma'\left[izv_{s1}'+\frac{e}{m}E_1(\mathbf{x}_s)+\Omega v_{s2}'\right]\times R(x,v\mid x',v';z)=v_{s1}f_N(x,v), \quad (4b)$$

where

$$\mathbf{E}(\mathbf{x}_r)=\sum_{n\neq r}(\partial/\partial\mathbf{x}_r)(e/|\mathbf{x}_r-\mathbf{x}_n|), \quad \text{and} \quad \Omega=eB/mc.$$

From these two equations and similar ones for the other components we get

$$i(z\pm\Omega)\int d\Gamma v_{r\pm}R(x,v\mid x',v';z)=\frac{e}{m}\int d\Gamma E_{\pm}(\mathbf{x}_r)R(x,v\mid x',v';z)+v_{r\pm}'f_N(x',v'), \quad (4c)$$

$$i(z\mp\Omega)\int d\Gamma' v_{s\pm}'R(x,v\mid x',v';z)=-\frac{e}{m}\int d\Gamma' E_{\pm}(\mathbf{x}_s')R(x,v\mid x',v';z)+v_{s\pm}f_N(x,v). \quad (4d)$$

To obtain $\sigma_{++}(z)$, we multiply Eq. (4c) (upper sign) by $v_{s+}'e^2\beta[i(z+\Omega)]^{-1}$, integrate over $d\Gamma$ and sum over r and s, we obtain

$$\sigma_{++}(z)=\frac{e^3\beta}{im(z+\Omega)}\times\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_+(\mathbf{x}_r)R(x,v\mid x',v';z)v_s'_+ +\frac{e^2\beta}{i(z+\Omega)}\sum_{r,s=1}^{N}\int d\Gamma' v_{r+}'v_{s+}'f_N(x',v'), \quad (z\neq-\Omega). \quad (5)$$

If we now make use of Eq. (4d) (upper sign) in the first term on the right-hand side of Eq. (5) we get

$$\sigma_{++}(z)=\frac{e^4\beta}{m^2}(z^2-\Omega^2)^{-1}\times\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_+(\mathbf{x}_r)R(x,v\mid x',v';z)E_+(\mathbf{x}_s') -e^2\beta\frac{i}{(z+\Omega)}\sum_{r,s=1}^{N}\int d\Gamma' v_{r+}'v_{s+}'f_N(x',v') -\frac{e^3\beta}{m}(z^2-\Omega^2)^{-1}\sum_{r,s=1}^{N}\int d\Gamma\, E_+(\mathbf{x}_r)v_{s+}f_N(x,v), \quad (z\neq\pm\Omega). \quad (6a)$$

Since

$$f_N(x,v)=\exp(-\beta H)=\exp[-\tfrac{1}{2}\sum_{i=1}^{N}mv_i^2+\sum_{i<j}V_{ij}(|\mathbf{x}_i-\mathbf{x}_j|)]$$

is an even function of velocities, the last two terms on the right-hand side of the above equation are zero. Therefore

$$\sigma_{++}(z)=\frac{e^4\beta}{m^2}(z^2-\Omega^2)^{-1}\times\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_+(\mathbf{x}_r)R(x,v\mid x',v';z)E_+(\mathbf{x}_s'), \quad (z\neq\pm\Omega). \quad (6b)$$

It should be noted that we started with an expression for $\sigma_{++}(z)$ which is written in terms of velocity correlation. By our reduction technique it now appears as an electric field correlation. In this process magnetic field dependence has been extracted out explicitly though not fully. The advantage of this reduction will be seen later. Following similar method one can bring out the reduction of $\sigma_{+-}(z)$ and $\sigma_{-+}(z)$. We shall give the final results without going through the details:

$$\sigma_{+-}(z)=\frac{e^4\beta}{m^2}\frac{1}{(z+\Omega)^2}\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_+(\mathbf{x}_r)R(x,v\mid x',v';z)E_-(\mathbf{x}_s')-e^2\beta\frac{i}{z+\Omega}\sum_{r=1}^{N}\int d\Gamma v_{r+}v_{r-}f_N(x,v), \quad (z\neq-\Omega), \quad (6c)$$

$$\sigma_{-+}(z)=\frac{e^4\beta}{m^2}\frac{1}{(z-\Omega)^2}\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_-(\mathbf{x}_r)R(x,v\mid x',v';z)E_+(\mathbf{x}_s')-e^2\beta\frac{i}{z-\Omega}\sum_{r=1}^{N}\int d\Gamma v_{r+}v_{r-}f_N(x,v), \quad (z\neq\Omega). \quad (6d)$$

Using Eq. (2b) we get the expressions for the Cartesian components

$$\sigma_{11}(z)=\frac{e^4\beta}{m^2}\frac{(z^2+\Omega^2)}{(z^2-\Omega^2)^2}\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_1(\mathbf{x}_r)R(x,v\,|\,x',v';x)E_1(\mathbf{x}_s')$$
$$+\frac{e^4\beta}{m^2}\frac{iz\Omega}{(z^2-\Omega^2)^2}\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma'[E_1(\mathbf{x}_r)R(x,v\,|\,x',v';z)E_2(\mathbf{x}_s')-E_2(\mathbf{x}_r)R(x,v\,|\,x',v';z)E_1(\mathbf{x}_s')]$$
$$-e^2\beta\frac{iz}{2(z^2-\Omega^2)}\sum_{r=1}^{N}\int d\Gamma v_{r\perp}^2 f_N(x,v)\qquad (v_{r\perp}^2=v_{r1}^2+v_{r2}^2,\ z\neq\pm\Omega),\tag{7a}$$

$$\sigma_{12}(z)=-\frac{e^4\beta}{m^2}\frac{2iz\Omega}{(z^2-\Omega^2)^2}\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_1(\mathbf{x}_r)R(x,v\,|\,x',v';z)E_1(\mathbf{x}_s')$$
$$+\frac{e^4\beta}{m^2}\frac{z^2}{(z^2-\Omega^2)^2}\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_1(\mathbf{x}_r)R(x,v\,|\,x',v';z)E_2(\mathbf{x}_s')$$
$$-\frac{e^4\beta}{m^2}\frac{\Omega^2}{(z^2-\Omega^2)^2}\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_2(\mathbf{x}_r)R(x,v\,|\,x',v';z)E_1(\mathbf{x}_s')$$
$$-e^2\beta\frac{\Omega}{2(z^2-\Omega^2)}\sum_{r=1}^{N}\int d\Gamma v_{r\perp}^2 f_N(x,v)\qquad (z\neq\pm\Omega),\tag{7b}$$

$$\sigma_{21}(z)=\frac{e^4\beta}{m^2}\frac{2iz\Omega}{(z^2-\Omega^2)^2}\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_1(\mathbf{x}_r)R(x,v\,|\,x',v';z)E_1(x_s')$$
$$-\frac{e^4\beta}{m^2}\frac{\Omega^2}{(z^2-\Omega^2)^2}\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_1(\mathbf{x}_r)R(x,v\,|\,x',v';z)E_2(\mathbf{x}_s')$$
$$+\frac{e^4\beta}{m^2}\frac{z^2}{(z^2-\Omega^2)^2}\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_2(\mathbf{x}_r)R(x,v\,|\,x',v';z)E_1(\mathbf{x}_s')$$
$$+e^2\beta\frac{\Omega}{2(z^2-\Omega^2)}\sum_{r=1}^{N}\int d\Gamma v_{r\perp}^2 f_N(x,v)\qquad (z\neq\pm\Omega).\tag{7c}$$

For σ_{31}, σ_{32}, σ_{13}, and σ_{33} we use two more equations,

$$\int d\Gamma\left[izv_{r3}-\frac{e}{m}E_3(\mathbf{x}_r)\right]R(x,v\,|\,x',v';z)=v_{r3}'f_N(x',v')$$

and

$$\int d\Gamma'\left[izv_{s3}'+\frac{e}{m}E_3(\mathbf{x}_s')\right]R(x,v\,|\,x',v';z)=v_{s3}f_N(x,v),$$

similar to Eqs. (4a) and (4b) and obtain

$$z^2\sigma_{33}(z)=\frac{e^4\beta}{m^2}\times\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_3(\mathbf{x}_r)R(x,v\,|\,x',v';z)E_3(\mathbf{x}_s')$$
$$-ie^2\beta z\sum_{r=1}^{N}\int d\Gamma v_{r3}^2 f_N(x,v),\tag{8a}$$

$$\sigma_{\mu\nu}(z)=\frac{e^4\beta}{m^2}\cdot(z^2-\Omega^2)^{-1}\times\sum_{r,s=1}^{N}\iint d\Gamma d\Gamma' E_\mu(\mathbf{x}_r)R(x,v\,|\,x',v';z)E_\nu(\mathbf{x}_s'),$$
$$(\mu=3;\nu=1,2;\mu=1,2;\nu=3).\tag{8b}$$

As remarked earlier, so far we have been partly successful in bringing out the explicit Ω dependence of the components of the conductivity tensor. This is because the explicit Ω-dependence of $R(x,v\,|\,x',v';z)$ is not known. However with the aid of Eq. (8a) it is possible to show that the integral $\iint dvdv'R(x,v\,|\,x',v';z)$ is independent of magnetic field. This is shown as follows: If we apply an external electric field $\mathcal{E}_3$ along the direction of the magnetic field, the current along the electric field is given by

$$J_3(z)=\sigma_{33}(z)\mathcal{E}_3(z).$$

We make use of the fact that this current is not affected by the magnetic field. Therefore $\sigma_{33}(z)$ must be independent of Ω. Expression for $\sigma_{33}(z)$ in Eq. (8a) clearly shows that it is possible only if $\iint dvdv'R(x,v\,|\,x',v';z)$ is independent of Ω because

$$\sum_{r=1}^{N}\int d\Gamma v_{r3}^2 f_N(x,v)$$

is independent of Ω. That $\iint dvdv'R(x,v\,|\,x',v';z)$ is independent of Ω can be shown in another way. In the absence of magnetic field this integral is

$\int\int dv^* dv^{*\prime} R(x^*, v^* \mid x^{*\prime}, v^{*\prime}; z)$ where $\mathbf{v}^* = \mathbf{v} - e\mathbf{A}/c$. It is readily seen that these two integrals are equal because the Jacobian of the transformation of the variables is unity.[4] Therefore the only Ω-dependence of the components of conductivity tensor are those that appear explicitly in the expressions given by Eqs. (7) and (8). We have thus achieved our goal of bringing out the explicit magnetic field dependence of the conductivity tensor.

Before ending this section we shall simplify the expressions for the conponents of the conductivity tensor by using the conclusion obtained above and the Onsager symmetry relation

$$\sigma_{\mu\nu}(z, \Omega) = \sigma_{\nu\mu}(z, -\Omega).$$

Since the integral $\int\int dv dv' R(x, v \mid x', v'; z)$ is independent of Ω, we must have

$$\sum_{r,s=1}^{N} \iint d\Gamma d\Gamma' E_1(\mathbf{x}_r) R(x, v \mid x', v'; z) E_2(\mathbf{x}_s') = \sum_{r,s=1}^{N} \iint d\Gamma d\Gamma' E_2(\mathbf{x}_r) R(x, v \mid x', v'; z) E_1(\mathbf{x}_s'),$$

$$\sum_{r,s=1}^{N} \iint d\Gamma d\Gamma' E_{1,2}(\mathbf{x}_r) R(x, v \mid x', v'; z) E_3(\mathbf{x}_s') = \sum_{r,s=1}^{N} \iint d\Gamma d\Gamma' E_3(\mathbf{x}_r) R(x, v \mid x', v'; z) E_{1,2}(\mathbf{x}_s'),$$

for the expressions (7b), (7c), and (8b) to be compatible with the above symmetry relation. Using these identities in Eqs. (7) we get the simplified expressions.

$$\sigma_{11}(z) = \frac{e^4\beta}{m^2} \frac{(z^2+\Omega^2)}{(z^2-\Omega^2)^2} \sum_{r,s=1}^{N} \iint d\Gamma d\Gamma' E_1(\mathbf{x}_r) R(x, v \mid x', v'; z) E_1(\mathbf{x}_s') - e^2\beta \frac{iz}{2(z^2-\Omega^2)} \sum_{r=1}^{N} \int d\Gamma v_{r\perp}^2 f_N(x, v), \tag{9a}$$

$$\begin{aligned}\sigma_{12}(z) = &-\frac{e^4\beta}{m^2} \frac{2iz\Omega}{(z^2-\Omega^2)^2} \sum_{r,s=1}^{N} \iint d\Gamma d\Gamma' E_1(\mathbf{x}_r) R(x, v \mid x', v'; z) E_1(\mathbf{x}_s') \\ &+ \frac{e^4\beta}{m^2} (z^2-\Omega^2)^{-1} \sum_{r,s=1}^{N} \iint d\Gamma d\Gamma' E_1(\mathbf{x}_r) R(x, v \mid x', v'; z) E_2(\mathbf{x}_s') - e^2\beta \frac{\Omega}{2(z^2-\Omega^2)} \sum_{r=1}^{N} \int d\Gamma v_{r\perp}^2 f_N(x, v) \\ &= \sigma_{21}(z, -\Omega).\end{aligned} \tag{9b}$$

But since $\sigma_{12}(z)$ should vanish for $\Omega = 0$, we must have

$$\iint d\Gamma d\Gamma' E_1(\mathbf{x}_r) R(x, v \mid x', v'; z) E_2(\mathbf{x}_s') = 0.$$

Hence

$$\sigma_{12}(z) = -\frac{e^4\beta}{m^2} \frac{2iz\Omega}{(z^2-\Omega^2)^2} \sum_{r,s=1}^{N} \iint d\Gamma d\Gamma' E_1(\mathbf{x}_r) R(x, v \mid x', v'; z) E_1(x_s') - e^2\beta \frac{\Omega}{2(z^2-\Omega^2)} \sum_{r=1}^{N} \int d\Gamma v_{r\perp}^2 f_N(x, v). \tag{9c}$$

III. MAGNETIC FIELD DEPENDENCE OF THE COMPLEX CONDUCTIVITY TENSOR

From these one can easily write down the expressions for the conductivity tensor in the absence of magnetic field. One can then see that the conductivity tensor in the presence of the magnetic field is related in a very simple way to the same tensor in the absence of the magnetic field. We give these relations below:

$$\sigma_{11}(z) = \frac{(z^2+\Omega^2)}{(z^2-\Omega^2)^2} \sigma_{11}^{\Omega=0}(z) + 2e^2\beta \frac{iz\Omega^2}{(z^2-\Omega^2)^2} \sum_{r=1}^{N} \int d\Gamma v_{r\perp}^2 f_N(x, v), \tag{10a}$$

$$\sigma_{12}(z) = -\frac{2iz^3\Omega}{(z^2-\Omega^2)^2} \sigma_{11}^{\Omega=0}(z) + e^2\beta \frac{(z^2+\Omega^2)\Omega}{(z^2-\Omega^2)^2} \sum_{r=1}^{N} \int d\Gamma v_{r\perp}^2 f_N(x, v), \tag{10b}$$

$$\sigma_{21}(z) = \frac{2iz^3\Omega}{(z^2-\Omega^2)^2} \sigma_{11}^{\Omega=0}(z) - e^2\beta \frac{(z^2+\Omega^2)\Omega}{(z^2-\Omega^2)^2} \sum_{r=1}^{N} \int d\Gamma v_{r\perp}^2 f_N(x, v), \tag{10c}$$

[4] A similar argument is made in proving Van Leeuwen's theorem on the absence of diamagnetism of a classical electron gas. See, for example, J. H. Van Vleck, *Theory of Electric and Magnetic Susceptibilities* (Oxford University Press, London, 1952).

and

$$\sigma_{13}(z)=\frac{z^2}{z^2-\Omega^2}\sigma_{13}{}^{\Omega=0}(z)=0.$$

Similarly $\sigma_{31}(z)$, $\sigma_{23}(z)$, $\sigma_{32}(z)$ all are zero because, conductivity tensor in the absence of magnetic field is diagonal.

ACKNOWLEDGMENT

The second author (BD) would like to express his gratefulness to the Department of Atomic Energy, Government of India for the grant of a senior fellowship for the investigations presented in this work.

PHYSICAL REVIEW VOLUME 160, NUMBER 1 5 AUGUST 1967

High-Frequency Waves in a Collisional Plasma with Magnetic Field

B. Buti*

National Aeronautics and Space Administration, Laboratory for Theoretical Studies, Goddard Space Flight Center, Greenbelt, Maryland

(Received 19 January 1967)

The damping of electron-plasma oscillations, in a hot plasma in an external uniform magnetic field in the presence of weak Coulomb collisions, is investigated by using the Fokker-Planck equation. The electron-ion collisions play the dominant role; nevertheless, the electron-electron collisions become important as the wavelength decreases from infinity. As far as the electron-ion-collision contribution is concerned, the frictional term exceeds the diffusion term; but in the electron-electron case, both the frictional and the diffusion contributions are of the same order. The two-body Coulomb collisions have a stabilizing effect on these plasma waves; the magnetic field, however, does not affect the longitudinal waves, but has a tendency to stabilize the left-handed polarized wave and to destabilize the right-handed polarized wave.

I. INTRODUCTION

THE small-amplitude oscillations in a fully ionized plasma in a uniform external magnetic field were studied by Bernstein.[1] He showed that in a collision-free plasma, the self-excitation of the waves around thermal equilibrium is not possible.

Comisar[2] and Buti and Jain[3] studied the high-frequency plasma waves in a hot plasma in the absence of any external magnetic field, but they took into account the weak Coulomb collisions by using the Fokker-Planck equation of Rosenbluth *et al.*[4] They found that the electron-ion collisions play a more important role in damping the longitudinal as well as the transverse waves; the electron-electron collisions have to be taken into account only if one is interested in finite-wavelength disturbances.

The wave motion in a plasma, where the collisions are too frequent and the applied magnetic field is strong, has been studied by Oppenheim[5] and Liboff[6] using the models known as the isotropic Fokker-Planck model and the Liboff-Krook model, respectively. Both predicted an infinite number of Larmor resonances; in addition, Oppenheim's model described the diffusion process in velocity space. In the cold-plasma regime and the long-wavelength-magnetohydrodynamic regime, to lowest order, these two models gave the same results.

Following Comisar and Buti and Jain, we consider the effect of an external uniform weak magnetic field on the plasma waves when the collisions are not too frequent, which allows us to neglect the many-body collisions. The magnetic field B_0 does not affect the nature of the collisions, provided the Larmor radius R_L is much larger than the Debye length λ_D, i.e., if the plasma frequency ω_p is much larger than the electron cyclotron frequency $\Omega=eB_0/mc$. For such small magnetic fields, the radiation is also negligible; so the Fokker-Planck coefficients remain unaltered and we can use the Fokker-Planck equation of Rosenbluth *et al.*[4] In this study, we take the contributions of the frictional and the diffusion terms separately, both for the electron-electron and the electron-ion collisions; in the former case both contributions are of the same order, but in the latter case frictional contribution is much larger than that of diffusion, which is comparable to the contributions caused by electron-electron collisions. The magnetic field as well as the collisions tend to stabilize the system under consideration.

It is, perhaps, proper to remark that the Fokker-Planck equation of Rosenbluth *et al.* is not strictly valid for high frequencies, particularly near the electron-plasma frequency.[7,8] It was shown by Price[9] that this equation is correct to the order $(1/\ln\Lambda)$. Strictly speak-

* National Academy of Sciences—National Research Council Resident Research Associate.

[1] I. B. Bernstein, Phys. Rev. **109**, 10 (1958).
[2] G. G. Comisar, Phys. Fluids **6**, 76 (1963); **6**, 1660 (1963).
[3] B. Buti and R. K. Jain, Phys. Fluids **8**, 2080 (1965).
[4] M. N. Rosenbluth, W. M. MacDonald, and D. L. Judd, Phys. Rev. **107**, 1 (1957).
[5] A. Oppenheim, Phys. Fluids **8**, 900 (1965).
[6] R. L. Liboff, Phys. Fluids **5**, 963 (1962).
[7] C. Oberman and J. Dawson, Phys. Fluids **5**, 517 (1962).
[8] C. Oberman, A. Ron, and J. Dawson, Phys. Fluids **5**, 1514 (1962).
[9] J. Price, Phys. Fluids **9**, 2408 (1966).