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Baryon Resonances as Orbital Excitations of the Baryons*

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A simple model of the baryon resonances is proposed in which internal spin–unitary-spin wave functions, belonging to a **56** of $SU(6)$, are coupled to internal orbital angular momentum. It is found that fourteen of the known baryon resonances may be accommodated in this scheme with maximal alignment of L and S. Decays of the resonances are described in the model by an $SU(6)_W$-invariant coupling to the meson **35**, with fitted phenomenological parameters representing the effects of internal structure. The agreement with experimental total and partial widths is reasonably good. Of special interest is the fact that consistency of the model calls for the assignment of $J^P=\frac{5}{2}^-$ to the $\Xi^*(1933)$, instead of $\frac{5}{2}^+$ as is usually assumed.

I. INTRODUCTION

A MAJOR impetus to the current activity in the application of symmetry groups to elementary-particle physics was provided by the successful classification of particle states into supermultiplets. The more recent approaches, such as current commutator algebras,[1] have reduced the emphasis on classification, but the problem remains an interesting one. The structure of the higher spin resonances is a particularly acute problem, for the higher symmetries now in vogue treat spin and isospin in an essentially equal fashion. Thus the simple assignment of the high spin resonances to representations of the symmetry group requires the use of multiplets of large multiplicity which include states of high isospin as well. There is currently no experimental evidence for resonances of large isospin, although this admittedly may be simply a reflection of experimental limitations, which may be aggravated by selection rules such as those suggested by Horn, Lipkin, and Meshkov.[2] A further problem is posed by acceptance of the Regge idea, however, for then higher spin states may be generated as Regge recurrences of states already classified. These new states, in turn, require the creation of other new states with high isospin, which would lie on new Regge trajectories. This sort of proliferation is hardly a desirable feature of a theory in the absence of experimental support. One way of avoiding the difficulty is the use of internal recoupling to angular momentum, which will be adopted in the present proposal.

The scheme suggested here is to regard the baryon resonances as structures with an internal spin–unitary-spin wave function coupled to an internal orbital angular momentum. The usual baryon octet and decuplet are taken as the s-wave ground states. The possibility of such a simple description of the baryon resonances is suggested by an energy-level chart, such as Fig. 1, which shows a number of baryon resonances as a function of "excitation energy" above the assumed ground state. These states fall into natural groupings which we identify as $L=1$, 2, and 3 excited states. We will consider here only states in which the internal spin and orbital angular momentum are maximally aligned. Note that in Fig. 1 some of the spin and parity assignments are only conjectures. As described in the remainder of this paper, some simple assumptions about the gross features of the internal structure permit us to calculate total and partial decay widths of the resonances, and to fit these calculations to experimental values.

II. DESCRIPTION OF THE MODEL

Contact is made with higher symmetries in the scheme outlined above by assuming that the internal spin–unitary-spin wave functions belong to the **56** representation of $SU(6)$. For example, the neutral $N^*(1688)$ with z component of spin equal to $+\frac{5}{2}$ would have the wave function

$$R(r)Y_2^2(\theta,\phi)n(\tfrac{1}{2}),$$

where $n(\frac{1}{2})$ has the spin and unitary-spin quantum numbers of a spin-up neutron. The internal states are assumed to be coupled to the meson **35** via an $SU(6)_W$-

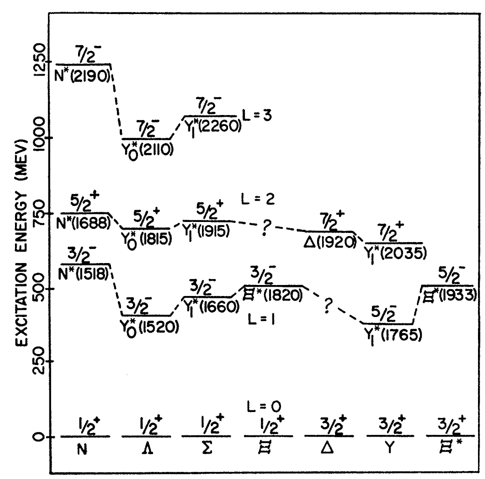

FIG. 1. Energy-level scheme for baryon resonances. The vertical scale gives the excitation energy, defined as the difference in mass between the resonance and its assumed ground-state baryon.

* This work supported in part by the U. S. Atomic Energy Commission.

[1] M. Gell-Mann, Physics 1, 63 (1964).

[2] D. Horn, H. J. Lipkin, and S. Meshkov, Phys. Rev. Letters 17, 1200 (1966).

TABLE I. The N and Λ relative couplings. For example, the entry in the Σ column and K row is the $\bar{N}\Sigma K$ coupling; while that in the Ξ column and K row is the $\bar{\Lambda}\Xi K$ coupling. The values correspond to Eq. (2) in the text, with spin and charge state sums and averages included as described there.

	N	Λ	Σ	Ξ	Δ	Y	Ξ^*
K	...	27	3	6	...	24	48
η	3	12	...	...	...	...	...
π	75	...	36	...	96	72	...
$\bar{K}$	54	...	...	...	...	...	...
ρ	177	...	72	...	192	144	...
K^*	...	81	33	66	...	48	96
$\bar{K}^*$	162	...	...	...	...	...	...

invariant[3] interaction. To describe the decays of the resonances, we adopt the formulas of Rushbrooke,[4] which are based on an effective local derivative coupling. We must assign a strength to this which consists of several independent factors. One is the couplings of the internal states to the mesons; we identify these with the dimensionless coupling constants in the Rushbrooke formulas. The relative couplings are determined by the $SU(6)_W$ symmetry and those to be used in the present study are summarized in Tables I–III. (These values include the usual sum and average over the initial and final spin states, and are appropriate to the decays of neutral resonances to all allowed charge states.) We are left with a single unknown coupling constant for all the decays to be considered.

The other factor in the effective local derivative coupling is a transition matrix element involving the nonlocal structure of the model; for example, an integral involving the radial wave function. This factor corresponds to the scale mass in the Rushbrooke formulas (which is required for dimensional reasons) and is analogous to an appropriate multipole moment in a nonrelativistic semiclassical treatment of a radiative transition. We assume, as part of the model, that all the resonances have essentially the same structure for a given L value, so that the scale mass does not depend on strangeness, isospin, or spin within such a level. We find empirically, however, that the scale mass must depend on the orbital angular momentum of the emitted meson (multipolarity of the emitted radiation), and of course on the L values of the initial and final states.

TABLE II. The Σ and Ξ relative couplings.

	N	Λ	Σ	Ξ	Δ	Y	Ξ^*
K	...	...	...	50	...	...	16
η	...	...	12	27	...	24	24
π	...	12	32	3	...	16	24
$\bar{K}$	2	3	75	...	64	24	...
ρ	...	24	136	33	...	32	48
K^*	...	33	...	118	...	...	32
$\bar{K}^*$	22	...	177	...	128	48	...

TABLE III. The Δ, Y, and Ξ^* relative couplings. Unless otherwise indicated, the symbols Y and Ξ^* refer to the $Y_1^*(1385)$ and the $\Xi^*(1530)$ throughout this paper.

	N	Λ	Σ	Ξ	Δ	Y	Ξ^*
K	...	...	24	8	...	30	40
η	...	...	12	12	15	...	15
π	24	12	8	12	75	40	15
$\bar{K}$	8	12	12	...	40	60	...
ρ	48	24	16	24	285	152	57
K^*	...	...	48	16	...	114	152
$\bar{K}^*$	16	24	24	...	152	228	...

We will treat only transitions to the ground state here and will need the following formulas. Let w, m, and ν be the masses of the parent resonance, daughter baryon, and meson, respectively. Define k and E as the center-of-mass momentum and energy of the daughter baryon. Let the spin and parity of the parent be $J^P=(n-\frac{1}{2})^{\pm}$ and define

$$C_n=\frac{2^n n!(n-1)!}{(2n)!}=\frac{(n-1)!}{(2n-1)!!},$$

$$m'=(-1)^n m.$$

Then we have, for the partial widths

$(n-\frac{1}{2})^{\pm}\to\frac{1}{2}^{+}+0^{-}$,

$$\Gamma=\gamma C_n\alpha_{n-1}{}^L\left(\frac{k}{m_\eta}\right)^{2n-2}\left(\frac{k}{w}\right)[E\pm m'];\qquad(1a)$$

$(n-\frac{1}{2})^{\pm}\to\frac{3}{2}^{+}+0^{-}$,

$$\Gamma=\gamma C_n\alpha_{n-2}{}^L\left(\frac{k}{m_\eta}\right)^{2n-4}\left(\frac{k}{w}\right)\times\left[\left(\frac{2n-1}{n-1}\right)(E\pm m')+\frac{2}{3}\frac{k^2}{m^2}(E\pm 2m')\right];\qquad(1b)$$

$(n-\frac{1}{2})^{\pm}\to\frac{1}{2}^{+}+1^{-}$,

$$\Gamma=\gamma C_n\alpha_{n-2}{}^L\left(\frac{k}{m_\eta}\right)^{2n-4}\left(\frac{k}{w}\right)(E\pm m')\left[\left(\frac{2n-1}{n-1}\right)+\frac{k^2}{\nu^2}\right].\qquad(1c)$$

In these formulas, m_η is the mass of the η meson and is introduced for numerical convenience. The other quantities are[5]

$$\gamma=3(g/g_0)^2,\qquad(2)$$

where g is the coupling constant for the decay of interest and g_0 is the coupling constant for $\bar{N}N\eta$; also

$$\alpha_l{}^L=(g_0{}^2/12\pi)(m_\eta/m_l{}^L)^{2l},\qquad(3)$$

where $m_l{}^L$ is the scale mass for a transition from the Lth level to the ground state with emission of a meson

[3] H. J. Lipkin and S. Meshkov, Phys. Rev. 143, 1269 (1966).
[4] J. G. Rushbrooke, Phys. Rev. 143, 1345 (1966).

[5] This is the quantity found in Tables I–III.

TABLE IV. Predicted partial widths of the strangeness-zero baryon resonances (MeV).

Resonance	$N\pi$	$N\eta$	ΛK	ΣK	Decay mode $\Delta\pi$	$\Delta\eta$	YK	$N\rho$	ΛK^*	ΣK^*	Total width
$N^*(1518)$	116.0	0.02	...	...	20.1	...	...	...	...	...	136.2
$N^*(1688)$	96.6	0.3	0.05	...	10.6	...	...	...	...	...	107.5
$N^*(2190)$	62.6	0.9	2.8	0.2	101.0	...	4.1	123.8	5.1	0.6	300.9
$\Delta(1700)$	158.3	...	...	0.01	16.5	...	...	...	...	...	174.8
$\Delta(1920)$	146.6	...	...	4.1	23.4	0.1	...	11.6	...	...	185.9

TABLE V. Predicted partial widths of the strangeness -1, isoscalar baryon resonances (MeV).

Resonance	$N\bar{K}$	$\Lambda\eta$	$\Sigma\pi$	Decay mode ΞK	$Y\pi$	Ξ^*K	$N\bar{K}^*$	$\Sigma\rho$	Total width
$Y_0^*(1520)$	3.2	...	3.1	...	1.3	...	...	...	7.6
$Y_0^*(1815)$	42.1	0.4	14.4	...	7.4	...	...	...	64.2
$Y_0^*(2110)$	11.4	0.4	3.6	0.02	26.2	0.3	24.0	3.1	69.0

having orbital angular momentum l. Our ignorance of the details of the internal structure of the baryon resonances is reflected in our treating the α_l^L as unknown parameters and adjusting them in an attempt to fit the data.

III. PREDICTIONS OF THE MODEL AND COMPARISON WITH EXPERIMENT

The states assigned to the $L=1$ level in Fig. 1 form a nearly complete set of excited states of the baryons.[6] There are two exceptions: First, there is no known $I=\frac{3}{2}$, zero strangeness resonance with spin $\frac{5}{2}^-$; and second, the identification of the $\Xi^*(1933)$ as the $L=1$ excited state of the $\Xi^*(1530)$ is contrary to the usual

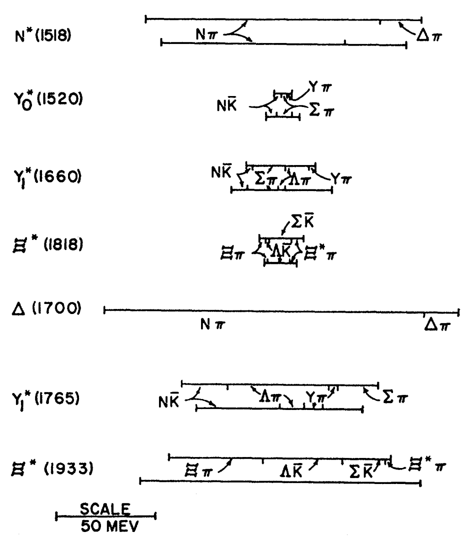

FIG. 2. Partial and total widths for the $L=1$ resonances. The upper bar is the prediction, the lower bar is the experimental value.

[6] We leave out the Ω^- and its conjectured excited states, since it is likely that the ideas presented here will be modified, refined, or rejected long before the properties of the latter are known experimentally.

assignment of $\frac{5}{2}^+$ to this state. The latter assignment, however, is based on assuming that the $\Xi^*(1933)$ is the Regge recurrence of the Ξ hyperon. Such an assumption is hard to reconcile with the large width of this state in any model which uses $SU(3)$ predictions for the couplings of the $\frac{5}{2}^+$ octet, as has been pointed out by Meshkov.[7] Therefore, until experimental measurements of the spin and parity of the $\Xi^*(1933)$ prove otherwise, the assignment of $\frac{5}{2}^-$ seems reasonable. As for the missing resonance, we will conjecture a state at a mass of 1700 MeV. There are three parameters to be adjusted in predicting the decays of the seven $L=1$ states. Values which give reasonable results are

$$\alpha_0^1=1.28\times10^{-3},\quad \alpha_1^1=0.65,\quad \alpha_2^1=0.17. \tag{4}$$

Inserting these values in (1), we get the predictions of Fig. 2. The actual numerical values for all the decays to be considered here are summarized in Tables IV–VII. In Fig. 2, as in later figures, the lower bars represent the experimental values.[8,9] On the whole, the results are encouraging. The three parameters do a fair job of reproducing the general pattern of the six experimentally known total widths, and with a few exceptions give reasonable branching ratios as well. The experimental values are rather poorly determined in most cases, so that such disagreement as is seen cannot be construed as a failure of the model as yet. In fact, the

[7] S. Meshkov (unpublished).

[8] The vaues for the zero-strangeness resonances are taken from A. H. Rosenfeld *et al.* [Rev. Mod. Phys. **37**, 633 (1965)]. All other values are from a recent revision of this work, Ref. 9. This revision also gives new values for the N^*'s; however, in many cases these are based less on new experimental data than on new estimates from the π-N phase shifts of P. Bareyre *et al.* [Phys. Letters **18**, 342 (1965)]. Since these phase shifts are by no means universally accepted, we do not consider the new estimates to carry more weight than the earlier ones. Since the latter are more congenial to our predictions, we use them. To minimize the admitted bias this introduces, we arbitrarily use Ref. 8 for all zero-strangeness resonances, and Ref. 9 otherwise.

[9] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

TABLE VI. Predicted partial widths of the strangeness −1, isovector baryon resonances (MeV).

Resonance	Decay modes $N\bar{K}$	$\Lambda\pi$	$\Sigma\pi$	$\Sigma\eta$	ΞK	$\Delta\bar{K}$	$Y\pi$	$Y\eta$	Ξ^*K	$N\bar{K}^*$	$\Lambda\rho$	$\Sigma\rho$	ΞK^*	Total width
$Y_1^*(1660)$	1.5	12.4	15.9	···	···	···	3.4	···	···	···	···	···	···	33.2
$Y_1^*(1765)$	23.1	50.2	20.9	0.08	···	0.09	3.1	···	···	···	···	···	···	97.4
$Y_1^*(1915)$	3.8	21.0	30.7	0.6	0.3	6.3	2.9	···	···	0.3	0.1	···	···	66.0
$Y_1^*(2035)$	38.0	65.8	29.2	6.2	1.4	5.7	8.6	···	···	3.3	1.8	0.1	···	160.1
$Y_1^*(2260)$	1.2	5.7	9.5	0.9	1.5	43.7	13.0	5.4	1.4	10.9	10.7	35.7	0.4	139.9

TABLE VII. Predicted partial widths of the strangeness −2 baryon resonances (MeV).

Resonance	Decay modes $\Lambda\bar{K}$	$\Sigma\bar{K}$	$\Xi\pi$	$\Xi\eta$	$Y\bar{K}$	$\Xi^*\pi$	$\Xi^*\eta$	$\Lambda\bar{K}^*$	$\Sigma\bar{K}^*$	$\Xi\rho$	Total width
$\Xi^*(1818)$	1.6	11.6	1.9	···	···	5.4	···	···	···	···	20.6
$\Xi^*(1933)$	39.6	20.5	47.0	1.0	0.4	1.3	···	···	···	···	109.8
$\Xi^*(2020)$	3.0	33.6	2.4	0.9	1.7	3.7	···	0.03	···	···	45.3
$\Xi^*(2460)$	2.2	34.3	1.5	4.1	24.5	27.5	9.1	25.3	88.4	18.9	235.7

experimental uncertainties enter not only in the final comparison of decay widths, but also in the masses inserted into Eq. (1). A change in the mass assumed for the resonance produces changes in the branching ratios as well as in the total width. In a few cases the prediction for a partial width is clearly unreasonable despite the experimental uncertainties. One could hope to invoke symmetry breaking here, and it is interesting that in nearly every case the predictions are off in the direction indicated by, for example, the work of Dashen, Dothan, Frautschi, and Sharp.[10]

The $L=2$ level in Fig. 1 is missing both Ξ^* members. We have taken the recently established[11] $Y_1(1915)$ as the Σ member. The three relevant parameters and their values are

$$\alpha_1^2=3.7\times10^{-3},\quad \alpha_2^2=0.45,\quad \alpha_3^2=7.7\times10^{-2}. \tag{5}$$

Inserting these values in Eq. (1) gives the results of Fig. 3. We have included a conjectured $\frac{5}{2}^+$ Ξ^* state at a mass of 2020 MeV. The five experimental total widths are fitted very nicely and the partial widths are rather good as well. It should be noted that the present model implies that the $N^*(1688)$ has the same F/D ratio as the nucleon in its coupling to mesons, in agreement with the suggestion of Heusch, Prescott, and Dashen.[12]

Only a few states are known at present which may be fitted into the $L=3$ level, and these may all be regarded as excited states of the octet baryons. For this reason, we can only estimate two of the three parameters required for this level, and from our estimates can only make complete predictions for the $\frac{7}{2}^-$ members of the level. Using the values

$$\alpha_2^3=6.97\times10^{-3},\quad \alpha_3^3=1.71\times10^{-2}, \tag{6}$$

gives the results of Fig. 4. We have conjectured a $\frac{7}{2}^-$ Ξ^* at a mass of 2460 MeV to complete the octet. The picture for this level has flaws, but is not completely discouraging. The masses of these states are not precisely known, and if the N^* and Y_0^* have masses of 2160 and 2140 MeV, respectively, then the total widths would be predicted to be about 250 and 90 MeV, which would improve the picture considerably.

IV. DISCUSSION

In the preceding sections we have proposed a conceptually simple model for a number of the baryon resonances and have applied its ideas to the calculation of their decay widths. While the results of the calculations were not in spectacular agreement with the experimental values, the general pattern of total and

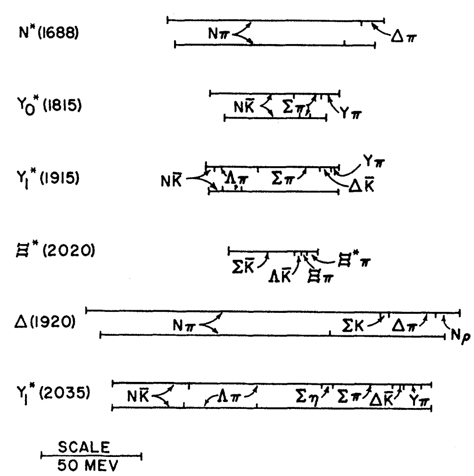

FIG. 3. Partial and total widths for the $L=2$ resonances.

[10] R. F. Dashen, Y. Dothan, S. C. Frautschi, and D. H. Sharp, Phys. Rev. **143**, 1185 (1966).
[11] R. L. Cool *et al.*, Phys. Rev. Letters **16**, 1228 (1966).
[12] C. A. Heusch, C. Y. Prescott, and R. F. Dashen, Phys. Rev. Letters **17**, 1019 (1966).

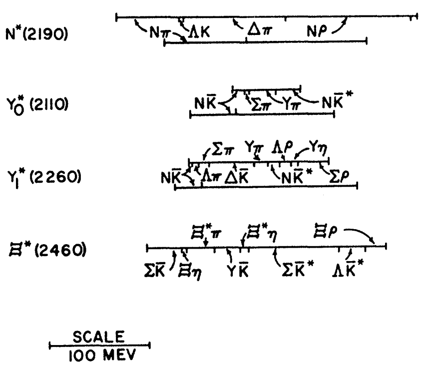

FIG. 4. Partial and total widths for the $L=3$ resonances.

partial widths was reproduced fairly well. Although the model admits a number of adjustable phenomenological parameters, it still places rather strong constraints on the branching ratios, and there are many predictions which are, in fact, independent of the model parameters. For example, the ratio of partial widths for the decays

$$Y_0^*(1520) \rightarrow N+\bar{K}$$

and

$$Y_1^*(1765) \rightarrow Y_1^*(1385)+\pi$$

is predicted to be 1.0, independent of the choice of α_1^1. This ratio is rather poorly determined experimentally, but may be estimated as 0.6 ± 0.4, which is in reasonable agreement. The real purpose of this model, however, is to provide a classification scheme for the baryon resonances which, it is hoped, will reveal the underlying structure of the baryon spectrum. The calculation of decay widths should be regarded primarily as a tool for checking the assignments of resonances to places in the scheme. In this sense, the results of Sec. III are very good, since the qualitative agreement found there makes the assignments of Fig. 1 seem quite plausible. Of particular note in this respect is the placement of the $\Xi^*(1933)$ in the $L=1$ level, for if this resonance were assigned the role of the $L=2$ excited state of the Ξ hyperon the model would give it a total width of the order of 10 MeV, in clear contradiction to the experimental facts. This assignment, therefore, seems to be the best test of the model presently in view and so we elevate it to a prediction: The $\Xi^*(1933)$ has $J^P=\frac{5}{2}^-$ and is *not* the Regge recurrence of the Ξ hyperon. If this prediction fails, it is unlikely that the present proposal will have any further usefulness. In the absence of an actual measurement of the spin and parity of the $\Xi^*(1933)$, the discovery of a fairly narrow ($\sim$50 MeV) Ξ^* at a mass of about 2020 MeV, decaying predominantly into $\Sigma+\bar{K}$ would provide some support to the ideas advanced here, since this new state would be a plausible candidate for the Regge recurrence of the Ξ hyperon and would fit neatly into the model.

The next area of interest is the collection of baryon resonances not yet accomodated in the model. Of those resonances regarded as "established" in Ref. 8, the model already accounts for all the Ξ^*'s and Y_1^*'s. It fails to include the $Y_0^*(1405)$, the $Y_0^*(1670)$, and a number of zero-strangeness states. The higher-mass examples of the latter may be accomodated as higher L excitations of the **56**. Some of the remainder might fit into the present scheme if we consider states in which L and S are not maximally aligned. It is clear, however, that this will prove insufficient to the task of classifying all the presently known resonances. For example, the η-N threshold resonance at about 1570 MeV has $J^P=\frac{1}{2}^-$, and might be regarded as an $L=1$ state of the nucleon with S and L antiparallel, but the sizeable coupling of this state to the η-N system most likely rules this out. It may be that an extension of this model will require the introduction of an additional set of ground-state baryons, perhaps belonging to a **70**$^-$ as suggested by Gyuk and Tuan.[13] A curious aspect of the set of states not as yet accommodated in the model is that four of them have essentially the same mass (about the mass of the Ω^-), and there is also evidence for a Ξ^* at this mass.

Our model has some relation to models making use of classification according to $SU(6)\times O(3)$ as discussed by Dalitz[14] and more recently by Gyuk and Tuan.[15] The states considered here would all lie in representations of the form (**56**, $2l+1$). Mitra[16] has proposed a dynamical model for baryon resonances based on quark-quark forces, in which these representations are dynamically suppressed in favor of others such as the **(20,3)**$^+$ and **(70,3)**$^-$. Our proposal makes no assumption about the composition of the internal states, which could be "elementary" objects belonging to a **56**, rather than quark combinations.

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The work described in this paper was carried out in part while the author was a guest of the Theoretical Physics Group of the Lawrence Radiation Laboratory, Berkeley. The author would like to express his sincere appreciation to Dr. Geoffrey Chew and the other members of the group for their cordial hospitality.

[13] I. P. Gyuk and S. F. Tuan, Phys. Rev. Letters **14**, 121 (1965). Note, however, that some of the states regarded by Gyuk and Tuan as members of the **70**$^-$ are already incorporated in the present model as excited states of the **56**$^+$.

[14] R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).

[15] I. P. Gyuk and S. F. Tuan, Phys. Rev. **151**, 1253 (1966).

[16] A. N. Mitra, Phys. Rev. **151**, 1168 (1966).