in order to satisfy the CPT theorem, $\mathcal{L}$ should have one of the following forms: (a) $\mathcal{L}=GA(J_\alpha J_\alpha\dagger)$ where A denotes antisymmetrization with respect to Fermi fields, (b) $\mathcal{L}=GN(J_\alpha J_\alpha\dagger)$, or (c) $\mathcal{L}=\frac{1}{2}G\{J_\alpha,J_\alpha\dagger\}$. Needless to say, if one regards the weak currents themselves as the fundamental entities, the alternative (c) seems to be the most natural choice; the currents being bosonlike, in the Lagrangian the product should be symmetrized with respect to the currents. On the other hand, the operations A and N in the first two alternatives have to be defined in terms of the fields rather than the currents themselves.

Note added in proof. If a nonlocal generalization of the commutator is used to describe CP violation, the conclusions in Ref. 2 that depend only on the symmetry properties of the commutator will still be valid for such a theory.

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Castillejo-Dalitz-Dyson Poles and Asymptotic Fields*†

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The meaning of Castillejo-Dalitz-Dyson poles (CDD poles) in the field-theoretic context of a boson separable-potential model is studied. Because the asymptotic-field operators for this model depend upon the D function, it is possible to insert CDD poles into these fields and to gauge their effect. It is found that CDD poles have a profound influence: Their presence prevents the in-fields from satisfying canonical commutation relations. A return to the free-particle algebra is possible, however, if new particles are inserted into the theory, one for every pole. These new particles are unstable, thus confirming the link between CDD poles and instability. A further consequence of this work is the development of a method for constructing Hamiltonians which, *a priori*, yield scattering amplitudes containing CDD poles.

I. INTRODUCTION

TRADITIONALLY, Castillejo-Dalitz-Dyson poles[1] (CDD poles) have been associated with unstable particles.[2] The reason is that unstable particles may be identified with the poles of the scattering amplitude in the lower half plane of the second Riemann sheet[3]; CDD poles can also cause these to appear. Of course not all such unstable particle poles can be attributed to CDD poles.[4] What is wanted is a precise connection between instability and the presence of a CDD pole.

The purpose of the present work is to examine this connection in the context of field theory. This is most conveniently done by a study of the infields of a field theoretic model. The program is to explore the effects that CDD poles have on the algebra of asymptotic fields.

In a previous paper[5] one of us investigated the asymptotic fields of a boson separable-potential model.[6,7] In Sec. II CDD poles are inserted into the $\theta_{\rm in}{}^\dagger$ fields of this model. It is found that the operator algebra is radically changed with CDD poles present; no longer do the in-fields obey canonical commutation relations. A return to free-boson algebra is possible, if additional particles are added to the theory. These new particles are shown to be unstable regardless of the existence of a pole in the scattering amplitude on the second Riemann sheet. Furthermore, a method of constructing Hamiltonians containing any number of CDD poles is found.

II. ASYMPTOTIC FIELDS OF THE SEPARABLE POTENTIAL MODEL AND CDD POLES

This model describes a system with two types of particles in it, a static heavy boson φ of mass M, and a boson θ which may move with three momentum $\mathbf{p}$; energy ω_p. The operators which create these particles are labeled $\varphi^\dagger$ and $\theta^\dagger(\mathbf{p})$; vector symbols for momentum indices on boson operators will hereafter be suppressed.

The Hamiltonian for this model in momentum space

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[1] L. Castillejo, R. H. Dalitz, and F. J. Dyson, Phys. Rev. **101**, 453 (1956).

[2] See, for instance, S. Mandelstam, *Weak Interactions and Topics in Dispersion Physics* (W. A. Benjamin, Inc., New York, 1963).

[3] R. E. Peierls, in *Proceedings of the Glasgow Conference on Nuclear and Meson Physics* (Pergamon Press, Ltd., London, 1955), p. 296.

[4] G. F. Chew, Lawrence Radiation Laboratory Report No. UCRL-9289, 1960 (unpublished).

[5] E. Kazes, Phys. Rev. **135**, B477 (1964).

[6] M. T. Vaughn, R. Aaron, and R. D. Amado, Phys. Rev. **124**, 1258 (1961).

[7] J. D. Childress and J. Urrechaga-Altuna, Phys. Rev. **148**, 1359 (1966).

may be written

$$H=\int d\mathbf{k}\,\omega_k\theta^\dagger(k)\theta(k)+M\varphi^\dagger\varphi+\lambda G^\dagger G\varphi^\dagger\varphi\,, \quad (2.1)$$

with

$$G^\dagger\equiv\int d\mathbf{p}\,f(\omega_p)\mathbf{q}^\dagger(p)\,,\quad \omega_p=(\mu^2+p^2)^{1/2}. \quad (2.2)$$

The following commutation laws are obeyed by the elementary-particle operators:

$$\begin{aligned}&[\varphi,\varphi^\dagger]_-=1\,,\\&[\theta(k),\theta^\dagger(p)]_-=\delta(\mathbf{k}-\mathbf{p})\,,\\&[\theta(k),\varphi^\dagger]_-=0.\end{aligned} \quad (2.3)$$

The asymptotic θ-field operator $\theta_{\text{in}}^\dagger(p)$, has been given in Ref. 5. It was constructed to obey

$$[H,\theta_{\text{in}}^\dagger(p)]_-=\omega_p\theta_{\text{in}}^\dagger(p)+i\epsilon[\theta_{\text{in}}^\dagger(p)-\theta^\dagger(p)] \quad (2.4)$$

in the limit of $\epsilon\to 0$. The solution to Eq. (2.4) is

$$\theta_{\text{in}}^\dagger(p)=\theta^\dagger(p)-\frac{\lambda\varphi^\dagger\varphi f(\omega_p)}{D^\dagger(\omega_p)}\int\frac{d\mathbf{p}'f(\omega_{p'})\theta^\dagger(p')}{\omega_{p'}-\omega_p-i\epsilon}, \quad (2.5)$$

where $D^+(\omega_p)$ is defined by

$$D(z)=1+\lambda\varphi^\dagger\varphi\int\frac{d\mathbf{q}\,f^2(\omega_q)}{\omega_q-z}. \quad (2.6)$$

$D^+(\omega_p)=D(z)$ with $z=\omega_p+i\epsilon$ in the limit of $\epsilon\to 0$. This D function is the same as would arise in the standard solution to the Lippmann-Schwinger equation for φ-θ scattering; here it is an operator because the $\theta_{\text{in}}^\dagger(p)$ field is an operator. It obeys the Low equation,

$$\frac{1}{D(z)}=1-\lambda\varphi^\dagger\varphi\int\frac{d\mathbf{q}\,f^2(\omega_q)}{|D^+(\omega_q)|^2(\omega_q-z)}, \quad (2.7)$$

in the absence of bound states.

Equation (2.7) allows solutions more general than Eq. (2.6),[8] however. A full solution to the Low equation is

$$D(z)=1-\lambda 4\pi^2\varphi^\dagger\varphi\sum_{j=1}^{j=n}\frac{a_j}{z-x_j}+\lambda 4\pi\varphi^\dagger\varphi\int_\mu^\infty\frac{d\omega_q\,f^2(\omega_q)q\omega_q}{(\omega_q-z)}, \quad (2.8)$$

where CDD pole terms have been added for completeness. Because Eq. (2.7) demands that $D(z)$ be proportional to an R function[1] (with proportionality constant λ), every a_j must be positive. The pole terms are labeled in order of increasing energy; depending on λ and $f(\omega_q)$ it is possible for x_1 to lie to the left of μ, but every other x_j must be in the cut (μ,∞).

[8] For a general methods of solution of Eq. (2.7) see R. Norton and A. Klein, Phys. Rev. **109**, 991 (1958).

Because, as will become apparent, CDD poles have such profound effects, the functions shall be labeled.

$$\begin{aligned}D(z)\text{ (with CDD poles)}&\equiv D_c(z)\,,\\ D(z)\text{ (no CDD poles)}&\equiv D(z).\end{aligned}$$

Similarly

$$\theta_{\text{in}}^\dagger(p)\text{ (with CDD poles)}\equiv\theta_{\text{in},c}^\dagger(p).$$

If CDD poles are absent, the in-fields may easily be shown to obey

$$[\theta_{\text{in}}(k),\theta_{\text{in}}^\dagger(p)]_-=\delta(\mathbf{p}-\mathbf{k}). \quad (2.9)$$

Furthermore the in-fields may be used to diagonalize the Hamiltonian, since (aside from terms proportional to ϵ)[5]

$$H=\int d\mathbf{k}\,\omega_k\theta_{\text{in}}^\dagger(k)\theta_{\text{in}}(k)+M\varphi^\dagger\varphi \quad (2.10)$$

by explicit calculation. Equation (2.10) defines the Hamiltonian when CDD poles are present.

If the D function contains CDD poles, however, the algebra of the asymptotic fields is changed. The boson commutation relation, Eq. (2.9), is not obtained for $\theta_{\text{in},c}^\dagger(p)$. Rather,

$$[\theta_{\text{in},c}(k),\theta_{\text{in},c}^\dagger(p)]_-=\delta(\mathbf{p}-\mathbf{k})-\frac{(\lambda\varphi^\dagger\varphi)^2 4\pi^2 f(\omega_k)f(\omega_p)}{D_c^-(\omega_k)D_c^+(\omega_p)}\times\sum_{j=1}^{j=n}\frac{a_j}{(\omega_p-x_j+i\eta)(\omega_k-x_j-i\epsilon)} \quad (2.11)$$

in the limit of $\epsilon\to 0$, $\eta\to 0$. For computational purposes $\theta_{\text{in},c}^\dagger(p)$ is to be defined in the limit of $\eta\to 0$, while $\theta_{\text{in},c}(k)$ is in the limit of $\epsilon\to 0$.

A return to boson commutation relations in the presence of CDD poles may be affected by the introduction of n new boson fields, labeled $\beta_j^\dagger$, such that they obey

$$\begin{aligned}&[\beta_i,\beta_j^\dagger]_-=\delta_{ij}\,,\\&[\beta_j,\varphi]_-=[\beta_j,\varphi^\dagger]_-=[\beta_j,\theta(k)]_-=0.\end{aligned} \quad (2.12)$$

We may now define a new in-field, $\theta_{\text{in}}^{\dagger\prime}(p)$,

$$\theta_{\text{in}}^{\dagger\prime}(p)\equiv\theta_{\text{in},c}^\dagger(p)+\sum_{j=1}^{j=n}\mathfrak{F}_j^*(\omega_p)\beta_j^\dagger\,, \quad (2.13)$$

where $\mathfrak{F}_j^*(\omega_p)$ is constructed so as to yield

$$[\theta_{\text{in}}'(k),\theta_{\text{in}}^{\dagger\prime}(p)]_-=\delta(\mathbf{k}-\mathbf{p}). \quad (2.14)$$

Substitution of Eq. (2.13) into Eq. (2.14) gives

$$\mathfrak{F}_j^*(\omega_p)=\lim_{\epsilon\to 0}\frac{\lambda 2\pi\varphi^\dagger\varphi(a_j)^{1/2}f(\omega_p)}{D_c^+(\omega_p)(\omega_p-x_j+i\epsilon)} \quad (2.15)$$

as a solution. $\mathfrak{F}_j^*(\omega_p)$ may be multiplied by an arbitrary phase factor which has been here set equal to $+1$. Thus a new asymptotic field has been constructed which obeys boson commutation relations. The effect of the CDD pole terms has been just cancelled by the new boson fields $\beta_j^\dagger$. Note that this would not have been possible if the residues of the poles, a_j, were negative.

In order to see what significance lies in this new in-field, or, rather, in $\beta_j^\dagger$, consider

$$H' \equiv \int d\mathbf{k}\, \omega_k \theta_{\text{in}}^{\dagger\prime}(k)\theta_{\text{in}}'(k) + M\varphi^\dagger\varphi. \quad (2.16)$$

Using Eqs. (2.7), (2.13), and

$$\lim_{\epsilon\to 0}(\lambda\varphi^\dagger\varphi)^2 a_j 4\pi^2 \int \frac{d\mathbf{q}\, f^2(\omega_q)}{|D_c^+(\omega_q)|^2|\omega_q - x_j + i\epsilon|^2} = 1 \quad (2.17)$$

(this relation is proved in the Appendix), one gets, aside from terms proportional to ϵ,

$$H' = H + \sum_j x_j\beta_j^\dagger\beta_j + \lambda\varphi^\dagger\varphi 4\pi^2 \sum_{i,j}(a_i a_j)^{1/2}\beta_j^\dagger\beta_i$$
$$+ \lambda\varphi^\dagger\varphi 2\pi \sum_j (a_j)^{1/2}\int d\mathbf{k}\, f(\omega_k)\beta_j^\dagger\theta(k)$$
$$+ \lambda\varphi^\dagger\varphi 2\pi \sum_j (a_j)^{1/2}\int d\mathbf{k}\, f(\omega_k)\theta^\dagger(k)\beta_j, \quad (2.18)$$

where H is given in Eq. (2.1) and the sums are over-all CDD pole terms.

One may calculate φ-θ scattering states for the model encountered thus far. The scattering states for the models which are written in terms of in-fields are obvious by construction. In the absence of CDD poles $\theta_{\text{in}}^\dagger(p)\varphi^\dagger|0\rangle$ is the scattering eigenstate for the H of Eq. (2.10) [not $\varphi^\dagger\theta_{\text{in}}^\dagger(p)|0\rangle$, however]—and with CDD poles present $\theta_{\text{in}}^{\dagger\prime}(p)\varphi^\dagger|0\rangle$ is the scattering state for the H' of Eq. (2.16). Note that Eq. (2.11) precludes $\theta_{\text{in},c}^\dagger(p)\varphi^\dagger|0\rangle$ from being the scattering state for the H of Eq. (2.10) even if Eq. (2.10) contained CDD poles. If the Hamiltonian are expressed in terms of bare fields the scattering states are not obvious. But, as expected, a Lippmann-Schwinger derivation gives $\theta_{\text{in}}^\dagger(p)\varphi^\dagger|0\rangle$ as the scattering state for the Hamiltonian of Eq. (2.1). The scattering amplitude will have no CDD poles. The new Hamiltonian, H', of Eq. (2.18) differs from that of Eq. (2.1) in one important respect: it yields, as a straightforward solution to the Lippmann-Schwinger equation, a scattering state, $\theta_{\text{in}}^{\dagger\prime}(p)\varphi^\dagger|0\rangle$, and a scattering amplitude which do contain CDD poles.

Because a favorite conjecture in the literature has been the association of CDD poles with unstable particles, we suspect that the β_j particles decay in time. In order to investigate this possible behavior, consider[9]

$$C_{Nj}(t) \equiv \langle(\varphi^\dagger(t))^N|\beta_j(t)\beta_j^\dagger(0)|(\varphi^\dagger)^N\rangle. \quad (2.19)$$

This function, $C_{Nj}(t)$, is thus defined as the probability amplitude of having a β_j particle present at time t, in a state with N φ-particles, if a β_j particle was originally present at time $t=0$. States containing N φ-particles are considered because the Hamiltonian of Eq. (2.18) shows that, while a β_j may go into a θ particle in the presence of a φ, a β_j by itself will always remain unchanged (having a rest energy, x_j).

To evaluate Eq. (2.19) $\beta_j^\dagger(t)$ must be found. Consider

$$[H', \beta_j^\dagger(t)]_- = -i\dot{\beta}_j^\dagger(t). \quad (2.20)$$

Therefore

$$-\dot{\beta}_j^\dagger(t) = -\lambda 2\pi\varphi^\dagger\varphi(a_j)^{1/2}\int \frac{d\mathbf{k}\, f(\omega_k)\omega_k\theta_{\text{in}}^{\dagger\prime}(k,t)}{D_c^-(\omega_k)(x_j-\omega_k+i\epsilon)}. \quad (2.21)$$

To get the time dependence of $\theta_{\text{in}}^{\dagger\prime}(k,t)$, note that

$$[H', \theta_{\text{in}}^{\dagger\prime}(k,t)_-] = \omega_k\theta_{\text{in}}^{\dagger\prime}(k,t) = -i\dot{\theta}_{\text{in}}^{\dagger\prime}(k,t). \quad (2.22)$$

Therefore

$$\theta_{\text{in}}^{\dagger\prime}(k,t) = \theta_{\text{in}}^{\dagger\prime}(k)e^{i\omega_k t}, \quad (2.23)$$

where

$$\theta_{\text{in}}^{\dagger\prime}(k) \equiv \theta_{\text{in}}^{\dagger\prime}(k,0).$$

Substituting Eq. (2.23) into Eq. (2.21) and integrating, yields,

$$\beta_j^\dagger(t) = \lambda 2\pi N(a_j)^{1/2}\int \frac{d\mathbf{k}\, f(\omega_k)(e^{i\omega_k t}-1)\theta^\dagger(k)}{D_c^-(\omega_k)(\omega_k - x_j - i\epsilon)}$$
$$+ \lambda^2 2\pi N^2(a_j)^{1/2}\int d\mathbf{k}\, f(\omega_k)\theta^\dagger(k)\int \frac{d\mathbf{q}\, f^2(\omega_q)(e^{i\omega_q t}-1)}{|D_c^+(\omega_q)|^2(\omega_q - x_j - i\epsilon)(\omega_q - \omega_k + i\epsilon)}$$
$$+ \lambda^2 4\pi^2 N^2(a_j)^{1/2}\sum_{m\neq j}(a_m)^{1/2}\int \frac{d\mathbf{q}\, f^2(\omega_q)(e^{i\omega_q t}-1)}{|D_c^+(\omega_q)|^2(\omega_q - x_j - i\epsilon)(\omega_q - x_m + i\epsilon)}\beta_m^\dagger(0)$$
$$+ \left\{\lambda^2 4\pi^2 N^2 a_j \int \frac{d\mathbf{q}\, f^2(\omega_q)(e^{i\omega_q t}-1)}{|D_c^+(\omega_q)|^2|\omega_q - x_j + i\epsilon|^2} + 1\right\}\beta_j^\dagger(0), \quad (2.24)$$

[9] M. Lévy, Nuovo Cimento **14**, 612 (1959).

where $N\equiv\varphi^\dagger\varphi$. Therefore

$$C_{N_j}(t)=\lambda^2 4\pi^2 N^2 a_j \times\int\frac{d\mathbf{q}\, f^2(\omega_q)(e^{-i\omega_q t}-1)}{|D_c^+(\omega_q)|^2|\omega_q-x_j+i\epsilon|^2}+1. \quad (2.25)$$

If $N=0$, $C_N(t)=+1$, i.e., the β_j particle does not decay if it is in a state without φ's.

With the aid of Eq. (2.17) one obtains, if $N\neq 0$,

$$C_{N_j}(t)=\lambda^2 4\pi^2 N^2 a_j\int\frac{d\mathbf{q}\, f^2(\omega_q)e^{-i\omega_q t}}{|D_c^+(\omega_q)|^2|\omega_q-x_j+i\epsilon|^2}. \quad (2.26)$$

Thus, by the Riemann-Lebesgue lemma,[10,11] every β_j is indeed unstable; the integral of Eq. (2.26) approaches zero as $t\to+\infty$. The asymptotic expansion of this term may be evaluated by integrating by parts,

$$C_{N_j}(t)\simeq\lambda^2 16\pi^3 N^2 a_j \times\left(\frac{\pi}{2}\right)^{1/2}\frac{e^{-i(\mu t-\pi/4)}f^2(\mu)\mu^{3/2}}{t^{3/2}|D_c^+(\mu)|^2|\mu-x_j|^2}+O(t^{-2}). \quad (2.27)$$

One may further evaluate the integral of Eq. (2.26) by familiar methods,[12] examining it for $t>0$ when the contour of integration is deformed into the lower half plane of the second Riemann sheet. An exponential time decay would require that the integrand have a (first-order) pole in this region. Such a pole depends upon the analytic continuation of $D_c(z)$. and, therefore, of $f^2(\omega_q)$. Regardless of the existence of a pole, however, the j particles are unstable by the Riemann-Lebesgue lemma.

III. SUMMARY

Because the in-fields of the separable-potential model contain D functions, it is possible to insert CDD poles into the theory and to exactly gauge their effect. The presence of these poles is found to produce a nonfree particle-field algebra for the θ asymptotic fields. Free-particle commutation relations may be retained, however, if a new in-field, containing new bosons is defined. Examination of these new particles shows that, in the context of a new Hamiltonian, they decay with time. This decay is present even if the scattering amplitude does not have a pole in the second Riemann sheet. Thus the usually conjectured link between CDD poles and the presence of unstable particles is confirmed—to every CDD pole there corresponds an unstable particle.

The new model H' in which the unstable bosons appear is one which *a priori* yields an amplitude for φ-θ scattering containing CDD poles, and the scattering state, $\theta_{\mathrm{in}}^{\dagger\prime}(p)\varphi^\dagger|0\rangle$, is one which is composed in part of unstable particle states. Since the method of construction of H' involves the use of asymptotic fields, it is applicable in general to other soluble models.[13]

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APPENDIX

In order to prove Eq. (2.17) consider

$$\int_C\frac{dz}{D_c(z)(z-x_j)^2}=0, \quad (A1)$$

where the contour of integration, C, is shown in Fig. 1. x_j lies in the cut for generality. The right hand side of Eq. (A1) is zero because no poles lie inside the region of

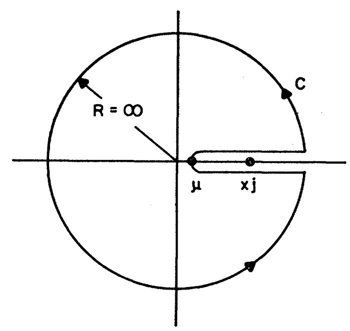

FIG. 1. Contour of integration for Eq. (A1).

integration since $D_c(z)$ is proportional to an R function. The Low equation, Eq. (2.7), shows that the only pole of the integrand is at x_j. Thus Eq. (A1) reduces to

$$P\int_\mu^\infty\frac{d\omega_q}{D_c^+(\omega_q)(\omega_q-x_j)^2}+P\int_\infty^\mu\frac{d\omega_q}{D_c^-(\omega_q)(\omega_q-x_j)^2}=\frac{-2\pi i}{\lambda 4\pi^2\varphi^2\varphi a_j}. \quad (A2)$$

Combining the two integrals of Eq. (A2) and realizing that, because the combined integrand has no pole, the principal value is equal to the total integral, which gives Eq. (2.17).

[10] S. Bochner and K. Chandrasekharen, *Fourier Transforms* (Princeton University Press, Princeton, New Jersey, 1949), p. 3.

[11] E. T. Copson, *Asymptotic Expansions* (Cambridge University Press, Cambridge, England, 1965), p. 24.

[12] See, for example, M. Lévy, Nuovo Cimento **13**, 115 (1959).

[13] See S. Jernow [doctoral thesis, Pennsylvania State University (unpublished)] for a discussion of other models.