Radiative Decays of Hadrons and Current Conservation

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Using electric current conservation, it is shown that in the series expansion of the amplitudes of radiative decays (with two particles at most in the final state, apart from the photon) in powers of the photon four-momentum, terms of order zero vanish unless they belong to photon pole diagrams.

I. INTRODUCTION

IN the present paper,[1] we derive from electric current conservation a low-energy theorem for radiative decays of hadrons. The theorem says: If the number of particles in the final state of the radiative decay is two at most, excluding the photon, then terms of order zero in the series expansion of the amplitude (in powers of the photon four-momentum k) are zero unless they belong to the photon pole diagrams [Fig. 2(a) and 2(b)]. It thus applies to the decays

$$K^{\pm}\to\pi^{\pm}+\pi^{0}+\gamma\,,$$
$$\Sigma^{+}\to p+\pi^{0}+\gamma\,,$$
$$\Sigma^{\pm}\to n+\pi^{\pm}+\gamma\,,$$

but *not* to

$$K^{\pm}\to\pi^{\pm}+\pi^{+}+\pi^{-}+\gamma\,.$$

We use the method developed by Low in 1958.[2,3] Low has shown that the amplitude for any collision process describing emission of a photon (bremsstrahlung) of for four-momentum k has terms of order k^{-1} (infrared divergent term) and of order zero in k, which depends only on the corresponding amplitude without bremsstrahlung and on the electromagnetic constants of the participating particles. This result comes from the fact that the divergence of the electric current is measurable.[3]

Let us consider the radiative decay

$$A\to B+\gamma\,,$$

where A and B are hadrons. The corresponding S-matrix element

$$\langle B\gamma|S|A\rangle=ie(2\pi)^{4}\delta^{(4)}(p_A-p_B-k)(2\pi)^{-3/2}(2k_0)^{-1/2}\times N_AN_B\epsilon^{\mu}M_{\mu}(k)\,. \quad (1)$$

Here p_A, p_B, and k are the respective four-momenta of A, B, and γ; N_A and N_B are the normalization factors of the particles represented by A (e.g., $K^{\pm}$) and B (e.g., $\pi^{\pm}+\pi^{0}$); ϵ_μ is the photon polarization, e is the proton electric charge, and $\epsilon^{\mu}M_{\mu}(k)$ is the bremsstrahlung matrix element.

$M_\mu(k)$ can be divided in two parts:

$$M_\mu(k)=M_\mu^{\rm ext}(k)+M_\mu^{\rm int}(k)\,. \quad (2)$$

$M_\mu^{\rm ext}$ is the sum of those terms when the photon is emitted from an external charged line [Fig. 2(a) and 2(b)]. $M_\mu^{\rm int}(k)$ is the sum of the other terms [Fig. 2(c)]. As $k\to 0$, $M_\mu^{\rm ext}\sim k^{-1}$, and $M_\mu^{\rm int}$ is a constant independent of how $k\to 0$. Therefore, if $M_\mu(k)$ is expanded with respect to k, then

$$M_\mu^{\rm int}(k)=M_\mu^{\rm int}(0)+O(k)\,. \quad (3)$$

$$M_\mu^{\rm ext}(k)=(\text{terms of order } k^{-1}) + (\text{terms of order zero})+O(k)\,. \quad (4)$$

Terms of order k^{-1} come from the propagators of the charged hadrons. $M_\mu^{\rm int}(k)$ has no term of the form k^{-1}, because here the photon vertex is never on a free line.

From the electric current conservation,

$$k^{\mu}M_\mu(k)=0\,, \quad (5)$$

and the relations (2), (3), and (4), the theorem already quoted follows. This will be proved in detail in Sec. II.

First, we shall treat radiative decays of K mesons when no difficulties from the spin arise. We will point out the difference between systems with two and three bodies (apart from the photon) in the final state, that is, here, between $K^{\pm}\to\pi^{\pm}\pi^{0}\gamma$ and $K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0}\gamma$. Afterwards, we shall treat the radiative pionic decays of hyperons. In Sec. III, we relate the pure radiative decay matrix elements to the corresponding weak vertex (e.g., $\Sigma^{+}\to p\gamma$ to $\Sigma^{+}\to p$; Figs. 3 and 4) in the limit of soft-photon emission,[4,5] and we relate the pion photoproduction to the corresponding strong vertex (e.g., $N+\gamma\to N'+\pi$ to $N\to N'+\pi$) in soft-pion emission.[6] The conclusions of the present paper are summarized in the last section.

II. RADIATIVE PIONIC DECAYS

(*a*) $K^{\pm}(p)\to\pi^{0}(q_1)+\pi^{\pm}(q_2)+\gamma(k)$. We have written in parentheses the respective four-momenta. Let $T(P^2,Q_1^2,Q_2^2)$ be the generalized amplitude for the corresponding *nonradiative* decay:

$$K^{\pm}(P)\to\pi^{0}(Q_1)+\pi^{\pm}(Q_2)\,.$$

The amplitude T conserves momentum and energy but

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[1] We use the conventions of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Company, New York, 1964).

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[3] S. L. Adler and Y. Dothan, Phys. Rev. **151**, 1267 (1966).

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[6] N. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954).

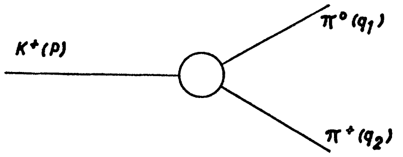

FIG. 1. Nonradiative decay with $K^+=A$ and π^0, $\pi^+=B$.

not mass. Then T is expressed as a function of generalized masses. The physical nonradiative decay amplitude is $T(m^2,m_1^2,m_2^2)$ (Fig. 1), where m, m_1, m_2 are, respectively, the masses of $K^\pm$, π^0, $\pi^\pm$. The nonradiative part of the process in Fig. 2(a) is described by $T((p-k)^2, m_1^2,m_2^2)$ which is off the mass shell. In Fig. 2(b), the corresponding amplitude is $T(m^2,m_1^2,(q_2+k)^2)$.

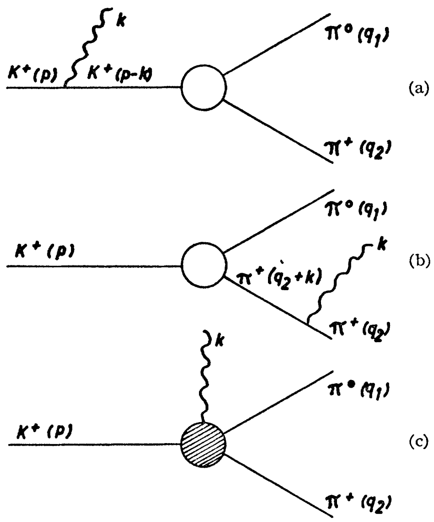

FIG. 2. (a,b) Photon pole diagrams of radiative decay with two bodies, excluding the photon, in the final state. (c) Direct contribution to the same decay.

Now, $M_\mu^{\text{ext}}(k)$ in terms of $T(P^2,Q_1^2,Q_2^2)$ becomes

$$M_\mu^{\text{ext}}(k)=\pm\left[\frac{(2q_2+k)_\mu}{(q_2+k)^2-m_2^2}T(m^2,\,m_1^2,\,(q_2+k)^2)+T((p-k)^2,\,m_1^2,\,m_2^2)\frac{(2p-k)_\mu}{(p-k)^2-m^2}\right]. \tag{6}$$

Here we use $\pm$ for $K^\pm$ decay. If we expand $M_{\text{ext}}^\mu(k)$ with respect to k and use the conditions $k^2=0$, $k\cdot\epsilon=0$, we find

$$M_\mu^{\text{ext}}(k)=\pm\left[\left(\frac{q_{2\mu}}{q_2\cdot k}-\frac{p_\mu}{p\cdot k}\right)T(m^2,m_1^2,m_2^2)+2q_{2\mu}\frac{\partial T(m^2,m_1^2,Q_2^2)}{\partial Q_2^2}\bigg|_{Q_2^2=m_2^2}+2p_\mu\frac{\partial T(P^2,m_1^2,m_2^2)}{\partial P^2}\bigg|_{P^2=m^2}\right]+O(k). \tag{7}$$

From (2), (3), (5), and (7), we obtain

$$M_\mu^{\text{int}}(0)=\pm\left[-2q_{2\mu}\frac{\partial T}{\partial Q_2^2}\bigg|_{Q_2^2=m_2^2}-2p_\mu\frac{\partial T}{\partial P^2}\bigg|_{P^2=m^2}\right] \tag{8}$$

and

$$M_\mu(k)=\pm\left[\frac{q_{2\mu}}{q_2\cdot k}-\frac{p_\mu}{p\cdot k}\right]T(m^2,m_1^2,m_2^2)+O(k). \tag{9}$$

(*b*) $K^\pm(p)\to\pi^\pm(q_1)\pi^0(q_2)\pi^0(q_3)\gamma(k)$. Again, we write the generalized amplitude of the nonradiative process $K^\pm(P)\to\pi^\pm(Q_1)\pi^0(Q_2)\pi^0(Q_3)$:

$$T(s=(P+Q_2)^2,\,t=(Q_2+Q_3)^2,\,P^2,\,Q_1^2,\,Q_2^2,\,Q_3^2).$$

Here there are other variables besides the masses. In the same way as in Sec. II(a), we can find the terms of order k^{-1} and order zero:

$$M_\mu(k)=\left[\pm\frac{q_{1\mu}}{q_1\cdot k}\mp\frac{p_\mu}{p\cdot k}\right]T((p+q_2)^2,\,(q_2+q_3)^2,\,m^2,\,m_1^2,\,m_1^2,\,m_1^2)\pm\left[\frac{2p_\mu}{p\cdot k}q_2\cdot k-2q_{2\mu}\right]\times\frac{\partial T((P+Q_2)^2,\,(q_2+q_3)^2,\,m^2,\,m_1^2,\,m_1^2,\,m_1^2)}{\partial s}\bigg|_{s=(p+q_2)^2}+O(k). \tag{10}$$

m and m_1 are, respectively, the masses of $K^\pm$ and π. We see that, in (10), terms of order zero appear, because (b) has three particles, excluding the photon, in the final state.

(*c*) $\Sigma^+(p_\Sigma)\to n(p_n)+\pi^+(q)+\gamma(k)$. $M_\mu^{\text{ext}}(k)$ is expressed as a function of $T(P_n^2,P_\Sigma^2,Q^2)$, the nonradiative decay amplitude off the mass shell (it is here a 4×4 matrix):

$$M_\mu^{\text{ext}}(k)=u(p_n)\left[-\frac{i\sigma_{\mu\nu}k^\nu\mu_n}{2m_n}\frac{1}{(p_n+k)-m_n}T((p_n+k)^2,\,m_\Sigma^2,\,m^2)+\frac{2q_\mu+k_\mu}{(q+k)^2-m}T(m_n^2,\,m_\Sigma^2,\,(q+k)^2)+T(m_n^2,\,(p_\Sigma-k)^2,\,m^2)\frac{1}{(p_\Sigma-k)-m_\Sigma}\left(\gamma_\mu-\frac{i\sigma_{\mu\nu}k^\nu\mu_\Sigma}{2m_\Sigma}\right)\right]u(p_\Sigma). \tag{11}$$

m_Σ, m_n, and m are, respectively, the masses of Σ^+, n, and π^+; μ_n and μ_Σ are the anomalous magnetic moments of n and Σ^+.

Let us note that $T(P_n^2,P_\Sigma^2,Q^2)$ contains terms which do not contribute, in the limit $k\to 0$, when multiplied by a spinor. The most general form for T can be written:

$$T(P_n^2,P_\Sigma^2,Q^2)=\frac{P_n+W_n}{2W_n}T^A(P_n^2,P_\Sigma^2,Q^2)\frac{P_\Sigma+W_\Sigma}{2W_\Sigma}+\frac{-P_n+W_n}{2W_n}T^B(P_n^2,P_\Sigma^2,Q^2)\frac{P_\Sigma+W_\Sigma}{2W_\Sigma}$$
$$+\frac{P_n+W_n}{2W_n}T^C(P_n^2,P_\Sigma^2,Q^2)\frac{-P_\Sigma+W_\Sigma}{2W_\Sigma},\quad(12)$$
$$W_n=(P_n^2)^{1/2},\quad W_\Sigma=(P_\Sigma^2)^{1/2}.$$

$\bar{u}(p_n)T^A(m_n^2,m_\Sigma^2,m^2)u(p_\Sigma)$ is the physical matrix element of the nonradiative process. On inserting (12) into (11) and using the energy projection operators, the last term of the second member of (11) becomes

$$T(m_n^2,\,(p_\Sigma-k)^2,\,m^2)\frac{1}{(p_\Sigma-k)-m_\Sigma}=\left[T^A(m_n^2,\,(p_\Sigma-k)^2,\,m^2)\frac{(p_\Sigma-k)+W_\Sigma}{2W_\Sigma}+T^C(m_n^2,\,(p_\Sigma-k)^2,\,m^2)\frac{-(p_\Sigma-k)+W_\Sigma}{2W_\Sigma}\right]$$
$$\times\left[\frac{(p_\Sigma-k)+W_\Sigma}{-2p_\Sigma\cdot k}+\frac{m_\Sigma-W_\Sigma}{-2p_\Sigma\cdot k}\right]=T^A(m_n^2,\,(p_\Sigma-k)^2,\,m^2)\frac{p_\Sigma-k+m_\Sigma}{-2p_\Sigma\cdot k}+\text{(remaining terms of order zero at least)}.\quad(13)$$

The remaining terms have *no* singularity in $k=0$ because

$$\frac{m_\Sigma-W_\Sigma}{2p_\Sigma\cdot k}=\frac{1}{m_\Sigma+W_\Sigma}=\frac{1}{2m_\Sigma}+O(k).$$

The effect of the remaining terms may be treated as in $M_\mu^{\text{int}}(k)$. With the help of (2)–(5), (11), and the identity

$$\frac{1}{(p_\Sigma-k)-m_\Sigma}ku(p_\Sigma)=-u(p_\Sigma),$$

we then find

$$M_\mu(k)=\bar{u}(p_n)\left[-\frac{i\sigma_{\mu\nu}k^\nu\mu_n}{2m_n}\frac{1}{(p_n+k)-m_n}T^A(m_n^2,m_\Sigma^2,m^2)+\frac{q_\mu}{q\cdot k}T^A(m_n^2,m_\Sigma^2,m^2)\right.$$
$$\left.+T^A(m_n^2,m_\Sigma^2,m^2)\frac{1}{(p_\Sigma-k)-m_\Sigma}\left(\gamma_\mu-i\frac{\sigma_{\mu\nu}k^\nu\mu_\Sigma}{2m_\Sigma}\right)\right]u(p_\Sigma)+O(k).\quad(14)$$

Following (9) and (14), we obtain the rule for finding radiative decay amplitudes $M_\mu(k)$ (with two bodies, apart from the photon, in the final state) to order k^{-1} and zero:

$$M_\mu(k)=\text{(photon pole diagrams with } \textit{physical} \text{ nonradiative decay)}+O(k).\quad(15)$$

III. PURE RADIATIVE DECAYS OF HADRONS AND PION PHOTOPRODUCTION

(*a*) *Two-body decays of hyperons:* $B\to B'+\gamma$, e.g., $\Sigma^+(p_\Sigma)\to p(p_p)+\gamma(k)$. Using the same method as in Sec. II(c) and replacing the generalized nonradiative amplitude $T(P_n^2,P_\Sigma^2,Q^2)$ by the weak vertex $T(P_\Sigma^2=P_p^2)$ (Fig. 3), it is easy to show that there is no direct contribution [Fig. 4(c)] to order k^{-1} and zero and that we have the property[5]

$$T^A(m_\Sigma^2)=T^A(m_p^2).$$

We obtain

$$M_\mu(k)=\bar{u}(p_p)\left[\left(\gamma_\mu-\frac{i\sigma_{\mu\nu}k^\nu\mu_p}{2m_p}\right)\frac{1}{(p_p+k)-m_p}T^A(m_p^2)+T^A(m_p^2)\frac{1}{(p_\Sigma-k)-m_\Sigma}\left(\gamma_\mu-\frac{i\sigma_{\mu\nu}k^\nu\mu_\Sigma}{2m_\Sigma}\right)\right]u(p_\Sigma)+O(k).\quad(16)$$

Thus a pole model[4,5,7] is justified in the limit of soft-photon emission.

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[7] R. H. Graham and S. Pakvasa, Phys. Rev. **140**, B1144 (1965); K. Tanaka, *ibid.* **151**, 1203 (1966).

Now, we are going to use a model to show that, unlike pure radiative decays of hadrons (e.g., $\Sigma^+ \to p+\gamma$), pure pionic decays of hadrons [e.g., $\Sigma^+(p_\Sigma) \nrightarrow p(p_p)+\pi^0(k)$] have a direct contribution [Fig. 4(c)].[8] Let us consider the identity

$$\int d^4x\, \partial_\mu[e^{ik\cdot x}\langle p_p|T(J_\mu(x),H_w(0))|p_\Sigma\rangle]=ik_\mu\int d^4x\, e^{ik\cdot x}\langle p_p|T(J_\mu(x),H_w(0))|p_\Sigma\rangle$$
$$+\int d^4x\, e^{ik\cdot x}\delta(x_0)\langle p_p|[J_0(x),H_w(0)]|p_\Sigma\rangle+\int d^4x\, e^{ik\cdot x}\langle p_p|T(\partial_\mu J_\mu(x),H_w(0))|p_\Sigma\rangle=0\,, \quad (17)$$

where $H_w(0)$ is the weak Hamiltonian density represented by a product of Cabibbo currents and $J_\mu(x)$ is either $T_5{}^3{}_\mu(x)$, the third component of the isoaxial current, or $J_\mu{}^{\mathrm{el}}(x)$, the electromagnetic current. Now, if we use the $SU(3)\otimes SU(3)$ current commutation relations, the partial conservation of axial-vector currents (PCAC)

$$\partial_\mu T_5{}^3{}_\mu(x)=c\pi^0(x)\,,$$

(π^0 is the neutral pion field), and the identity (17), then we obtain in the limit of soft-pion emission[8]

$$\frac{ic}{m_\pi{}^2}\langle p_p,k|H_w(0)|p_\Sigma\rangle(k_0)^{1/2}=i\lim_{k\to 0}k_\mu\int d^4x\, e^{ik\cdot x}\langle p_p|T(T_5{}^3{}_\mu(x),H_w(0))|p_\Sigma\rangle+\tfrac{1}{2}\langle p_p|H_w(0)|p_\Sigma\rangle\,, \quad (18)$$

where $\langle p_p,k|H_w(0)|p_\Sigma\rangle$ is the decay amplitude of $\Sigma^+\to p+\pi^0$ in the soft-pion limit, and $\langle p_p|H_w(0)|p_\Sigma\rangle$ is equal to the weak vertex $\bar{u}(p_p)T^A(m_\Sigma{}^2)u(p_\Sigma)$ (Fig. 3). We observe that the limit procedure is unambiguous because we have seen that $T^A(m_\Sigma{}^2)=T^A(m_p{}^2)$. On the other hand, if we use the electromagnetic current conservation

$$\partial_\mu J_\mu{}^{\mathrm{el}}(x)=0\,,$$

the $SU(3)\otimes SU(3)$ current commutation relations, and the identity (17), then we obtain

$$\lim_{k\to 0}k_\mu\int d^4x\, e^{ik\cdot x}\langle p_p|T(J_\mu{}^{\mathrm{el}}(x),H_w(0))|p_\Sigma\rangle=0\,, \quad (19)$$

which is nothing other than the electric current conservation relation [Eq. (5)] in the soft-photon limit.

We see that pure pionic decay amplitudes of hyperons [Eq. (18)] have contributions coming from pole diagrams [Figs. 4(a) and 4(b)] as well as a direct contribution [Fig. 4(c)]. The first term of Eq. (18) corresponds to pion pole diagrams where the meson is emitted by the initial (Σ^+) or final baryon (p).[8] The second term is the direct contribution given by current algebra.[8] This latter appears because $T_5{}^3{}_0$ does not commute with H_w. We have no such term in (19) because $J_0{}^{\mathrm{el}}$ commutes with H_w. That is why we have no direct contribution in the soft-photon limit [Eq. (16)]. On the other hand, it has been shown[8] that $\langle p_p|H_w(0)|p_\Sigma\rangle$ is a scalar. So $\Sigma^+\to p+\gamma$ has only a p-wave contribution in the soft-photon limit.

(*b*) *Pion photoproduction:* $N+\gamma\to N'+\pi$. Using the same method as in Sec. II(c), we can find the pion photoproduction amplitude in the limit of soft-pion emission (threshold), i.e., the Kroll-Ruderman result.[6] It is given by the pole diagrams because there is no direct contribution (e.g., no seagull term) to order -1 and zero in the photon four-momentum.

Thus, if $T(P_n{}^2,P_p{}^2,Q^2)$ is the amplitude of the generalized strong vertex $p(P_p)\to n(P_n)+\pi^+(Q)$, then the amplitude of

$$p(p_p)+\gamma(k)\to n(p_n)+\pi^+(q)$$

is written

$$M_\mu(k)=\bar{u}(p_n)\left\{\frac{i\sigma_{\mu\nu}k^\nu\mu_n}{2m_n}\frac{1}{(p_n-k)-m_n}T^A(m_n{}^2,m_p{}^2,m^2)-\frac{q_\mu}{q\cdot k}T^A(m_n{}^2,m_p{}^2,m^2)+T^A(m_n{}^2,m_p{}^2,m^2)\right.$$
$$\left.\times\frac{1}{(p_p+k)-m_p}\left(\gamma_\mu+\frac{i\sigma_{\mu\nu}k^\nu\mu_p}{2m_p}\right)\right\}u(p_p)+O(k)\,, \quad (20)$$

where $T^A(m_n{}^2,m_p{}^2,m^2)$ is defined from the generalized strong vertex, as in Eq. (12).

[8] H. Sugawara, Phys. Rev. Letters **15**, 870, 997 (1965); M. Suzuki, *ibid.* **15**, 986 (1965); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* **16**, 380 (1966); L. S. Brown and C. M. Sommerfield, *ibid.* **16**, 751 (1966).

IV. SUMMARIES AND CONCLUSIONS

(1) We have proved that no terms of order zero, excluding the photon pole diagrams, appear in the radiative amplitudes with one or two bodies in the final state, apart from the photon. Thus, this theorem is

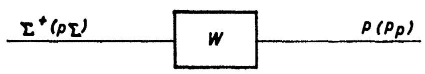

FIG. 3. Weak vertex $\Sigma^+ \rightarrow p$.

valid for the following decays:

$$B \rightarrow B' + \pi + \gamma\,,$$
$$\eta \text{ or } K \rightarrow \pi + \pi + \gamma\,,$$
$$\pi \text{ or } K \rightarrow l + \nu_l + \gamma\,.$$

(2) The *physical* nonradiative amplitude must be used to construct the photon pole terms correctly. In this way we have a radiative amplitude which conserves the electric current and which is exact to order k^{-1} and zero.

(3) Direct computations[9] made independently by Li and by Young, Sugawara, and Sakuma have shown, that the radiative-decay matrix elements of hyperons in the pole approximation of Feldmann, Matthews, and Salam[10] become essentially the same as those obtained from effective weak-interaction Hamiltonians of non-derivative or derivative type,[11] provided the photon momentum is not too large. Equation (14) shows that this result is completely general and *model-independent*. Therefore, because of electric current conservation, the radiative pionic decays of hyperons can give information about the structure of weak interactions only when the photon momentum is nearly at its maximum.

(4) In the limit where the four-momentum of the pion is zero, pure pionic decay amplitudes of hyperons[8] have contributions coming from pole diagrams [Figs. 4(a) and 4(b)] as well as a direct contribution [Fig. 4(c)]. On the other hand, for the soft-photon emission, pure radiative decay amplitudes of hyperons have no direct contribution. This has been shown in Sec. III(a).

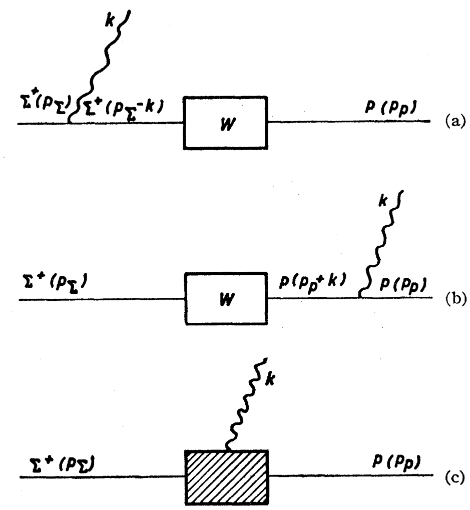

FIG. 4. (a,b) Pole diagrams of Σ^+ decay; the wavy line represents either π^0 or γ. (c) Direct contribution to the same decay.

(5) Because the decay $\eta \rightarrow \pi^+ + \pi^-$ cannot occur electromagnetically, but only weakly, the inner-bremsstrahlung term gives only a negligible contribution to the amplitude of the decay $\eta \rightarrow \pi^+ + \pi^- + \gamma$. Thus the leading term is here proportional to the photon four-momentum.

After completing this text, we found that H. Chew[12] had already derived, by a slightly different method, the low-energy theorem cited in the Introduction, for radiative decays of K mesons.

ACKNOWLEDGMENTS

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[9] R. D. Young, M. Sugawara, and T. Sakuma, Phys. Rev. **145**, 1181 (1966); Ming Chiang Li, *ibid.* **141**, 1328 (1966).

[10] G. Feldman, P. T. Matthews, and A. Salam, Phys. Rev. **121**, 320 (1961).

[11] S. Barshay and R. E. Behrends, Phys. Rev. **114**, 931 (1959); S. Iwa and J. Leitner, Nuovo Cimento **22**, 904 (1961); S. Barshay, U. Nauenberg, and J. Schultz, Phys. Rev. Letters **12**, 76 (1964).

[12] H. Chew, Phys. Rev. **123**, 377 (1961).