magnitude and sign of $\mu_{n\Sigma}$, so that the Cabibbo angle can be affected.

IV. SUMMARY

The partial conservation of tensor current (PCTC) hypothesis has been introduced with a projection operator so that the partial derivative of the tensor current connects the magnetic terms of $\langle B|j_\mu^i|A\rangle$ to the vector mesons with the appropriate quantum numbers. Without this device, the PCTC combined with the asymptotic property of the tensor current at $k\to 0$ given by the quark model leads to a vanishing charge form factor.[4] There is no strong reason to preserve this property of the quark model, but it appears desirable to do so for consistency. The consequences of PCTC are equivalent to those of the vector-meson dominance models.

Sum rules of the magnetic moments of baryons are obtained on the basis of PCTC. The value $\mu_{\Xi^-} = -(1.4\pm1.5)$ nm is found from one of the sum rules, μ_{Σ^+}, and μ_n.

It is found that the requirement of reproducing the magnetic-moment relations that result from $SU(3)$ or $SU(6)$ symmetry, broken by electromagnetic interaction, leads to the D/F ratios ($\bar{B}BV$ coupling) of $D/F=3.4$ in $SU(3)$ and $D/F=\frac{3}{2}$ in $SU(6)$.

Sum rules for the transition magnetic moments μ_{BA} of $A\to B+e^-+\bar{\nu}$ are obtained as a consequence of PCTC. The contribution of the magnetic terms of $\langle B|J_\mu^i|A\rangle$ to various semileptonic decay rates of hyperons were estimated on the basis of $SU(3)$ symmetry and found to be negligible.

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Some Connections between Sum Rules and Symmetries

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The question of the existence of consistency conditions arising from the combination of sum rules with internal symmetries is discussed further. The sum rules considered are the forward-scattering Compton and photoproduction rules. A new photoproduction rule is stated. The notion of the support of a sum rule is introduced: viz., the set of states such that saturation with these states does not generate null solutions for low-energy parameters. Different sum rules generally have distinct supports because of differences in the consistency conditions from rule to rule. Applications of saturation methods to the axial D/F ratio, full and anomalous magnetic moments, and the $\pi^0\to 2\gamma$ lifetime are discussed.

I. INTRODUCTION

It has been clear for some time that higher symmetries of the $SU(6)$ type cannot be of the kinematical variety but must rather be looked upon as dynamical in nature.[1] Thus, if symmetries such as $SU(6)$ are to be understood better than as a neat device to codify a few empirical facts, one must find dynamical equations such that $SU(6)$ appears as a symmetry of some of their approximate solutions.[2] Because we have so far no idea as to what are the equations of motion in the domain of strong interactions, we are perforce obliged to take as general a starting point as is profitably possible.

The saturation of equal-time charge commutators was a first attempt in this direction,[3,4] the idea being that dynamical symmetries could be induced by restricting the sum over intermediate states to a select set. It is not clear, however, by what dynamical principle such a selection can be justified, as the mass of an intermediate state has no apparent bearing on its importance in this sum over states.

An alternative procedure for the induction of dynamical symmetries consists[5] in applying the saturation procedure to physical sum rules, i.e., relationships between physical amplitudes (or continuations thereof) and low-energy parameters such as coupling constants, magnetic moments, etc. Since one is now making approximations in the calculation of physical amplitudes, there is perhaps more reason to expect that the restriction to a few low-lying intermediate states may be a genuine dynamical approximation.

[1] For the distinction between kinematical and dynamical symmetry see A. Pais [Rev. Mod. Phys. 38, 215 (1966)] especially Sec. II B.
[2] See Ref. 1, Sec. IV I.
[3] B. W. Lee, Phys. Rev. Letters 14, 676 (1965).
[4] R. F. Dashen and M. Gell-Mann, Phys. Letters 17, 142 (1965).
[5] V. A. Alessandrini, M. A. B. Bég, and L. S. Brown, Phys. Rev. 144, 1137 (1966).

In order to illustrate the procedure, we consider the Adler-Weisberger sum rules[6] in the $SU(3)$ limit.

$$1=g_A^2+c\int\frac{d\omega}{\omega}[\sigma(\pi^-p)-\sigma(\pi^+p)],$$

$$1=g_A^2(1-2\alpha)^2+c\int\frac{d\omega}{\omega}[\sigma(K^-n)-\sigma(K^+n)], \tag{1.1}$$

$$2=2g_A^2(1-2\alpha+\tfrac{4}{3}\alpha^2)+c\int\frac{d\omega}{\omega}[\sigma(K^-p)-\sigma(K^+p)].$$

Here

$$c=2M_N^2g_A^2/\pi G_{\pi N}^2,\quad g_A\equiv G_A/G_V, \tag{1.2}$$
$$\alpha=g_D/(g_D+g_F),$$

g_D and g_F being the D-type and F-type axial coupling constants.

If we assume that the dispersion integrals are saturated by a decuplet, we obtain[7]

$$g_A^2-\tfrac{2}{3}\Delta=1,$$
$$g_A^2(1-2\alpha)^2+\tfrac{1}{3}\Delta=1, \tag{1.3}$$
$$g_A^2 2(1-2\alpha+\tfrac{4}{3}\alpha^2)+\tfrac{1}{6}\Delta=2,$$

where Δ measures the decuplet contributions. Hence, we have the solutions

$$\alpha=\tfrac{3}{5}\quad\text{or}\quad g_D/g_F=\tfrac{3}{2}, \tag{1.4}$$
$$g_A^2=25/9,\quad \Delta=(24/25)g_A^2.$$

These are the standard $SU(6)$ predictions[8] for D/F, G_A/G_V, and the N^* width. The above derivation may well provide us with a significant clue about the dynamical origins of $SU(6)$, viz., the tendency of the low-lying baryon octet and decuplet to form a closed system in a manner reminiscent of bootstrap ideas.[9]

It should be noted, however, that these $SU(6)$ results for axial-vector constants follow from the prescription: saturate with a **10**. It is not necessary, in principle, to specify any further properties of this **10** such as mass or its spin. In this connection it is also important to note that the D/F ratio $\frac{3}{2}$ (*not* the value for g_A^2) is stable for saturation with a wider set of states, as is discussed further in Sec. V.

On the other hand, no explanation exists so far for the $SU(6)$ ratio[10] $\mu(p)/\mu(n)=-\frac{3}{2}$ for the full magnetic moments. Attempts were made[3,4] to get this result from an algebra of magnetic-dipole operators with the additional assumption of closure between baryon octet and decuplet states. However, closer scrutiny revealed[11] that under these conditions one actually obtains the null solution $\mu(p)=\mu(n)=0$. Such null solutions have meanwhile turned up in a variety of saturation problems.

It appears, therefore, that one is in a somewhat erratic situation in regard to the applicability of saturation methods to sum rules. We do not claim to understand these problems in every respect. However, and this is the main purpose of this paper, we would like to explain for a class of sum rules what is the detailed nature of the saturation problems. We shall also have to raise the question whether saturation prescriptions may constitute sufficient input to obtain the full magnetic-moment ratio of the nucleons.

To start with, we recapitulate in Sec. II the sum rules connected with Compton scattering and the assumptions that go into their derivation. Amongst these rules is the one obtained by Drell and Hearn.[12] It was recently noted[13] that this rule, when applied in conjunction with an internal symmetry, implies the existence of consistency conditions which hold rigorously in the strong interactions and to order e^2 in the electromagnetic interactions. In Sec. III these considerations are extended to other Compton scattering sum rules. We find that the different sum rules have distinct consistency properties. In particular, it turns out that the Cabibbo-Radicati sum rule,[14] unlike the Drell-Hearn rule, is free of constraints. For the linear magnetic-moment sum rule,[15–17] on the other hand, there does exist a consistency condition which turns out to be an integral form of a weak Johnson-Treiman relation.[18]

We find that the existence of null solutions generated by nontrivial saturations is intimately connected with the existence of these consistency conditions. In particular for the Cabibbo-Radicati sum rule, where no constraint exists, no null-saturations exist either.

It is noted that the Adler-Weisberger sum rule is likewise of the constraint-free type. It should be recalled that the derivation of this rule is based on more assumptions than appear in the Compton case. In particular, an extrapolation to zero mass for the external pseudoscalar mesons is involved. The discussion of constraints is in reference to this zero-mass limit. The

[6] S. Adler, Phys. Rev. Letters **14**, 1051 (1965); Phys. Rev. **140**, B736 (1965); W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965); Phys. Rev. **143**, 1302 (1966).

[7] See, e.g., H. J. Schnitzer, Phys. Letters **20**, 539 (1966).

[8] F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters, **13**, 299 (1964); M. A. B. Bég and A. Pais, *ibid.* **14**, 51 (1965).

[9] The relationship between the bootstrap ideas and $SU(6)$ was noted by R. H. Capps, Phys. Rev. Letters **14**, 31 (1965); J. G. Belinfante and R. E. Cutkosky, *ibid.* **14**, 33 (1965); V. Singh and B. M. Udgaonkar, Phys. Rev. **139**, B1585 (1965).

[10] M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964); **13**, 650(E) (1964); B. Sakita, *ibid.* **13**, 643 (1966).

[11] B. W. Lee, Phys. Rev. Letters **14**, 850(E) (1965).

[12] S. D. Drell and A. C. Hearn, Phys. Rev. Letters **16**, 908 (1966).

[13] A. Pais, Phys. Rev. Letters **18**, 17 (1967). As in this paper, we reserve the phrase "consistency condition" for linear and homogeneous relationships between the real parts of scattering amplitudes which are nontrivial, in the sense that they neither follow from the internal symmetry alone nor from a low-energy theorem alone.

[14] N. Cabibbo and L. A. Radicati, Phys. Letters **19**, 697 (1966).

[15] M. A. B. Bég, Phys. Rev. **150**, 1276 (1966), Eq. (1.2); see also Ref. 17.

[16] K. Kawarabayashi and M. Suzuki, Phys. Rev. **150**, 1181 (1966).

[17] M. A. B. Bég, Phys. Rev. Letters **17**, 333 (1966).

[18] K. Johnson and S. B. Treiman, Phys. Rev. Letters **14**, 189 (1965).

same is true for the treatment in Sec. IV of the photoproduction sum rules. In this category belong: (a) a sum rule involving the anomalous magnetic moments.[19] The consistency properties of this rule are such that one of the two constraints of the Drell-Hearn sum rule has a direct analog also in this case, while the other has no photoproduction counterpart. (b) A new linear sum rule intimately related to the Kroll-Ruderman low-energy theorem.[20] The structure of this rule is algebraically the same as that of the linear magnetic moment sum rule. Accordingly, its consistency condition has again the structure of a weak Johnson-Treiman relation.

In Sec. V we turn to a fairly detailed discussion of the saturation in the $SU(3)$ limit for all the sum rules mentioned above. We introduce the notion of support of a sum rule, by which we mean such selections of saturating states that null solutions for low-energy parameters are avoided. Of course, the idea is to find sufficiently simple supports such that the dynamical approximations implied thereby lead to specific values for some of the physical parameters. In regard to saturating states in the s and u channels we consider only three-triplet states (**10**, **8**, and **1**) while for the t channel we consider dominance by **1** and **8** states only, guided by current ideas on quark models and on Regge-pole dominance.

We have attempted to find a common support for all the mentioned sum rules which is simple enough to have predictive power and which gives reasonable answers. To the best of our knowledge such a simple common support does not exist and this appears to be largely due to the distinct structure of the consistency conditions for the various sum rules. We then divide the sum rules into two groups. The first group comprises those sum rules which are antisymmetric under the exchange of the unitary spin labels of the external bosons. These we call the antisymmetric sum rules. The remaining ones are denoted as symmetric sum rules. Some degree of coherence was obtained by examining the possibility of a common support for the antisymmetric rules and likewise, but separately for the symmetric ones.

(a) Antisymmetric case, Sec. VA. The simplest common starting point turns out to be: use a **10** in the s, u channels and an **8** in the t channel. Here one finds the axial D/F ratio $\frac{3}{2}$ and we study the stability of this number under inclusion of a further s, u channel **8**, representative of the intermediate (N^{**}) energy region. The linear magnetic moment and photoproduction sum rules generate equalities between low-energy D/F ratios and similar high-energy ones, the latter referring to the t-channel octet; see Eqs. (5.9) and (5.10) below. It appears that no further information can be extracted from these rules, unless one supplements the saturation prescription with a dynamical assumption extraneous to the sum rules as well as to the choice of support. We show in Sec. V A. that if one assumes, for the forward scattering amplitudes concerned, that high-energy additivity holds within the framework of a quark model with nonrelativistic "internal" motion, then the ratio $\mu(p)/\mu(n) = -\frac{3}{2}$ follows.

It should be stressed immediately that the present authors do not consider this derivation of the magnetic-moment ratio to be quite satisfactory. As will be discussed in detail in Sec. V, the additional assumption of additivity for these high-energy forward but spin-dependent amplitudes appears to us to be too model-dependent. We believe that the conclusions drawn in this way may well be qualitatively correct, but would much prefer to see an eventual derivation based on weaker assumptions. It is our present conjecture, however, that such assumptions will have to remain extraneous in the sense defined above. At any rate, the present derivation does not need to make use of the **56** structure of a baryon wave function as an input.

(b) Symmetric case, Sec. V B. The simplest common feature of the supports in these cases appears to be the use of at least a **10** and a **1** in the s, u channels. The anomalous baryon moments and a $\pi^0 \longrightarrow 2\gamma$ lifetime formula are discussed under this heading.

Our results lead us to the conjecture that the dynamical basis for $SU(6)$ is to be found in the antisymmetric sum rules referring to strictly forward scattering. The symmetric sum rules appear to have no obvious connection with $SU(6)$.

We conclude the Introduction with the following remarks:

(1) The present treatment is limited to the case of $SU(3)$ symmetry. Our discussion of the full and the anomalous magnetic moments is of some physical interest only if this symmetry limit does not introduce qualitative deviations for these quantities.

Until we have a means of discussing support questions in the presence of symmetry breaking, a detailed comparison cannot be made between saturation methods on the one hand and the "experimental" evaluation of dispersion integrals on the other. Nevertheless, it is somewhat encouraging to note (see Sec. V) that at least some qualitative features of the "experimental" method appear to have their counterparts in the saturation approach.

(2) The saturations discussed do not depend on narrow-width approximations for the continuum states. Also, in what follows a statement such as: "saturate with a **10**" may in principle be taken to refer to any set of states with transformation properties of a **10**.

(3) The saturation arguments given here do not involve a full specification of the spins of the saturating states. For this purpose one will have to distinguish between "spin-symmetric" (non-spin-flip) rules for

[19] S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **40**, 1171 (1965).

[20] N. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954).

which the saturation is formally independent of spin; and "spin-antisymmetric" (not quite correctly called spin-flip) rules, where the spin of the saturating states does play a role. A few brief comments on spin aspects will be found in Sec. V.

(4) Our treatment does not include a full discussion of overlapping between saturating states with distinct $SU(3)$ properties.

(5) A crucial assumption made in the derivation of all sum rules considered here is that the respective amplitudes each satisfy an unsubtracted dispersion relation. No proof of this is known for any of these amplitudes. If the ansatz turns out to be false for some of them, then much of the subsequent argument will need revision. In particular, it may be recalled[17] that the linear magnetic-moment rule [Eq. (2.17) below] is incompatible with a power-series expansion in the strong couplings. The same is true for the linear photoproduction rule, Eq. (4.9), below. It is true that this incompatibility would be eliminated if one subtraction were made. But, of course, that does not prove that a subtraction is indeed necessary. In this connection it should be noted that Singh has shown[21] the following:

(a) The dispersion integral in the linear magnetic-moment rule has the same convergence properties as the one in the Cabibbo-Radicati rule if a Regge-pole model for high-energy behavior is used in both cases. At this point it may also be appropriate to recall an earlier remark[10] that the nucleon magnetic moment ratio $-\frac{3}{2}$ is hard to understand from any weak-coupling approach.
(b) Further sum rules, based like the present ones on low-energy theorems, may be obtained by differentiation of amplitudes with respect to the momentum transfer. We have no complete answer as to whether the supports discussed in the present paper are also of use for such additional rules. However, one such case will be discussed in Sec. V. This is a sum rule due to Pagels[22]; we shall discuss the saturation for this rule and its connection with an approximate formula for the $\pi_0 \rightarrow 2\gamma$ lifetime.

Finally we note the following. It will be obvious that the concept of support is at best meaningful only within a framework of dynamical approximations, and that it plays no role when one employs a sum rule in order to verify whether experimental information on low-energy parameters can be properly correlated with experimental information on the cross sections which appear in the dispersion integrals.

II. COMPTON SCATTERING SUM RULES

The topic of this section is a group of sum rules which we call the Compton scattering rules. They comprise: (a) the Drell-Hearn (DH) sum rule[12] for anomalous magnetic moments as well as the intimately related rules for the separate isoscalar and isovector parts of these moments[17]; (b) the Cabibbo-Radicati (CR) rule[14]; (c) the "linear rule" for full magnetic moments.[15–17]

This group of rules stands out amongst the many recently derived sum rules because of the relatively minimal nature of the assumptions that go into their derivation. These are:

(α) The low-energy theorem for Compton scattering.[23] This theorem is rigorously valid to all orders in the strong interactions and to order e^2 in electromagnetic couplings. (In this paper we shall not be concerned with the interesting question whether the theorem can be extended to higher orders in e^2. Weak interactions are of course neglected.)

(β) The validity of unsubtracted dispersion relations for each of those parts of the Compton scattering amplitude which enters in the respective sum rules mentioned above.

(γ) Where an internal symmetry is brought to bear [either isospin $SU(2)$ or $SU(3)$], it is further assumed that the no-subtraction ansatz remains valid in the symmetry limit. There is at least no logical objection to such an assumption. As we shall discuss in Sec. III, the simultaneous implementation of the conditions α-γ leads to new kinds of constraints on the theory.

Before we come to this, it is helpful to discuss briefly first the case in which one disregards any internal symmetry. Consider, for example, proton Compton scattering, described by a covariant amplitude[24] $M_{\mu\nu}(k',k)$. Here k and ν are the initial photon momentum and polarization in the laboratory system. k' and μ refer likewise to the outgoing photon. The relations

$$k_\mu' M_{\mu\nu}(k',k) = M_{\mu\nu}(k',k)k_\nu = 0 \qquad (2.1)$$

are readily seen to follow from current conservation. In the forward direction ($\omega = |\mathbf{k}|$),

$$e^2(2\pi)^3 \times M_{mn}(k,k) = S_1(\omega^2)\delta_{mn} + \omega S_2(\omega^2) \times \tfrac{1}{2}[\sigma_m,\sigma_n]. \qquad (2.2)$$

The DH rule follows[12] from the low-energy theorem for $S_2(0)$ and an unsubtracted dispersion relation for $S_2(\omega^2)$.

In order to extend Eqs. (2.1) and (2.2) so as to include internal symmetry, it is convenient to introduce[15,17] the device of a multiplet of massless vector mesons, a triplet for $SU(2)$ or an octet for $SU(3)$, coupled to the respective vector currents which are conserved in the symmetry limit. The purpose of this device is the following: Consider the Compton

[21] V. Singh (private communication).
[22] H. R. Pagels, Phys. Rev. (to be published).

[23] F. E. Low, Phys. Rev. **96**, 1428 (1954); M. Gell-Mann and M. L. Goldberger, *ibid.* **96**, 1433 (1954).
[24] $M_{\mu\nu}$ is the scattering amplitude apart from an over-all energy-momentum δ function and some numerical factors. The precise structure of $M_{\mu\nu}$ can be read off for example from Eq. (1) in Ref. 17 by dropping all α, β superscripts.

scattering of these "photons" on some target baryon. As these "photons" are only used to lowest non-vanishing (i.e., second) order, they appear only *externally*. (For *internal* virtual vector mesons, zero-mass extrapolations are neither necessary nor are they ever used.) Because of the zero-mass nature of these external particles, it now becomes possible, just as was done in Eq. (2.2) to consider the zero-frequency limit $\omega \to 0$ and to derive low-energy theorems by a straightforward extension[15,17] of known methods. We state the analogs of Eqs. (2.1) and (2.2) for this more general situation:

$$k_\mu' M_{\mu\nu}{}^{\alpha\beta}(k',k) = M_{\nu\lambda}{}^{\alpha\beta}(k',k)k_\lambda = if^{\alpha\beta\gamma}\langle p'|V_\nu{}^\gamma(0)|p\rangle . \quad (2.3)$$

Here $\beta(\alpha)$ denotes the internal symmetry index of the incoming (outgoing) photon, α, β, $\gamma = 1-3$ for $SU(2)$, $1-8$ for $SU(3)$. $f^{\alpha\beta\gamma}$ are the structure constants of the symmetry group. The $V_\nu{}^\gamma$ are the conserved vector currents. p and p' are the incoming (outgoing) baryon momenta.

Instead of the previous $M_{\mu\nu}(k',k)$ we now must deal with quantities $M_{\mu\nu}{}^{\alpha\beta}(k',k)$, where α, β are internal symmetry indices. Put

$$M_{\mu\nu}{}^{\alpha\beta}(k',k) = M_{\mu\nu}{}^{\{\alpha,\beta\}}(k',k) + M_{\mu\nu}{}^{[\alpha,\beta]}(k',k) , \quad (2.4)$$

where the first and second terms on the right-hand side are symmetric and antisymmetric in (α,β), respectively. The extension of (2.2) is

$$e^2(2\pi)^3 M_{mn}{}^{\{\alpha,\beta\}} = S_1{}^{\alpha\beta}(\omega^2)\delta_{mn} + \omega S_2{}^{\alpha\beta}(\omega^2) \times \tfrac{1}{2}[\sigma_m,\sigma_n] , \quad (2.5)$$

$$e^2(2\pi)^3 M_{mn}{}^{[\alpha,\beta]} = \omega A_1{}^{\alpha\beta}(\omega^2)\delta_{mn} + A_2{}^{\alpha\beta}(\omega^2) \times \tfrac{1}{2}[\sigma_m,\sigma_n] . \quad (2.6)$$

Thus M has now been decomposed into its even and odd parts with respect to both spin and internal spin. The separate parts play the following roles:

(1) (Even, even), $S_1{}^{\alpha\beta}$. This is the Thomson-type scattering amplitude. As is well known, the dispersion relation for S_1 needs a subtraction. The assumptions β, γ stated earlier refer to no subtractions for S_2, A_1, A_2.

(2) (Odd, even), $S_2{}^{\alpha\beta}$. This is the amplitude which gives rise to the DH rules.

(3) (Even, odd), $A_1{}^{\alpha\beta}$. This gives the CR rule.

(4) (Odd, odd), $A_2{}^{\alpha\beta}$. This gives the linear rule.

To conclude this section, we give the various sum rules in their $SU(3)$ form.

DH *rules.*

$$(2\pi^2\alpha/M^2)(\kappa_F F_\alpha + \kappa_D D_\alpha)^2 = X_P(\gamma^\alpha) - X_A(\gamma^\alpha) , \quad (2.7)$$

$$X_{P \text{ or } A}(\gamma^\alpha) = \int \frac{d\omega}{\omega} \sigma^\alpha{}_{P \text{ or } A}(\omega) ,$$

where the meaning of the symbols is as follows. M is the baryon mass, κ_F and κ_D are the F and D parts of the anomalous moments; in particular,

$$\kappa(p) = \kappa_F + \tfrac{1}{3}\kappa_D , \quad (2.8)$$
$$\kappa(n) = -\tfrac{2}{3}\kappa_D .$$

Further, $D_\alpha = \frac{2}{3} d_{\alpha\beta\gamma} F_\beta F_\gamma$ and normalizations are such that $F_3 = D_3 = \frac{1}{2}$ $(-\frac{1}{2})$ for the proton (neutron). γ^α denotes a "photon" of unitary kind α with F-type coupling strength eF_α. $\sigma^\alpha(\omega)$ is the total cross section for the scattering of a γ^α photon on the baryon in question. The subscript P (A) means, that the photon helicity is parallel (antiparallel) to the baryon spin.

Equation (2.7) is to be read as an 8×8 diagonal matrix equation in baryon octet space. The same is true for the additional sum rules to follow.

Examples. To get the isovector moments, put $\alpha = 3$, γ^3 is the isovector photon. To get the isoscalar moments, put $\alpha = 8$ and multiply both sides of Eq. (2.7) by $\frac{4}{3}$. One gets

$$\frac{2\pi^2\alpha}{M^2}\left[\kappa_F Y + \frac{2\kappa_D}{3}\left(I(I+1) - \frac{Y^2}{4} - \frac{F^2}{3}\right)\right]^2 = 4X_P(\gamma^Y) - 4X_A(\gamma^Y) , \quad (2.9)$$

where γ^Y is the isoscalar photon with F-type coupling strength $Y/2$. Y = hypercharge, I = isospin, F = unitary spin. For the representation (p,q) of $SU(3)$, $F^2 = (p^2+q^2-pq+3p)/3$. [For the octet, $(p,q) = (2,1)$ and $F^2 = 3$.]

The total moments are obtained by putting the linear combinations $F_3 + F_8/\sqrt{3}$ on the left-hand side of (2.7), and one gets[13]

$$\frac{2\pi^2\alpha}{M^2}\left[\kappa_F Q - \frac{2\kappa_D}{3}\left(U(U+1) - \frac{Q^2}{4} - \frac{F^2}{3}\right)\right]^2 = X_P(\gamma) - X_A(\gamma) , \quad (2.10)$$

where U is the U-spin and γ is "the" photon.

The isovector sum rule is

$$\frac{2\pi^2\alpha}{M^2}[\kappa_F F_3 + \kappa_D D_3]^2 = X_P(\rho^0) - X_A(\rho^0) = \tfrac{1}{2}[X_P(\rho^+) + X_P(\rho^-) - X_A(\rho^+) - X_A(\rho^-)] , \quad (2.11)$$

where $\rho^{0\pm}$ means that we deal with the scattering of a massless external vector meson of the $\rho^{0\pm}$ variety.

CR rule.

$$\frac{2\pi^2\alpha}{M^2}[AF_3 + BD_3 + iC\theta_{12}] = X(\rho^+) - X(\rho^-) , \quad (2.12)$$
$$X(\rho^\pm) = X_P(\rho^\pm) + X_A(\rho^\pm) .$$

The symbols X are as defined in Eq. (2.7). Furthermore,

$$A=\mu_F^2+\mu_D^2-1+8M^2\left(\frac{\partial G_E^F}{\partial q^2}\right)_{q^2=0}, \quad (2.13)$$

$$B=2\mu_F\mu_D+8M^2\left(\frac{\partial G_E^D}{\partial q^2}\right)_{q^2=0}, \quad (2.14)$$

$$C=\tfrac{2}{3}\mu_D^2. \quad (2.15)$$

μ_F, μ_D are the F, D parts of the total magnetic moments, and G_E^F, G_E^D are the F, D parts of the charge form factor. Both the μ and the G quantities are related to the corresponding nucleon quantities by relations of the type Eq. (2.8). $\theta_{\alpha\beta}$ is defined by

$$[D_\alpha,D_\beta]=if_{\alpha\beta\gamma}F_\gamma-\tfrac{2}{3}\theta_{\alpha\beta}, \quad (2.16)$$
$$(\theta_{\alpha\beta})_{ab}=\delta_{\alpha a}\delta_{\beta b}-\delta_{\alpha b}\delta_{\beta a}.$$

The only baryon state for which θ_{12} has nonvanishing expectation are Σ^+ and Σ^- for which $i\theta_{12}=-1$ and $+1$, respectively.

Linear rule.[25]

$$\frac{4\pi^2\alpha}{M}(\mu_F F_3+\mu_D D_3)=Z, \quad (2.17)$$

$$Z=Z(\rho^+)-Z(\rho^-),$$

$$Z(\rho^\pm)=\int[\sigma_P(\rho^\pm)-\sigma_A(\rho^\pm)]d\omega, \quad (2.18)$$

Finally, it should again be stressed that the introduction of a photon octet is nothing but a formal trick. For example, in Eqs. (2.12) and (2.18), any reference to a massless $\rho^\pm$ can be eliminated by noting the identity $\sigma(\rho^+)-\sigma(\rho^-)=\sigma(\frac{3}{2})-2\sigma(\frac{1}{2})$, if the target has isospin $\frac{1}{2}$, where $\sigma(\frac{3}{2})$ and $\sigma(\frac{1}{2})$ are total cross sections for absorption of a conventional isovector photon in the total isospin states $\frac{3}{2}$ and $\frac{1}{2}$, respectively.

III. CONSISTENCY CONDITIONS

In a recent discussion[13] of the DH rules it was noted that the requirements α-γ stated in Sec. II lead to consistency conditions which take the form of integral relations between cross sections. From the formulation of the assumption α-γ it follows, in particular, that these conditions are exact to all orders in the strong interactions and to order e^2 in electromagnetism. It is an unusual situation that conditions α and β which have reference to space-time aspects on the one hand and conditions γ which refer essentially to internal symmetry aspects on the other constrain each other when intercombined. In general, the implications of Compton sum rules can be understood only when these consistency conditions are taken into account along with the rules themselves. In this section we shall extend the previous considerations by discussing also the consistency conditions for the CR and the linear rules.

In order to get a clear picture of the origin of these conditions, it is useful to return to Eq. (2.4). The α, β symmetric and antisymmetric parts of M satisfy distinct divergence conditions:

$$k_\mu' M_{\mu\nu}^{\{\alpha,\beta\}}(k',k)=M_{\mu\nu}^{\{\alpha,\beta\}}(k',k)k_\nu=0, \quad (3.1)$$

$$k_\mu' M_{\mu\nu}^{[\alpha,\beta]}(k',k)=M_{\nu\lambda}^{[\alpha,\beta]}(k',k)k_\lambda =if^{\alpha\beta\gamma}\langle p'|V_\nu^\gamma(0)|p\rangle. \quad (3.2)$$

We consider first the case of isospin only.

Internal symmetry SU(2). The amplitude $M_{mn}^{\{\alpha,\beta\}}(k,k)$ contains the (odd, even) amplitude $S_2(\omega^2)$ which enters in the DH rule. Consider the case of a target with arbitrary isospin and corresponding isospin operator t^α. The low-energy theorem tells us that

$$M_{mn}^{\{\alpha,\beta\}} \text{ is proportional to } \{t^\alpha,t^\beta\}=T^{\alpha\beta}+S^{\alpha\beta}, \quad (3.3)$$
$$T^{\alpha\beta}=\{t^\alpha,t^\beta\}-\tfrac{2}{3}\delta^{\alpha\beta}(t^\gamma t^\gamma),$$
$$S^{\alpha\beta}=\tfrac{2}{3}\delta^{\alpha\beta}t^\gamma t^\gamma,$$

where T and S are tensor operators for isospin 2 and 0, respectively. Thus the proportionality given in Eq. (3.3) means that the photon-target scattering can be considered as coming about by transmitting a *prescribed mixture* of isospin 2 and 0 in the t channel. It is the fixed proportion of this mixture which causes consistency conditions to arise, except if the target has isospin $\frac{1}{2}$ or 0. For then the $I=2$ transmission in the t channel is forbidden anyway and $I=0$ transmission is in fact the most general case. This then gives an alternative way of seeing how the $SU(2)$ conditions given before[13] come about.

Consider next the antisymmetric part of M. We have

$$M_{mn}^{[\alpha,\beta]} \text{ proportional to } [t^\alpha,t^\beta]=i\epsilon^{\alpha\beta\gamma}t^\gamma, \quad (3.4)$$

corresponding to pure $I=1$ transmission in the t channel. This is at once seen to be the most general situation for the antisymmetric part. We have therefore the:

Theorem. The conditions α-γ for the case of isospin symmetry do not imply any constraints for the CR or the linear sum rules, regardless of the isospin of the target.

Remark. Here and in the following we restrict ourselves to the forward amplitude, i.e., $\mathbf{k}'=\mathbf{k}$. The present considerations can of course be extended also to the nonforward case.

Internal symmetry SU(3). The origin of the constraints can be stated in similar terms. We have a prescribed mixture of **1**, **8**, **27** in the t channel for the

[25] Equations (2.12) and (2.17) each are members of corresponding larger sets, just as the isovector DH rule is a special case of Eq. (5). We do not write out these larger sets because in the present context one learns nothing new that way.

symmetric part and of **8**, **10**, **10*** for the antisymmetric part of M. The antisymmetric part generates the CR and linear rules. We next discuss the consistency conditions for these cases, always for a baryon octet target.

CR rule. One finds the conditions by eliminating the three parameters A, B, C in Eq. (2.12). Define

$$R=X(\rho^+)-X(\rho^-). \tag{3.5}$$

Let $R(p)$ denote the cross-section integral for scattering on a proton, etc:

$$R(n)+R(\Xi^0)=2R(H)\,,\quad H=\tfrac{1}{2}(\Sigma^0-\Lambda\sqrt{3}). \tag{3.6}$$

However, Eq. (3.6) is a consequence of U spin itself. Indeed Ξ^0, H, and n are the $U_3=1$, 0, -1 members, respectively, of a U-spin triplet, while ρ^+ may in part be characterized by $U=U_3=\frac{1}{2}$ and ρ^- by $U=\frac{1}{2}$, $U_3=-\frac{1}{2}$. It is then readily checked that Eq. (3.6) is nothing but a combination of triangular relations between U-spin scattering amplitudes. Thus, unlike the DH case, the combination of the assumptions α-γ of Sec. II does not impose any constraints at all for an octet target.

At this point we digress to point out that the same is true for the Adler-Weisberger (AW) rule. For this purpose it is convenient to recast Eq. (1.1) in an equivalent form which is closely similar to Eq. (2.12).

$$A'F_3+B'D_3+iC'\theta_{12}=\tfrac{1}{2}c[X(\pi^+)-X(\pi^-)]. \tag{3.7}$$

The notations are as follows: c is given by Eq. (1.2).

$$X(\pi^\pm)=\int d\omega\,\omega^{-1}\sigma(\pi^\pm)\,,$$

cf., Eqs. (2.7), (2.12), and

$$A'=g_F{}^2+g_D{}^2-1,\quad B'=2g_Fg_D,\quad C'=\tfrac{2}{3}g_D{}^2. \tag{3.8}$$

From the algebraic similarity between Eqs. (2.12) and (3.7), it follows at once that the AW rule is likewise constraint free.

Linear rule. From a comparison of Eqs. (2.12) and (2.17) one sees that the linear rule has a similar structure as the CR rule but with $C=0$. Hence, the constraints are stronger for the linear rule. One finds for this case again an integral cross-section consistency condition. It has the following form:

$$Z(p)+Z(\Xi^0)=Z(\Sigma^+). \tag{3.9}$$

This condition can be written in an alternative form by using $SU(3)$ transformations. Introduce "photons" of the type $K^{*\pm}$. Then Eq. (3.9) is equivalent to

$$\begin{aligned}Z(K^{*+},p)-Z(K^{*-},p)\\ =Z(K^{*+},n)-Z(K^{*-},n)+Z(\rho^+,p)-Z(\rho^-,p).\end{aligned} \tag{3.10}$$

That is to say, the consistency condition (3.9) or (3.10) which is a consequence of the linear rule is an integral form of a weak Johnson-Treiman relation[18] for massless vector mesons. One can verify that this relation comes about because the left-hand side of the linear rule corresponds to pure octet transmission in the t channel.

Remark. The corresponding integral form of the strong Johnson-Treiman relation, namely, Eq. (3.10) *and* $Z(K^{*+},n)-Z(K^{*-},n)=Z(\rho^+,p)-Z(\rho^-,p)$, cannot be true as it would imply a vanishing magnetic moment for the neutron.

IV. PHOTOPRODUCTION SUM RULES

We discuss these sum rules first on the level of isospin and thereafter cast them into $SU(3)$ form. The forward amplitude for photoproduction of massless pions by isovector photons may be written as

$$M^{\alpha\beta}=\{\delta^{\alpha\beta}f_S{}^V(\omega)+\tfrac{1}{2}[\tau^\alpha,\tau^\beta]f_A{}^V(\omega)\}i(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})\,, \tag{4.1}$$

where $\boldsymbol{\varepsilon}$ is the photon-polarization vector, ω the photon frequency, and nucleon spinors are understood, but not displayed, on the right-hand side. The amplitude $f_S(\omega)$ is an odd function of ω, whereas $f_A(\omega)$ is an even function.[26] Under the assumption of no subtractions, these functions satisfy the dispersion relations

$$f_S{}^V(\omega)=\frac{2\omega}{\pi}\int_0^\infty d\omega'\frac{\operatorname{Im}f_S{}^V(\omega')}{\omega'^2-\omega^2-i\epsilon}\,, \tag{4.2}$$

$$f_A{}^V(\omega)=\frac{2}{\pi}\int_0^\infty d\omega'\frac{\omega'\operatorname{Im}f_A{}^V(\omega')}{\omega'^2-\omega^2-i\epsilon}\,. \tag{4.3}$$

The low-energy behavior of $f_A(\omega)$ is given by the Kroll-Ruderman theorem[20] to be

$$f_A{}^V(\omega)=(eG_{\pi N}/2M)+O(\omega^2). \tag{4.4}$$

Equations (4.3) and (4.4) lead to the antisymmetric sum rule

$$\frac{eG_{\pi N}}{2M}=\frac{2}{\pi}\int_0^\infty\frac{\operatorname{Im}f_A{}^V(\omega)}{\omega}d\omega. \tag{4.5}$$

This sum rule may be compared with the symmetric sum rules of Fubini *et al.*[19]

$$\frac{e(\kappa(p)-\kappa(n))}{2M}G_{\pi N}=\frac{2}{\pi}\int_0^\infty\frac{\operatorname{Im}f_S{}^V(\omega)}{\omega^2}d\omega\,, \tag{4.6}$$

$$\frac{e(\kappa(p)+\kappa(n))}{2M}G_{\pi N}=\frac{2}{\pi}\int_0^\infty\frac{\operatorname{Im}f_S{}^S(\omega)}{\omega^2}d\omega\,, \tag{4.7}$$

where $f^S(\omega)$ is the amplitude for photoproduction by isoscalar photons. Eqs. (4.6) and (4.7) have been derived in the literature by the prescription of taking the limit: (massless) pion four-momentum $\to 0$ independently of the limit: photon four-momentum $\to 0$.

[26] For details on crossing properties of photoproduction amplitudes see G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

Note that Eq. (4.5) contains only one power of ω in the denominator of the integrand; at first sight this might lead one to believe that this sum rule is less likely to converge than the last two. One observes, however, that $f_A{}^V(\omega)$ is an isospin flip amplitude and is therefore expected to be damped faster than, say, $f_S{}^V(\omega)$. No further statements about convergence are possible without going into specialized models.

The $SU(3)$ form of the sum rules (4.6), (4.7), and (4.5) is

$$\frac{e}{M}\{(\kappa_F F_\alpha+\kappa_D D_\alpha),(G_F{}^P F_\alpha+G_D{}^P D_\alpha)\}_+ =\frac{2}{\pi}\int\frac{d\omega}{\omega^2}\operatorname{Im}f_S(\omega), \quad (4.8)$$

$$\frac{e}{M}(G_F{}^P F_\alpha+G_D{}^P D_\alpha)=\frac{2}{\pi}\int\frac{d\omega}{\omega}\operatorname{Im}f_A(\omega), \quad (4.9)$$

where $G_F{}^P$ and $G_D{}^P$ are, respectively, the F and D part of the pseudoscalar coupling constant. Note that, in the sense of a power-series expansion, $f_A(\omega)$ develops an imaginary part to order G^3 only. Hence the linear rule (4.9), like the linear rule (2.17), is incompatible with a power-series expansion in G, as was already discussed in Sec. I.

The left-hand sides of Eqs. (2.17) and (4.9) have the same algebraic structure. Correspondingly, the sum rule (4.9) has a consistency condition similar to Eq. (3.9). The left-hand side of Eq. (4.8) has a structure similar to but not identical with that of the DH rule, Eq. (2.7). The distinction lies in that the former contains four, the latter only two low-energy parameters. As a result, the consistency conditions for Eq (4.9) are weaker than those for Eq. (2.7), as will be discussed in more detail in the next section.

V. SUPPORT QUESTIONS AND SATURATION PROBLEMS

The various sum rules and consistency conditions that we have met are formally valid independently of the detailed dynamics of the dispersion integrals, as long as the assumptions α–γ of Sec. II are met. We will discuss next a number of saturation approximations applied to these rules and their constraints.

It is important to note that the various sum rules under consideration have distinct supports. By the support of a sum rule we shall mean such sets of saturating states which do not lead to a null solution for the low-energy parameters involved. As we shall illustrate by a number of examples, the differences in support are due to the fact that, as we have seen, the respective rules have distinct constraint properties.

These distinctions indicate that one will not necessarily find a valid leading dynamical approximation by seeking a common support for all sum rules. Indeed, a fairly detailed search which we made did not enable us to find satisfactory approximations to the various low-energy parameters.

We therefore proceeded on what appears to be the next most general approach. Namely, as already mentioned in Sec. I, we treated the antisymmetric and the symmetric sum rules separately. We next discuss the results so obtained.

A. Antisymmetric Sum Rules

In an attempt to extract useful information from the rules we start out by making the following dynamical approximation: (a) For low energies the integrals are dominated by the decuplet in the s, u channels. (b) The residual or "high-energy" contribution may be adequately treated by putting octets in the t channel. One may attempt to justify this high-energy ansatz by an appeal to Regge-pole models. Alternatively, within the framework of a quark model in which one assumes additivity in the high-energy region, one can only have **8**'s in the t channel for antisymmetric rules. After having discussed this saturation we shall then ask for the degree of stability of the results if a further saturating "B^{**} octet" is introduced to account for the intermediate energy region.

(I) AW rule. Using the notations of Eqs. (3.7) and (3.8) we get

$$\frac{A'}{B'}=\frac{\frac{1}{3}C_{10}+2\gamma_F}{C_{10}+2\gamma_D}, \quad (5.1)$$

$$\frac{C'}{B'}=\frac{\frac{1}{2}C_{10}}{C_{10}+2\gamma_D}, \quad (5.2)$$

(where the possibility $g_D=0$ has been excluded). C_{10} is the decuplet contribution. γ_F and γ_D represent the contributions from the **8** in the crossed channel inasfar as its coupling to baryons is of the F and D type.

(1) For $\gamma_F=\gamma_D=0$, $|g_A|=5/3$ and

$$g_D/g_F=\tfrac{3}{2}. \quad (5.3)$$

(2) For this non-spin-flip amplitude we find from a quark model that *any* quark-model wave function leads to $\gamma_D=0$ if additivity is assumed. In terms of quark states Q^A, the antisymmetric boson coupling is of the form

$$Q_A O Q^B[M_C{}^A\bar{M}_B{}^C-\bar{M}_C{}^A M_B{}^C], \quad (5.4)$$

where M and $\bar{M}$ are the incoming and outgoing boson, respectively. The structure of the operator O should be appropriate to non-spin flip, and hence it is essentially a constant if one assumes nonrelativistic internal quark motion. From this and the additivity ansatz $\gamma_D=0$ follows. Observe that $\gamma_D=0$ may also be justified theoretically by considering the t-channel octet as a Regge pole (with quantum numbers of the ρ) which connects with the baryon without momentum transfer.[27]

[27] See, e.g., S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962).

We now observe that for $\gamma_D=0$, $\gamma_F\neq 0$, Eq. (5.3) is still true but not $|g_A|=5/3$, because of the appearance of γ_F in Eq. (5.1). This is a remark originally due to Schnitzer.[7]

However, it is known[28] that the $N^{**}(1520)$ region contributes sizably to $|g_A|$ and therefore it is physically not justified to consider deviations from $|g_A|=5/3$ to be due to the high-energy region only.[29] In this connection we note that the inclusion of a $B^{**}-\mathbf{8}$ in the s, u channels changes Eqs. (5.1) and (5.2) (with $\gamma_D=0$) into

$$\frac{A'}{B'}=\frac{-4C_{dd}-4C_{ff}+\frac{1}{3}C_{10}+2\gamma_F}{-8C_{df}+C_{10}}, \tag{5.5}$$

$$\frac{C'}{B'}=\frac{-(8/3)C_{dd}+\frac{1}{2}C_{10}}{-8C_{df}+C_{10}}. \tag{5.6}$$

Here C_{dd}, C_{ff}, and C_{df} denote, respectively, the pure d, pure f, and df interference contributions from the added $\mathbf{8}$. From Eq. (5.6) it follows that Eq. (5.3) remains valid provided the d/f ratio of the B^{**} octet in its coupling to the baryon–pseudoscalar-meson system is equal to $\frac{3}{2}$. A first attempt to obtain this d/f ratio from experiment has recently been made.[30] On the other hand, it follows from Eq. (5.5) that the B^{**} does contribute to g_A.

We conclude that the axial D/F ratio $\frac{3}{2}$ may well be considerably more stable under inclusion of additional saturating states than the ratio g_A/g_V. For later purposes we note that Eq. (5.3) combined with the Goldberger-Treiman relation yields

$$G_D^P/G_F^P=\tfrac{3}{2}. \tag{5.7}$$

(II) CR rule. With a similar saturation as for the AW case, we again obtain equations of the form (5.1) and (5.2) for the quantities A, B, C defined in Eqs. (2.13)–(2.15). The previous argument for $\gamma_D=0$ applies here as well. This yields $C/B=\frac{1}{2}$ or

$$\mu_D(\mu_D-\tfrac{3}{2}\mu_F)(4M^2)^{-1}=\tfrac{3}{2}(\partial G_E^D/\partial q^2)_{q^2=0}, \tag{5.8}$$

which is not well obeyed.[28] We next include a B^{**} $\mathbf{8}$. Then C/B is given by an equation of the form (5.6). But now the d/f ratio refers to a different process, namely $B^{**}\to B+\gamma$. Evidence has been quoted for $d/f=3$ for this case (pure isovector radiative transition). Inserting this value we see that Eq. (5.8) is *not* stable for this d/f ratio. Thus dynamical approximation methods appear to be less transparent for the CR than for the AW case. The "experimental" method happens to indicate likewise[28] a more complicated origin of the contributions to the CR sum rule.

[28] See, e.g., F. J. Gilman and H. J. Schnitzer, Phys. Rev. **150**, 1362 (1966).

[29] Further deviation from $|g_A|=5/3$ due to the N^* region in the *broken* $SU(3)$ description cannot be discussed with the present methods.

[30] M. Goldberg, J. Leitner, R. Musto, and L. O'Raifeartaigh, Nuovo Cimento (to be published).

(III) The linear sum rules Eqs. (2.17) and (4.9) are both of the antisymmetric type. Again we attempt to saturate them with a $\mathbf{10}$ in the s, u channels and an $\mathbf{8}$ in the t channel. We denote by γ_D/γ_F the D/F ratio of the coupling between this $\mathbf{8}$ and the baryons inasfar as the Compton sum rule is concerned. The corresponding ratio in the photoproduction case will be called γ_D'/γ_F'. These saturations yield

$$\mu_D/\mu_F=\gamma_D/\gamma_F \tag{5.9}$$

and

$$G_D^P/G_F^P=\gamma_D'/\gamma_F', \tag{5.10}$$

respectively. For these rules the consistency condition (3.9) and its photoproduction analog must be enforced, which gives

$$C_{10}=C_{10}'=0, \tag{5.11}$$

where C_{10} and C_{10}' denote the decuplet contributions to the sum rules (2.17) and (4.9), respectively. Equation (5.11) implies that our approximation scheme puts the principal contributions to these sum rules on the "high-energy" region and Eqs. (5.9) and (5.10) relate the D/F ratios for low-energy parameters to the high-energy D/F ratios of t-channel octets. Preliminary numerical estimates[16] indicate that indeed the high-energy region (beyond the N^{**}) is relatively more important for the sum rule (2.17) than appears to be the case for the AW and the CR rules. As before, we have also examined what happens upon inclusion of an additional s, u-channel octet but this does not appear to lead to any clear-cut results.

We shall therefore concentrate on the Eqs. (5.9) and (5.10). The next question is then what one can say about the high-energy D/F ratios which appear on the right-hand sides of these equations. For this purpose we appeal once more to a quark model with additivity and the corresponding unitary-spin-flip coupling to quarks given by Eq. (5.4). If one again assumes nonrelativistic internal motion, then the rest frame of the nucleon is essentially the same as the rest frame of the individual quark under consideration. As a result, the operator O in Eq. (5.4) is essentially $\boldsymbol{\sigma}\cdot(\boldsymbol{\varepsilon}_f\times\boldsymbol{\varepsilon}_i)$ for the Compton case and $\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}$ for the photoproduction case, where the $\boldsymbol{\varepsilon}$'s are the respective photon polarization vectors in the problem. From this it follows that

$$\gamma_D/\gamma_F=\gamma_D'/\gamma_F'. \tag{5.12}$$

This relation when combined with Eqs. (5.7), (5.9), and (5.10) yields

$$\mu_D/\mu_F=\tfrac{3}{2}, \tag{5.13}$$

which is the $SU(6)$ result.

At this point we would like to stress that we have reservations about the derivation of Eq. (5.12), which is an essential link to obtain the answer (5.13). We believe that Eq. (5.12) may be substantially correct, indicating a "universal" high-energy D/F ratio in the t channel for spin flip. However, it would clearly be preferable to have a derivation for it which is less model-dependent. In this connection it should be emphasized that it is out of the question to find any argument for Eq. (5.12) based on the Regge-pole picture only. Indeed if we consider the t-channel octets for these spin-flip sum rules as actual Regge octets, then these octets have opposite G parity for the Compton as compared to the photoproduction case.

However this may be, no appeal has been found necessary to the **56** structure of the baryons to arrive at Eq. (5.13). In a baryon model with nonrelativistic internal quark motion a general base is provided by a superposition of a **56**, two kinds of **70**, and a **20**. One can now ask a converse question. Considering the magnetic moment ratio (5.13) as given, to what extent does this constrain this general linear superposition? Curiously enough, the answer is that one gets uniquely a pure **56**.

Finally we add a comment about the role of spin in these saturation problems. As was noted in Sec. I, the spin of the saturating states is irrelevant in the "spin-symmetric" sum rules, i.e., the AW and the CR rules. The saturation for the "spin-anti-symmetric" linear magnetic-moment rule does depend on the spin of these states as can be illustrated by the following example: Assume a saturation of Z, Eq. (2.17) with a spin-$\frac{1}{2}$ octet and a spin-$\frac{3}{2}$ decuplet, both in the s, u channels. Then the condition (3.9) leads to the result (all quantities referring to a proton-target)

$$Z_{10}(\rho^+)-Z_{10}(\rho^-)=-\frac{8}{3}\left(\frac{d}{d+f}\right)^2[Z_8(\rho^+)-Z_8(\rho^-)],$$

where d and f are the (relative) strengths of the d and f coupling between the saturating octet, the baryon octet, and the vector-meson octet. From the spin properties of these multiplets it is seen that the right-hand side of this relation is negative definite whereas the left-hand side is positive definite. Both sides must therefore be zero.

The spin situation with respect to photoproduction sum rules can only be clarified on the basis of detailed multipole expansions which will not be considered here.

B. Symmetric Sum Rules

We have attempted saturations with a **10** in the s, u channels and an **8** in the t channel also for this group of rules, but did not find satisfactory results this way. The same is true if the saturations to be described below are applied to the antisymmetric rules.

Our starting approximation is here the following: (a) The low-energy region is dominated by a **10** and a **1** in the s, u channels.[13] (b) The high-energy region may be treated by putting a **8** and a **1** in the t channel. (The t-channel contributions actually can often be dispensed with.) In addition we examine again the stability under inclusion of a B^{**} octet.

(IV) DH rules. The three rules Eqs. (2.10), (2.9), and (2.11) for the total, the isoscalar, and the isovector anomalous moments can be obtained from each other by rotations in the (F_3,F_8) plane. They have therefore a common support and we shall discuss Eq. (2.9) for definiteness.

Denote by X the right-hand side of Eq. (2.9) and by $X(N)$ the case of a nucleon target, etc. Then we have (excluding the uninteresting case $\kappa_F=0$)

$$\frac{\kappa_D}{\kappa_F}=-3\frac{X(\Sigma)}{X(N)-X(\Xi)}. \tag{5.14}$$

The consistency conditions for this rule are

$$X(\Sigma)=X(\Lambda), \tag{5.15}$$

$$[X(N)+X(\Xi)-X(\Sigma)]^2=4X(N)X(\Xi). \tag{5.16}$$

Under the stated saturation, Eqs. (5.14)–(5.16) become

$$\frac{\kappa_D}{\kappa_F}=3\frac{C_{10}+\gamma_1+\frac{2}{3}\gamma_D}{C_{10}-2\gamma_F}, \tag{5.17}$$

$$C_1=C_{10}+\tfrac{4}{3}\gamma_D, \tag{5.18}$$

$$(\gamma_1-\tfrac{4}{3}\gamma_D)^2=4(\gamma_1+\gamma_F-\tfrac{1}{3}\gamma_D)\times(C_{10}+\gamma_1-\gamma_F-\tfrac{1}{3}\gamma_D), \tag{5.19}$$

where C_{10}, C_1, (γ_D,γ_F), and γ_1, respectively, measure the contributions of the states in the order mentioned above. We are interested in saturations which yield

$$\kappa_D=3\kappa_F\rightarrow\kappa(p)+\kappa(n)=0. \tag{5.20}$$

(1) Equation (5.20) follows from a saturation with a **10** and a **1** only. In that case (5.18) reduces to $C_1=C_{10}$ and (5.19) is identically satisfied. This is the case discussed before.[13]

The DH rule is of the "spin-antisymmetric" type, so that the spins of the saturating states do come into play. We note that the saturation with **10** and **1** is of the non-null kind if and only if the spins of both **10** and **1** are larger than $\frac{1}{2}$.

(2) More generally, Eq. (5.19) is satisfied provided

$$\gamma_D+\gamma_F=0,\quad \gamma_1=\tfrac{4}{3}\gamma_D, \tag{5.21}$$

together with Eq. (5.18).

(3) The inclusion of the B^{**} octet with[31] $d/f\equiv3$ leaves Eq. (5.20) stable. Thus, $\kappa(p)+\kappa(n)=0$ remains a good approximation when we include these intermediate-energy contributions.

(V) The photoproduction sum rule, Eq. (4.8). For the same reasons as under (IV) we can confine our attention to $\alpha=\beta=8$. For this case write Eq. (4.8) as

$$(2e/M)(\kappa_F F_8+\kappa_D D_8)(G_F{}^P F_8+G_D{}^P D_8)=Y \tag{5.22}$$

and denote by $Y(N)$ the integral concerned if the target is a nucleon, etc.

We now have the one and only consistency condition,

$$Y(\Sigma)=Y(\Lambda), \tag{5.23}$$

[31] See Ref. 28, p. 1370, and footnote 25.

which is the analog of Eq. (5.15). The DH condition (5.16) on the other hand has no analog for photoproduction. The reason for this weaker constraint situation lies in the larger number of low-energy parameters in the photoproduction case. Thus the support for the DH sum rule is more limited than for Eq. (5.22) as can be illustrated by simple examples.[32]

Using the same saturation and a corresponding notation as for the DH rule we have

$$C_1=C_{10}+\tfrac{4}{3}\gamma_D \qquad (5.24)$$

as the only constraint. If we now furthermore use Eq. (5.20) as input, we have in addition

$$\gamma_1+\gamma_F-\tfrac{1}{3}\gamma_D=0 \qquad (5.25)$$

and

$$\frac{2G_D{}^P}{3G_F{}^P+G_D{}^P}=\frac{C_{10}-\gamma_F+\gamma_D}{C_{10}-2\gamma_F}, \qquad (5.26)$$

which leads to the following situation:

(1) Using C_{10} only gives a null solution for the G's.

(2) Using C_{10} and C_1 only gives the bad result $G_D{}^P=3G_F{}^P$.

(3) The experimental method applied to this rule shows[33] that an important contribution arises from a low-energy s-wave pion-production mechanism. In $SU(3)$ language, this is due to an octet transmission in the t channel, including rescattering effects. Note that we may put Eq. (5.26) to use for a qualitative description of this situation provided we associate γ_D and γ_F with such a *low*-energy octet. In particular, if we saturate with $(C_{10},C_1,\gamma_F,\gamma_D)$ only, we have $\gamma_D/\gamma_F=3$ while furthermore Eq. (5.26) is compatible with (but does not force us to) $2G_D{}^P=3G_F{}^P$. One easily sees that the same would not be true if we put $C_1=0$. This then is the closest to the simple (C_{10},C_1) saturation for the DH rule.[34]

(VI) Pagels's sum rule[22] for $\pi_0\rightarrow 2\gamma$ decay. We shall discuss the saturation problems for this sum rule in the derivation of which the low-energy theorem once again plays a crucial role, but now applied to not strictly forward directions. We write the Pagels rule in the following form, in the $SU(3)$ limit:

$$F_\pi\{G_F{}^PQ-\tfrac{2}{3}G_D{}^P[U(U+1)-\tfrac{1}{4}Q^2-1]\}+T_1 =\frac{e^2}{4m}(2Q\kappa+\kappa^2)+Z, \qquad (5.27)$$

$$Z=\frac{2}{\pi}\int\frac{\mathrm{Im}A_3(s)ds}{s-m^2}.$$

κ is the anomalous magnetic moment of the baryons,

$$\kappa=\kappa_FQ-\tfrac{2}{3}\kappa_D[U(U+1)-\tfrac{1}{4}Q^2-1].$$

F_π is essentially the $\pi_0\rightarrow 2\gamma$ decay amplitude; for its precise definition see Ref. 22, Eq. (2.13). A_3 is the U-spin scalar member of the same Hearn-Leader amplitude employed by Pagels. As is discussed in Ref. 22, this amplitude has Feynman poles in the t channel because of the 2γ decay of pseudoscalar mesons. The term proportional to F_π on the left-hand side of Eq. (5.24) is just that pole term which corresponds to this decay mode for the neutral members π^0 and η of the usual pseudoscalar octet.

The T_1 term which we have inserted on the left-hand side of Eq. (5.24) represents a similar pole term (or set of terms) which may arise from the existence of one (or more) unitary singlet pseudoscalar mesons with an important 2γ decay mode. At least one possible candidate for contributions to T_1 is the η' meson.

A main assumption which is made in the derivation of Eq. (5.27) is that the amplitude A_3 satisfies an unsubtracted dispersion relation after all pole terms corresponding to 2γ decays have been isolated. Let us suppose that formally this has been done in Eq. (5.27). Then we can make the following statements:

(1) A saturation of Z by **10** and **1** in the s, u channels is again a simple example of a possible consistent treatment of the dispersion integral.

(2) In particular, such a saturation is compatible with

$$\kappa_D=3\kappa_F \quad\text{and}\quad 2G_D{}^P=3G_F{}^P.$$

(3) With this saturation the Z integral does not contribute to $\pi_0\rightarrow 2\gamma$ decay as a result of which one has the approximate formula

$$\tau(\pi^0)^{-1}=\left(\frac{m_\pi}{m}\right)^2\frac{4\pi\alpha^2}{G_{\pi N}{}^2}[2\kappa(p)+\kappa^2(p)-\kappa^2(n)]^2\frac{m_\pi}{64} \cong(2.2\times10^{-16}\mathrm{sec})^{-1}, \qquad (5.28)$$

which is in approximate agreement with experiment as has been discussed by Pagels.[35]

(4) The result (5.28) could be invalidated if along with the term T_1 in Eq. (5.27) there would be a sizable "T_8" term corresponding to contributions of higher pseudoscalar octets.

(VII) The sum rule for differences of Thomson scatterings appears to give qualitatively reasonable answers, too, if one saturates with **10** and **1** in the s, u channels and if one uses the constraint imposed between C_{10} and C_1 by this saturation.[36]

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[32] Thus, a saturation of the DH rule with a pure F-type t-channel octet yields a null solution, while the same saturation applied to Eq. (5.22) gives $\kappa_DG_D{}^P=\kappa_FG_F{}^P=0$ which is not a null solution.

[33] S. L. Adler and F. J. Gilman, Phys. Rev. **152**, 1460 (1966).

[34] Alternatively one can use $(C_{10},\gamma_1,\gamma_F,\gamma_D)$ where $2G_D{}^P=3G_F{}^P$ implies $5\gamma_D=-3\gamma_F$. For other discussions of this sum rule see M. Cini, M. de Maria, and B. Taglienti (Rome report, unpublished).

[35] As was noted in Ref. 22, this leading term is also present in the work of M. Goldberger and S. B. Treiman [Nuovo Cimento **9**, 451 (1958)] apart from some numerical corrections necessary in that paper.

[36] See H. R. Pagels, Phys. Rev. Letters **18**, 316 (1967); and the closely related work by H. Harari, *ibid.* **18**, 319 (1967).