Conspiracy and Superconvergence in Pion Photoproduction*

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Pion photoproduction, especially at $t=0$, is studied, with particular reference to the conspiracy condition and the superconvergence relation that should apply at that point.

I. INTRODUCTION

OUR basic interest in this paper is pion photoproduction at $t=0$—and the conspiracy[1] condition and superconvergence[2] relation that should apply at that point. Our main conclusions are: (a) The conspiracy relation, although its derivation is complicated by the presence of unequal masses, is similar in structure to the conspiracy found originally by Gribov and Volkov[1] in nucleon-nucleon scattering. We argue on physical grounds that the conspirators are not any of the well-known particles, and that conspirator intercepts are low. If the $\delta(965)$ turns out to have the quantum numbers of the pion, a natural explanation of this would be a conspiracy between its trajectory and the one on which the $\pi_V(1003)$ lies. (b) The large ν behavior[3] of the $A_1^{(-)}$ amplitude at $t=0$ is governed by the exchange of conspirator trajectories only, and hence should satisfy a superconvergence relation which converges rapidly in the asymptotic region. (c) Numerical discussion of the low-energy data, including S waves, $N_{3/2}^*(1238)$, and $N_{1/2}^*(1520)$ fails to saturate the superconvergence relation by an order of magnitude. Because our data certainly stop far short of the asymptotic region, and because higher resonances in photoproduction do not have the usual unitarity limitations, we see no reason why the higher resonances cannot make up the difference.

The order of presentation of the material is as follows: We begin in Sec. II by using conventional Regge arguments to derive a momentum-transfer-parametrized family of superconvergence relations which express the pion-nucleon coupling as an integral over $A_1^{(-)}+tA_2^{(-)}$ above threshold. These conventional arguments indicate good convergence of the $t=0$ form of the superconvergence relation (i.e., for $A_1^{(-)}$). In Sec. III, we tighten these arguments in the course of considering the conspiracy in photoproduction at $t=0$. We point out that the high-energy behavior of $A_1^{(-)}$ is governed entirely by conspirator exchange, and we argue physically that the conspirators have low-lying intercepts. Conjectures are made about actual recurrences of the conspirator trajectories. In Sec. IV, we evaluate the superconvergence relation to find the somewhat surprising result that the data mentioned above fail to saturate the relation by an order of magnitude. Some discussion is given of this, including the possibility of a fixed singularity at $J=0$ (Ref. 4) and/or a Regge cut with the quantum numbers of the pion. Finally, the possibility of the pion conspiring, together with a dynamical damping mechanism for the other conspirators at physical points, is mentioned.

II. DERIVATION AND DISCUSSION OF THE SUPERCONVERGENCE RELATIONS

We take the relation between the invariants in the direct channel and those relevant to the t channel as[5]

$$\begin{aligned}
G_1&=-(p_tk_t/M)(A_1+tA_2)\,,\\
G_2&=-(p_tk_t/M)t^{1/2}A_3\,,\\
G_3&=(k_t/2M)[t^{1/2}-2M][A_1+t^{1/2}A_4]\,,\\
G_4&=(k_t/2M)(2MA_1-tA_4)\,,
\end{aligned}\tag{2.1}$$

where

$$k_t=(t-\mu^2)/2t^{1/2}\,,\quad p_t=\tfrac{1}{2}(t-4M^2)^{1/2}\tag{2.2}$$

are, respectively, the magnitude of the photon and nucleon momenta in the barycentric system of the t channel. μ and M are, respectively, the pion and nucleon mass.

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[1] D. V. Volkov and V. N. Gribov, Zh. Eksperim. i Teor. Fiz. **44**, 1068 (1963) [English transl.: Soviet Phys.—JETP **17**, 720 (1963)].

[2] S. Fubini and S. Segrè, Nuovo Cimento **45**, 641 (1966); V. de Alfaro, S. Fubini, G. Furlan, and C. Rosetti, Phys. Letters, **21**, 576 (1966); A. P. Balachandran, Phys. Rev. **134**, B197 (1964); Ann. Phys. (N. Y.) **30**, 476 (1964).

[3] Where applicable we use the notation of G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957). Our A_i, $i=1\cdots4$ correspond to their A, B, C, D.

[4] Non-Reggeization of a weak amplitude was first noted by K. Bardakci, M. B. Halpern, and G. Segrè, Phys. Rev. **158**, 1544 (1967). Recent elaboration of this point in terms of fixed complex angular momentum poles has appeared in the work by J. B. Bronzan, I. S. Gerstein, B. W. Lee, and F. E. Low, Phys. Rev. Letters **18**, 32 (1967); V. Singh, *ibid.* **18**, 36 (1967) [**18**, 300(E) (1967)].

[5] Our G's are those of G. Kramer and P. Stichel [Z. Physik **178**, 519 (1964)] multiplied by i. The relation between our G's and those of J. S. Ball [Phys. Rev. **124**, 2014 (1961)] is $[G]_{\text{Ball}}=(M/4\pi t^{1/2})[G]_{\text{here}}$. There are two typographical errors in Ball: The right-hand side of his (4.8) should be multiplied by -2, and the right side of his (4.10) should have an extra minus. These errors also appear in G. Zweig [Nuovo Cimento **32**, 689 (1964)].

The conventional partial-wave analysis in the t channel is

$$G_1=-\sum_J (J+\tfrac{1}{2})b_J^-(t)P_J'(z_t),$$

$$z_t=-\nu M/p_t k_t,$$

$$G_2=-\tfrac{1}{2}\sum_J \{a_J^-(t)[JP_{J+1}''+(J+1)P_{J-1}''] -(2J+1)a_J^+P_J''\}, \quad (2.3)$$

$$G_3+G_4=-\sum_J (J+\tfrac{1}{2})b_J^+(t)P_J'(z_t),$$

$$G_4=-\tfrac{1}{2}\sum_J \{a_J^+(t)[JP_{J+1}''+(J+1)P_{J-1}''] -(2J+1)a_J^-(t)P_J''\},$$

where z_t is the scattering angle in the t channel. The amplitudes a_J^+, b_J^+ are excited by triplet nucleon-antinucleon states of total angular momentum J and parity $(-1)^J$. a_J^- is excited by triplet states of parity $(-1)^{J+1}$, and b_J^- by singlet states of parity $(-1)^{J+1}$. Thus A_1+tA_2 receives contributions only from pion exchange (or exchange of an axial vector meson with $C=-1$). For the rest of this section, we shall concentrate our attention on this particular combination in the isospin $(-)$ configuration.

From Ref. 3 we know the dispersion relation

$$A_1^{(-)}(\nu,t)+tA_2^{(-)}(\nu,t)=-\tfrac{1}{2}f_r\frac{\mu^2+t}{\mu^2-t}\left[\frac{1}{\nu_B-\nu}-\frac{1}{\nu_B+\nu}\right] +\frac{1}{\pi}\int_{\nu_0}^{\infty}d\nu'\,[a_1^{(-)}(\nu',t)+ta_2^{(-)}(\nu',t)] \times\left[\frac{1}{\nu'-\nu}-\frac{1}{\nu'+\nu}\right] \quad (2.4)$$

in which the small letters denote imaginary parts and f_r is the pion-nucleon coupling, taken as $f_r^2/4\pi\cong 0.08$. The large-ν limit of Eq. (2.4) is

$$A_1^{(-)}+tA_2^{(-)}\to\frac{1}{\nu}\left\{f_r\frac{\mu^2+t}{\mu^2-t} -\frac{2}{\pi}\int_{\nu_0}^{\infty}d\nu'\,[a_1^{(-)}(\nu',t)+ta_2^{(-)}(\nu',t)]\right\}. \quad (2.5)$$

If the pion is a Regge particle, we expect this amplitude to go down like $\nu^{\alpha_\pi(t)-1}$ for large ν.[6] Because the pion trajectory is negative below $t=\mu^2$, we have the superconvergence relation (for $t<\mu^2$)

$$f_r\frac{\mu^2+t}{\mu^2-t}=\frac{2}{\pi}\int_{\nu_0}^{\infty}d\nu'\,[a_1^{(-)}(\nu',t)+ta_2^{(-)}(\nu',t)]d\nu'. \quad (2.6)$$

At $t=0$, this reduces to the particularly simple form

$$f_r=\frac{2}{\pi}\int_{\mu^2+\mu^2/4M}^{\infty}d\nu'\,a_1^{(-)}(\nu',0), \quad (2.7)$$

because the A's are free of kinematic singularities.[7]

Pion Trajectory and Pion Pole

We can get some insight into how fast this family of sum rules may be expected to converge by seeing how the pole at $t=\mu^2$ arises on the right of (2.6). The conventional way to Reggeize the pion is as follows. Set $G_2=G_3=G_4=0$ in (2.1). This implies that $A_1=0$, and hence

$$\sum_J (J+\tfrac{1}{2})b_J^-(t)P_J'(z_t)=(tp_tk_t/M)A_2, \quad (2.8)$$

Assuming that b_J^- has a moving pole at $J=\alpha_\pi(t)$, and the usual threshold behavior near the pole,

$$b_{\alpha_\pi}^-(t)=t\tilde{b}_\pi(t)(p_tk_t)^{\alpha_\pi}, \quad (2.9)$$

we find (doing a Watson-Sommerfeld transform) that the pion trajectory contributes to $A_2^{(-)}$ in the form

$$tA_2^{(-)}(\nu,t)\sim\frac{t\alpha_\pi(t)\tilde{\beta}_\pi(t)}{\sin\pi\alpha(t)}\left(\frac{1+e^{-i\pi\alpha_\pi(t)}}{2}\right)\left(\frac{\nu}{\nu_0}\right)^{\alpha_\pi(t)-1}. \quad (2.10)$$

The extra factor of α_π comes from the derivative of P_J. To guarantee that $A_2^{(-)}$ has a pion pole, we must take the residue at the pole singular at $t=\mu^2$; then

$$tA_2^{(-)}(\nu,t)\sim\frac{t\alpha_\pi(t)\hat{\beta}_\pi(t)}{2(t-\mu^2)}\left(\frac{1+e^{-i\pi\alpha_\pi(t)}}{\sin\pi\alpha_\pi(t)}\right)\left(\frac{\nu}{\nu_0}\right)^{\alpha_\pi(t)-1}. \quad (2.11)$$

Notice now that, although the imaginary part

$$ta_2^{(-)}(\nu,t)\sim\frac{t\alpha_\pi(t)\hat{\beta}_\pi(t)}{2(t-\mu^2)}\left(\frac{\nu}{\nu_0}\right)^{\alpha_\pi(t)-1} \quad (2.12)$$

has no pion pole, the integral over this Regge "tail" does:

$$\int_{\nu_0}^{\infty}d\nu'\,ta_2^{(-)}(\nu',t)=\frac{t\hat{\beta}_\pi(t)}{2(t-\mu^2)}. \quad (2.13)$$

This then matches the explicit pole on the left side of the superconvergence relation. Note that (2.11) is essentially the pion contribution used by Zweig.[8]

We learn that near, $t=\mu^2$ the continuum contribution to our sum rule will be dominated by a large "tail,"

[6] The angle in the cross channel is not necessarily large for small t, but we assume that daughter trajectories [see D. Z. Freedman and J. M. Wang, Phys. Rev. **153**, 1596 (1967)] allow us to use the usual asymptotic forms. We shall return to note the intertion of daughters and conspirators in Sec. III.

[7] See J. S. Ball, Ref. 5.

[8] See Ref. 5.

and one would not expect to saturate with low-energy data. Of course, as t goes away from μ^2, toward and into negative values, one would expect the low-energy data to be more important. In particular, at $t=0$, the pion pole [contributing only to $A_2^{(-)}$] evidently fails to contribute at all.

What about possible contributions to $A_1^{(-)}(\nu,0)$ from the exchange of the trajectories corresponding to the particles $A_2(1300)$, $A_1(1080)$ which are allowed in the $(-)$ configuration? In the usual Reggeization of a particle which excites a_J^+, b_J^+, or a_J^-, one starts by setting G_1 to zero, thus guaranteeing no contribution to A_1+tA_2 for all t (and, in particular, no contribution to A_1 at $t=0$).

We see that conventional Regge arguments indicate good convergence of the $t=0$ sum rule (2.7). Actually, although this conclusion is correct, more sophistication is necessary because $t=0$ is a point of conspiracy,[1] and the Reggeization at $t=0$ should take this into account. As we shall see in Sec. III, the high-energy behavior of $A_1^{(-)}$ (indeed, all the A_1's) is governed by the exchange of conspirator trajectories only, all of which are low.

III. CONSPIRACY

It has just been mentioned that, in the usual Reggeization of a particle which excites a_J^+, b_J^-, or a_J^-, one starts by setting G_1 to zero, thus guaranteeing no contribution to A_1 ($t=0$). That is, for example, although exchange of the particles $A_2(1300)$, $A_1(1080)$ is allowed in the "minus" configuration, their contribution to $G_3^{(-)}$, $G_4^{(-)}$, and hence to

$$A_1^{(-)}=-\frac{M}{2p_t^2k_t}\{(t)^{1/2}G_3^{(-)}-[2M-(t)^{1/2}]G_4^{(-)}\} \quad (3.1)$$

is usually taken to vanish at $t=0$. On the other hand, however, setting G_1, G_3, and G_4 to zero at $t=0$ is really not consistent with the data; we know both from the sum rule (2.7) and the data[9] that $A_1(\nu,0)\neq 0$, and we see from Eq. (2.1) that, near $t=0$

$$G_1\to\frac{\mu^2A_1}{2|t|^{1/2}},\quad G_3\to\frac{\mu^2A_1}{2i|t|^{1/2}},\quad G_4\to-\frac{\mu^2A_1}{2i|t|^{1/2}}. \quad (3.2)$$

Thus, we learn that any Regge poles whose contributions are to survive in $A_1(\nu,0)$ must satisfy the "conspiracy" equations[10]

$$G_1(t=0)-iG_3(t=0)=0,\quad G_3(t=0)+G_4(t=0)=0. \quad (3.3)$$

[9] The (extrapolated) data for $A_1^{(-)}(\nu,0)$ are discussed in Sec. IV. The data for $A_1^{(+)}$ at $t=0$ are essentially those given in S. L. Adler and F. J. Gilman, Phys. Rev. **152**, 1460 (1966).

[10] Notice that $t=0$ is not in the physical region for photoproduction. This is typical of conspiracy points in the presence of unequal mass, and makes the physical interpretation of the additional symmetry more difficult than in the case of Ref. 1 (where it is a matter of angular momentum conservation): As $\nu\to\infty$, $t=0$ approaches the forward direction, so that, at least for large ν, the additional symmetry at $t=0$ is clear.

Presumably these relations hold for all three isospin configurations, although our primary interest will be in the $(-)$, as it is relevant to our sum rule. It is worth emphasizing that, at this point, we do not know whether the more common particles A_2, A_1, π, etc. conspire or whether the conspirators must be more exotic.

Solution

From Eqs. (2.2) and (2.3) we see that $z_t=0$ at $t=0$. Because z_t is fixed at a point, we cannot obtain the more convenient difference equations in the J plane by a simple partial-wave analysis in z_t. This difficulty is typical of conspiracy relations in the presence of unequal mass, and requires a small trick: We imagine writing the conspiracy relations (3.3) for small t with a right-hand side which vanishes as $t\to 0$. For any finite t, z_t is free to vary (with ν), and we can partial-wave analyze, using Eqs. (2.3). Then, in the limit $t=0$, the right-hand side vanishes. The assumption that we can interchange the order of partial-wave analysis and $t\to 0$ is evidently an assumption of regularity at $t=0$, which should be guaranteed by the daughter trajectories.[11]

In this way, it is straightforward to show that the second conspiracy relation of (3.3) implies

$$b_J^+(t=0)=0 \quad (3.4)$$

for all J, or, in the helicity notation of Ball,

$$T_J(+,+,1)=-T_J(-,-,1), \quad (3.5)$$

at $t=0$. That is to say, the conspiracy suppresses one of the two kinds of magnetic-radiation-initiated transitions at $t=0$.

The first identity of (3.3) implies the more interesting relation (redefining $ia^+\to a^+$)

$$(J-1)a_{J-1}^{+}+(J+2)a_{J+1}^{+}+b_{J-1}^{-}-b_{J+1}^{-}=(2J+1)a_J^-, \quad (3.6)$$

whose structure and solutions are very similar to that found by Gribov and Volkov in nucleon-nucleon scattering.[12] There are two solutions with a finite number of poles:

$$1.\ b_J^-=\frac{b^-}{J-J_0},\quad a_J^+=\frac{a^+}{J-J_0},\quad a_J^-=\frac{a^-}{J-J_0+1},\quad a^+=-\frac{1}{J_0}b^-,\quad a^-=-\frac{2J_0+1}{J_0(2J_0-1)}b^-, \quad (3.7)$$

$$2.\ b_J^-=\frac{b^-}{J-J_0},\quad a_J^+=\frac{a^+}{J-J_0},\quad a_J^-=\frac{a^-}{J-J_0-1},\quad a^+=\frac{1}{J_0+1}b^-,\quad a^-=\frac{3J_0+1}{(2J_0+3)(2J_0+1)}b^-. \quad (3.8)$$

[11] D. Z. Freedman and J. M. Wang (see Ref. 6).

[12] Presumably this is because, in both problems, the cross channel involves the nucleon-antinucleon system.

TABLE I. Contributions to superconvergence relation (in units of μ^{-2}).

Multipole	$\frac{2}{\pi}\int d\nu'\,(\nu')^{-1}a_1^{(+)}$	$\frac{2}{\pi}\int d\nu'\,(\nu')^{-1}a_1^{(-)}$	$\frac{2}{\pi}\int d\nu'\,a_1^{(-)}$
$E_{0+}^{(1/2)}$	0.027	0.027	0.07
$E_{0+}^{(3/2)}$	0.040	−0.020	−0.045
$M_{1+}^{(3/2)}$	0.206	−0.103	−0.25
$E_{1+}^{(3/2)}$	−0.044	+0.022	0.05
$M_{2-}^{(1/2)}$	−0.015	−0.015	−0.08
$E_{2-}^{(1/2)}$	+0.021	+0.021	0.12

In the minus configuration, only particles with $G=-1$, $I=1$ can be exchanged, i.e., particles with the quantum numbers of A_1, A_2, and π, call them A_1', A_2', and π'. We can say immediately on grounds of parallel trajectories that π' cannot be the pion, for, if it were, the A_2' trajectory with the same intercept as the π' would generate a $J^{PG}=0^{+-}$, $I=1$ particle with essentially the mass of the pion. The only candidate at the moment for such a particle is the so-called $\pi_V(1003)$.[13] If this is the first recurrence of the A_2' trajectory, its intercept, along with π', would be around (-1). Then there should be a particle with the quantum numbers of the pion near π_V. The only likely candidate for this is the $\delta(965)$ meson.[13,14] If the A_1' is taken above the π', A_2' (solution 2), it could conceivably be the A_1 itself.

If one allows an infinite number of poles, there are many solutions to (3.6).[15] For example, one can remove the A_1' pole or the A_2' pole if the π' has a family of daughter trajectories spaced downward two units at a time. In any event, all solutions with parent π', A_2' have them with degenerate intercepts, and all solutions with parent π', A_1' have the A_1' one above or below π', so there is, in the asymptotic sense, not much difference between the finite- and infinite-pole solutions.

The result that the commonly known particles are not conspiring can be extended to the other two isospin configurations. In the (+) configuration, only particles with $G=C=-1$ can be exchanged, which means $I=0$ in the nucleon-antinucleon system (e.g., ϕ and ω can be exchanged). According to (3.6), if the ϕ were to conspire (and keep $G_1\neq 0$), it must do so with a ${}^1(J)_J$ trajectory (i.e., b_J^-). In fact, it would have to be an odd-signature trajectory to obtain $G=C=-1$. Hence, a conspiring ϕ would imply a $J^{PGC}=1^{+--}$ meson at about the ϕ mass. Because such a particle is not observed, the ϕ is probably not conspiring. The same arguments apply to the ω. One can only say then that if another ω is discovered at higher energy, and it appears near a 1^{+--} meson, one would suspect these of being on conspirator trajectories. In the isospin (0) configuration, only particles with $G=I=1$ can be exchanged (e.g., the ρ and the 1^{++-} B meson). If the ρ conspired, it would have to do so with a trajectory physical at the B quantum numbers (which excites b_J^-). Because any 1^{++-} meson is much heavier than the ρ, we conclude that neither the ρ, nor the B if it exists, is a conspirator.

Asymptotics

At $t=0$, we know from (3.1) and (3.3) that

$$A_1=\frac{M^2}{p_t^2k_t}G_4. \qquad (3.9)$$

Using the partial-wave expansion (2.1) for G_4, doing a Watson-Sommerfeld transform, and taking $a_J^\pm$ in the pole approximation of the conspiracy solution 1, we find

$$A_1=\frac{\pi M^2}{2p_t^2k_t}\frac{a^+}{\sin\pi J_0}\{J_0P_{J_0+1}''+(3J_0+2)P_{J_0-1}''\}\,, \qquad (3.10)$$

where the coefficient of the second Legendre polynomial is a sum of the terms in a^+, a^- of that form. Because of the daughter trajectories, we should be able to take the large z_t limit of Ref. 11 formally, and have the daughters cancel off any terms singular at $t=0$. That is, as $\nu\to\infty$ [now exhibiting the $(-)$ explicitly, and with proper signature]

$$A_1^{(-)}\sim\frac{\pi M^2}{2p_t^2k_t}\frac{a^+}{\sin\pi J_0}\frac{2^{J_0+1}}{(\pi)^{1/2}}\frac{\Gamma(J_0+\frac{3}{2})}{\Gamma(J_0)}\times\left(\frac{1+e^{-i\pi J_0}}{2}\right)(z_t)^{J_0-1}. \qquad (3.11)$$

This comes only from the A_2' trajectory. With the usual threshold behavior assumed for a^+,

$$a^+=(p_tk_t)^{J_0}\tilde{a}^+\,, \qquad (3.12)$$

the asymptotic form (3.11) is finite at $t=0$. It is the responsibility of the daughter of the A_2' (really the second daughter, as odd-numbered daughters do not couple in photoproduction) to cancel the terms singular at $t=0$ in P_{J_0-1}''. Notice that the A_2' daughter is thus subtracting away both the singular term of the A_2' and the singular A_1'. [The singular residue of the daughter can easily be calculated from (3.10).] Similar analysis can be given for the other solutions to the conspiracy equations.

We conclude then that $A_1(\nu,0)$ is quite convergent for large ν. For example, in the case of $A_1^{(-)}$, we have reasoned that A_2' has an intercept near or below -1, so that

$$A_1^{(-)}(\nu,0)\sim\nu^{-2}\,, \qquad (3.13)$$

[13] See the particle data tables of A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[14] S. F. Tuan and T. T. Wu [Phys. Rev. Letters **18**, 349 (1967)] have recently been led to suspect, on other grounds, that the δ may have the quantum numbers of the pion.

[15] We are informed by D. Z. Freedman and by G. Chew (private communications) that, when conspiracy is approached through $O(4)$ symmetry, only infinite-pole solutions are allowed. Moreover, the towerlike solution which has all three parents keeps the A_1' below the other two.

and one expects the sum rule (2.7) to be quite convergent once the asymptotic region is reached.

IV. EVALUATION OF THE SUPERCONVERGENCE RELATIONS

We shall content ourselves here with a rough estimate for $\int a_1^{(-)}(\nu',0)d\nu'$ based on the detailed calculation of $\int a_1^{(+)}(\nu')^{-1}d\nu'$ done by Adler and Gilman.[16] These authors list the contributions to $\int a_1^{(+)}(\nu')^{-1}d\nu'$ from $E_{0+}^{(1/2)}$, $E_{0+}^{(3/2)}$, $M_{1+}^{(3/2)}$, $E_{1+}^{(3/2)}$, $M_{2-}^{(1/2)}$, $E_{2-}^{(1/2)}$, the non-S-wave multipoles being dominated by $N_{3/2}^*(1238)$ and $N_{1/2}^*(1520)$. The first column in Table I lists their results divided by 2, to compensate for their normalization of the invariant amplitudes. The second column is calculated from the first by an isospin rotation, using the relations

$$A^{(+)}=\tfrac{1}{3}A^{(1/2)}+\tfrac{2}{3}A^{(3/2)}\,,\qquad A^{(-)}=\tfrac{1}{3}A^{(1/2)}-\tfrac{1}{3}A^{(3/2)}\,. \tag{4.1}$$

That is, for $I=\frac{1}{2}, \frac{3}{2}$, we use, respectively,

$$\left[\frac{A^{(-)}}{A^{(+)}}\right]^{(1/2)}=1\,,\quad \left[\frac{A^{(-)}}{A^{(+)}}\right]^{(3/2)}=-\tfrac{1}{2}\,. \tag{4.2}$$

The third column is calculated from the second by a narrow resonance approximation; that is, we have multiplied the contributions dominated by $N_{3/2}^*(1238)$ by ν evaluated at that mass, etc. Because we trust the S-wave models more at low energy, we have multiplied them by $\nu(1238)$, but our results are insensitive to reasonable variations in this. Adding the contributions in the third column, we find

$$\frac{2}{\pi}\int_{\mu^2+\mu^2/4M}^{\infty} d\nu'\, a^{(-)}(\nu',0)\sim -0.13 \tag{4.3}$$

to be compared with $f_r\approx 1.0$. We see that these data fail to saturate the superconvergence relation by an order of magnitude.

Our integration has only gone up to about 2.25 $(\text{BeV})^2$ (center-of-mass energy squared), which is certainly not asymptotic, so the poor saturation is not yet in contradiction to our Regge analysis. Moreover, because the unitarity statement for photoproduction is linear (to lowest order in electromagnetism), there is no ordinary unitarity limitation on the higher resonances. Thus, it is not unreasonable to hope that the higher resonances will saturate the superconvergence relation. This, of course, remains to be seen.

This is, on the other hand, the appropriate place to mention certain "diseases" of a theoretical nature which might prevent the sum rules (2.6) from being correct in the first place, namely, a fixed singularity at $J=0$ and/or a Regge cut with pionic quantum numbers, or, finally, a dynamical mechanism that would allow the pion to conspire while damping the other conspirators at physical points.

Fixed Pole

A fixed pole[4] is always a possibility in scattering amplitudes with linear unitarity, and certainly there is a fixed pole at $J=0$ in the (gauge-invariant) Born term for pion photoproduction. Although some theoretical arguments exist for not having a fixed singularity in a matrix element of only one weak current,[17] we want to discuss briefly the possibility that this pole does not begin to move.

We begin conventionally, as in Sec. II, setting $G_2=G_3=G_4=0$, which results in Eq. (2.8). We then do the Watson-Sommerfeld trick and assume (near the respective poles)

$$b_J{}^-=\tilde{b}_J(k_tp_t)^J\,,\quad \tilde{b}_J=\frac{tp(t)}{J}+\frac{tg(t)}{J-\alpha_\pi(t)}\,. \tag{4.4}$$

This results in the large ν behavior of $tA_2^{(-)}$

$$tA_2^{(-)}(\nu,t)\rightarrow 2\pi\left[\frac{tp(t)}{\nu}+\frac{\pi tg(t)\alpha_\pi(t)}{\sin\pi\alpha_\pi(t)}\times\nu^{\alpha_\pi(t)-1}\left(\frac{1+e^{-i\pi\alpha_\pi(t)}}{2}\right)\right]. \tag{4.5}$$

Taken with Eq. (2.5), this implies the sum rule

$$f_r\frac{\mu^2+t}{\mu^2-t}-\frac{2}{\pi}\int_{\nu_0}^{\infty} d\nu'\,(a_1^{(-)}+ta_2^{(-)})=2\pi tp(t) \tag{4.6}$$

instead of Eq. (2.6) To match the pion pole on the left of (4.6), we must have either $p(t)$ or $g(t)$ (or both) singular at $t=\mu^2$ For simplicity, consider $p(t)$ singular

$$p(t)=\tilde{p}(t)/(t-\mu^2)\,. \tag{4.7}$$

Requiring that the coefficients of the pion pole on both sides of (4.6) are equal, we can rewrite the sum rule as

$$f_r\left[\frac{\mu^2+t}{2}-\frac{t\tilde{p}(t)}{\tilde{p}(\mu^2)}\right]=\frac{\mu^2-t}{\pi}\int_{\nu_0}^{\infty}[a_1^{(-)}+ta_2^{(-)}]d\nu'\,. \tag{4.8}$$

The first thing to note about this sum rule is that, at $t=0$, it reduces to the same sum rule as does the superconvergence relation, namely (2.7), so that the lack of convergence of (2.7) is not a test for a fixed singularity of this kind. Only the $t\neq 0$ form of (4.8) can sense its presence. Of course, this is because we have taken the conventional Reggeization, namely setting $G_2=G_3=G_4=0$ to Reggeize G_1. [It will thus be interesting to evaluate both (2.6) and (4.8) for negative $t\neq 0$, as a test for the fixed singularity.] The way to construct

[16] See Ref. 9.

[17] R. F. Dashen and S. C. Frautschi, Phys. Rev. 143, 1171 (1966).

sum rules that differ from (2.7) at $t=0$ is to assume that the fixed singularity has nonvanishing residue at $t=0$, i.e., that it enters into the conspiracy equations (conspiring fixed singularity). At the moment, we do not feel that the lack of saturation demonstrated above merits such analysis here.

The second thing to note about (4.8) is that it has the form of the sum rule written down by Bardakci, Halpern, and Segrè,[18]

$$f_r[\tfrac{1}{2}(\mu^2+t)-tF_\pi(t)]=\frac{\mu^2-t}{2}\int_{\nu_0}^{\infty}(a_1^{(-)}+ta_2^{(-)})d\nu', \quad (4.9)$$

where $F_\pi{}^+(t)$ is the π^+NN form factor (with π off the mass shell) in a particular field theory of pions, nucleons, and photons. The relation was criticized by these authors on the basis of perturbation theory and/or pure Regge behavior; we learn here that this form of sum rule is implied if there is a fixed singularity at $J=0$. The implication is strong that, if there is a fixed singularity at $J=0$, its residue [more precisely $t\tilde{p}(t)/\tilde{p}(\mu^2)$] could be considered as a natural definition of the πNN form factor.

It is worth underscoring that the fixed singularity discussion given above applies only to the special type of fixed pole assumed, whereas the type suggested by perturbation theory in fact does not vanish at $t=0$. This is because the (gauge-invariant) Born term, which already has the fixed pole, contributes both to A_1 and A_2. The resulting sum rule is Eq. (4.8) with $t\tilde{p}(t)$ replaced by $\mu^2\tilde{p}(t)$. Note that this does *not* have the form of the Bardakci-Halpern-Segrè sum rule—and the residue at the fixed pole is *not* the πNN form factor (although it can be shown that it still has the phase of the form factor). As mentioned above, moreover, a fixed singularity of this form would conspire, according to Eq. (3.6), with infinite sets of fixed singularities for $J\leq 0$.

Regge Cut

It has been pointed out[19,20] that Regge cuts should exist in relativistic theory. We only want to note here that a Regge cut with the quantum numbers of the pion can be made by putting together a ρ trajectory and an ω trajectory. Its leading edge should be at

$$\alpha_c(0)=\alpha_\rho(0)+\alpha_\omega(0)-1, \quad (4.10)$$

which may be above zero. If this is so, then our sum rules (2.6) do not strictly hold. Moreover, if the discontinuity across the cut is large, then they do not hold even in some approximate sense. We do not at the moment consider the lack of saturation of (2.7) as necessarily indicative of a cut.

Dynamical Damping of Physical Conspirators

Recall that our arguments against a conspiring pion were based on not seeing a 0^+ particle near the pion mass (from the conspiring A_2' trajectory). On the other hand, if a mechanism were operative to damp out the A_2' conspirator just as it became physical,[21] then the pion could conspire. Moreover, the conspiring A_2' trajectory, with the same $t=0$ intercept as the pion would add a large asymptotic tail $\nu^{\alpha_\pi(0)-1}$ to $A_1^{(-)}(\nu,0)$—in which case poor convergence of the $t=0$ sum rule would certainly be expected. Again, we emphasize that our present low-energy analysis is not adequate to imply that such a thing is happening.

Note added in proof. Equations (3.3) should be combined to read

$$G_1(t=0)+iG_4(t=0)=0.$$

That is, there is only one conspiracy condition obtainable from our methods. We wish to thank Ling Lie Wang for pointing this out. Strictly speaking, then, Eqs. (3.4) and (3.5) do not follow. On the other hand, Mitter [University of California at Santa Barbara report (unpublished)] has recently claimed that (3.4) and (3.5) are true anyway by $O(4)$ considerations. It should also be noted that the conspiracy solution emphasized here is a Class III solution in the language of $O(4)$, as has recently been discussed by Mitter and by Frautschi and Jones [California Institute of Technology report (unpublished)].

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[18] See Ref. 4.
[19] D. Amati *et al.*, Phys. Letters **1**, 29 (1962); S. Mandelstam, Nuovo Cimento **30**, 1127 (1963); **30**, 1148 (1963).
[20] I. Muzinich, Phys. Rev. Letters **18**, 381 (1967).
[21] For example, a mechanism like that conjectured for daughters by L. Durand, Phys. Rev. Letters **18**, 58 (1967). See also, in this connection, R. J. Oakes, Phys. Letters **24B**, 154 (1967).