Hyperon Decays and Partial Conservation of Tensor Current*

K. TANAKA
Department of Physics, The Ohio State University, Columbus, Ohio
and
Center for Theoretical Studies, University of Miami, Coral Gables, Florida
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On the basis of the partial conservation of tensor current (PCTC) hypothesis, sum rules for the magnetic moments of baryons are obtained, one of which yields $\mu_{\Xi^-}=-1.4\pm1.5$ nm. The ratio of D-type coupling to F-type coupling D/F is obtained from the magnetic-moment relations in broken unitary symmetry in $SU(3)$ and $SU(6)$. Sum rules for the transition magnetic moments μ_{BA} of $A\rightarrow B+e^-+\bar{\nu}$ are obtained as a consequence of PCTC. The contribution of the magnetic term to the various semileptonic decay rates of hyperons is estimated on the basis of $SU(3)$ and found to be less than 8% of that of the charge term for the vector current.

I. INTRODUCTION

THE partial conservation of axial-vector current (PCAC) hypothesis together with the algebra of current commutation relations has played an important role in our understanding of particle physics.[1] The PCAC is formulated by the relation

$$\partial_\mu A_\mu{}^i(x)=c^i\phi_5{}^i(x)\,, \tag{1}$$

where $A_\mu{}^i(x)$ is the axial-vector current, $\phi_5{}^i(x)$ is the pseudoscalar field, c^i is a known constant, and i is the $SU(3)$ index of $A_\mu{}^i(x)$ that forms an octet.

In view of the fact that the pseudoscalar mesons $\phi_5(x)$ and vector mesons $\phi_\mu(x)$ form a **35** representation of the group $SU(6)$, it is natural to assume a corresponding relation

$$\partial_\nu T_{\nu\mu}{}^c(x)=Pd^c\phi_\nu{}^c\,, \tag{2}$$

which expresses the partial conservation of tensor current (PCTC) hypothesis,[2,3] where $T_{\nu\mu}{}^c$ is a tensor current, d^c is a known constant, and P is a projection operator. When we consider matrix elements with respect to spin-$\frac{1}{2}$ particles, $P=(1-P_5)/2$, where P_5 carries out a γ_5 transformation so as to project out the magnetic term.[4] This device is introduced to avoid an undesirable result concerning the charge term, which is discussed later.

The application of PCTC to electromagnetic interaction has been shown to reproduce for the nucleon electromagnetic form factors the pole approximation in which the form factors are expressed in terms of the poles due to the vector mesons ρ^0, ω^0, and ϕ^0.[5]

In order to examine further consequencies of PCTC, we consider in this paper (a) the magnetic moments of baryons in the framework of broken $SU(3)$, (b) the transition magnetic moments of the $\Delta S=0$ decay modes $\Sigma^0\rightarrow\Lambda+\gamma$, $\Sigma^-\rightarrow\Lambda+e^-+\bar{\nu}$ that are related by the vector mesons ρ^0, ρ^+, and ρ^-, and (c) the transition magnetic moments of the $|\Delta S|=1$ decay modes $\Sigma^+\rightarrow p+\gamma$ and $A\rightarrow B+e^-+\bar{\nu}$ that are related by the vector mesons K^{*0} and K^{*+}, where S is the strangeness quantum number. One notes that PCTC leads in a natural way to the isotriplet vector current hypothesis[6] that relates the three decays in (b) and an isodoublet vector current hypothesis that relates the two decays in (c).

The matrix elements of $j_\mu{}^c(0)$ and $\partial_\nu T_{\nu\mu}{}^c(0)$ between spin-$\frac{1}{2}$ particle states A and B must have the following form from proper Lorentz invariance, space inversion, conserved vector current hypothesis $\partial_\mu j_\mu{}^c(x)=0$, and $\partial_\mu\partial_\nu T_{\nu\mu}{}^c=0$

$$\langle B(q)|j_\mu{}^c(0)|A(p)\rangle =\bar{u}(q)\bigg[\bigg(i\gamma_\mu-k_\mu\frac{\Delta}{k^2}\bigg)g_{cBA}{}^FK_c{}^V(k^2) -i\sigma_{\mu\nu}k_\nu g_{cBA}K_c{}^T(k^2)\bigg]u(p)\,, \tag{3}$$

and

$$\langle B(q)|\partial_\nu T_{\nu\mu}{}^c|A(p)\rangle=\bar{u}(q)\bigg[\bigg(i\gamma_\mu-k_\mu\frac{\Delta}{k^2}\bigg)G_{BA}{}^{cV}(k^2) -i\sigma_{\mu\nu}k_\nu G_{BA}{}^{cT}(k^2)\bigg]u(p)\,, \tag{4}$$

where $k=q-p$, $\Delta=M_A-M_B$, $\sigma_{\mu\lambda}=(\gamma_\mu\gamma_\lambda-\gamma_\lambda\gamma_\mu)/2i$, the baryon–baryon–vector-meson ($\bar{B}BV$) coupling

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[1] M. Gell-Mann, Physics **1**, 63 (1964).

[2] R. Dashen and M. Gell-Mann, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, 1966); S. Fubini, G. Segre, and J. D. Walecka, Ann. Phys. (N. Y.) **39**, 381 (1966).

[3] W. Krolikowski, Nuovo Cimento **42A**, 435 (1966).

[4] The operator P is not unique and depends on the expression of the matrix element of the current operators with respect to baryon states. It is necessary to include this projection operator in order to avoid an undesirable consequence of PCTC and the asymptotic condition of the matrix elements of the tensor current that follows from the quark model. W. Krolikowski, International Centre for Theoretical Physics, Trieste, Report No. IC/66/70, IC/66/15 (unpublished).

[5] W. Krolikowski, Nuovo Cimento **44A**, 745 (1966). The $\bar{B}BV$ coupling constants here are taken differently compared to the present article.

[6] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); S. S. Gerstein and Ya B. Zeldovitch, Zh. Eksperim. i Teor. Fiz., **29**, 698 (1955) [English transl.: Soviet Phys.—JETP **2**, 576 (1957)].

constant $g_{cBA}{}^F$ would be F type and g_{cBA} would be D and F type in $SU(3)$. It is noted here that the form factors $K_c{}^V$ and $K_c{}^T$ do not depend on the initial and final particles but only on the vector meson. The first and second terms on the right-hand side of Eqs. (3) and (4) are called the charge term and the magnetic term, respectively.

The form factors in Eqs. (3) and (4) are related by PCTC [Eq. (2)]

$$\begin{aligned}\langle B(q)|\partial_\nu T_{\nu\mu}{}^c|A(p)\rangle &= Pd^c\frac{1}{k^2+M_c{}^2}\langle B(q)|j_\mu{}^c(x)|A(p)\rangle \\ &= d^c\frac{1}{k^2+M_c{}^2}\bar{u}(q)[-i\sigma_{\mu\nu}k_\nu K_c{}^T(k^2)]u(p)\,, \quad (5)\end{aligned}$$

where $(-\Box+M_c{}^2)\phi_\mu{}^c=j_\mu{}^c$ was used. We obtain from Eqs. (3)–(5), at $k^2=0$

$$G_{BA}{}^{cV}=0\,, \quad (6)$$

$$G_{BA}{}^{cT}=d^c g_{cBA}K_c{}^T/M_c{}^2. \quad (7)$$

The asymptotic limit $k\to 0$ of the left-hand side of Eq. (4) from the quark model is

$$\lim_{k\to 0}\langle B|\partial_\nu T_{\nu\mu}{}^c(0)|A\rangle=\bar{u}(q)i\sigma_{\mu\nu}k_\nu G_{BA}{}^{cT}u(p)=0\,, \quad (8)$$

which is consistent with Eq. (6). By the device of inserting the projection operator P in the PCTC relation (2), the undesirable result $K_c{}^V=0$ which would forbid the vertex $A\to B+\rho$ is avoided. This is an alternative way to that proposed by Krolikowski.[4]

For the electromagnetic decay such as $\Sigma^0\to\Lambda+\gamma$, and both electromagnetic and weak decay such as $\Sigma^+\to p+\gamma$, one has the following relation for the tensor current $T_{\lambda\mu}{}^i(x)$:

$$-\tfrac{1}{2}\langle B(q)|T_{\lambda\mu}{}^i|A(p)\rangle f_{\lambda\mu} = \langle B(q)|\partial_\lambda T_{\lambda\mu}{}^i(x)|A(p)\rangle e_\mu\,, \quad (9)$$

where $i=3$, 6, 7, 8 and

$$f_{\lambda\mu}=i(k_\lambda e_\mu-k_\mu e_\lambda)\,,\quad k_\mu e_\mu=0. \quad (10)$$

It would be tempting to put

$$\partial_\lambda T_{\lambda\mu}{}^i=Pj_\mu{}^i\,, \quad (11)$$

so that Eq. (8) is satisfied, where P is the projection operator of Eq. (2).

Combining Eqs. (4), (7), and (11), we obtain

$$F_{BA}{}^{iT}=d^c g_{cBA}K_c{}^T/M_c{}^2\,, \quad (12)$$

where $F_{BA}{}^{iT}$ is the magnetic term of the matrix element of $j_\mu{}^i$. The right-hand side of Eq. (12) for $i=3$, 6, 7, 8 will be identified with the corresponding vector meson c. Equation (12) is applicable to the remaining components of i because of the isotriplet and isodoublet vector current hypothesis. Further, note that[3]

$$\lim_{k\to 0}\langle B|j_0{}^c(0)|A\rangle=0$$

for $M_A\neq M_B$ so that the source vector current of the vector meson field cannot be a current of a unitary transformation generator between the states A and B.

Magnetic moments of baryons are considered in Sec. II, $|\Delta S|=1$ decays in Sec. III, and a summary is given in Sec. IV.

II. MAGNETIC MOMENTS

In order to relate the magnetic moments, take $i=3$, $c=\rho$; $i=8$, $c=\phi_0$, and suitable initial and final baryons in Eq. (12), and obtain

$$F_{pp}{}^{3T}=d^\rho g_{\rho pp}K_\rho{}^T/M_\rho{}^2=(\mu_p-\mu_n)/4M\,, \quad (13)$$

$$F_{\Sigma\Sigma}{}^{3T}=d^\rho g_{\rho\Sigma\Sigma}K_\rho{}^T/M_\rho{}^2=(\mu_{\Sigma^+}-\mu_{\Sigma^-})/4M\,, \quad (14)$$

$$F_{\Lambda\Sigma}{}^{3T}=d^\rho g_{\rho\Lambda\Sigma}K_\rho{}^T/M_\rho{}^2=\mu_{\Lambda\Sigma}/2M\,, \quad (15)$$

$$F_{\Xi\Xi}{}^{3T}=d^\rho g_{\rho\Xi\Xi}K_\rho{}^T/M_\rho{}^2=(\mu_{\Xi^0}-\mu_{\Xi^-})/4M\,, \quad (16)$$

$$F_{pp}{}^{8T}=d^\phi g_{\phi pp}K_\phi{}^T/M_\phi{}^2=(\mu_p+\mu_n)/4M\,, \quad (17)$$

$$F_{\Sigma\Sigma}{}^{8T}=d^\phi g_{\phi\Sigma\Sigma}K_\phi{}^T/M_\phi{}^2=(\mu_{\Sigma^+}+\mu_{\Sigma^-})/4M\,, \quad (18)$$

$$F_{\Xi\Xi}{}^{8T}=d^\phi g_{\phi\Xi\Xi}K_\phi{}^T/M_\phi{}^2=(\mu_{\Xi^0}+\mu_{\Xi^-})/4M\,, \quad (19)$$

where M is the nucleon mass, ϕ_0 is a fictitious vector meson that would transform like the $I=S=0$ member of an octet in $SU(3)$, and the anomalous magnetic moments are given in an obvious notation.

Equations (13)–(19) are consequences of PCTC. It is evident that a vector-meson dominance model also yields these relations. Since the value of the coupling constants that appear on the left-hand side of Eqs. (13)–(19) are not known, one cannot calculate the right-hand sides, but can obtain sum rules of the magnetic moments.

Sum rules for the baryon–vector-meson vertex $B\to B'+V$ were obtained by Lai[7] to the first-order symmetry breaking of $SU(3)$, employing algebra of currents and PCTC. A few sum rules of the coupling constants g_{cBA} that have both D- and F-type couplings are reproduced here

$$\begin{aligned} g_{\rho\Sigma\Sigma}+\sqrt{3}g_{\rho\Sigma\Lambda}&=2g_{\rho NN}\,,\\ g_{\rho\Xi\Xi}+\sqrt{3}g_{\rho\Sigma\Lambda}&=g_{\rho NN}\,,\\ -g_{\phi\Xi\Xi}-g_{\phi\Sigma\Sigma}&=g_{\phi NN}.\end{aligned} \quad (20)$$

The combination of Eqs. (13)–(20) yields the following sum rules of the magnetic moments:

$$(\mu_{\Sigma^+}-\mu_{\Sigma^-})+2\sqrt{3}\mu_{\Lambda\Sigma}=2(\mu_p-\mu_n)\,, \quad (21)$$

$$(\mu_{\Xi^0}-\mu_{\Xi^-})+2\sqrt{3}\mu_{\Lambda\Sigma}=(\mu_p-\mu_n)\,, \quad (22)$$

$$-(\mu_{\Xi^0}+\mu_{\Xi^-})-(\mu_{\Sigma^+}+\mu_{\Sigma^-})=(\mu_p+\mu_n). \quad (23)$$

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[7] C. S. Lai, Phys. Rev. **155**, 1562 (1967).

Rearrangement of Eqs. (21)–(23) leads to

$$\mu_{\Xi^-}=-\mu_n-\mu_{\Sigma^+}, \tag{24}$$

$$\mu_{\Xi^0}=-\mu_p-\mu_{\Sigma^-}. \tag{25}$$

When the values[8] $\mu_{\Sigma^+}=(3.3\pm1.5)$ nm and $\mu_n=-1.91$ nm are substituted in Eqs. (24), one obtains

$$\mu_{\Xi^-}=-(1.4\pm1.5)\text{ nm}. \tag{26}$$

The value of μ_{Ξ^-} is different from the value of $\mu_{\Xi^-}=-(\mu_p+\mu_n)=0.12$ nm obtained in $SU(3)$, provided the $SU(3)$ result $\mu_{\Sigma^+}=\mu_p$ does not hold experimentally.

It is interesting that the D/F ratio of $\bar{B}BV$ coupling can be determined by the requirement that the magnetic-moment relations that follow from some symmetry, such as $SU(3)$ or $SU(6)$, broken by electromagnetic interaction, be reproduced. The difference in the D/F ratio of $SU(3)$ compared to $SU(6)$ is explained.

We invoke $SU(3)$ symmetry of the coupling constants and express the $g_{\rho BA}$ in terms of D- and F-type couplings in Eqs. (13) and (14), and also the right-hand sides of the equation in terms of the $SU(3)$ results, and get

$$D+F=(\mu_p-\mu_n)/4M, \tag{27}$$

$$2F=(\mu_{\Sigma^+}-\mu_{\Sigma^-})/4M=(2\mu_p+\mu_n)/4M. \tag{28}$$

Equations (27) and (28) are satisfied by

$$D/F=-3\mu_n/(2\mu_p+\mu_n)=3.4, \tag{29}$$

where $\mu_p=1.79$ nm and $\mu_n=-1.91$ nm. It is emphasized that Eq. (29) is obtained from $SU(3)$ without PCTC.

The D/F ratio in Eq. (29) is that of the magnetic term. The total magnetic moment is the sum of the intrinsic moment that comes from the F-type charge term and the anomalous magnetic moment that comes from the D- and F-type magnetic terms. In $SU(6)$, the F in Eq. (29) can be regarded as including the contributions from the charge and magnetic terms so that Eq. (29) should hold for the total magnetic moments. Putting the $SU(6)$ result[9] $\mu_p/\mu_n=-\frac{3}{2}$ in Eq. (29), one finds

$$D/F=\tfrac{3}{2}, \tag{30}$$

which is the D/F ratio in $SU(6)$. In other words, the difference in the values of the D/F ratio given in Eqs. (29) and (30) is due to their different meaning.

From the isotriplet vector current hypothesis and Eq. (12), we also have for $\Sigma^+\to\Lambda$ and $\Sigma^-\to\Lambda$

$$F_{\Lambda\Sigma}{}^{3T}=\mu_{\Lambda\Sigma}/2M. \tag{31}$$

It is known that the isotriplet vector current hypothesis applied to the charge term leads to the result that the decays $\Sigma^+\to\Lambda+e^-+\bar{\nu}$ do not depend on the charge term.[10,11] The magnetic term with the aid of Eq. (31) gives a negligible contribution to the decay rate of $\Sigma^\pm\to\Lambda+e^\pm+\bar{\nu}$.[10]

[8] A. D. McInturff and C. E. Roos, Phys. Rev. Letters **13**, 246 (1964).

[9] B. Sakita, Phys. Rev. Letters **13**, 643 (1964); M. A. Baqi Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964).

III. $\Delta S=1$ DECAYS

The hyperon decay $A\to B+e^-+\bar{\nu}$ in the familiar description has contributions from the vector and the axial-vector current.[12] The contributions from the magnetic terms of the matrix elements of these currents are not considered. Before an estimate of the contribution of the magnetic terms is made, sum rules of the magnetic terms or transition magnetic moments μ_{BA} of the matrix elements of the vector currents is obtained.

Take $i=4$, 5 and $c=K^{*+}$ in Eq. (12) and obtain

$$(F_{BA}{}^{4T}-iF_{BA}{}^{5T})/2=d^{K^*}g_{K^*BA}K_{K^*}{}^T/M_{K^*}{}^2 =\mu_{BA}/2M. \tag{32}$$

When $A=\Sigma^-$, $B=n$; $A=\Lambda$, $B=p$; $A=\Xi^-$, $B=\Lambda$; and $A=\Xi^-$, $B=\Sigma^0$, respectively, are substituted in Eq. (32), one has

$$d^{K^*}g_{K^*n\Sigma}K_{K^*}{}^T/M_{K^*}{}^2=\mu_{n\Sigma}/2M, \tag{33}$$

$$d^{K^*}g_{K^*p\Lambda}K_{K^*}{}^T/M_{K^*}{}^2=\mu_{p\Lambda}/2M, \tag{34}$$

$$d^{K^*}g_{K^*\Lambda\Xi}K_{K^*}{}^T/M_{K^*}{}^2=\mu_{\Lambda\Xi}/2M, \tag{35}$$

$$d^{K^*}g_{K^*\Sigma\Xi}K_{K^*}{}^T/M_{K^*}{}^2=\mu_{\Sigma\Xi}/2M. \tag{36}$$

The combination of Eqs. (33)–(36) and the following sum rules for the $\bar{B}BV$ vertex[7]

$$\sqrt{3}g_{K^*p\Lambda}-(g_{K^*n\Sigma}/\sqrt{2})+2g_{K^*\Sigma\Xi}=0, \tag{37}$$

$$\sqrt{2}g_{K^*n\Sigma}+\sqrt{3}g_{K^*\Lambda\Xi}-g_{K^*\Sigma\Xi}=0 \tag{38}$$

yield the sum rules for μ_{BA}

$$\sqrt{3}\mu_{p\Lambda}-(\mu_{n\Sigma}/\sqrt{2})+2\mu_{\Sigma\Xi}=0, \tag{39}$$

$$\sqrt{2}\mu_{n\Sigma}+\sqrt{3}\mu_{\Lambda\Xi}=\mu_{\Sigma\Xi}=0. \tag{40}$$

Equations (39) and (40) are consequences of PCTC and are valid to the first-order symmetry breaking in $SU(3)$.

In order to relate $\mu_{n\Sigma}$ of $\Sigma^-\to n+e^-+\bar{\nu}$ to the transition magnetic moment $\mu_{p\Sigma}$ of $\Sigma^+\to p+\gamma$, we introduce the isodoublet vector current hypothesis, which states that the $\Delta S=\Delta Q=1$ current operator $j_\mu{}^{K^{*+}}$ and the $\Delta S=1$, $\Delta Q=0$ current operator $j_\mu{}^{K^{*0}}$, or

$$j_\mu{}^{K^{*+}},\quad j_\mu{}^{K^{*0}}/ef, \tag{41}$$

transform, respectively, like the $I_z=\frac{1}{2}$, $-\frac{1}{2}$ members of a doublet, where e is the electric charge and f is a reduction factor.[13] Then, it follows from Eqs. (12),

[10] C. C. Carlson, Phys. Rev. **152**, 1433 (1966); N. Cabibbo and R. Gatto, Nuovo Cimento **15**, 159 (1960). The author thanks N. Cabibbo for a discussion.

[11] R. H. Dalitz, in *Lectures at the International School of Physics "Enrico Fermi" on Weak Interactions* (Italian Physical Society, Varenna, 1964).

[12] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[13] The reduction factor is given in K. Tanaka, Phys. Rev. **151**, 1203 (1966); **140**, B463 (1965).

(32), and (41) that

$$\mu_{n\Sigma}=\mu_{p\Sigma}, \tag{42}$$

where $(F_{p\Sigma}{}^{6T}-iF_{p\Sigma}{}^{7T})/2=\mu_{p\Sigma}/2M$. The charge term of the matrix element $\langle p|j_\mu{}^{K^{*0}}|\Sigma^+\rangle$ that appears in the rate of the decay $\Sigma^+\to p+\gamma$ does not contribute because of gauge invariance.

The transition magnetic moment $\mu_{p\Sigma}{}^T$ in the framework of a pole model has two contributions: (i) the inner bremsstrahlung contribution $\mu_{p\Sigma}{}^b$, in which the initial Σ^+ changes into a p by a weak vertex and the photon is emitted by Σ^+ or p; and (ii) the structure contribution $\mu_{p\Sigma}$, in which the Σ^+ goes into a p and K^{*0} and the K^{*0} changes effectively into a photon by a weak vertex. On the basis of PCTC, the latter contribution is suggested analogously to the related decay mode $\Sigma^0\to\Lambda+\rho\to\Lambda+\gamma$. For the validity of PCTC, it is necessary that the structure contribution dominate the inner bremsstrahlung contribution.

The latter is estimated from the procedure of relating the radiative matrix element $\mathfrak{M}$ $(\Sigma^+\to p+\gamma)$ to that of $\mathfrak{M}_0$ $(\Sigma^+\to p)$.[14] One then has

$$\mathfrak{M}=i\frac{\mathfrak{M}_0}{\Delta}\bar{u}(q)\left(\frac{\mu_{\Sigma^+}}{2M}-\frac{\mu_p}{2M}\right)\sigma_{\mu\nu}k_\nu u(p)A_\mu, \tag{43}$$

where $\Delta=M_\Sigma-M_p$, from which one can put

$$\mu_{p\Sigma}{}^b=\mu_{\Sigma^+}-\mu_p. \tag{44}$$

When the values[8] $\mu_{\Sigma^+}=3.3\pm1.5$ nm and $\mu_p=1.79$ nm are substituted in Eq. (44),

$$\mu_{p\Sigma}{}^b=(1.5\pm1.5)\text{ nm}. \tag{45}$$

In order to estimate $\mu_{p\Sigma}$, we invoke $SU(3)$ in which case $F_{BA}{}^{cT}$ is expressible in terms of two constants D and F. Then, Eqs. (27), (33), and (42) yield

$$\mu_{p\Sigma}/2M=\sqrt{2}(D-F), \tag{46}$$

$$(\mu_p-\mu_n)/4M=D+F. \tag{47}$$

When

$$D=-\tfrac{3}{2}\mu_n/4M,\quad F=[\mu_p+(\mu_n/2)]/4M, \tag{48}$$

which result from Eqs. (27) and (28), are put in Eq. (46), one has[15]

$$\mu_{p\Sigma}=(-2\mu_n-\mu_p)/\sqrt{2}=1.44\text{ nm}. \tag{49}$$

This rough estimate gives $\mu_{p\Sigma}\approx\mu_{p\Sigma}{}^b$, which is unfortunately not negligible. In the following, the contribution due to $\mu_{p\Sigma}{}^b$ is ignored.

The μ_{BA} of the $\Delta S=\Delta Q=1$ decay modes of hyperons can be obtained from Eqs. (33)–(36), (42), (49), and $SU(3)$

$$\mu_{n\Sigma}=\mu_{p\Sigma}=1.44\text{ nm}, \tag{50}$$

$$\mu_{p\Lambda}=\mu_{p\Sigma}[-(D+3F)/\sqrt{6}(D-F)]=-1.57\text{ nm}, \tag{51}$$

$$\mu_{\Lambda\Xi}=\mu_{p\Sigma}[-(D-3F)/\sqrt{6}(D-F)]=-0.01\text{ nm}, \tag{52}$$

$$\mu_{\Sigma\Xi}=\mu_{p\Sigma}[(D+F)/\sqrt{2}(D-F)]=1.86\text{ nm}. \tag{53}$$

The decay rate of $A\to B+e+\nu$ due to the vector current is given to order Δ^7 as[10]

$$\Gamma_{BA}=\frac{G^2}{60\pi^3}\Delta^5\left[1-\frac{3}{2}\frac{\Delta}{M_A}+\frac{6}{7}\frac{\Delta^2}{M_A{}^2}+\frac{3}{7}\mu_{BA}\frac{\Delta^2}{MM_A}+\frac{\mu_{BA}{}^2}{7}\frac{\Delta^2}{M^2}\right], \tag{54}$$

where $\Delta=M_A-M_B$, M is the nucleon mass, and $G=10^{-5}/M_p{}^2$. The first three terms on the right-hand side of Eq. (54) are from the change term, the fourth term is due to the interference between the charge and magnetic terms, and the last term is from the magnetic term. The decay rate (54) is modified in the Cabibbo theory by a factor $\sin^2\theta$ and suitable $SU(3)$ Clebsch-Gordan coefficients which follow from the assumption of octet currents.

It is of interest to consider the ratio $R_{BA}=\Gamma_{BA}{}^m/\Gamma_{BA}{}^c$, where $\Gamma_{BA}{}^m$ is the decay rate due to the magnetic term, and $\Gamma_{BA}{}^c$ is that due to the charge term. The sign of μ_{BA} is not known so the sign of $\mu_{p\Sigma}$ is taken to be positive.

The ratio R_{BA} of $\Sigma^-\to n+e^-+\bar\nu$, $\Lambda\to p+e^-+\bar\nu$, $\Xi^-\to\Lambda+e^-+\bar\nu$, and $\Xi^-\to\Sigma^0+e^-+\bar\nu$ are calculated with the aid of Eqs. (50)–(54) to be

$$R_{n\Sigma}=8\times10^{-2},$$
$$R_{p\Lambda}=1\times10^{-2},$$
$$R_{\Lambda\Xi}=2\times10^{-4},$$
$$R_{\Sigma\Xi}=2\times10^{-2}.$$

The contributions from the magnetic term are less than 8% of those rates due to the charge term of the vector current.[16] This result indicates that on the basis of $SU(3)$, which is a stronger assumption than the isodoublet hypothesis, the corrections to the Cabibbo theory in strangeness-changing hyperon decays are small. When it is noted that there is also a contribution to the semileptonic decays due to the axial vector current, perhaps the present rough estimate which gave a negligible result is acceptable, except for $\Sigma^-\to n+e^-+\bar\nu$, although $\mu_{BA}{}^b$ was ignored and $SU(3)$ was used to obtain μ_{BA}. The ratio $R_{n\Sigma}$ depends on the

[14] H. Chew, Phys. Rev. **123**, 377 (1961).

[15] The decay rate of $\Sigma^+\to p+\gamma$ with the value $\mu_{p\Sigma}=2$ nm and $f^2=3.17\times10^{-4}$ leads to the value $W(\Sigma^+\to p+\gamma)/W(\Sigma^+\to p+\pi^0)=(0.37\pm0.08)10^{-2}$ reported in a recent publication [M. Bazin *et al.*, Phys. Rev. Letters **14**, 154 (1965)]. Therefore the present value of $\mu_{p\Sigma}$ given by Eqs. (45) and (49) is consistent with previous calculations discussed in Ref. 13, and also with Ming Chiang Li (to be published) based on a tadpole model.

[16] H. Courant, H. Filthuth, P. Franzini, A. Minguzzi-Ranzi, A. Segar, R. Engelmann, V. Hepp, E. Kluge, R. A. Burstein, T. B. Day, R. G. Glasser, A. J. Herz, B. Kehoe, B. Sechi-Zorn, N. Seeman, G. A. Snow, and W. Willis, Phys. Rev. **136**, B1791 (1964); N. Brene, B. Hellesen, and M. Roos, Phys. Letters **11**, 344 (1964).

magnitude and sign of $\mu_{n\Sigma}$, so that the Cabibbo angle can be affected.

IV. SUMMARY

The partial conservation of tensor current (PCTC) hypothesis has been introduced with a projection operator so that the partial derivative of the tensor current connects the magnetic terms of $\langle B|j_\mu{}^i|A\rangle$ to the vector mesons with the appropriate quantum numbers. Without this device, the PCTC combined with the asymptotic property of the tensor current at $k\to 0$ given by the quark model leads to a vanishing charge form factor.[4] There is no strong reason to preserve this property of the quark model, but it appears desirable to do so for consistency. The consequences of PCTC are equivalent to those of the vector-meson dominance models.

Sum rules of the magnetic moments of baryons are obtained on the basis of PCTC. The value $\mu_{\Xi^-}=-(1.4\pm1.5)$ nm is found from one of the sum rules, μ_{Σ^+}, and μ_n.

It is found that the requirement of reproducing the magnetic-moment relations that result from $SU(3)$ or $SU(6)$ symmetry, broken by electromagnetic interaction, leads to the D/F ratios ($\bar{B}BV$ coupling) of $D/F=3.4$ in $SU(3)$ and $D/F=\frac{3}{2}$ in $SU(6)$.

Sum rules for the transition magnetic moments μ_{BA} of $A\to B+e^-+\bar{\nu}$ are obtained as a consequence of PCTC. The contribution of the magnetic terms of $\langle B|J_\mu{}^i|A\rangle$ to various semileptonic decay rates of hyperons were estimated on the basis of $SU(3)$ symmetry and found to be negligible.

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Some Connections between Sum Rules and Symmetries

M. A. B. Bég and A. Pais
The Rockefeller University, New York, New York

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The question of the existence of consistency conditions arising from the combination of sum rules with internal symmetries is discussed further. The sum rules considered are the forward-scattering Compton and photoproduction rules. A new photoproduction rule is stated. The notion of the support of a sum rule is introduced: viz., the set of states such that saturation with these states does not generate null solutions for low-energy parameters. Different sum rules generally have distinct supports because of differences in the consistency conditions from rule to rule. Applications of saturation methods to the axial D/F ratio, full and anomalous magnetic moments, and the $\pi^0\to 2\gamma$ lifetime are discussed.

I. INTRODUCTION

It has been clear for some time that higher symmetries of the $SU(6)$ type cannot be of the kinematical variety but must rather be looked upon as dynamical in nature.[1] Thus, if symmetries such as $SU(6)$ are to be understood better than as a neat device to codify a few empirical facts, one must find dynamical equations such that $SU(6)$ appears as a symmetry of some of their approximate solutions.[2] Because we have so far no idea as to what are the equations of motion in the domain of strong interactions, we are perforce obliged to take as general a starting point as is profitably possible.

The saturation of equal-time charge commutators was a first attempt in this direction,[3,4] the idea being that dynamical symmetries could be induced by restricting the sum over intermediate states to a select set. It is not clear, however, by what dynamical principle such a selection can be justified, as the mass of an intermediate state has no apparent bearing on its importance in this sum over states.

An alternative procedure for the induction of dynamical symmetries consists[5] in applying the saturation procedure to physical sum rules, i.e., relationships between physical amplitudes (or continuations thereof) and low-energy parameters such as coupling constants, magnetic moments, etc. Since one is now making approximations in the calculation of physical amplitudes, there is perhaps more reason to expect that the restriction to a few low-lying intermediate states may be a genuine dynamical approximation.

[1] For the distinction between kinematical and dynamical symmetry see A. Pais [Rev. Mod. Phys. **38**, 215 (1966)] especially Sec. II B.
[2] See Ref. 1, Sec. IV I.
[3] B. W. Lee, Phys. Rev. Letters **14**, 676 (1965).
[4] R. F. Dashen and M. Gell-Mann, Phys. Letters **17**, 142 (1965).
[5] V. A. Alessandrini, M. A. B. Bég, and L. S. Brown, Phys. Rev. **144**, 1137 (1966).